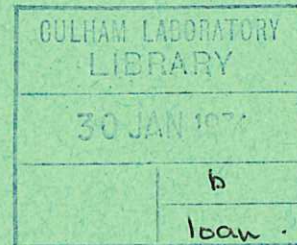


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Preprint

A MODEL FOR FUEL-COOLANT INTERACTIONS

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A MODEL FOR FUEL-COOLANT INTERACTIONS

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ABSTRACT

A model for fuel-coolant interactions is proposed. The interaction is divided into five stages: an initial perturbation triggers the interaction and causes a vapour bubble to form at the fuel-coolant interface; bubble expansion and collapse with jetting; penetration of the fuel by the liquid jet; heat transfer from the fuel to the jet; the formation of a new bubble. The process repeats itself cyclically from stage 2. The most important result is that the strength of the interaction is reduced as the external pressure is increased and can be inhibited entirely if the pressure is large enough.

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1. Introduction

It is well known that when a hot liquid comes into contact with a cold vaporisable liquid an explosion of considerable violence may occur. This type of explosion is called alternatively a vapour explosion, a thermal interaction or fuel-coolant interaction (FCI), the hot liquid being the fuel and the cold liquid the coolant. These interactions are not the result of chemical change; the energy source is the excess heat in the fuel. FCIs have been observed between cold water and various molten metals (e.g. tin, indium, steel and aluminium), in the nuclear field between molten uranium dioxide (the fuel) and liquid sodium (the coolant), and in the chemical industry between liquefied natural gas, LNG, (the coolant) and water (the fuel). The term 'FCI' has, in the past, been used only of interactions between reactor fuel and reactor coolant but we use it generically to cover all interactions. The interaction between UO_2 and Na has occurred only in very specialized circumstances and there is much interest in determining whether FCIs are physically possible in Na/ UO_2 reactors. Witte et al (1970) quote many examples of FCIs. In addition, these authors as well as Brauer et al (1968) and Groenveld (1972) give resumés of some proposed mechanisms for FCIs, however, none of these qualitative descriptions is really satisfactory. Numerous other references may be found in those papers.

Several authors [Katz and Sliepcevich (1971), Katz (1972), Enger et al (1972) and Yang (1973)] have recently discussed a model for the LNG/water interactions based on the theory of homogeneous nucleation (Döring 1937). It is proposed that the water heats the LNG to the superheated, metastable state. The violent explosions occur when the LNG reaches the homogeneous nucleation temperature and sudden vaporisation occurs. However, such a scheme cannot completely account for all FCIs. Firstly, to transfer sufficient energy from fuel to coolant within the observed timescale of the explosion (a few tens of

milliseconds for metal/water explosions) an extremely large contact area between fuel and coolant is needed. Witte et al (1970) have shown that in the case of metal/water explosions heat transfer rates three orders of magnitude greater than those that occur during boiling processes are required. This result implies that the contact area must increase by a factor of 10^3 . In the case of LNG/water explosions it is just conceivable that the LNG spreads by a sufficient amount (LNG and water have similar densities) but for metal/water explosions no spreading occurs. Secondly, although the homogeneous nucleation temperature of water is about 290°C , explosions have been observed (Dullforce and Buchanan 1973) between water and tin at an initial temperature below 290°C . An additional drawback to the superheating model exists in the case of metal/water explosions. Water is notoriously difficult to superheat. For example, experiments with water more than 10 degrees superheated require careful preparation. The water must be boiled for several hours and then kept under vacuum. In industrial explosions, where, of course, the water has not been specially treated, it is unlikely that highly superheated water is produced unless some mechanism in the interaction itself reduces the number of nucleation sites. It must be recognized, however, that it might be possible for water to be transiently superheated to a high degree.

Since FCI's may present a safety problem in the nuclear field as well as in the smelting and liquid natural gas industries much theoretical effort has been spent analysing pressure-time histories and the damage that is likely to be caused by a particular history. However, a really adequate treatment of the problem requires that the basic physics of the situation be understood. Because of the limitations of the superheating model we propose an alternative model for FCIs. We give first an outline of the model and then a detailed description of the calculations and results. A preliminary description of the model was given in an earlier paper [Buchanan and Dullforce (1973)].

