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THE MICROWAVE EMISSIVITY OF TURBULENT PLASMA

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THE MICROWAVE EMISSIVITY OF TURBULENT PLASMA

by

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A B S T R A C T

The ZETA plasma is believed to contain field-aligned irregularities in density, loosely called turbulence. Such irregularities cause scattering of a microwave beam traversing the plasma, and will also affect the emission of thermal microwave noise. To calculate the effect on the emissivity, a naive model of the turbulence is taken in which the individual turbulence elements are considered to be parallel cylinders, the density profile within the cylinders being parabolic. A ray theory treatment of the emissivity of a cylinder is combined with a statistical treatment of the radiation mean free path, and the time-averaged emissivity of the plasma is calculated in terms of element size, spacing and central density, the plasma electron temperature and the size and reflectivity of the containing vessel. It is found that the turbulence can increase or decrease the emissivity several-fold according to the values of the various parameters, and that the range of variation is sufficient to encompass the discrepancies between observed and calculated emissivities previously reported.

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1. INTRODUCTION

The emission of microwave noise by the ZETA plasma has been previously reported (HARDING, 1958), and in some circumstances it is possible to interpret the observed radiation temperatures in terms of plasma density and temperature, using a comparatively elementary theory applying to a quasi-uniform plasma (WORT, 1962). The theory shows that the emissivity at any given wavelength is governed by the steepness of the density gradient in the plasma, the electron temperature, and, as an external factor, the reflectivity of the walls surrounding the plasma; calculation yields values for the emissivity of the ZETA plasma which are quite small, typically about one-tenth of black body emissivity. However, the observed noise emission from ZETA often appears to be substantially greater than the elementary theory predicts, the discrepancy amounting almost to an order of magnitude. Similarly there are conditions under which the emission is unexpectedly small, requiring steeper density gradients than would be expected if the plasma is quasi-uniform. Thus it appears that the elementary theory is inadequate, and that the plasma emissivity depends on some additional parameters, hitherto unrecognised.

There is an increasing body of evidence indicating that the ZETA plasma is not uniform, but suffers from field-aligned density irregularities whose characteristic size (across the magnetic field) is a few centimetres (BURTON, 1961). It is evident that such irregularities will have a profound effect on the transmission of a microwave signal through the plasma if the mean plasma frequency is not too far below the signal frequency, and experimentally it is found that such effects do occur even when the expected plasma frequency is about one quarter of the signal frequency. This in turn suggests that the hills and valleys in the plasma density are pronounced and steep-sided.

It now becomes of interest to determine the effect of such irregularities upon the plasma emissivity. These irregularities will be loosely called 'turbulence', but this term is used solely for convenience, and should not be taken to imply any particular properties of the plasma other than those postulated below.

2. THE CYLINDER MODEL

A very elementary model of the turbulent plasma will be taken, in which the turbulence is represented by a parallel array of infinitely long plasma cylinders, surrounded by vacuum. These cylinders are large compared with the wavelength of observation, so that diffraction effects can be ignored. The cylinders are in random motion, and only time averaged effects are considered here.

If a narrow (compared to cylinder radius) beam of radiation is sent into such a system, it will bounce around from one cylinder to another, being deflected by refraction at each cylinder. Ultimately it will random-walk its way to the wall, but it will by then have suffered the cumulative effects of the slight absorption occurring at each encounter with a cylinder. Thus to obtain the effective absorption coefficient of the plasma, it is necessary to know the number of encounters the radiation suffers, and the absorption at each encounter.

If we assume that the density of plasma within the cylinders is sufficient to give a large deflection at each encounter ('large' is defined later) the process by which the radiation reaches the wall is a true random walk. The step length of this random walk is $\bar{\lambda}$, which represents a mean free path for the radiation between successive encounters with turbulence elements. Assume that the radiation is incident in a direction normal to the axis of the turbulence elements, and that these elements lie parallel to the axis of a cylindrical containing vessel (the problem has now become essentially two-dimensional). If the cylindrical turbulence elements have radius r_0 and an average centre-centre spacing S, then we have

$$\bar{\lambda} = \frac{S}{2r_0} \cdot \frac{S\sqrt{3}}{2} = \frac{S^2\sqrt{3}}{4r_0} \qquad \text{... (1)}$$

The random walk starts at a distance $\bar{\lambda}$ inside the wall of the container, and ends at the wall. If the container radius is R, the mean square displacement is

$$\bar{d}^2 = R^2$$

irrespective of the starting point of the random walk. Thus the mean number of encounters during the random walk is

$$\bar{N} = \frac{\bar{d}^2}{\bar{\lambda}^2} = \frac{16r_0^2 R^2}{3S^4} \qquad \text{... (2)}$$

If now the mean absorption produced by one cylinder is $\bar{\eta}_K$ (the reason for the notation, in particular the suffix K, will become apparent later), the overall absorption suffered by the radiation will be

$$\begin{aligned} \eta_T &= 1 - (1 - \bar{\eta}_K)^{\bar{N}} \\ &\simeq \bar{N}\,\bar{\eta}_K \quad \text{if} \quad \bar{N}\,\bar{\eta}_K \ll 1 \end{aligned} \qquad \text{... (3)}$$

which is usually the case. The value of $\bar{\eta}_K$ depends on the exact configuration within the turbulence elements, and in order to obtain some insight into the properties of this quantity we will consider a specific model.

3. THE PARABOLIC CYLINDER

It is mathematically convenient to consider a density profile within individual cylinders which is parabolic, that is the density is given by

$$n = K \cdot n_c \left[1 - \left(\frac{r}{r_o} \right)^2 \right] \qquad \ldots (4)$$

where

n_c is the critical density

r_o is the cylinder radius

K is a parameter describing the scale of the distribution.

The properties of such a density profile have been fully described elsewhere (WORT, 1963) but the essential results required for this work will be recapitulated. These results all depend on the use of ray theory, and also assume that the static magnetic field impressed on the plasma is low, so that the electron cyclotron frequency is far below the observation frequency. Alternatively, the E-vector of the observed radiation must lie along the magnetic field.

