CLM-P 36

This document is intended for publication in a journal, and is made available on the understanding that extracts or references will not be published prior to publication of the original, without the consent of the author.

United Kingdom Atomic Energy Authority

RESEARCH GROUP

Preprint

EQUILIBRIUM AND STABILITY OF PLASMA IN ARBITRARY MIRROR FIELDS

J. B. TAYLOR

Culham Laboratory,
Culham, Abingdon, Berkshire
1964

EQUILIBRIUM AND STABILITY OF PLASMA IN ARBITRARY MIRROR FIELDS

by

J. B. TAYLOR

(Submitted for publication in Physics of Fluids)

ABSTRACT

It is shown that an appropriate choice of variables can greatly simplify the discussion of equilibrium and stability of low-pressure plasma in arbitrary mirror fields. One specifies by α, β the line of force on which a particle is moving and also specifies its adiabatic invariants μ, J; the energy of the particle is then determined as a function $K(\mu,J,\alpha,\beta)$ which plays the role of a Hamiltonian. Any equilibrium distribution can then be written in the form $F\{\mu,J\ K(\alpha,\beta,\mu,J)\}$ and it is shown that a sufficient criterion for such distributions to be stable against interchanges is $\left(\frac{\partial F}{\partial K}\right)_{\mu J} < 0$. Necessary and sufficient criteria are also derived. When approached in this way, the exact form of the field configuration only enters the problem through the determination of the function K, which may be easily calculated. In general a comprehensive view of plasma behaviour, convenient for the discussion of equilibrium, confinement and stability, can be obtained from the structure of the $K(\alpha,\beta,\mu,J)$ = constant contours. An example of the application of this approach to a Ioffe stabilised mirror is described; this confirms the existence of stable plasma equilibria in this field configuration.

U.K.A.E.A. Research Group,
Culham Laboratory,
Nr. Abingdon,
Berks.

January, 1964

(C/18 IMG)

C O N T E N T S

1. INTRODUCTION

In an earlier paper[1], henceforth referred to as I, the equilibrium and stability of certain special plasma distributions in combined mirror-cusp fields were discussed. These fields are such that $|B|$ possesses a (non-zero) minimum and the distributions are those in which the pressure tensor $\underline{\underline{P}}$ depends only on $|B|$. In the present work a general method is described for the analysis of equilibrium and stability in any form of low-β adiabatic mirror machine, including hybrid mirror-cusp systems such as Ioffe's stabilised mirror[2], the stuffed cusp, the systems discussed by Furth[3] and by Andreoletti[4] and similar devices. Some of these devices do not possess the high degree of symmetry of the elementary mirror but it will be shown that if the proper variables are employed it is nevertheless possible to express the conditions for equilibrium and for stability in forms which are both simple and general.

In considering these problems it has been customary to consider a confined plasma (i.e. one localized in a limited region) and to apply criteria for equilibrium (often automatically ensured by symmetry) and for stability. However, in I it was more convenient to use a description in which equilibrium and stability could be discussed first and only later did one discuss confinement. In other words instead of taking a confined (localized) distribution and applying a stability criterion one considered stable distributions, with $\underline{\underline{P}}$ expressed as a function of $|B|$, and then applied a confinement criterion. The advantage of this approach is that the actual form of the magnetic field only enters the problem through the confinement criterion, and furthermore it enters in a very simple way (e.g. in I it reduced to the question 'do the surfaces $|B|$ = constant form a closed nested set?' In the present paper we adopt a similar viewpoint but we will now employ the 'particle-drift' description of a low-β plasma instead of a fluid description.

In section 2 these particle drifts are discussed. The instantaneous drift velocity is well known but a more relevant concept is the <u>average</u> drift motion over several oscillations between mirrors. The equations for this average motion can be put in a simple form if the appropriate co-ordinates are used[5]; this is because the adiabatic invariants $\mu = mv_\perp^2/B$ and $J = \oint v_\parallel ds$ are constant during the drift motion. The key to the problem is to note that if one specifies the line of force on which the particle is moving (by co-ordinates α, β) and also specifies its μ, J then its energy is determined. This allows one to regard the energy of a particle, not as an independent variable but as a known function of μ, J, α, β. Then this function $K(\mu, J, \alpha, \beta)$ plays the role of a Hamiltonian[5], and the equations for the average drift motion are:

$$\dot{\alpha} = -\frac{c}{e}\frac{\partial K(\alpha,\beta,\mu,J)}{\partial\beta} \quad , \qquad \ldots (1.1)$$

$$\dot{\beta} = +\frac{c}{e}\frac{\partial K(\alpha,\beta,\mu,J)}{\partial\alpha} \quad . \qquad \ldots (1.2)$$