Similar models involving feedback have been considered by other authors [Potter and Jakeman (1972a,b); Roberts (1972); Board et al (1972)] but no detailed calculations have been performed. Peckover and Roberts (see Roberts 1972) have developed a simple model in which part of the energy released as each new element of surface is created goes into the turbulent velocity field, which in turn creates new surface. They find that the energy released is infinite at some finite time. However, Peckover and Roberts recognize that their model proposes no mechanism for the feedback. In the model discussed here the feedback mechanism is discussed.

2. The Model

The energy source for an FCI is the excess heat in the fuel [Lipsett (1966)]; however, calculation shows that the surface area of contact between fuel and coolant must somehow increase extremely rapidly if the interaction is to be complete in the observed time with anything like observed laboratory heat-transfer rates. Thus any proposed mechanism must provide a means for creating a large surface area within a few milliseconds. Small-scale experiments by Dullforce and Buchanan (1973) and Board et al (1972) indicate that the explosion proceeds in a series of cycles. These oscillations may be observed directly from a high speed movie or by measuring the pressure as a function of time. In addition, examination of the debris after an interaction indicates that explosions have occurred within the lump of fuel (Fig.1) and also that the fuel has been penetrated by jets of coolant (Fig.2). It has been shown [Brauer et al (1968), Board et al (1972) and Dullforce and Buchanan (1973)] that there exists a threshold value for the initial coolant temperature above which no interaction occurs. Board et al (1972), however, have shown that above the threshold temperature a mechanical disturbance (in the form of a hammer blow) precipitates an interaction. These experimental

facts suggest the following model.

The interaction is divided into five stages, the last four of which occur cyclically.

Stage 1: as a result of some triggering mechanism (for example, the onset of transition boiling as the hot liquid cools) the liquids come into intimate contact and a vapour bubble is formed.

Stage 2: the vapour bubble expands and then collapses as a result of condensation in the subcooled coolant. Because of the density and/or viscosity difference between fuel and coolant (and also possibly because a solid crust has formed on the fuel) the bubble collapse is asymmetric and a high velocity jet of liquid coolant, directed towards the fuel, is formed.

Stage 3: the jet of liquid coolant penetrates the fuel and, since it has a high velocity, its subsequent disintegration and hence increase in fuel coolant contact area is extremely rapid.

Stage 4: as the jet of coolant penetrates and breaks up heat is transferred from the surrounding fuel to the jet and since the surface area is increasing rapidly the total heating rate also increases rapidly.

Stage 5: when the jet has been heated to a certain temperature it suddenly vaporizes and a high pressure vapour bubble forms. The rapid expansion of this bubble disperses the surrounding fuel

into the coolant in an analogous manner to the spray of water produced by a depth charge. The process now proceeds from stage 2 again.

Because of the cyclic scheme some of the energy released from the fuel during each cycle is fed into subsequent cycles and it is this feedback that enables a small initial perturbation to grow. Note that the model provides a mechanism for increasing the contact area between fuel and coolant which is essential to explain the observed rapid heat transfer.

Under certain conditions the second and subsequent cycles may be weaker than the preceding ones and in these conditions the initial perturbation (stage 1) is stable and does not lead to an explosion. However, under other conditions subsequent cycles are more energetic than the preceding ones and in this case an explosion occurs. We now give a detailed description of each stage.

3. Calculations

Stage 1.

Experiment has shown that the interaction may be triggered in a number of seemingly different ways. The automatic triggering if the coolant temperature is below the threshold and the mechanical disturbance experiment are two examples. We therefore regard stage 1 solely as a means of supplying the initial perturbation which causes the first bubble to form adjacent to the fuel surface. All our calculations in this paper will start with the assumption that this bubble has been formed. Although stage 1 is independent of the rest of the proposed mechanism nevertheless many of the experimental results can be explained qualitatively in terms of it [Dullforce and Buchanan (1973)].

We shall see how, having triggered an initial perturbation, the interaction proceeds explosively.

Stage 2.

We assume that the initial condition for the i^{th} cycle is a spherical bubble, initial pressure P_i and radius R_i , which expands in the surrounding incompressible coolant of density ρ_c and ambient pressure P_o ($P_i > P_o$). We assume that the bubble expands until the maximum radius is reached and that during this expansion phase no heat transfer occurs and hence that the mass of vapour in the bubble is constant. When the maximum radius is reached we assume that all the vapour suddenly condenses due to the surrounding subcooled liquid. A cavity now exists in the coolant and this cavity collapses under the external pressure P_o . Clearly this is an idealised description of this stage - heat transfer does occur during the expansion phase; however, the simple assumptions made here result in an elementary calculation and since, at this stage, we seek only to postulate a possible mechanism the refined calculation is not necessary.