If a ray impinges on a cylinder making an angle φ_o with the radius vector at the point of impact ($\varphi_o = 0$ for a central impact), the deviation it will suffer is given by

$$\psi = \frac{\pi}{2} + 2\alpha - 2\varphi_o,$$

where

$$\alpha = \frac{1}{2} \operatorname{Sin}^{-1} \left[- \frac{1 - K - 2 \operatorname{Sin}^2 \varphi_o}{\sqrt{(1-K)^2 + 4K \operatorname{Sin}^2 \varphi_o}} \right].$$

During its passage through the cylinder it will suffer a power absorption

$$\eta(K, \varphi_o) = 1 - \exp \left[- \frac{4 r_o}{\lambda_c} \cdot Q \right] \qquad \ldots (5)$$

where $\lambda_c = \frac{c}{\nu}$; ν is collision frequency for electrons at a plasma density equal to the critical density for the observation frequency. Q is a geometrical factor governed by the continuous refraction within the cylinder:

$$Q = \frac{3K^2 + 2K(1 + 2 \operatorname{Sin}^2 \varphi_o) + 3}{32\sqrt{K}} \cdot \operatorname{Tanh}^{-1} \left[\frac{2\sqrt{K} \operatorname{Cos} \varphi_o}{1 + K} \right] - \frac{3}{16} (1 + K) \operatorname{Cos} \varphi_o,$$

this expression referring to a plasma which is isothermal, and in which the dominant collision process occurs between electrons and ions. Parameters applicable to a succession of random encounters may be obtained by integrating ψ^2 and Q for all values of the impact pa ameter, and we thereby obtain mean values $\sqrt{\overline{\psi}^2(K)}$ and $\overline{Q}(K)$ which depend only on K. Figs.1 and 2 show these quantities.

Returning to the question of 'large' deflections, we shall take it that the random walk theory is an adequate approximation if $\sqrt{\bar{N}\,\bar{\psi}^2}$, the total mean deflection suffered by the radiation, is greater than π. This could be used to give a rather complicated criterion for K, but is best treated as a final check of the method in any particular application.

The emissivity of the plasma is thus given by

$$\eta_T = \bar{N}\,\bar{\eta}_K ,$$

and

$$\bar{\eta}_K = 1 - \exp\left[- \frac{4r_o}{\lambda_c}\,\bar{Q}(K) \right] ,$$

$$\simeq \frac{4r_o}{\lambda_c}\,\bar{Q}(K) \quad \text{if} \quad \bar{\eta}_K \ll 1 \qquad \ldots (6)$$

Thus the plasma emissivity depends on three factors: $r_o\,\bar{N}$ which depends on the overall geometry of the turbulence, $\bar{Q}(K)$ which depends on the geometry of the individual turbulence element, and $\frac{4}{\lambda_c}$ which depends on the properties of the plasma within the turbulence elements.

If the quantity $\sqrt{\bar{N}\,\bar{\psi}^2}$ is $\ll \pi$, the random walk model is obviously not applicable. We may now treat the absorption as though it arose from the undeflected passage of the radiation through the array of cylinders. Crudely, the cylinders may be arranged in rows, the separation of cylinders in a row being S and the separation of the rows $\frac{S\sqrt{3}}{2}$. The number of rows in the container will be $\frac{4R}{S\sqrt{3}}$, and the mean absorption in a row will be $\frac{2r_o}{S}\,\bar{\eta}_K$. Thus the total absorption will be

$$\eta_t = \frac{4R}{S\sqrt{3}}\;\frac{2r_o}{S}\,\bar{\eta}_K \qquad \ldots (7)$$

The factor $N_o = \frac{8r_oR}{S^2\sqrt{3}}$ has replaced $\bar{N}$ previously obtained, and if $\bar{\lambda} \ll R$, we have $\bar{N} = \frac{1}{4}\,N_o^2$. Evidently the loss of the random walking may considerably reduce the emissivity.

4. COMPARISON WITH NON-TURBULENT PLASMA

To ascertain the effect of turbulence, it is necessary to compute the emissivity of the same total quantity of plasma arranged in some quasi-uniform (non-turbulent) distribution. The total quantity of plasma per unit length of a turbulence element with parabolic density distribution is

$$\bar{n} = \frac{1}{2}\,\pi\,r_o^2\,K\,n_c .$$

The total number of turbulence elements within the container is $\frac{2\pi R^2}{S^2\sqrt{3}}$.

Thus the total quantity of plasma per unit length of the tube ('line density') is the product of these two expressions.

It is not unrealistic to rearrange this plasma into a parabolic distribution which fits the container. This will have scale parameter K_R, and equating line densities we find

$$\frac{1}{2}\pi R^2 \; K_R \; n_c = \frac{2\pi R^2}{S^2\sqrt{3}} \cdot \frac{1}{2}\pi \, r_o^2 \; K \; n_c \; ,$$

or

$$K_R = \frac{2\,\pi\, r_o^2}{S^2\sqrt{3}} \cdot K \qquad \ldots (8)$$

The emissivity of the quasi-uniform distribution may be calculated using the Q-expression (5), with $\varphi_o = 0$, giving

$$\eta_o = 1 - \exp\left[-\frac{4R}{\lambda_c} \cdot Q_o\right] \qquad \ldots (5')$$

Fig.3 shows Q_o as a function of K. Alternatively, for $K > 1$, the emissivity may be calculated by the gradient method, (WORT, 1962) giving

$$\eta_o = 1 - \exp\left[-\frac{8}{3}\frac{\ell}{\lambda_c}\right] \; ,$$

where, for a parabolic distribution,

$$\ell = \frac{R}{2\sqrt{K(K-1)}} \; .$$

Neither method is rigorously correct for a parabolic distribution, but unless $0.9 < K < 1.3$ the error will be small, and probably negligible when compared with the errors introduced by taking a rather artificial turbulent model.

