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THE POSSIBILITY OF ELECTROMAGNETIC SELF-EXCITATION IN LIQUID METAL FLOWS IN FAST REACTORS

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THE POSSIBILITY OF ELECTROMAGNETIC SELF-EXCITATION IN LIQUID METAL FLOWS IN FAST REACTORS

by

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ABSTRACT

The possibility that the liquid metal flows in fast reactors may form electromagnetically self-exciting systems and the potential consequences are examined. Simple calculations indicate that the smaller prototypes are unlikely to self-excite, but there is a possibility that the larger ones might. The pumps appear more likely to self-excite than the reactor flow systems as a whole. Lengthy calculations would be needed to establish definitely whether the flow régimes in the reactors are self-exciting or not.

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INTRODUCTION

As engineering devices are made larger and their operating conditions uprated, unexpected and sometimes embarrassing physical effects can occur. The purpose of this note is to draw attention to one such effect that may occur in liquid metal cooled fast reactor systems, namely electromagnetic self-excitation. In the present state of knowledge it is not possible to specify critical reactor sizes or conditions, but the indications are that self-excitation may occur in reactors of the size of a Commercial Fast Reactor but is unlikely, although conceivable, in the prototypes.

Whether self-excitation occurs in a moving conducting fluid is determined by the flow pattern and the flow speed, measured in terms of a non-dimensional number called the magnetic Reynolds' number. In brief the process can be visualized as follows* . Suppose a liquid of conductivity σ is moving in an externally imposed magnetic field. If B is the typical field strength and v the typical velocity, then electric fields of order vB and current densities of order $j = \sigma v B$ will be induced. If a typical dimension over which the motion takes place is L then the current j will itself induce a magnetic field of order $\mu j L$ or $\mu \sigma v L B$, where μ is the magnetic permeability. Thus while the magnetic Reynolds' number $R_m = \mu \sigma v L \ll 1$ it is a measure of the size of a small perturbation in the imposed field, but when $R_m = O(1)$ it becomes conceivable that the perturbation field could replace the original field first assumed. In this case the liquid will maintain a magnetic field with no external source, except perhaps for a small seed field to start the process. The situation is in some ways similar in a conventional self-exciting, series-wound dynamo, but there the currents are carefully constrained by insulation so that the field they produce is correctly oriented. In a homogeneous liquid they distribute themselves according to Ohms law and may not produce a magnetic field in the right direction to react with the flow velocity even if the magnetic Reynolds' number is large. The situation is also more complicated because the flow pattern and self-induced magnetic field may be highly non-uniform, and the field may be oscillatory rather than steady.

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*Roberts [1] contains a more detailed explanation.

Asymmetrical and non-uniform motions are thought more likely to self-excite since their asymmetry may compensate for the homogeneity of the conductivity. Although axisymmetric motions can self-excite* it is known that certain motions cannot, no matter how high the magnetic Reynolds' number. Any toroidal motion (i.e. one in which the velocity is always perpendicular to the radius vector measured from a fixed point) in a system with spherically symmetric electrical geometry cannot produce a steady magnetic field (Bullard and Gellman, see Roberts[1 or 2]). A single spinning rotor is a simple example of such a motion.

From the rather limited evidence available it is also thought that even in promising motions the magnetic Reynolds' number must be rather high. It has been shown theoretically (see Roberts[2]) that for a motion in a sphere of radius a with maximum velocity v_{max} and maximum shear in the velocity S_{max} that the magnetic Reynolds' numbers $\mu \sigma v_{max} a > \pi$ and $\mu \sigma S_{max} a^2 > \pi^2$ are necessary conditions for self-excitation. In practice these seem to be far from sufficient. At present the theoretical results, many of which assume a particular flow pattern thought to be promising, are numerical and not always conclusive, but generally indicate considerably higher values. Lilley, for example, (Roberts[2]) assumed a motion which was a combination of a strong toroidal flow and a rather weaker poloidal flow and found evidence of self-excitation when the magnetic Reynolds' number based on the maximum velocity was about 80. G.O. Roberts[6] has done numerical calculations assuming an axisymmetric cellular poloidal flow in a sphere. A self-excited unsteady magnetic field occurred at a magnetic Reynolds' number of about 100.