As a consequence of this canonical form for the drift motion, any equilibrium distribution can be expressed in the form

$$F_{eq}[\mu,J,\alpha,\beta,] \equiv F[\mu,J,K(\alpha,\beta,\mu,J)] \quad , \qquad \ldots (1.3)$$

where again one is not regarding K as a variable but as a known function of α, β, μ, J.

The question of stability is considered in section 4 where it is shown that once the equilibrium has been expressed in the special form (1.3) one can obtain a simple criterion for stability against interchanges (i.e. motions in which flux tubes are interchanged and the magnetic field is unaltered - these are the most important of the possible instabilities at low-β). In fact the system is stable against interchanges if

$$\frac{\partial}{\partial K} F(\mu,J,K) < 0 \quad . \qquad \ldots (1.4)$$

Equation (1.3) states that, for each μ, J, an equilibrium distribution is constant along contours $K(\alpha,\beta) =$ constant, so that confinement (i.e. localization) of a distribution is easily discussed in terms of the topology of these K contours. For example if F is non-zero for a particular $K = K_0$ then the K_0 contour and its associated flux surface must be closed within the region of interest. A more significant observation is that in a confined plasma F must decrease toward the periphery and if it is also to satisfy the criterion (1.4) it must decrease with increasing K; for this to be possible K itself must increase toward the periphery, i.e. $K(\alpha,\beta)$ must possess a minimum. It is important, therefore, to note that this requirement is fulfilled in magnetic fields in which $|B|$ increases toward the periphery. Such fields have a large class of stable equilibrium distributions, among which are the special distributions found in I.

It is apparent that the function $K(\alpha,\beta,\mu,J)$ is of supreme importance in understanding the behaviour of low-β plasma in arbitrary adiabatic mirror systems and in searching for systems possessing stable confined equilibria. Once it is known a complete description of particle drifts, of equilibrium and of confinement is available, and so is a partial description of stability which encompasses the most important types of possible motion. It is only in the determination of this function K that the specific form of the magnetic field enters the problem and it does so in such a way that the determination of K is not difficult. An example of the application of the present approach to a

Ioffe-type stabilised mirror is described in section 5.

2. CANONICAL EQUATIONS FOR AVERAGE GUIDING CENTER DRIFT

When the Larmor radius is small compared to the scale of the variations in magnetic field the motion of a particle can be regarded as a rapid gyration about a guiding center. In the course of this motion the magnetic moment $\mu = mv_\perp^2/B$ is a constant and as a result the particle is confined, between magnetic mirrors, to the region where $\mu B < E - e\varphi$ (where E is the particle energy and φ the electrostatic potential).

In this event the guiding center itself oscillates rapidly along a line of force between the two mirror points and at the same time it 'drifts' more slowly across the field. The instantaneous drift velocity is well known, being given by

$$v_d = \frac{c\underset{\sim}{n}}{B} \times \left\{ \nabla\varphi + \frac{\mu\nabla B}{e} + \frac{m}{e} v_\parallel^2 \frac{\partial \underset{\sim}{n}}{\partial s} \right\} , \qquad \ldots (2.1)$$

where $\underset{\sim}{n}$ is the unit vector along $\underset{\sim}{B}$. It is assumed that the potential and field gradients are such that $v_d \ll v_\parallel$. However, in view of the rapid oscillation along the line of force this instantaneous drift velocity is of less significance than the <u>average</u> guiding center drift over a period of the oscillation between mirrors. The equations for this average drift can be put in a partcularly simple (canonical) form which was first given by Northrop and Teller(5). They considered the drift process in detail and actually constructed the average of (2.1) over the period of an oscillation between mirrors - a lengthy procedure Here the canonical equations will be derived directly, without recourse to (2.1), by a canonical transformation.