Thus we may now compare η_T and η_t with η_o and hence ascertain the effect of turbulence.

To obtain the radiation temperature it is necessary to allow for the reflectivity of the container wall, which will increase the η for an isolated plasma to an apparent η' given by

$$\eta' = \frac{\eta}{1-\rho(1-\eta)} \qquad \ldots (9)$$

where ρ is the power reflectivity of the wall. The radiation temperature will be $T_r = \eta' T_e$ where T_e is the electron temperature.

5. CLOSE-PACKED TURBULENCE

Evidence obtained from spatial correlation of magnetic and electrostatic fields in ZETA indicates that the cylinder model would best represent the ZETA plasma if the cylinders were assumed to be in contact so that $S = 2\ r_o$ (RUSBRIDGE, 1963 - Private Communication).

This special case, of close-packed turbulence elements, has resulted in the elimination of one parameter and makes it possible to appreciate the effects of turbulence by inspection.

In this situation we find, using (2),

$$\bar{N} = \frac{R^2}{3r_o^2} ,$$

so that, from (3) and (6),

$$\eta_T \simeq \frac{R^2}{3r_o^2} \cdot \frac{4r_o}{\lambda_c} \bar{Q}(K) .$$

Thus η_T varies inversely as r_o. We also find in this case, using (8),

$$K_R = \frac{\pi}{2\sqrt{3}} \cdot K = 0.907\ K ,$$

so that, from (5′),

$$\eta_o = \frac{4R}{\lambda_c} \cdot Q_o(0.907\ K) .$$

Thus

$$\frac{\eta_T}{\eta_o} = \frac{R}{3r_o} \cdot \frac{\bar{Q}(K)}{Q_o(0.907\ K)} .$$

Fig.4 shows $\frac{\eta_T}{\eta_o}$ as a function of $\frac{r_o}{R}$, for various values of K. evidently different values of K could either refer to different plasma densities at one particular observation frequency, or to the use of different frequencies to observe one particular plasma.

Application of the $\sqrt{\bar{N}\ \bar{\psi}^2}$ criterion leads to the upper limits of 0.058 and 0.130 for $\frac{r_o}{R}$ when K = 0.4 and 0.8 respectively. In the close-packed situation, we find, using (2),

$$\eta_t = \eta_T \cdot \frac{2r_o\sqrt{3}}{R}$$

so that $\frac{\eta_t}{\eta_o}$ is independent of $\frac{r_o}{R}$,

$$\frac{\eta_t}{\eta_o} = \frac{2}{\sqrt{3}}\ \frac{\bar{Q}(K)}{Q_o(0.907\ K)} .$$

The two lines for $\frac{\eta_t}{\eta_o}$ when K = 0.4 and 0.8 are also shown on Fig.4. Note $\frac{\eta_T}{\eta_o} > 1$ for the range on the graph, whereas $\frac{\eta_t}{\eta_o} < 1$. There must be some sort of smooth transition between η_T and η_t as $\frac{r_o}{R}$ or K are changed, but the behaviour in the transition region, lying between two statistical realms is obscure. For $K > 1$, it is found that $\bar{\psi}^2$ becomes greater than unity, so that the process is always a random walk unless the mean free path becomes comparable with R, in which case the statistical treatment will break down.

It may be remembered that the ratio $\frac{\eta_T}{\eta_o}$ refers to a plasma which is either isolated in space or surrounded by a cold non-reflecting wall, and that the presence of a reflecting wall will considerably modify the results. In general, unless η_o and η_T are $\ll 1-\rho$, the reflectivity will lower the ratio of the radiation temperatures as compared with $\frac{\eta_T}{\eta_o}$. Furthermore, it has been assumed that the plasma parameter λ_c is large (hot plasma), so that the individual emissivities are sufficiently small to permit the approximation to the exponentials to be valid.

6. SPACED TURBULENCE

We will now re-introduce the spacing parameter S, although it is not yet clear whether 'spaced turbulence' has any physical reality. Probably the most straight-forward way of appreciating the effect of S is to choose a few values of $\frac{S}{r_o}$ and then determine the values of $\frac{\eta_T}{\eta_o}$ as before. First, let $S = 3r_o$. Now we find, using (2)

$$\bar{N} = \frac{16}{243} \cdot \frac{R^2}{r_o^2} ,$$

and using (8)

$$K_R = \frac{2\pi}{9\sqrt{3}} K = 0.403 \ K .$$

Thus

$$\frac{\eta_T}{\eta_o} = \frac{16}{243} \cdot \frac{R}{r_o} \cdot \frac{\bar{Q}(K)}{Q_o(0.403 \ K)} .$$

Fig.5 shows $\frac{\eta_T}{\eta_o}$ as a function of $\frac{r_o}{R}$ for various values of K. These K values have been chosen so that the total amount of plasma per unit length of container is the same as for the three curves in Fig.4 (same K_R values). Thus the effect of spacing the turbulence and at the same time increasing the density within the turbulence elements can be seen by comparing Figs.4 and 5. For the lowest density curve, spacing has hardly affected the overall emissivity, because the increased density within the elements has compensated for gaps between them. However, the other curves have been greatly lowered, because the emissivity of the individual elements has fallen as their central density now considerably exceeds the critical density.

The $\sqrt{\bar{N}\,\bar{\psi}^2}$ criterion shows that for $\frac{r_o}{R} > 0.067$, $K = 0.9$, η_t should be taken. For $S = 3r_o$ we find

$$\frac{\eta_t}{\eta_o} = \frac{8\sqrt{3}}{27} \cdot \frac{\bar{Q}(K)}{Q_o(0.403 \ K)}$$

and the line for η_t is shown in Fig.5. Unlike the previous example in which the turbulence was close-packed, this plasma has yielded a value of $\frac{\eta_t}{\eta_o}$ which is greater than unity.