Experimental evidence is also scarce, mainly because of the difficulty in building apparatus large enough. Lowes and Wilkinson[4] have built a series of model dynamos consisting of two solid rotors made of material with a high magnetic permeability and embedded in a block of the same material. The rotors occupy about a third of the volume of the block and good electrical contact is ensured using a thin seal of mercury. Although it is solid the model may be regarded as a fluid with a particular flow pattern. The critical magnetic

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*Cowling's theorem only rules out axisymmetric fields, see Roberts[1 or 2]

Reynolds' number based on the rotor radius and speed (when both rotors have the same speed) is found to be very sensitive to the positions of the rotors. The earliest version of the model gave a value of 1,000, estimated by extrapolation. In later versions values of 200 and more recently 100 (private communication) have been experimentally achieved.

As a very rough rule of thumb based on limited evidence it could be said that where the magnetic Reynolds' number calculated from a typical dimension and velocity is less than, say, 3, self-excitation is theoretically impossible; between 3 and 30 it has not been shown to be theoretically impossible but has not been achieved; between 30 and 300 it is possible but not likely unless the flow pattern is suitable; and above 300 it becomes more and more likely unless the flow pattern is one which is incapable of self-excitation.

ANALOGY WITH A SIMPLE CONVENTIONAL SELF-EXCITING DYNAMO

Let I be the current circulating through the armature and field coils which are in series, with total resistance R. The magnetic field B produced across the armature is proportional to I, say $B = \alpha I$. If the residual field is B_o, and the armature speed ω, the driving voltage will be $\beta\omega\,(B_o + B)$ where β is another constant. If the inductance of the system is L then

$$IR + L\frac{dI}{dt} = \beta\omega(B_o + B)$$

$$= \beta\omega B_o + \alpha\beta\omega I \qquad (1)$$

with an initial condition, say, of $I = 0$ at time $t = 0$. The solution to (1) is :

$$I = I_s\left[1 - e^{-\frac{R}{L}(1 - R_m)t}\right] \qquad (2)$$

where

$$R_m = \frac{\alpha\beta\omega}{R} \qquad (3)$$

and

$$I_s = \frac{\beta\omega B_o}{R[1 - R_m]}\,. \qquad (4)$$

Thus if the equivalent of the magnetic Reynolds' number $R_m < 1$, the current, and corresponding field, reach a finite value given by (4) but when $R_m > 1$ the current grows without limit. At $R_m = 1$ (4) gives an infinite value for I_s unless $B_o = 0$. This is the self-exciting condition, since the residual field necessary to maintain I is now arbitrarily small. For fields much bigger than the residual field $R_m = 1$ (almost) and the value of the current is determined by the power supplied which must balance the resistive losses I^2R. Attempts to increase the speed (i.e. R_m) by increasing the torque will, on the steady state assumption, be unsuccessful, but the current and power dissipation will increase to balance the increased power input.

In practice if the system was rotating with a steady torque at $R_m = 1$ with a self-excited current and field, increasing the torque would increase the speed temporarily. The current would increase at a rate $\frac{dI}{dt}$ given by (1) and the speed decrease until the new steady state with increased torque and self-excited field had been reached, in a time of order L/R. In a practical dynamo the self-exciting speed usually varies with the current delivered since the field system is partially saturated, that is α in (3) is a function of I.

ESTIMATES OF THE MAGNETIC REYNOLDS' NUMBER IN LIQUID METAL COOLED FAST REACTOR

Unless the magnetic Reynolds' number of the flow satisfies a necessary numerical condition, like that given above, self-excitation is impossible whatever the flow pattern. We show that the larger fast reactors have magnetic Reynolds' numbers sufficiently large for dynamo action if the flow pattern is suitable (a big proviso), whereas the smaller ones do not. The consequences of dynamo action are considered later.

The estimates are given for the D.F.R., P.F.R. and C.F.R. and are based on a number of assumptions. A simplified model of the P.F.R. is shown in figure 1. The reactor is treated as an isolated vessel whose volume is that of the total liquid metal coolant. The internal pipe work etc. is assumed to have the same conductivity, or to be insignificant electrically, and to be non-magnetic. Table I lists the various parameters. R_m is calculated using the conductivity of the hot liquid metal: when the P.F.R. and C.F.R. are running cold the values of R_m would be increased by a factor of up to 3 (3 at 100° C).