During the expansion phase the equation of motion is (Lord Rayleigh 1917; Lamb 1932) (assuming spherical symmetry)

$$R \ddot{R} + \frac{3}{2} \dot{R}^2 = \frac{P-P_o}{\rho_c} \quad . \quad (3.1)$$

R and P are the radius and pressure of the bubble at time t . Assuming that the expansion is adiabatic and that the γ value is $4/3$ (the value for water vapour) we find that the maximum radius of the bubble is given by

$$R_m = R_i (1 + z_r) \quad (3.2)$$

where z_r is the real solution of the cubic equation

$$z^3 + 4z^2 + 6z - 3(P_i/P_o - 1) = 0 \quad (3.3)$$

(There is only one real solution of Eq. 3.3).

The time taken to expand to R_m is

$$t_g = R_i \sqrt{\frac{3\rho_c}{2P_o}} \int_0^{z_r} \frac{(1+z)^2 dz}{\{3(P_i/P_o - 1)z - z^4 - 4z^3 - 6z^2\}^{\frac{1}{2}}} \quad (3.4)$$

This integral is an elliptic integral and can, after some tedious algebra, be reduced to a combination of the three canonical forms, the elliptic integrals of the first, second and third kinds respectively [Whittaker and Watson (1927)]. However, for our purposes it is adequate and easier to evaluate the integral numerically, the only minor problem being that the integrand diverges at both limits. The singular parts are subtracted out as follows:

Near $z = 0$ the integrand behaves like $1/\sqrt{za}$ where $a = 3(P_i/P_o - 1)$.

Near $z = z_r$ let $z = z_r - \epsilon$.

By expanding in rising powers of ϵ we find that near z_r the integrand behaves like

$$\frac{(1+z_r)^2 (z_r - z)^{-\frac{1}{2}}}{(3z_r^3 + 8z_r^2 + 6z_r)^{\frac{1}{2}}}$$

Thus if we add and subtract these two limiting forms to the integrand we find that the integral is given by

$$\int_0^{z_r} F(z) dz + 2 \sqrt{\frac{z_r}{a}} + \frac{2(1+z_r)^2}{(3z_r^2+8z_r+6)^{\frac{1}{2}}}$$

where

$$F(z) = \frac{(1+z)^2}{(az-z^4-4z^3-6z^2)^{\frac{1}{2}}} - \frac{z^{-\frac{1}{2}}}{\sqrt{a}} - \frac{(1+z_r)^2 (z_r-z)^{-\frac{1}{2}}}{(3z_r^3+8z_r^2+6z_r)^{\frac{1}{2}}}$$

$F(z)$ is a perfectly well behaved analytic function in the region $[0, z_r]$ and thus its numerical integration presents no difficulty. A Gaussian formula is quite convenient. At $z = 0$ and at $z = z_r$, $F(z)$ has the values

$$F(0) = \frac{-(1+z_r)^2}{z_r(3z_r^2+8z_r+6)^{\frac{1}{2}}}$$

$$F(z_r) = \frac{-1}{\sqrt{az_r}}$$

When the maximum radius is reached the vapour condenses and the bubble collapses under the external pressure P_0 . The equation of motion is (assuming spherical symmetry)

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{P_0}{\rho_c} \quad (3.5)$$

and the time for collapse to take place is [Lord Rayleigh (1917)]

$$t_c = R_m \sqrt{\frac{\rho_c}{3P_0}} \frac{\Gamma(5/6)\Gamma(1/2)}{\Gamma(4/3)} = 0.915 R_m \sqrt{\frac{\rho_c}{P_0}} \quad (3.6)$$

where Γ is the gamma function. Thus the total time taken for the bubble to expand and collapse is

$$t_b = t_g + t_c \quad (3.7)$$

The expansion and collapse of the bubble takes place adjacent to the fuel surface. It is well known that when an initially spherical cavity collapses adjacent to a solid wall the collapse is axi-symmetric and a jet of liquid, directed towards the wall, is produced. Plesset and Chapman (1971) have done a numerical simulation of this situation and they find that the final jet velocity scales as $(\Delta P/\rho_c)^{1/2}$ where $\Delta P (=P_o$ in this case) is the pressure difference causing the collapse. The final jet dimensions scale as the initial (pre-collapse) cavity radius. Thus the jet dimensions are determined by R_m and consequently by R_i and P_i . The velocity, length and diameter of the jet when it strikes the fuel surface are

$$\left. \begin{aligned} V_o &= V_c \left(\frac{\Delta P}{\rho_c} \right)^{1/2} \\ L_o &= L_c R_m \\ d_o &= d_c R_m \end{aligned} \right\} \quad (3.8)$$