The simplicity of the final form of the drift equations is made possible by using a representation of the magnetic field which allows the field lines to be used as one element of a co-ordinate grid*. One writes

$$\underset{\sim}{B} = \nabla\alpha \times \nabla\beta ,$$

then clearly α, β are constant along a field line so that α, β can be regarded as the co-ordinates of that field line. More specifically, if we consider any surface S cut by field lines and draw on this surface the lines α = constant, β = constant then these lines

---

* In incompressible hydrodynamics a similar representation to this is sometimes used and has been referred to as a Clebsch Transformation. In their present context the α,β co-ordinates were introduced by H. Grad and H. Rubin. Proceedings second U.N. Conference, Geneva, 1958, paper 383.

form a co-ordinate grid on which the lines of force are located by the α, β value of their intersection with S.

The scale of the co-ordinate α, β can be chosen so that the flux through any part ΔS of the surface S is numerically equal to

$$\iint_{\Delta S} d\alpha\, d\beta \; .$$

In terms of this representation of the field the vector potential can be written

$$\underset{\sim}{A} = \alpha \nabla \beta \; .$$

Once the field lines have been specified by the α, β co-ordinate system, any point P in space can be located by co-ordinates (α, β, χ), where α, β are the co-ordinates of the field line on which P lies and χ is the magnetic potential along that field line from P to the reference surface S, i.e.

$$\chi = \int_S^P \underset{\sim}{B} \cdot d\underset{\sim}{s} \; .$$

We now return to the problem of describing the average guiding center motion. As the magnetic moment of the particle is constant the guiding center moves as if it were a particle of charge e, mass m, and magnetic moment $\mu \underset{\sim}{n}$. The Lagrangian for such a particle is

$$L = \frac{1}{2} mv^2 + \frac{e}{c} \underset{\sim}{v} \cdot \underset{\sim}{A} - e\varphi - \mu B \; , \qquad \ldots (2.2)$$

and the total energy is

$$E = \frac{1}{2} mv^2 + e\varphi + \mu B \; . \qquad \ldots (2.3)$$

We now express the Lagrangian in the (α, β, χ) co-ordinate system and use the corresponding $\alpha \nabla \beta$ representation of $\underset{\sim}{A}$, then

$$L = \varepsilon + \frac{m\dot{\chi}^2}{2B^2} + \frac{e}{c} \alpha \dot{\beta} - e\varphi(\alpha,\beta,\chi) - \mu B(\alpha,\beta,\chi) \; , \qquad \ldots (2.4)$$

where ε is the kinetic energy associated with the transverse drift and is negligible compared to the other terms in(2.4) when the drift velocity is small compared to the actual particle velocity. The conjugate momenta to α, β, χ are then

$$p_\alpha = 0 \; , \quad p_\beta = \frac{e\alpha}{c} \; , \quad p_\chi = \frac{m\dot{\chi}}{B^2} \; , \qquad \ldots (2.5)$$

and we note that there is, in fact, no momentum conjugate to α; instead $\frac{e\alpha}{c}$ is itself conjugate to β. The Hamiltonian function is then

$$H = \frac{B^2}{2m} p_\chi^2 + e\varphi(\alpha,\beta,\chi) + \mu B(\alpha,\beta,\chi) \; . \qquad \ldots (2.6)$$

In order to obtain the equations of motion for the average drift we should solve the equations of motion in the χ direction and then average the transverse $(\dot{\alpha},\dot{\beta})$ equations over this motion as was done by Northrop and Teller. However we can obtain the same result directly if we eliminate χ from the Hamiltonian by an appropriate canonical transformation to Action-Angle variables. To do this we introduce as a co-ordinate the Action conjugate to χ i.e.

$$J = \oint p_\chi \, d\chi = \oint [2m(H - e\varphi - \mu B)]^{\frac{1}{2}} \, ds \quad , \qquad \ldots (2.7)$$

where the integral is along a particular α, β field line and is over one period of the oscillation between mirrors.

This equation implicitly defines H as a function of the new variables α,β,J,μ and also preserves the form of Hamilton's equations of motion. When the Hamiltonian (Energy) is expressed in terms of α,β,J,μ through (2.7) we will denote it by K. Then recalling that $\frac{e\alpha}{c}$ is conjugate to β and that $K(\alpha,\beta,J,\mu)$ is now the Hamiltonian function, we can write

$$\dot{\alpha} = -\frac{c}{e} \frac{\partial K(\alpha,\beta,J,\mu)}{\partial \beta} \quad , \qquad \ldots (2.8)$$

$$\dot{\beta} = +\frac{c}{e} \frac{\partial K(\alpha,\beta,J,\mu)}{\partial \alpha} \quad , \qquad \ldots (2.9)$$

$$\dot{J} = 0 \quad . \qquad \ldots (2.10)$$

It is important to note that these simple equations are only true when the motion is expressed in the α,β co-ordinate system and when K is expressed in terms of α,β,μ,J by means of (2.7), i.e.