The behaviour of the emissivity as the spacing of the turbulence elements varies is shown in Fig.6, which gives $\frac{\eta_T}{\eta_o}$ as a function of $\frac{S}{r_o}$ for the condition $\frac{r_o}{R} = 0.1$. These curves have been computed for two constant values of K_R, so that along either curve there is the same quantity of plasma in the tube. As $\frac{S}{r_o}$ increases, the turbulence elements remain the same size, but become denser and more separated. The effect of the two competing factors, non-absorbing gaps and elements of varying absorption, can be clearly seen, for as the density of the plasma within the turbulence elements increases, the increasing absorptivity of each element at first outweighs the effect of the increasing gaps between them. The emissivity reaches a maximum when the central density of the element attains the critical value ($K = 1$), but thereafter the cylinders become increasingly reflecting and the overall emissivity falls rapidly. For $K_R = 0.3$, η_t should be taken for $\frac{S}{r_o} < 3.5$, and the $\frac{\eta_t}{\eta_o}$ line is shown.

As a final example, the effect of varying r_o whilst S remains constant is shown in Fig.7. Again K_R is constant along either of the two curves, and $\frac{S}{R}$ is taken to be $\frac{1}{5}$ so that the cylinders come into contact at $\frac{r_o}{R} = 0.1$. For the $K_R = 0.6$ curve, the expression for η_T is always valid, but for $K_R = 0.3$ and $\frac{r_o}{R} > 0.057$, η_t must be taken as indicated on the graph. Again $\frac{\eta_T}{\eta_o}$ can be greater or less than unity, according to the scale of the turbulence.

7. RANDOM-SIZED TURBULENCE

So far it has been assumed that the three parameters describing the geometry of the turbulence, r_o, S and K, each have a unique value, whereas a completely general description of a turbulent plasma, based on this model, would randomise these parameters by giving each of them some sort of statistical spread. A rigid analysis of the random-sized system is beyond the scope of this treatment, but a few qualitative remarks about the effect of the randomising may be made. The parameter S affects the step-length of the random-walk, and the overall effect will be changed very little if S is allowed to spread, providing that the spread in S is small compared to the total path length. S alone cannot be randomised if the turbulence is strictly close packed, but this difficulty is removed if r_o is also randomised.

If r_o is randomised, then the turbulence can never be close packed, and there must be some value of $\overline{S}$ which must exceed $2\overline{r}_o$. Both r_o and K have an effect on the emissivity of an individual turbulence element, but it is evident that if the ranges of variation of r_o and K are not too large, the overall effect could be obtained by a suitably weighted sampling operation on the curves in Figs.4 - 7. The most obvious effect would be to 'smear out' the peaks and discontinuities in the curves. In view of the artificial description of the turbulence element which has been adopted here, any quantitative discussion seems pointless.

8. TIME-RESOLVED MEASUREMENTS

The model proposed here is one in which discrete turbulence elements are in random motion, and this random motion is used to average out all the possible configurations of turbulence elements which might be seen by an aerial. If the resolving time of the receiver is sufficiently short, however, the variation of emissivity between different configurations could be observed, and such variations might provide useful information about the turbulence.

One characteristic time interval which should appear in the noise output will be given by the mean element separation S divided by the RMS velocity with which the elements move. If the aerial is looking squarely into the gap between the two nearest turbulence elements, it will receive roughly the mean level of noise power, for this is a configuration which would give a high absorption to any radiation sent into the plasma via the aerial. The emissivity will thus be close to the value of η_T calculated (obviously special configurations will give higher emissivities, but these configurations, depending on the exact placing of many turbulence elements, would only persist for a very short time). If now a turbulence element with $K > 1$ arrives squarely in front of the aerial the noise output will be reduced, for the radiation from the bulk of the plasma can now only reach the aerial after two reflections - one from the wall of the container, and one from the turbulence element (we are, strictly speaking, postulating an aerial aperture which is small compared to r). Thus the radiation temperature will be reduced from

$$T_r = \frac{\eta_T T_e}{1 - \rho(1 - \eta_T)} ,$$

to

$$T'_r = \rho(1 - \bar{\eta}_K) \cdot T_r .$$

This effect will only occur for $K > 1$, for only then can the necessary refractive deviation exceed $\frac{\pi}{2}$, so that the radiation has to arrive, in part at least, via a wall reflection.

9. DISCUSSION

This crude theory indicates that turbulence can modify the emissivity of a plasma, although the effect is not large. If the plasma line density is low and the turbulence is such that the central density in the turbulence elements approaches the critical density, the emissivity may be increased by a few times, and this effect becomes more marked as the number of elements in the cross section of the container increases. On the other hand, a plasma of high line density, or a system of turbulence giving element central densities exceeding the critical density, generally decreases the emissivity, again by a few times. These modifications are sufficient to encompass the discrepancy between the elementary theory and observations of the ZETA plasma, reported previously (WORT, 1962), although there is as yet insufficient experimental data to allow quantitative comparison. Such a comparison may not be meaningful in ZETA, or any other pinch device, because the geometry of the turbulence will be more complex than the model taken here (see below) but at least the results should be qualitatively similar.

In a pinch device the magnetic field lines are, in general, helical. The turbulence elements will also be arranged in a similar helical array, and this helicity will lead to axial scattering of radiation, so that the problem is no longer two dimensional. The mean square displacement between container walls will be increased (unless the container is roughly spherical), so that the number of encounters made with the turbulence elements by any given ray as it finds its way from wall to wall will be increased, although the absorption suffered at each encounter will be lower because the impact will in general be oblique. There will thus be two effects acting in opposition, the overall effect is complex, and any particular configuration would have to be treated individually.

However, mirror machines and other devices with magnetic fields which are primarily axial provide a geometry in which this theory is applicable (compare IOFFE, 1961), provided that the limitations imposed by the high magnetic field in these devices are remembered.