The phenomenon of jetting is due to the lack of spherical symmetry in the local surroundings and thus the constants V_c , L_c and d_c are determined by the degree of departure from spherical symmetry. For bubble collapse adjacent to a solid wall

$$\left. \begin{aligned} V_c &= 13 \\ L_c &= 0.493 \\ d_c &= 0.237 \end{aligned} \right\} \quad (3.9)$$

If the adjacent layer of fuel is liquid then the collapse will still be axi-symmetric (due to the density difference and /or viscosity difference) but the numerical values of the constants will be different. In addition, the

values of the constants will also be different if the bubble is not tangential to the fuel surface but is instead positioned some distance from the surface. However, for the purpose of these calculations we use the values given in Eq. 3.9. Although the collapse is not spherically symmetric the collapse time, t_c , is not greatly altered. Plesset and Chapman's results (1971) indicate that the coefficient in Eq. 3.6 is still of order unity. It should be noted that the effect of the fuel surface has been neglected during the expansion phase where spherical symmetry has been assumed.

Stage 3.

The jet of coolant (diameter d_o and length L_o) enters the fuel with velocity V_o . [A solid crust may exist on the fuel surface but Buchanan (1973a) has shown that provided the bubble radius prior to collapse is greater than a certain minimum size then penetration will still occur. In these calculations we shall always assume that penetration will occur]. Christiansen and Reynolds (1971) have done a numerical simulation of this situation using the computer code VORTEX [Christiansen (1970)]. They show that the length of the jet (the calculation is two-dimensional) increases exponentially with a time constant proportional to d_o/V_o . Fig.3 illustrates their results

$$L = L_o e^{t/\tau} \quad (3.10)$$

where

$$\tau = f d_o/V_o \quad (3.11)$$

(t is measured from the time the jet starts to penetrate the fuel).

Christiansen and Reynolds' results indicate that f has the value $11/8$, however, VORTEX can only handle fluids of equal density and so f will have a different value from that above since fuel and coolant have different densities.

Taylor's (1963) results suggest that $f = \frac{11}{8} \left(\frac{\rho_f}{\rho_c} \right)^{\frac{1}{2}}$ where ρ_f is the density of

the fuel. Peckover (1971) has explained Christiansen and Reynolds' results

by showing that the local vorticity increases exponentially. Batchelor (1952) has investigated the effect of homogeneous turbulence on material lines and surfaces, and he also finds that the length of a line increases exponentially. In addition, he has shown that the area of a material surface increases exponentially with a time constant double that for the length increase. Thus we assume that the surface area of contact between the fuel and the jet is given by

$$\left. \begin{aligned} A &= A_0 e^{t/\tau} \\ \tau &= \frac{11}{4} \left(\frac{\rho_f}{\rho_c} \right)^{\frac{1}{2}} \frac{d_0}{V_0} \end{aligned} \right\} \quad (3.12)$$

A_0 is the initial surface area of the jet. Whilst the jet remains liquid its volume is almost constant. Consequently, since the area increases exponentially the average distance, s , between material surfaces must decrease exponentially:

$$s = s_0 e^{-t/\tau} \quad (3.13)$$

There are three factors limiting the area increase. Firstly, if the fuel surrounding the jet solidifies then no further increase in contact area can occur. Secondly, if the jet vaporizes then the conditions are radically altered. A high pressure bubble now exists and this blows out the surrounding fuel. Thirdly, the minimum distance between surfaces may be determined by a critical Weber number [Hinze (1948)]. In this case surface tension limits the value of s . We denote the minimum value of s by s_m .

Stage 4.