$$J = \oint [2m\{K - e\varphi(\alpha,\beta,s) - \mu B(\alpha,\beta,s)\}]^{\frac{1}{2}} \, ds \quad .$$

3. EQUILIBRIUM DISTRIBUTION

Once the equations of motion have been put into the canonical form the construction of the equilibrium distribution is obvious. Let $F[\alpha,\beta,J,\mu,t]$ be the particle density in (α,β,J,μ) space, then

$$-\frac{\partial F}{\partial t} = \frac{\partial}{\partial \alpha}(F\dot{\alpha}) + \frac{\partial}{\partial \beta}(F\dot{\beta}) \quad , \qquad \ldots (3.1)$$

and, using the values of $\dot{\alpha},\dot{\beta}$ given by (2.8),(2.9), a stationary state exists if and only if

$$\frac{\partial K}{\partial \beta}\frac{\partial F}{\partial \alpha} - \frac{\partial K}{\partial \alpha}\frac{\partial F}{\partial \beta} = 0 \quad , \qquad \ldots (3.2)$$

that is if F is a function of α,β only through the quantity $K(\alpha,\beta,J,\mu)$. Any equilibrium can therefore be written

$$F_{eq} = F\{\mu,J,K(\alpha,\beta,\mu,J)\} \ . \qquad \dots (3.3)$$

As μ, J, K are all constants of the motion this merely says that the equilibrium distribution is a function of the constants of the motion - a well known result. It should be remembered that $F\{\mu,J,K\}$ is defined so that

$$F\{\mu,J,\ K(\alpha,\beta,J,\mu)\}\ d\mu\ dJ\ d\alpha\ d\beta \ , \qquad \dots (3.4)$$

is the number of particles in the element $d\mu\ dJ\ d\alpha\ d\beta$ and not as if

$$F\{\mu,J,K\}\ d\mu\ dJ\ dK \ , \qquad \dots (3.5)$$

were the number in $d\mu\ dJ\ dK$. The difference arises because of the existence of the surfaces of constant (μ,J,K). The function F is constant over such a surface so that one of the space co-ordinates does not really enter into the specification of F. (This is the general analogue of the fact that for equilibrium distributions in an axisymmetric system the azimuthal angle is redundant.)

4. STABILITY

In a low-β system in which the magnetic field is the vacuum field due to external conductors, 'interchanges' of flux tubes are the most important form of instability. Indeed these are the only quasi-hydrodynamic, i.e. adiabatic, instabilities possible at low-β. It is an important result of this paper, therefore, that a very simple criterion can now be obtained for the stability of equilibria such as (3.3) against these interchanges.

We will suppose that the equilibrium is stationary and that there is no electric field in the equilibrium state. Then K is defined as a function of J,μ,α,β, by

$$J = \oint [2m(K - \mu B(\alpha,\beta,s))]^{\frac{1}{2}}\ ds \ . \qquad \dots (4.1)$$

An 'interchange' motion is one in which particles initially on the same flux tube remain on the same flux tube. It results from the "$\underset{\sim}{E} \times \underset{\sim}{B}$" drift associated with an electric field transverse to the magnetic field. We will consider a possible interchange in which particles on a flux tube $(\alpha_1,\ \beta_1)$ are interchanged with those on an equivalent flux tube $(\alpha_2\ ,\ \beta_2)$. In this motion the invariants μ and J of each particle are conserved, but its energy may alter.

The total energy of the particles on the two flux tubes concerned before the interchange was

$$W_i = \int d\mu \; dJ \; \{ F(1)\, K(1) + F(2)\, K(2) \} \quad , \qquad \ldots (4.2)$$

where

$$F(1) \equiv F\{ \mu, J, K(\mu, J, \alpha_1, \beta_1) \} \quad , \qquad \ldots (4.3)$$

and

$$K(1) \equiv K(\mu, J, \alpha_1, \beta_1) \quad , \qquad \ldots (4.4)$$

and $K(2)$ and $F(2)$ are similarly defined.