The observation of fluctuations in the noise level would require very sophisticated techniques, as otherwise any such fluctuations will be overwhelmed by the inherent noise of the microwave receiver. This effect is particularly significant in a ZETA-like plasma, for it is necessary to use a millimetre wave receiver to observe at a frequency in the region of the mean critical frequency for the plasma, and in general millimetre wave receivers become rapidly more noisy as the wavelength is decreased. Furthermore, transmission measurements on ZETA indicate that typical fluctuation times are less than one microsecond, and the wide post-detector bandwidth required to respond to such fluctuations implies a very high receiver noise level. Quantitatively, it would be impossible to detect fluctuations of radiation temperature from ZETA if they were less than about 15% of the average temperature level (receiver noise figure 20 db, I.F. bandwidth 30 mc/s, output time-constant 1 μsecond). The only circumstance in which such fluctuations might become readily detectable would be if the wall reflectivity ρ were zero, as then T_r' (see above) would fall to zero (strictly to the wall temperature of the container). As the ZETA liner has $\rho = 0.92$ at 2 mm wavelength it is not possible to observe any fluctuations in noise temperature caused by turbulence in this device.

The choice of a parabolic density distribution within the turbulence elements is rather arbitrary, and is only used for its mathematical convenience. There is as yet no experimental evidence to guide the choice of distribution, but the general effect will not be very sensitive to changes in shape because most of the absorption in any element arises from a narrow region lying close to the region of critical density. The magnitude of the absorption depends on the density gradient at this region (WORT, 1962) which will not be greatly changed for different shapes - provided that anything very bizarre is excluded. This does not apply to elements whose central density is lower than the critical density, but the absorption now has to be averaged across the cross section of the element, and again the exact profile will not be very important.

It has been assumed that the space between the turbulence elements is a perfect vacuum, and it may be more realistic to have a background of plasma. To some extent this can be taken into account by allowing the element spacing S to become less than the element diameter $2r_0$, so that the elements overlap. The full treatment allowing for a plasma background is not difficult, but rather too complicated for the purpose of this paper. A further implied assumption is that the plasma is isothermal, there being no variation of temperature between regions of varying density. In practice turbulence may be adiabatic, so that the turbulence elements are hotter than the background plasma, and the elements themselves are hotter towards the centre. The overall absorptivity and apparent radiation temperature of such an element, in which both the local absorption coefficient and temperature depend on radius, may be deduced by methods similar to those used before (WORT, 1962), but the calculation is complicated, and the integrals which arise can no longer be evaluated in terms of elementary functions. Thus whilst it is possible to extend the scope of this theory without making any basic changes, the present state of experimental knowledge does not justify the introduction of the necessary additional parameters.

10. CONCLUSIONS

Turbulence may either increase or decrease the thermal emissivity of a plasma, according to the parameters governing the turbulence. The emissivity shows a maximum when the central density in the turbulence elements approaches the critical density for the observation frequency, and the enhancement may then approach an order of magnitude. If the turbulence elements are over-dense, their decreased absorptivity overwhelms the effect of the increased radiation path length within the plasma as a whole, and the emissivity is in general lowered. The model on which these conclusions are based assumes that the turbulence elements are cylinders of isothermal plasma whose density profile is parabolic. This choice of density profile is arbitrary, but the overall effect of turbulence is not very sensitive to the exact details of the distribution, and the model is by its simplicity well suited to the present lack of experimental data. Moreover, preliminary results indicate that the behaviour of the ZETA plasma is in semi-quantitative accord with the predictions of this theory, although the assumption that the cylinders are parallel makes the theory geometrically inappropriate in a pinch device.

It is probably unrealistic to expect any theory with so few governing parameters to yield fully quantitative results for experimental confirmation, but it is felt that the methods outlined above afford an adequate qualitative description of the effect of turbulence upon plasma emissivity.