As the jet penetrates the fuel heat transfer occurs. Fig. 4. represents a cross-sectional view of the situation sometime after the initial impact. We assume that heat transfer takes place one-dimensionally across each element of fuel-coolant-fuel as shown in Figs. 4 and 5. The thickness of the element

of coolant, $s(t)$, is a decreasing function of time until the minimum size s_m is attained. Thereafter, the thickness is constant. Buchanan (1973b) has discussed a variety of methods of treating the problem of heat transfer across a hot-wall/liquid interface. We shall use the ODE method described in that paper. In this method it is assumed that the temperature and pressure are uniform within some region of the coolant. Then

$$c\rho_c x \frac{dT}{dt} = r_1 - r_2 T \quad . \quad (3.14)$$

r_1 and r_2 are parameters specifying the rates of heating and cooling per unit area. x is the length of the coolant region being heated and may be either a constant or a function of time. In this case x takes the following functional forms in turn.

Initially, the region of coolant effectively heated has a length $2\sigma\sqrt{t}$ at both ends of the slab of fuel-coolant-fuel. $\sigma^2 = k/c\rho_c$, k and c are the thermal conductivity and specific heat of the coolant. Thus in Eq. 3.14 we use the variable mass approximation [Buchanan (1973b)] with $x = 4\sigma\sqrt{t}$. The solution of Eq. 3.14 is then

$$T = \frac{r_1}{r_2} - \left(\frac{r_1}{r_2} - T_0 \right) \exp \left(- \frac{r_2}{2\rho_c c \sigma} \sqrt{t} \right) \quad . \quad (3.15)$$

When $4\sigma\sqrt{t}$ is equal to the thickness of the element, s , the coolant is heated throughout and the value of x that must be used in Eq. 3.14 is given by Eq. 3.13. The solution of Eq. 3.14 in this case is

$$T = \frac{r_1}{r_2} + F \exp \left\{ - \frac{r_2 \tau}{\rho_c c s_0} e^{t/\tau} \right\} \quad (3.16)$$

where F is a constant such that at time t' Eqs. 3.15 and 3.16 give the same value for T . t' is the solution of the equation

$$4\sigma\sqrt{t} = s_o e^{-t/\tau} . \quad (3.17)$$

Eventually $s(t)$ is equal to the minimum value and in this case x is equal to the constant s_m . The solution is then

$$T = \frac{r_1}{r_2} - \left(\frac{r_1}{r_2} - T_1 \right) \exp \left(- \frac{r_2}{\rho_c c s_m} t \right) . \quad (3.18)$$

T_1 is the temperature given by Eq. 3.16 with $t = t''$ where t'' satisfies

$$s_m = s_o e^{-t''/\tau} . \quad (3.19)$$

Thus the temperature is given initially by Eq. 3.15 and then by either Eq. 3.16 and then Eq. 3.18 or else by Eq. 3.18 alone. Which of the latter two possibilities is the case depends on the times t' and t'' . If $t' < t''$ the former case applies.

Stage 5.

When the jet of coolant has been heated to its saturation temperature T_{sat} , it, the jet, vaporizes provided nucleation sites are available (heterogeneous nucleation). An estimate of the time required to supply the latent heat can be made using the latent heat equivalent temperature ΔT_L :

$$\Delta T_L = L/c . \quad (3.20)$$

L is the latent heat of vaporization of the coolant. The time required to heat the coolant to a temperature ΔT_L above T_{sat} is an estimate of the time to supply the latent heat. If no nucleation sites are available the jet must be heated to the homogeneous nucleation temperature before vaporization occurs. For water the homogeneous nucleation temperature, T_{hn} , is about 290°C at an external pressure of 1bar and increases as the pressure increases. When the jet starts to vaporize by this process the rate of vaporization is so rapid

[$\sim 10^{-6}$ sec, Rodebush (1952)] that the latent heat cannot be supplied by normal heat transfer from the surrounding fuel. Instead the heat is supplied by the jet itself. Suppose the mass of vapour is m_v . Then the heat required to vaporize a further amount dm_v is Ldm_v and the temperature drop in the remaining part of the liquid jet is $Ldm_v / [(M_j - m_v) c]$ where M_j is the initial mass of the jet. Thus the temperature of the liquid jet after m_v is formed is

$$T = T_{hn} - \int_0^{m_v} \frac{Ldm}{(M_j - m)c}$$

i.e. $T = T_{hn} - \frac{L}{c} \ln \frac{M_j}{M_j - m_v}$ (3.21)

where we have assumed L and c to be constant. This process stops when $T = T_{sat}$ and thus m_v is given by

$$\frac{m_v}{M_j} = 1 - \exp \left\{ - \frac{c}{L} (T_{hn} - T_{sat}) \right\} \quad (3.22)$$

For water at one atmosphere the fraction $\beta = m_v / M_j$ has the value 0.33. As the external pressure increases, $(T_{hn} - T_{sat})$ decreases and thus β decreases.

