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DISTRIBUTIONS OF PLASMA DENSITY AND BETA IN HIGH BETA PINCHES

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DISTRIBUTIONS OF PLASMA DENSITY AND BETA IN HIGH BETA PINCHES

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ABSTRACT

Predictions of density distributions for theta pinches in radial pressure equilibrium, with the electron drift speed (j/ne) less than or equal to the sound speed ($\sqrt{kT_e/m_i}$), are compared to and found to agree with results obtained from six experiments. A high beta is only obtained when high line densities are used. The relevance of this result to high beta fusion plasma is discussed.

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In recent years interest in pinch discharges has been stimulated by the need to contain high beta plasma for fusion purposes. Particular distributions of magnetic fields and pressure within the plasma are required for MHD stability so that the penetration of fields into the pinch during its formation is of some importance. For high beta a substantial part of the plasma current must flow perpendicular to the magnetic field, and this is in part influenced by the perpendicular resistivity.

The linear theta pinch, having a neutrally stable and well defined configuration, is ideal for investigations of $\eta_\perp$ since only $j_\perp$ currents are present. Detailed measurements of diffusion have been made from which it is deduced that in the formation stage, lasting typically 1 to 2 μsec, the effective resistivity exceeds that due to simple coulomb collisions by a large factor[1]; at later times fully classical behaviour is seen until axial loss processes complicate the interpretation [2].

It has been shown that the formation stage, which includes the implosion, can be understood if an anomalously high $\eta_\perp$ is present when, and only when, the electron drift speed ($V_D = j/n_e e$) exceeds the sound velocity ($V_s = \sqrt{kT_e/m_i}$) [3]. Thus when $V_D > V_s$ rapid diffusion occurs and the current sheet is broadened, reducing V_D. The associated dissipation increases V_s until $V_D = V_s$, when the diffusion becomes classical and relatively slow. Consequently on this model the final plasma distribution has $V_D \lesssim V_s$ everywhere.

A simple analytic theory was described and solutions were given for the radial distributions of density, $n_e(r)$, and beta on axis, β_o, in an isothermal theta pinch with $V_D \lesssim V_s$. The β_o and the form of $n_e(r)$ were found to depend on N^1, the ratio of line density

$\left[N = 2\pi \int_o^{r_{wall}} n_e\,(r)rdr\right]$ to a critical value, $N_c = (5.3\pi m_i/2\mu_o e^2)\left[1 + T_i/T_e\right]^{\frac{1}{2}}$, which is about 0.2 to 1.0 × 10^{19} m^{-1}. This critical value is related to the ion plasma wavelength $\lambda_{io} = (m_i/\mu_o n_e(o)\ e^2)^{\frac{1}{2}}$ since $N_c \sim \pi\ \lambda_{io}^2\ n_e(o)$, where $n_e(o)$ is the density on axis. From this simple model it was shown that high beta plasmas were always bounded by a sheath of thickness $\sim 2\ L_e$, where $L_e \sim \lambda_{io} \left[\beta_o/2\ (1 + T_i/T_e)\right]^{\frac{1}{2}}$, and for unity beta on axis an $N^1 > 1$ is required. A significantly large central region of $\beta \sim 1$ exists only if $N^1 >> 1$.

Predictions of the above theory have been compared with results from six theta pinch experiments[2-8] in which N^1 covered the range from 0.6 to 14. Data was selected for a time during each experiment after the implosion but before axial electron thermal conduction became significant. Thus the measured plasma properties were determined only in the formation stage. Figure 1 shows the predicted and measured density distributions plotted on a radius scale normalised to L_e; good agreement can be seen. Approximately Gaussian distributions of $n_e(r)$ are seen when $N^1 < 1$ and distributions with a uniform central region are observed only for $N^1 > 1$. There is no case with $n_e(r)$ lying significantly above the corresponding theoretical value.