11. ACKNOWLEDGEMENTS

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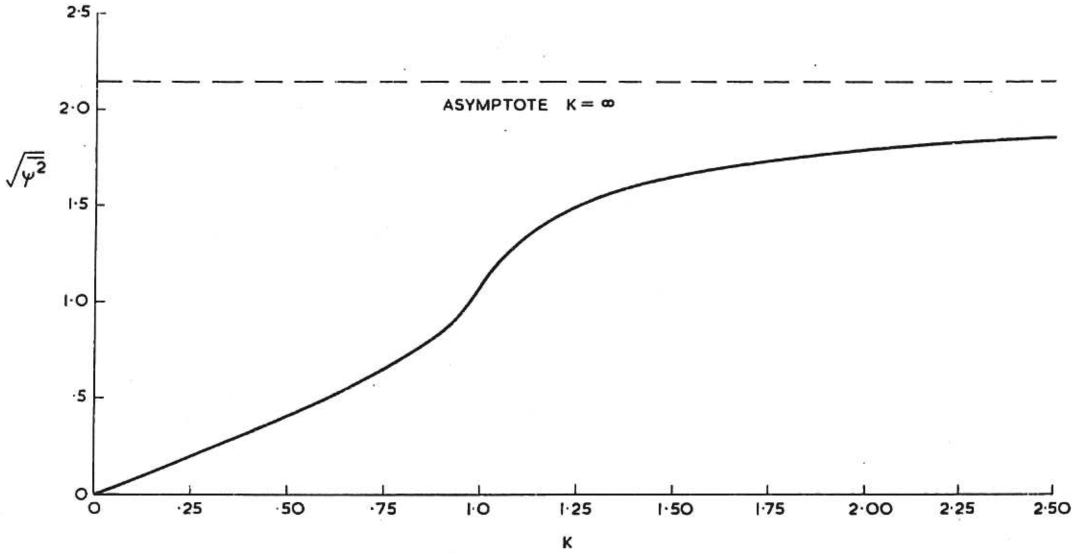
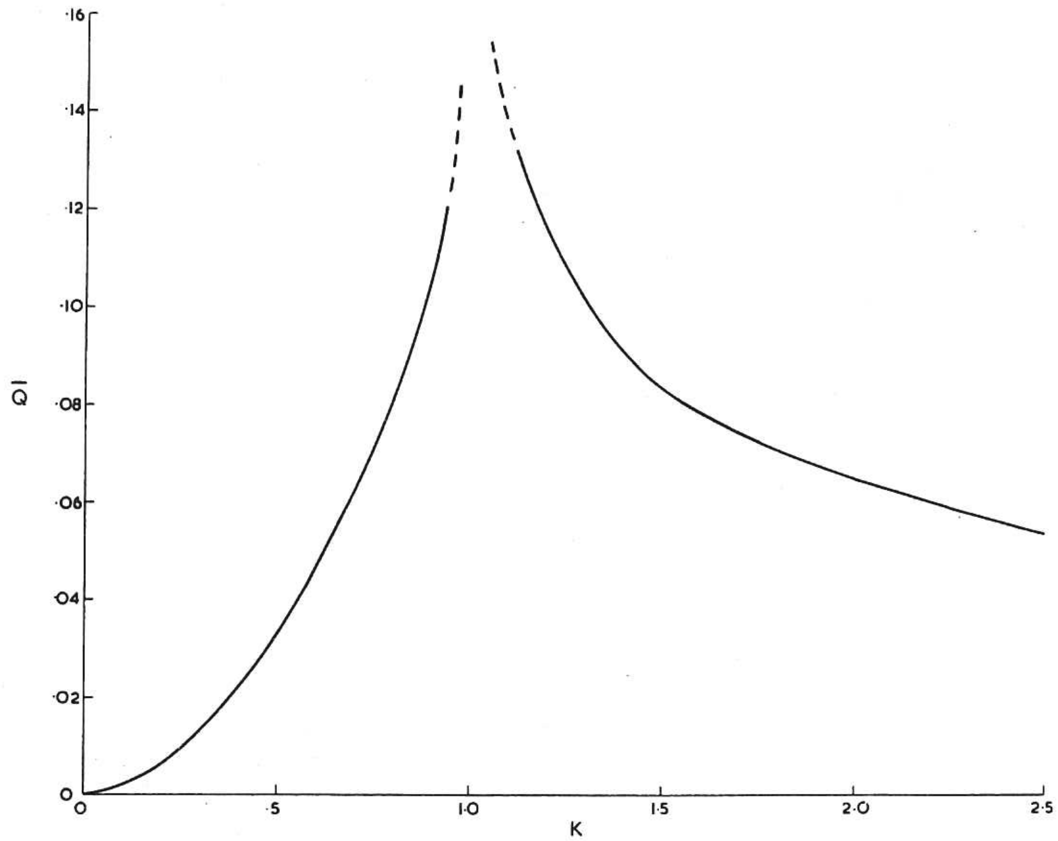
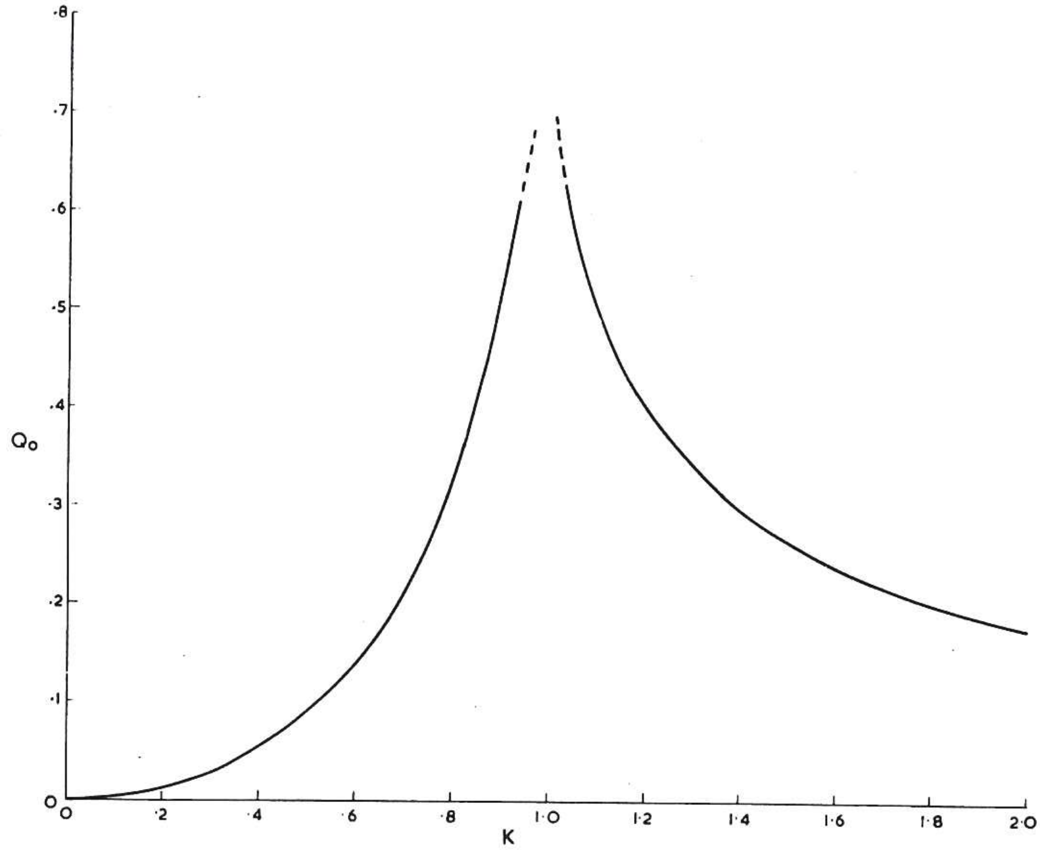