The value of β depends on the vaporization process, being unity for heterogeneous nucleation. Let the temperature at which the vapour is formed be T_v and the corresponding saturated vapour density be ρ_v . The vapour pressure is P_v . The volume occupied by the vapour is much larger than that just occupied by the liquid. Since the density cannot change instantaneously from ρ_ℓ to ρ_v we suppose that the vapour is initially formed at the saturated liquid density, ρ_ℓ , (and is therefore at a high pressure) and then expands adiabatically. The initial high pressure P_i is given by

$$P_i \rho_\ell^{-\gamma} = P_v \rho_v^{-\gamma} \quad (3.23)$$

Clearly the value of P_i calculated by this method is an upper limit. An initial temperature T_i can also be calculated on the basis of the ideal gas law $P_i / \rho_\ell = GT_i$ and if T_i is greater than the initial fuel temperature

then the initial expansion process cannot be exactly as depicted here.

Nevertheless, the method adopted allows us to examine the consequences of the model. Thus the initial state after vaporization is a high-pressure, high-density gas bubble. To completely define the initial state we need the dimensions of the bubble. If we assume that the bubble is spherical with a radius R_i then R_i is given by

$$\frac{4}{3} \pi R_i^3 \rho_l = \beta M_j \quad (3.24)$$

This high-pressure bubble expands and also sends out a shock wave. If $P_i \ll \rho c_0^2$ where ρ is the density and c_0 the velocity of sound then the subsequent motion can be described using weak shock wave theory. For water $\rho c_0^2 \sim 22$ kbar whilst for a liquid metal ρc_0^2 is an order of magnitude greater. The maximum value of P_i that arises from Eq. 3.23 occurs when the jet is vaporized by heterogeneous nucleation. In this case $P_i \sim 19$ kbar. The initial motion takes place in the fuel (by virtue of penetration) and thus, since the pressure will have decreased before water is involved, weak shock wave theory is probably a reasonable approximation.

We consider first the shock wave and then the motion of the gas sphere. Cole (1965) has shown that the pressure at the point r at time t (r measured from the centre of the bubble and t from the time when the radius of the bubble is R_i , i.e. just formed) is given by

$$P(r, t) - P_0 = \frac{a}{r} [P(a, \tau') - P_0] \quad (3.25)$$

where τ' is the retarded time

$$\tau' = t - \frac{r-a}{c_0} \quad (3.26)$$

Thus the pressure at the point r at time t is determined by the pressure at the point a at a time $(r-a)/c_0$ earlier. Normally a is chosen to be the boundary of the bubble and the liquid and hence the pressure at a is simply the pressure of the vapour in the bubble. Since the initial radius of

the bubble is R_i the initial pressure at r is proportional to R_i/r . Note that we have neglected the effect of the liquid-coolant liquid-fuel boundary.

If the initial pressure in the bubble is as high as 19kbar the Rayleigh equation, Eq. 3.1, does not describe accurately the early motion of the bubble, however, the pressure is lowered by a factor of 16 when the bubble doubles its radius and for these pressures the Rayleigh equation is quite accurate. We shall assume, therefore, that the Rayleigh equation is valid throughout the bubble's motion. Thus the vapour bubble is described by stage 2.