After the interchange the particles which were on α_1, β_1 have moved to α_2, β_2 and so have energy $K(2)$ and vice-versa. The energy after the interchange is therefore

$$W_f = \int d\mu \; dJ \; \{ F(1)\, K(2) + F(2)\, K(1) \} \quad . \qquad \ldots (4.5)$$

The change in energy resulting from the interchange is thus

$$(W_f - W_i) = - \int d\mu \; dJ \; \{ [F(2) - F(1)] \; [K(2) - K(1)] \} \quad . \qquad \ldots (4.6)$$

It will be noted that we have so far made no restriction that the change $[F(2) - F(1)]$ need be small but we now make the usual assumption that the displacements are infinitesimal and calculate the energy change to second order in displacement. If the displacement of the flux tubes is measured by $\delta\alpha$, $\delta\beta$ we have

$$\delta^2 W = - \int d\mu \; dJ \left(\frac{\partial F}{\partial \alpha}\, \delta\alpha + \frac{\partial F}{\partial \beta}\, \delta\beta \right) \left(\frac{\partial K}{\partial \alpha}\, \delta\alpha + \frac{\partial K}{\partial \beta}\, \delta\beta \right) . \qquad \ldots (4.7)$$

However, since F depends on α, β only through K this becomes

$$\delta^2 W = - \int d\mu \; dJ \left(\frac{\partial K}{\partial \alpha}\, \delta\alpha + \frac{\partial K}{\partial \beta}\, \delta\beta \right)^2 \left(\frac{\partial F}{\partial K} \right)_{\mu J} . \qquad \ldots (4.8)$$

It is now apparent that $\delta^2 W$ must be positive for all $\delta\alpha$, $\delta\beta$ if

$$\frac{\partial F}{\partial K} < 0 \quad ,$$

therefore a criterion which is sufficient for stability against 'interchanges' is

$$\left(\frac{\partial F}{\partial K} \right)_{\mu J} < 0 \; , \qquad \ldots (4.9)$$

for all μ, J, K.

Criteria which are both necessary and sufficient can be obtained in terms of the appropriate averages of $\frac{\partial F}{\partial K}$. Thus if

$$\lambda_{\alpha\alpha} \equiv \int d\mu \; dJ \left(\frac{\partial K}{\partial \alpha} \right)^2 \left(\frac{\partial F}{\partial K} \right) , \qquad \ldots (4.10)$$

$$\lambda_{\beta\beta} \equiv \int d\mu \; dJ \left(\frac{\partial K}{\partial \beta} \right)^2 \left(\frac{\partial F}{\partial K} \right) , \qquad \ldots (4.11)$$

and

$$\lambda_{\alpha\beta} \equiv \int d\mu \; dJ \left(\frac{\partial K}{\partial \alpha} \right) \left(\frac{\partial K}{\partial \beta} \right) \left(\frac{\partial F}{\partial K} \right) ,$$

then a necessary and sufficient set of conditions is

$$\lambda_{\alpha\alpha} > 0 \; , \quad \lambda_{\beta\beta} > 0 \; , \quad [\lambda_{\alpha\beta}]^2 > \lambda_{\alpha\alpha} \cdot \lambda_{\beta\beta} \; . \qquad \ldots (4.12)$$

The simple condition (4.9) demands that F should decrease with increasing K while confinement of plasma requires that F should decrease toward the periphery of the system so that (4.9) and confinement are compatible only if K has the general form of a 'potential-well' in the α, β space, that is if $K(\alpha,\beta)$ possesses a minimum within the region of interest. If the magnetic field itself possesses a minimum then $K(\alpha,\beta)$ will possess a minimum for a wide range of μ, J so that many classes of stable equilibria can be constructed in these 'minimum-B' fields. Among these are the equilibria discussed in I which do indeed satisfy (4.9).

It must again be emphasised that the simplicity of the result (4.9) arises solely from the correct choice of variables - it is only correct when F is expressed in the form

$$F\{ \mu, J, K(\mu,J,\alpha,\beta) \} \; .$$

It would not be correct if, for example, one had expressed F in the more usual variables (μ,K,x) employed by Rosenbluth and Rostoker[6] and by Kruskal and Oberman[7].

5. EXAMPLE OF METHOD

The actual form of the magnetic field has not been mentioned in the theory given above. This is because it enters the problem only in the determination of K and as K is defined by the single integral (4.1) it is not difficult to compute K once the field is given. The calculation of K for several fields of interest has been carried out on the A.W.R.E. I.B.M. 7030, (Stretch) computer by F.M. Larkin and an example is illustrated below.