In Figure 2 the average beta $<\beta>$ (ref.9 Appendix describes the averaging) for theory and experiment are compared. As predicted, $<\beta>$ rises asymptotically to unity as N^1 is increased. There is no case of the measured $<\beta>$ exceeding its corresponding theoretical value. Details of the experiments are summarised in the accompanying table.

It should be emphasised that the detailed mechanism responsible for the resistance anomaly is neither assumed nor revealed in the above comparison. Work on collision-free shocks demonstrates the existence of a number of streaming-induced microinstabilities which could enhance the resistivity [10]. An alternative model based on a rigid rotor distribution also yields a critical line density and similar $n_e(r)$ for $N^1 < 1$ [6]. However it cannot give the large central regions when $N^1 \gg 1$ as seen on the existing experimental timescales.

The assumption that $V_D \lesssim V_s$ may be applied to other types of discharge and for example in the Bennett Pinch leads to a corresponding critical line density slightly larger than that for theta pinch. Useful axial current discharges are complicated by MHD effects and necessarily have $j_{\parallel}$ currents. Also electric fields can be sustained at the boundary to maintain $V_D > V_s$ provided $N^1 < 1$. Despite this complexity anomalous resistivity effects can be seen and in the early Zeta work [11] a critical line density $\sim 3 \times 10^{19}\ m^{-1}$ was observed. When $N^1 \gg 1$ thin current skins are produced which are inconsistent with MHD stability unless special conditions prevail. It should be noted that all hot, stable, long-lived toroidal discharges studied to date exist with $N \lesssim 3 \times 10^{19}\ m^{-1}$ where resistivity anomalies could contribute to the sheath broadening during their formation. Alternative sheath broadening mechanisms, not accompanied by large energy loss, must be sought for fusion devices which require diffuse magnetic field distributions and $N \sim 10^{21}\ m^{-1}$.

In conclusion it is noted that the analytic model of a theta-pinch equilibrium based on a uniform electron drift velocity in the sheath equal to the sound speed leads to density distributions which depend on the line density. A central high beta region is obtained at high line density and a sheath of a few ion plasma wavelengths is always present. Distributions measured in six different experiments covering a wide range of line density are in agreement with predictions of the model and this result supports the assumption that drift and sound speeds are equal in the theta pinch sheath. The universal nature of the sheath width and the critical line density effect highlights the need of alternative sheath broadening mechanisms for fusion plasmas which are to operate with diffuse current distributions at very high line densities.

The analysis of the theta pinch is due to the late J. McCartan who first pointed out that the model explains the density and β distributions observed in theta pinches. This work has not been previously published and much of the information is from his notebooks. He would have acknowledged fruitful discussions with many of the authors referenced.

Table I — PARAMETERS OF THETA PINCHES USED IN THE COMPARISON

Device	Line Density $\times 10^{18}$ m^{-1}	$\frac{T_i}{T_e}$	Density on axis $n_o \times 10^{22}$ m^{-3}	Gradient scale length $\times 10^{-2}$ m	Normalised Line Density N^1	Calculated beta		Measured beta	
						β_o	β average	β_o	β average
CENTAUR * (CULHAM) (5)	30	1.5	1.0	0.35	14.3	1	0.97	0.99	0.9
1 METRE (4) (CULHAM)	16	2	1.2	0.35	6.3	1	0.95	0.95	0.88
8 METRE (2) (CULHAM)	3.3	1	1.6	0.25	2	1	0.75	0.9	0.5
SCYLLA IV (6) LOS ALAMOS	6.2	5	6.0	0.22	1.21	1	0.6	0.8	0.4
ISAR GARCHING (7)	4.6	5	2	0.38	0.91	1	0.55	-	0.5
SCYLLA III LOS ALAMOS (8)	4.3	7.5	2.6	0.35	0.6	0.851	0.5	0.6	0.3

* A small negative bias field was used in this experiment. Other experiments had zero bias field.