4. Results

In this section we examine some of the consequences of the proposed mechanism. These are illustrative calculations in which the coolant is assumed to be water and the fuel density seven times that of water. The peak pressure, $P_i(r)$, (measured at r) for the i^{th} cycle is given by (Eq. 3.25)

$$P_i(r) - P_o = \frac{R_i}{r} [P_i - P_o] \quad (4.1)$$

Now R_i is given by Eq. 3.24 and M_j is the mass of the jet of coolant hence we have

$$\frac{4}{3} \pi R_i^3 \rho_\ell = \beta \frac{\pi}{4} d_c^2 L_c R_{m_{i-1}}^3 \rho_c \quad (4.2)$$

$R_{m_{i-1}}$ is the maximum radius of the bubble during the $(i-1)^{\text{th}}$ cycle and is given by Eq. 3.2 Thus

$$R_i = \left(\frac{3}{16} \beta d_c^2 L_c \frac{\rho_c}{\rho_\ell} \right)^{1/3} (1+z_r) R_{i-1} \quad (4.3)$$

Consequently the ratio of the peak pressures at r due to successive cycles is

$$\frac{P_i(r) - P_o}{P_{i-1}(r) - P_o} = \left(\frac{3}{16} \beta d_c^2 L_c \frac{\rho_c}{\rho_\ell} \right)^{1/3} (1+z_r) \quad (4.4)$$

For an external pressure of 1 bar Eq. 4.4 gives

$$\frac{P_i(r) - P_0}{P_{i-1}(r) - P_0} = \begin{cases} 6.673 ; & (\text{heterogeneous nucleation } \beta = 1) \\ 2.899 ; & (\text{homogeneous nucleation } \beta \approx 0.33) . \end{cases} \quad (4.5)$$

Thus the model predicts a series of pressure pulses at the point r (Fig 6). The shape of each pulse can be calculated using Eq. 3.25 and the indefinite form of the integral in Eq. 3.4. In addition, the initial size of the bubble increases as the cycle number increases, thus, from Eqs. 3.2 - 3.4 the time between successive pressure peaks at r also increases.

In Eq. 4.4 z_r is dependent on the initial bubble pressure P_i and the ambient pressure P_0 . As P_0 increases z_r decreases and so the ratio of successive peaks decreases. At some critical value of P_0 this ratio is unity and for larger values of P_0 successive peaks are smaller than preceding ones. Let P_{th} be the pressure at which this ratio is unity. Then, below P_{th} an initial perturbation is damped out. At P_{th}

$$z_r = \frac{1}{\alpha} - 1 \quad (4.6)$$

where

$$\alpha^3 = \frac{3}{16} \beta d_c^2 L_c \frac{\rho_c}{\rho_l} . \quad (4.7)$$

z_r also satisfies Eq. 3.3 and so

$$P_{th} = \frac{3\alpha^3}{1 + \alpha + \alpha^2} P_i . \quad (4.8)$$

Eq. 4.8 is an implicit equation for P_{th} since P_i and β , in the case of homogeneous nucleation, depend on the external pressure. We find

$$P_{th} = \begin{cases} 67.5 \text{ bar} ; & (\text{heterogeneous nucleation}) \\ 13.0 \text{ bar} ; & (\text{homogeneous nucleation}) . \end{cases} \quad (4.9)$$

After N cycles the total mass of coolant heated during the jet penetration stage is

$$M_c = \sum_{i=1}^N M_{j_i} \quad (4.10)$$

where M_{j_i} is the mass of the i^{th} jet and since $M_{j_i} = \frac{\pi}{4} \rho_c d_c^2 L_c R_{m_i}^3$

we find

$$M_c = \frac{\pi}{4} \rho_c d_c^2 L_c \frac{\xi^N - 1}{\xi - 1} R_{m_1}^3 \quad (4.11)$$

where $\xi = \alpha^3 (1+z_r)^3$ and R_{m_1} is the radius just prior to collapse of the initial perturbing bubble. If the mass of fuel required to heat each jet is μM_{j_i} , where μ is a proportionality constant, then the total mass of fuel whose heat causes the interaction is $M_f = \mu M_c$. An estimate of μ will enable the thermal efficiency of the interaction to be calculated. Of course, it is possible that μ is a function of the cycle number; in this case an average thermal efficiency must be calculated.

The ratio of the kinetic energy of the jet just as it starts to penetrate, E_j , to the energy of the bubble, E_b , calculated as the work done by the bubble expanding to its maximum radius, is independent of cycle number and is given by

$$\frac{E_j}{E_b} = \frac{d_c^2 L_c V_c^2 \Delta P (1+z_r)^4}{32 z_r P_i} \quad (4.12)$$

Now $\Delta P = P_0$ and by using Eq. 3.3 we find

$$\frac{(1+z_r)^4}{z_r P_i} = \frac{3}{P_0} + \frac{1}{P_i} + \frac{1}{P_i z_r} \quad (4.13)$$

thus neglecting quantities $O(1/P_i)$ we find

$$\frac{E_j}{E_b} = \frac{3}{32} d_c^2 L_c V_c^2 \quad (4.14)$$

i.e. $E_j/E_b = 0.439$.