First we note that we can reduce $K(\alpha,\beta,\mu,J)$ to a function of three variables only:

$$\frac{J}{\mu^{1/2}} = \oint \left[2m \left\{ \frac{K}{\mu} - B(\alpha,\beta,s) \right\} \right]^{1/2} ds \; , \qquad \ldots (5.1)$$

so that $\frac{K}{\mu}$ is a function of α, β and $\frac{J}{\mu^{1/2}}$ only.

The arrangement of the calculation is roughly as follows; given the magnetic field one selects a convenient surface S on which to locate the α, β co-ordinate system and thereby to label each line by the α, β value of its point of intersection with S. Then for each of a number of representative field lines (i.e. of α, β points) the quantity $(\frac{J}{\mu^{\frac{1}{2}}})$ is computed for some value of $(\frac{K}{\mu})$ by integration along the appropriate field line to the mirror points. By interpolation in the surface S one then obtains the contours $\frac{1}{\mu} K(\alpha, \beta, \frac{J}{\mu^{\frac{1}{2}}}) =$ constant for a number of values of $(\frac{J}{\mu^{\frac{1}{2}}})$. The results are plotted automatically on a Benson-Lehner Model-J graph plotter. A typical calculation takes 10-15 minutes of computer time.

In the example shown the field is an elementary form of the configuration used in Ioffe's stabilized mirror experiments and is produced by two circular coils and four infinite straight conductors, (Fig.1). The coils are of radius R and separation 2R and carry a current I/2. The straight conductors are distant $R\sqrt{2}$ from the common axis of the two circular coils and adjacent conductors carry a current I in opposite directions. The field is thus a superposition of an orthodox mirror and an $\ell = 2$ multipole cusp.

The α, β plane for this calculation was chosen to be the mid-plane of the system, perpendicular to the common axis of the circular coils, then because of the symmetry of the conductors the $K(\alpha,\beta,J,\mu)$ contours have eight-fold symmetry. Only one quadrant of the α, β plane is shown. The figures cover the central area out to a radius of about $\frac{R}{3}$ and the contours are labelled with the values of $\frac{K}{\mu}$ in arbitrary units.

One may interpret these diagrams somewhat as one interprets contour heights on a geographic map. For example one may note such items as the following:

(i) As the K surfaces are also particle drift surfaces one sees immediately where the particles drift, but also from equations (1.1, 1.2) one gets a picture of the speed of drift from the separation between contours (just as one pictures the gradient from the separation between height contours on a map).

(ii) As the K surfaces are surfaces of constant $F_{equil.}$ they can also be visualised as density contours for this function. These points are elementary and do not utilize the theory given in this paper. However one can also see some more important points concerning stability.

(iii) Thus the example shown has a minimum in K at the center of the system so that confined distributions stable against interchange by (4.9) can be set up in this region of the field. However one also sees that the region of such stable confinement is small (remember the diagram shows only the central part of the system out to about one third of a coil radius.)

(iv) There are also other closed K-contours centered about a point X on the 45° axis (this is the axis passing through one of the straight conductors) so that other confined equilibria exist in this region. However, as the point X corresponds to a maximum rather than a minimum in K, such equilibria cannot satisfy the criterion (4.9).

The existence of a minimum in K, which ensures the existence of stable confined equilibria is a general feature of fields in which $|B|$ itself possesses a minimum as in the present example. It also occurs in the mirror configurations of Furth[3] and Andreoletti[4], but it does not usually occur for simple mirrors though it may presumably do so for some very special values of $(\frac{J}{\mu^{1/2}})$.

6. CONCLUSIONS

It is clear from the above example that the discussion of equilibrium, stability and confinement of low-β plasma in adiabatic mirror traps is, indeed, much simplified if the problem is approached in the way described in this paper. Far reaching results can often be obtained with little effort. The method involves using the field lines themselves as co-ordinates (α,β) and expressing the particle distribution function in the phase space of α,β,μ,J where μ,J are the two adiabatic invariants. The energy K is not treated as an independent variable but is <u>defined</u> by

$$J = \oint \left[2m\left\{K - \mu B\,(\alpha,\beta,s)\right\}\right]^{\frac{1}{2}} ds \, . \qquad \ldots (6.1)$$

(Note that this is the reverse of the usual procedure, in which K is regarded as a variable and J is defined by (6.1).)