The flow pattern is idealised to a loop inside the reactor passing through a pump, heat exchanger and the reactor core. The secondary circuit of sodium in the heat exchanger should strictly be taken into account but has been ignored. Using the parameters in table I, an R_m based on the maximum velocity is calculated for a strict application of the necessary condition to see if dynamo action is at all possible. The first set of values of R_m, which are large enough for possible dynamo action, must grossly over-estimate the possibility since the condition was derived with the understanding of some global flow with a maximum velocity, whereas only a small proportion (a few per cent) of the liquid is moving in a fast reactor. This is taken into account in the second set of values of R_m which are based on an effective velocity calculated on the crude assumption that the total flow rate q flowing in a pipe loop of a given length ℓ has been averaged out over the whole volume of the liquid metal. These figures are much lower, and are probably pessimistic in the sense that the original necessary condition no longer strictly applies and can no longer rule out dynamo action in the concentrated flow in the original loop. The length of the loop is a smaller proportion of the size of the D.F.R. than of the other reactors since in the D.F.R. the coolant is taken outside to the heat exchangers. The flow in the breeding section of the D.F.R. has also been ignored. This second calculation of R_m still does not rule out dynamo action, except in the case of the D.F.R., though they are not high enough to make dynamo action probable, unless by accident the flow pattern is one of the most favourable.

There remains the possibility that even if the system as a whole is not self-exciting, that various parts of it might be. The obvious candidates are the pumps and heat exchangers. In the case of the D.F.R. these lie outside the central vessel and the pumps are electromagnetic so that the following discussion is not relevant.

A pump and heat exchanger have the same flow in order of magnitude inside them, but in a counter flow heat exchanger with secondary coolant flowing in the opposite direction the net velocity locally averaged at each position will be much lower. Although the high shear in the flow might encourage self-excitation we assume a heat exchanger is less likely to self-excite than a pump. We therefore consider the pump only and calculate R_m in two different ways. The first calculation is based on the assumption that the flow through the pump occurs at a typical speed equal to the mean velocity in the pipe work (6 m/sec in both reactors), and that the required flow passes through a circular area representing the centre of the pump and back through an annulus (figure 2). The outer radius of this annulus and the velocity is used to calculate R_m. The second, much higher, value of R_m is based on the quoted radius and tip speed of the impeller. The flow in the pump would be a combination of the toroidal flow set up when the impeller was rotating at full speed without any flow through the pump, and the poloidal flow of variable strength depending on the flow rate through the pump. Although the toroidal flow has the highest value of R_m, it is a flow pattern that cannot self-excite, with a steady field at any rate, in spherically symmetric electrical geometries. The addition of a variable amount of poloidal flow (largely axisymmetric, though there would be some azimuthal variation due to the guide vanes and more especially the diffuser outlets) makes self-excitation much more likely, though whether the induced magnetic field would be steady or oscillating is not obvious. The flow in the pump might be somewhat similar to the flows assumed by Lilley in the calculations (criticized by Roberts[2]) cited above for which R_m based on the maximum velocity in the toroidal flow was 80.

We conclude that the P.F.R. pumps have a marginal possibility of self-exciting when running hot, and are more likely to do so when running cold. The C.F.R. pumps, with values of R_m about twice as large, are more likely to self-excite. The reactor flow systems as a whole are less likely to self-excite than the pumps.* We cannot say more than this without a detailed knowledge of the flow pattern and proper calculations of the critical magnetic Reynolds' numbers. Such calculations would not be easy as the lack of firm theoretical predictions of critical Reynolds' numbers shows - however recent approaches using variational methods to give approximate values (e.g. Lerche[5]) might be possible.

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*Two or more pumps in a large vessel of liquid sodium might form a dynamo of the Herzenberg type (Roberts 1 or 2). However very high values of R_m would be required even if their relative orientation was favourable, and parallel orientation of the rotors is not.

THE CONSEQUENCES OF SELF-EXCITATION

If self-excitation occurs, the effect depends on the constraints on the flow pattern. If, for example, the whole reactor loop forms a self-exciting system at some critical flow rate and if the flow is entirely constrained in direction by the pipework, leaving the flow rate as the only variable, then it is probable that at the critical flow rate currents and magnetic fields would be induced which would keep the flow rate at its critical level. If in this case the pressure were increased in an attempt to increase the flow rate, the strength of the induced fields would increase just sufficiently to balance the increased pressure, but the flow rate would remain the same, because the flow pattern cannot be distorted. This situation is analogous to that in the conventional dynamo discussed above.