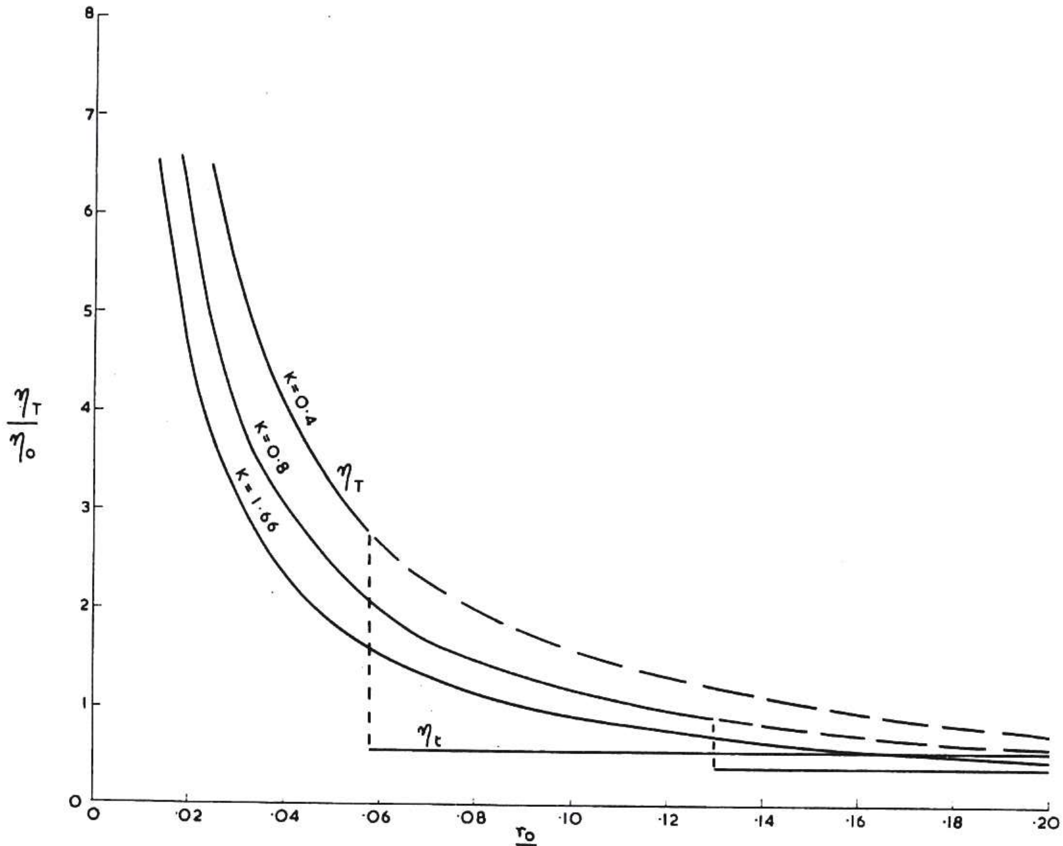
CLM - P 34 Fig. 1
Root mean square angular deviation of a ray, as a function of central density.



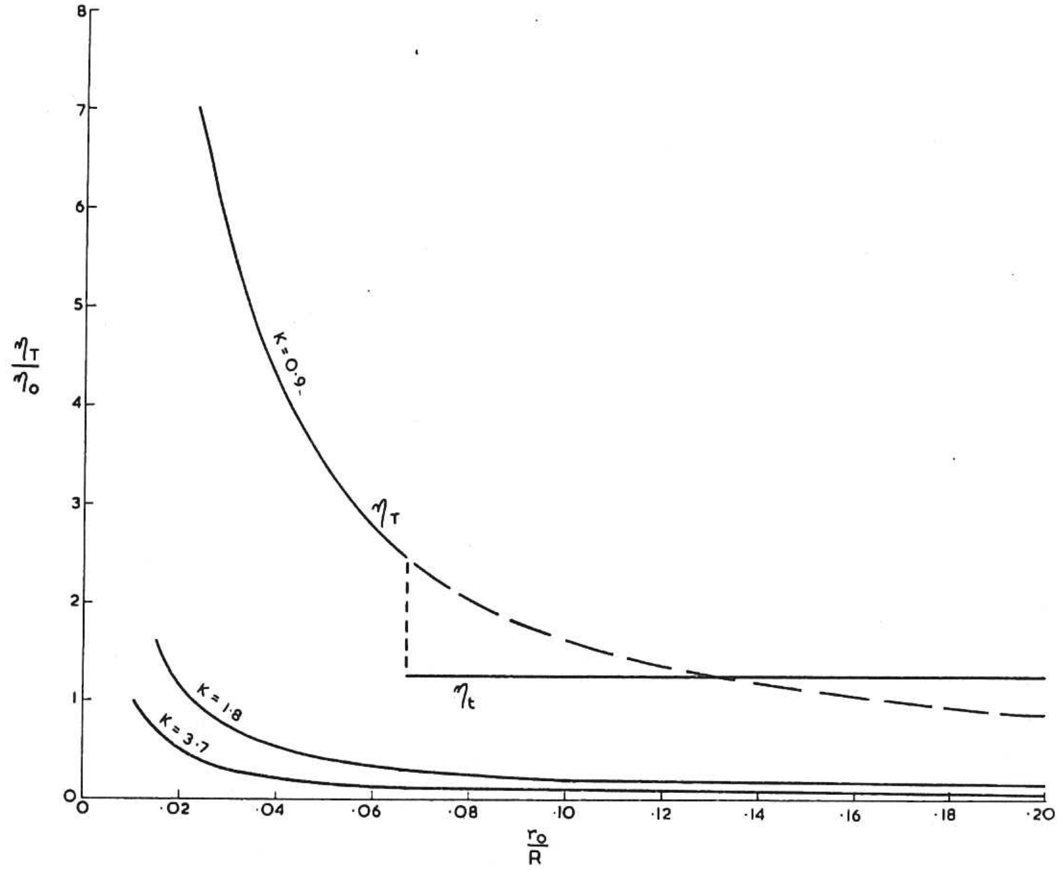
CLM - P 34 Fig. 2
Mean emissivity coefficient as a function of central density.