In terms of these variables the equilibrium distribution function is of the form

$$F = F\left\{\mu,J,\ K(\alpha,\beta,\mu,J)\right\} \quad , \qquad \ldots (6.2)$$

and a sufficient condition for stability is

$$\left(\frac{\partial F}{\partial K}\right)_{\mu J} < 0 \, . \qquad \ldots (6.3)$$

Necessary and sufficient conditions are given by (4.12).

The confinement (localization) of the distribution is determined by the topology and location of the K = constant contours which are easily computed. Confined equilibria satisfying the stability criterion (6.3) can always be found if the $K(\alpha,\beta)$ function possesses a minimum in the region of interest. This will be the case if $|B|$ itself possess a minimum.

7. ACKNOWLEDGEMENTS

The author would like to thank members of the Culham Computing Unit, particularly Mr. F.M. Larkin, for the calculations described in section 5.

8. REFERENCES

1. TAYLOR, J.B. Phys. Fluids, vol.6, p.1529, (1963).

2. GOTT, Yu.B., IOFFE, M.S. and TELKORSKY, V.C. Nuclear Fusion, Suppl. 3, p.1045, (1962).

   BAIBORODOV, Yu.T., IOFFE, M.S., PETROV and SOBOLEV, R.I. Atomnaya Energiya, vol.14, p.443, (1963).

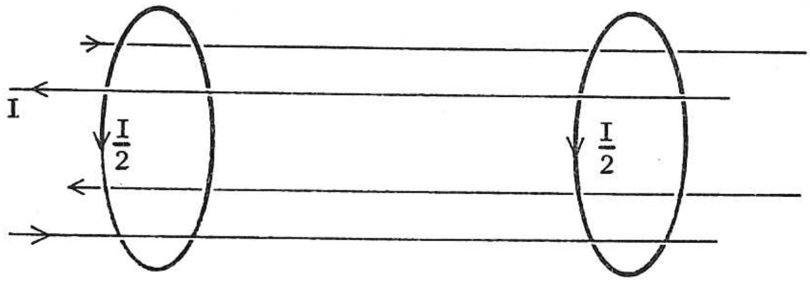
3. FURTH, H.P. Phys. Rev. Lett., vol.11, p.308, (1963).

4. ANDREOLETTI, J. Euratom Meeting. Fontenay aux Roses (1963). (Unpublished).

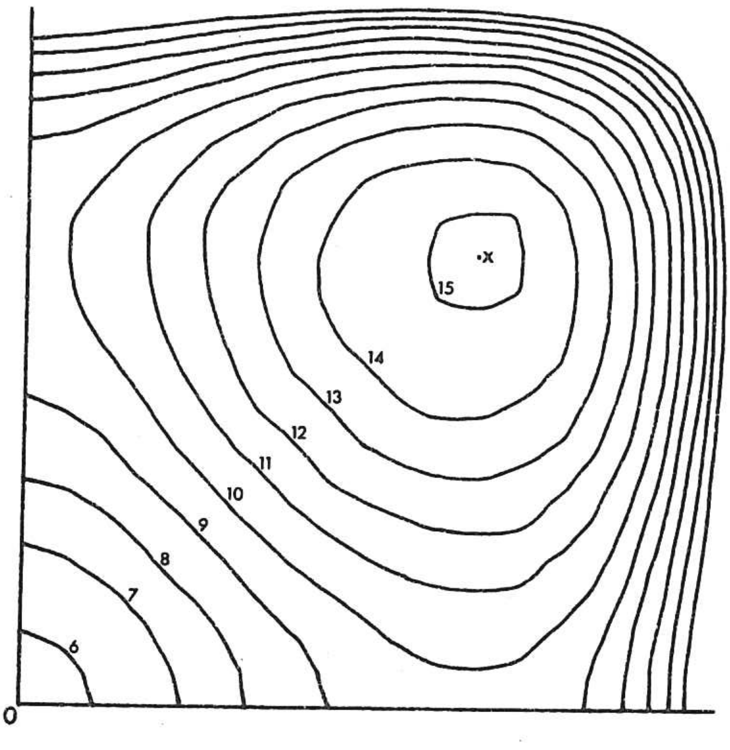
5. NORTHROP, T.G. and TELLER, E. Phys. Rev. vol.117, p.215, (1960).