The reactor loop resembles a constrained flow in most places, but the return flow from the heat exchanger via the reactor vessel to the pump is unconstrained, and dictated by fluid mechanical forces. As self-excitation takes place and the induced electromagnetic forces increase, they might modify the flow pattern in such a way that a higher flow rate was possible, albeit with increased pressure losses. How much the flow rate could be increased before the extra pressure loss became intolerable would depend on the detailed situation, but in principle the loop might be able to carry flows greater than the initial critical flow by progressively modifying the flow pattern so that it was always just critical. The theory of motions which are not specified, but are determined by the interaction between the fluid mechanic and electromagnetic equations is even more difficult than the theory of dynamos where the flow pattern is fixed.

In the pumps, if they self-excite as systems by themselves, the situation is the same. One might expect a pump not to self-excite when running unloaded owing to the symmetry of the flow pattern, but to do so at a critical flow rate, which could not be exceeded. The prospects of the induced electromagnetic forces altering the flow pattern inside the pump to accommodate an increased flow rate would appear to be limited by the constraints put on the flow pattern by the pump geometry.

Other possibilities exist. It might turn out that even if the pump self-excited at a certain flow rate, it would de-excite at a higher flow rate (since the flow pattern would vary with flow rate in a definite way, being the combination of the flow caused by the spinning rotor without pumping and that caused by pumping). There would then be the problem of passing through the forbidden region of the flow rates. Another possibility is that the self-exciting mode might favour an oscillatory magnetic field rather than a steady one as has been tacitly assumed until now. This would result in oscillatory electromagnetic forces and the additional pressure drops would be oscillatory, which might cause mechanical problems.

Incipient self-excitation is difficult to detect. A characteristic value of the magnetic Reynolds' number and a corresponding magnetic field distribution exist, rather like the characteristic frequency and mode in a vibration problem, and nothing happens until the critical conditions are reached. If the characteristic mode, in this case the magnetic field pattern, is known, an estimate of the critical magnetic Reynolds' number might be made by imposing that field pattern on the system and measuring the field enhancement at a lower Reynolds' number. Even if the characteristic mode is unknown measurements of the distortion of an arbitrary field, such as the Earth's field, might provide useful indications of self-excitation. If the reactor as a whole self-excites measurements outside the reactor vessel would probably suffice, but to test the pumps measurements as near as possible to them would be preferable.

CONCLUSIONS

The liquid metal flows in the D.F.R., P.F.R. and C.F.R. have been examined to see if they may be electromagnetically self-exciting systems. Any self-excitation would result in the spontaneous growth of currents and magnetic fields in the liquid metal. These fields might either limit the flow rate in the circuit to a certain critical value if the flow pattern is sufficiently constrained, or else disturb the flow pattern in such a way that the required flow rate is achieved at the expense of increased pressure losses.

It has been shown that the necessary (but not sufficient) condition - that the magnetic Reynolds' number be large enough - is

(i) barely satisfied in the D.F.R.,

(ii) might be satisfied in the P.F.R.

and (iii) is more likely to be satisfied in the C.F.R.

In all cases self-excitation is more likely to occur in a cold reactor since the electrical conductivity of the liquid coolant is higher. It appears that the pumps in the P.F.R. and C.F.R. are more likely to self-excite than the flow circuit as a whole. Experimental evidence has been quoted which shows that if the flow pattern is suitable self-excitation can happen at magnetic Reynolds' numbers that will occur in the P.F.R. and C.F.R.

Complexities due to realistic geometries and inhomogeneous material properties have been ignored in this elementary account. More definite conclusions could not be reached without considerable computational work.

ACKNOWLEDGEMENTS

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Table I Fast reactor magnetic Reynolds' numbers

Reactor	typical values	D.F.R.	P.F.R.	C.F.R.
Half-dimension a	m	2	6	12
Maximum velocity v	m/s	5	6	6
Coolant		NaK 70/30	Na	Na
σ_{hot}	mho/m	2×10^6 (400°C)	3.3×10^6 (550°C)	3.3×10^6 (550°C)
σ_{cold}	mho/m	3×10^6	10^7 (100°C)	10^7 (100°C)
$R_m = \mu \sigma_{hot} v_{max} a$		25	140	290
Flow loop length ℓ	m	8	30	60
Total reactor flow q	m^3/s	0.43	3.8	15
Total coolant volume V	m^3	60	1000	5000
Averaged velocity $\frac{q\ell}{V}$	m/s	0.06	0.11	0.18
$R_m(\text{averaged}) = \mu \sigma_{hot} \frac{q\ell}{V} a$		0.3	3	9
Pumps		Electromagnetic	Mechanical rotating	Mechanical rotating
Pump flow rate	m^3/s	-	1.2	3.5
Typical through flow velocity	m/s	-	6	6
Calculated radius	m	-	0.35	0.6
R_m (through flow, hot)		-	8	14
Impeller radius	m	-	0.4	0.6
Impeller tip speed	m/s	-	42	50
R_m (impeller, hot)		-	67	120