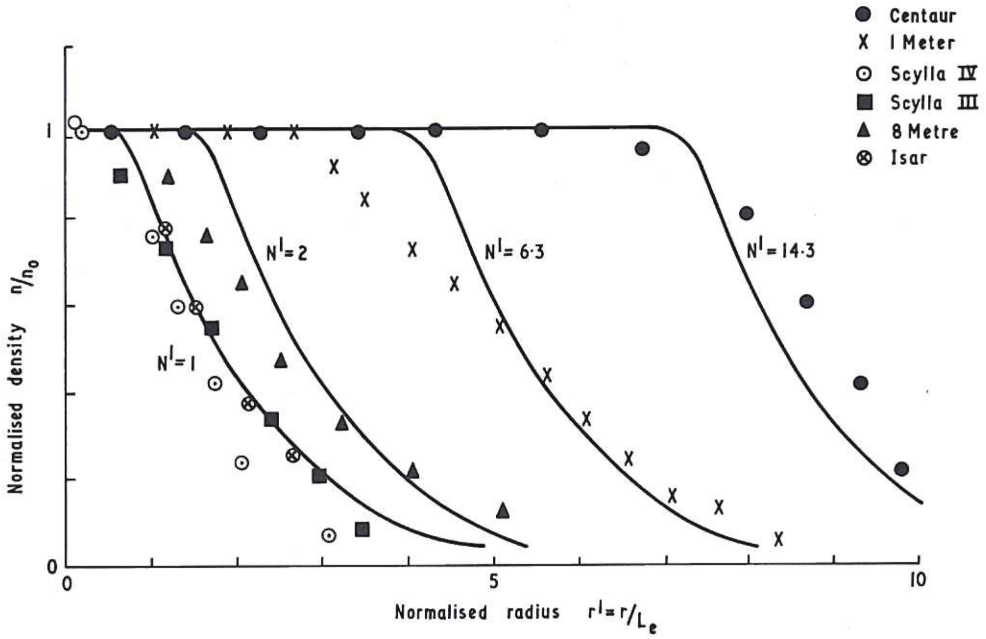

Fig. 1 Radial density distributions predicted using the analytic model. Points show results from experiments. Solid lines show predictions of the theory for some values of normalized line density, N^1.

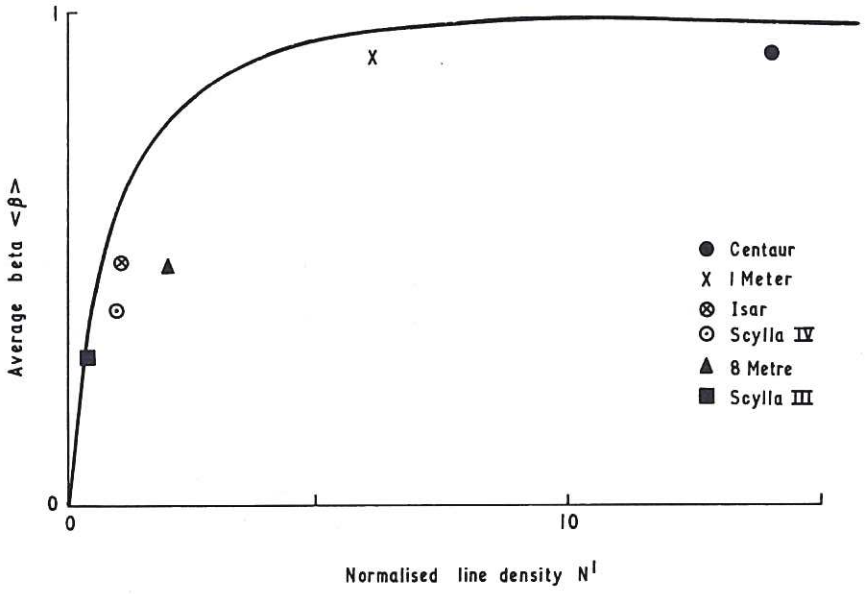

Fig. 2 Average beta predicted using the analytic model vs. normalized line density. Points show results from experiments.

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