6. ROSENBLUTH, M.N. and ROSTOKER, N. Phys. Fluids, vol.2, p.23, (1959).

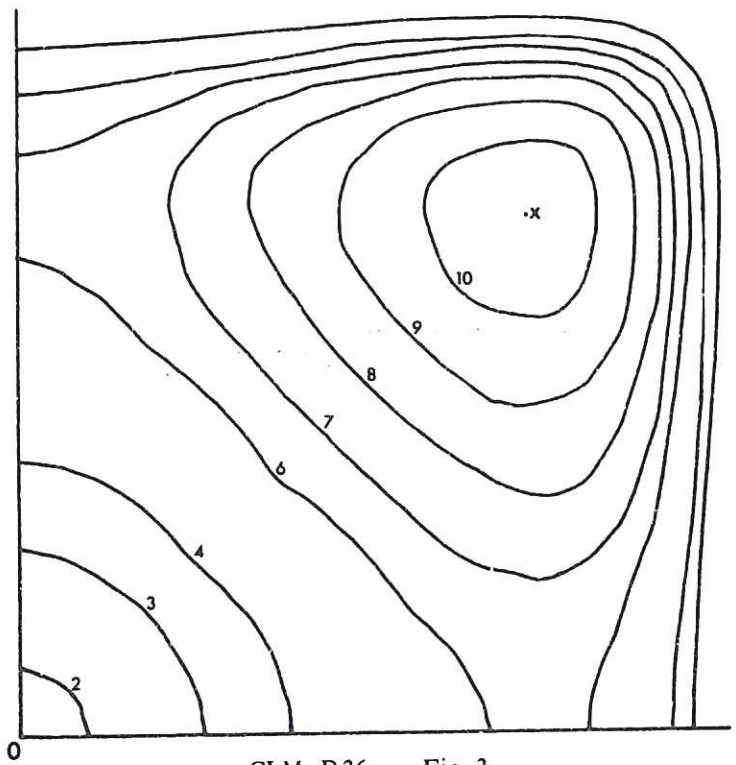
7. KRUSKAL, M.D. and OBERMAN, C.R. Phys. Fluids, vol.1, p.275, (1958).



CLM - P 36 Fig. 1
Coil arrangement.

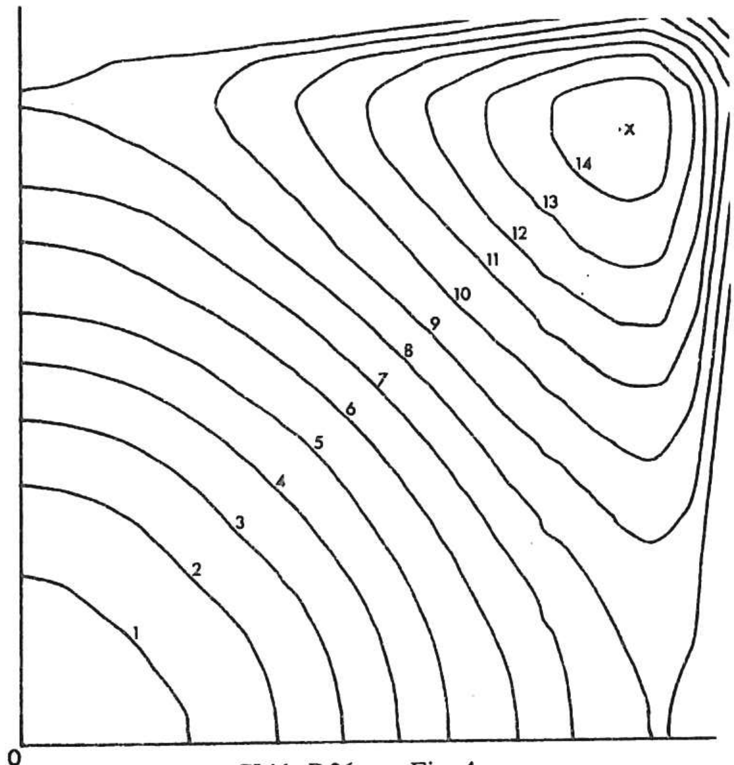


CLM - P 36 Fig. 2
Constant K contours for $\left(\frac{I}{\mu^{1/2}}\right) = 0.616$.



CLM - P 36 Fig. 3

Constant K contours for $\left(\frac{J}{\mu^{1/2}}\right) = 0.464$.



CLM - P 36 Fig. 4

Constant K contours for $\left(\frac{J}{\mu^{1/2}}\right) = 0.305$.