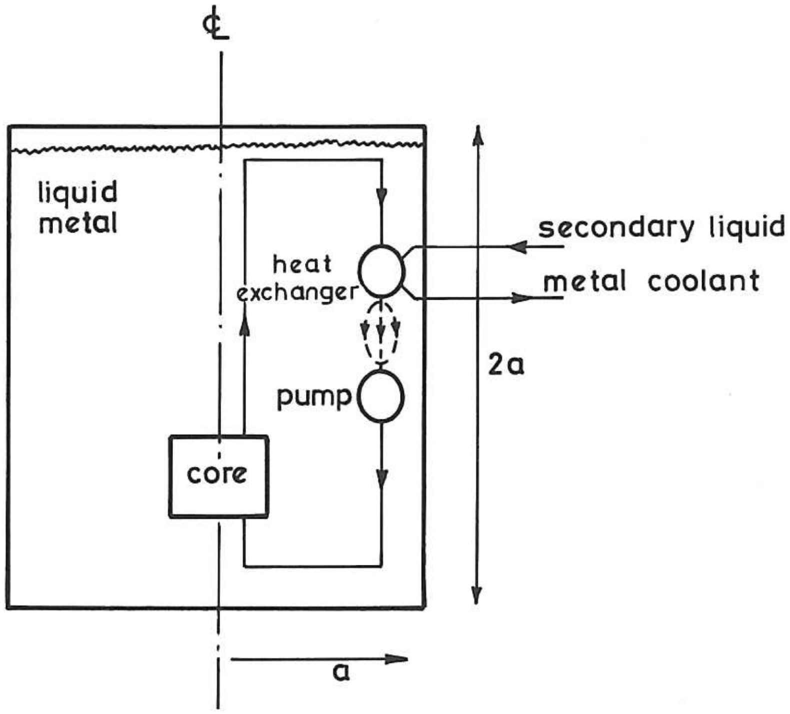

Fig.1 A SIMPLIFIED FAST REACTOR

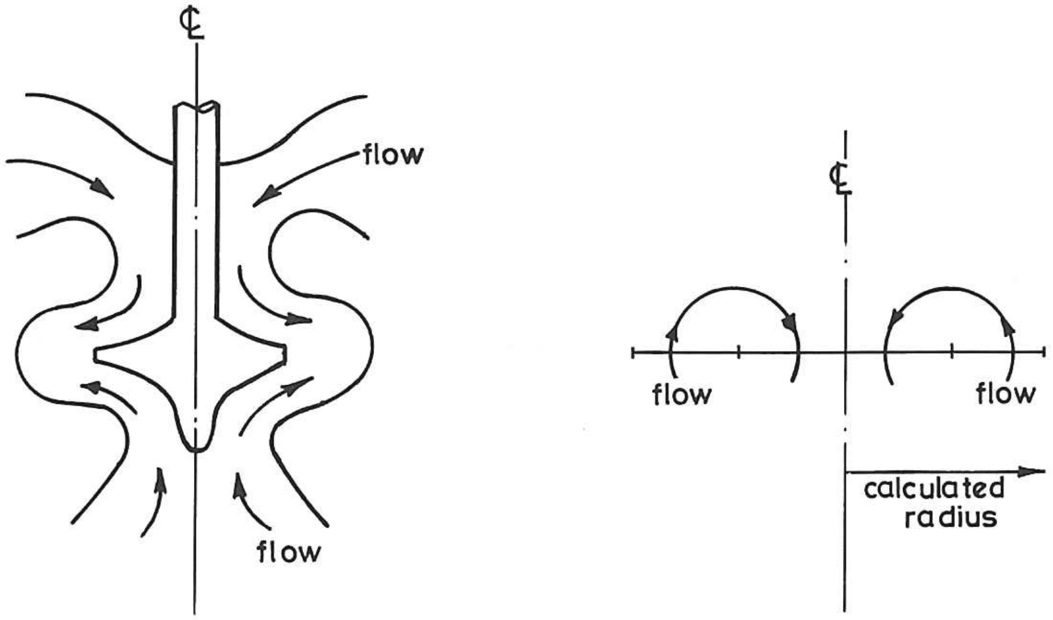

Fig.2 SKETCH OF P.F.R. PUMP