CLM - P 34 Fig. 3
Radial emissivity coefficient as a function of central density.

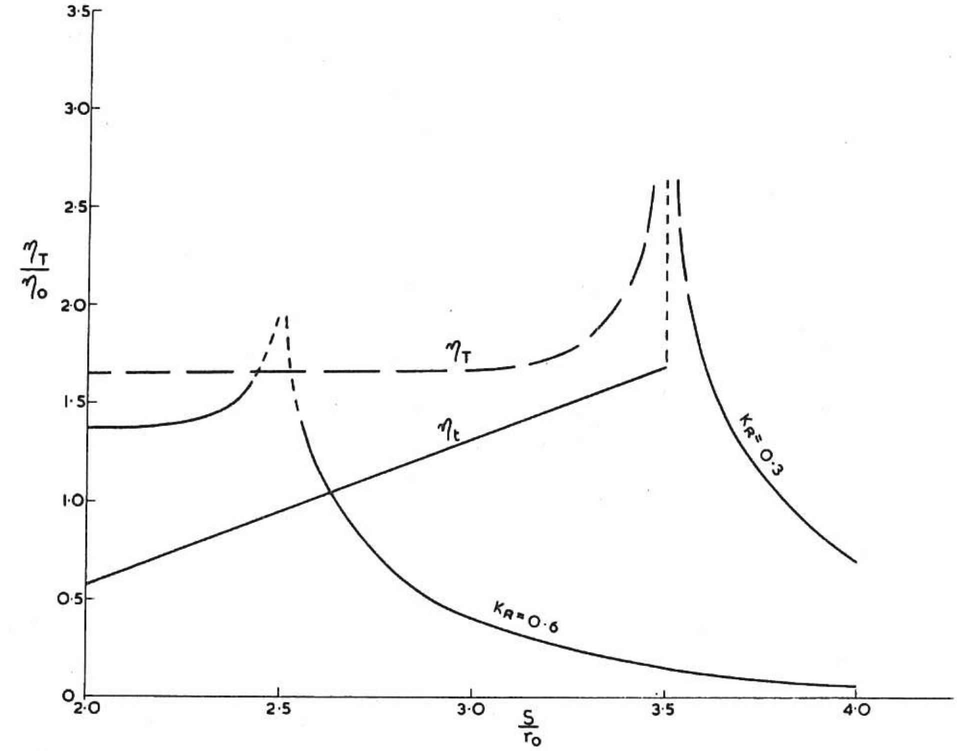


CLM - P 34 Fig. 4
Ratio of turbulent to non-turbulent emissivity as a function
of element size for close-packed turbulence.



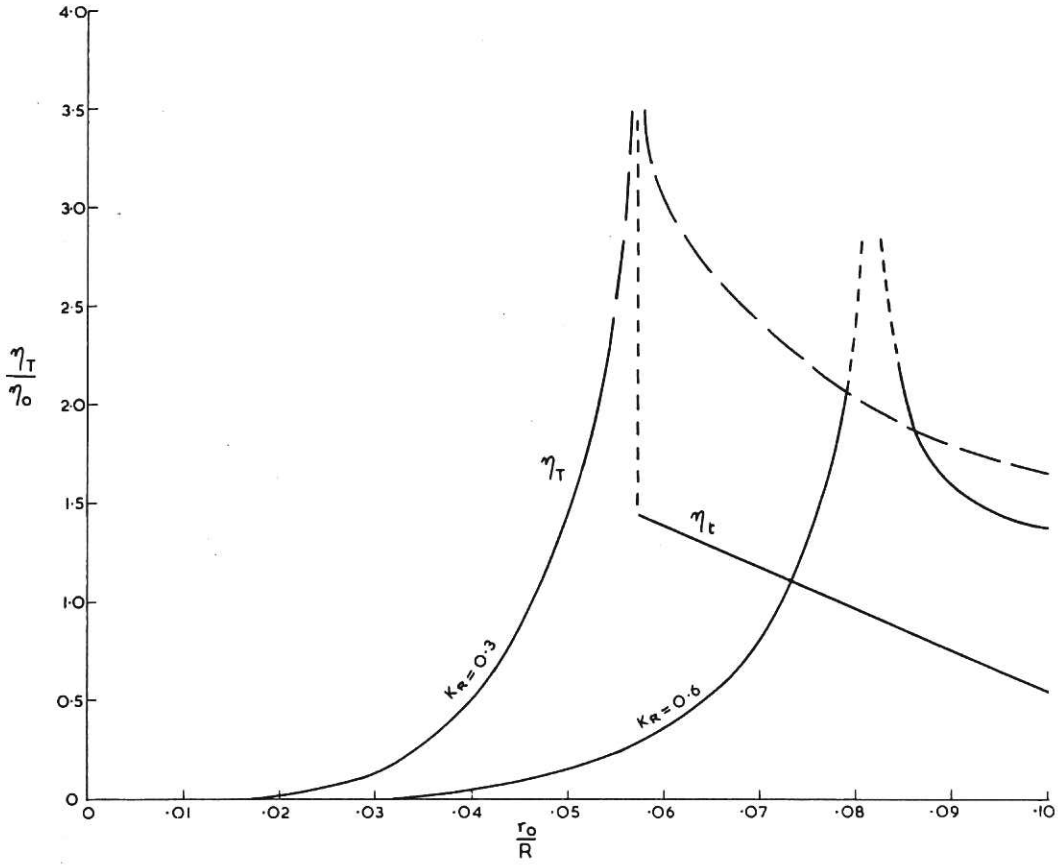
CLM - P 34 Fig. 5

Ratio of emissivities as a function of element size when $S = 3r_0$.



CLM - P 34 Fig. 6

Ratio of emissivities as a function of element spacing, for $\frac{r_0}{R} = 0.1$.



CLM - P 34 Fig. 7

Ratio of emissivities as a function of element size, for $\frac{S}{R} = 0.2$.