An essential feature of Eqs. 4.4, 4.9, 4.11 and 4.14 is their dependence on V_c , L_c and d_c . The values quoted in Eq. 3.9 for these constants are for an empty cavity collapsing adjacent to a solid wall. The effect of gas in the

cavity is to reduce the amount of jetting, thus L_c and d_c decrease and this affects the interaction dramatically. For example, if L_c and d_c are reduced to half of their present values the threshold pressure is reduced by a factor of order 1/8, the ratio of successive pressure peaks is half the present value and thus after five cycles the pressure is only 1/32 of its present value. Clearly the determination of V_c , L_c and d_c under a variety of conditions is required.

The analytical results expressed in Eqs. 4.4, 4.9, 4.11 and 4.14 provide useful checks on the computer code that has been written to solve the model equations. The results of the computations are presented in Figs. 7-12 and Tables 1-3. All the calculations are started with a cavity of radius $10^{-4}m$. The cavity collapses and the interaction proceeds. The heterogeneous nucleation process is used for vaporization. Table 1 shows how an initially small perturbation can produce large amounts of energy in just a few cycles. $E(i)$ is the work done by the bubble expanding during the i^{th} cycle. $T(i)$ is the time taken to complete i cycles. $P_i(r)$ and $I_i(r)$ are the peak pressure and impulse due to the i^{th} cycle at a point r from the interaction. In all these calculations r is 10 cm. Figs. 7-11 show the effect of varying the external pressure. It may be seen that even an increase of a few bars in pressure has a dramatic damping effect on the interaction. Tables 2 and 3 and Fig. 12 show the effects of varying r_1 , r_2 and s_m . Initially as r_1 is increased only the time is altered but as r_1 is increased further the energy $E(i)$ is reduced. At such large values of r_1 the heating rate is so great that not all the jet is vaporized, i.e. vaporization occurs with Eq. 3.15 still operative. (The maximum observed laboratory heat transfer rates are of the order $10^7 J m^{-2} s^{-1}$). An increase in r_1 represents an increase in the efficiency of energy transfer and hence the elapsed time is reduced, but because it is the product $r_1 A$ that gives the total heating rate and A is exponentially increasing, the elapsed time is only slightly altered for

quite large changes in r_1 . Variation of r_2 affects the time slightly. Fig. 12 shows the effect of s_m . For small values of s_m the time for the interaction is independent of s_m since the jet is vaporized before s_m is attained, however, as s_m increases the time for the interaction increases since the minimum size is reached before vaporization occurs.

5. Conclusions.

A model for fuel-coolant interactions has been proposed. By using a feedback mechanism we are able to show how large quantities of energy can be released in a short time from a small perturbation. We have also been able to show that the strength of the interaction is reduced as the external pressure is increased and can be inhibited entirely if the pressure is large enough. If the pressure resulting from the initial few cycles is not relieved (by, for example, enclosing the interacting fluids in a strong container) then the sustained pressure will inhibit further cycles. That is, FCIs are self-limiting if the resulting pressure is maintained.

The effect of noncondensable gas is not included specifically in the model, nevertheless, as we have pointed out, its presence within the vapour bubbles can be taken into account via the constants V_c , L_c and d_c . Pockets of noncondensable gas throughout either the fuel or coolant or both will tend to cushion any high pressures, thus we expect the interaction to be damped in the presence of noncondensable gas.

The model presented here has applications not only in industrial processes but also in naturally occurring phenomena. For example, Peckover et al (1973) have applied the theory to the interaction of sea water with hot molten lava from a submarine volcano and have shown that volcanic hydro-explosions in the form of FCI do not occur at sea depths greater than some critical value. In addition, Buchanan et al (1973) have shown that FCIs may occur in a cosmological context with the interaction of matter and anti-matter.

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Cycle No. i	Energy of bubble E (i) j	Final radius of bubble R_m (i) m	Elapsed time T (i) secs	Mass of coolant used kg	Peak pressure at r $P_i(r) - P_0$ bars	Impulse at r $I_i(r)$ n/m ² s
1	1.261×10^{-4}	6.673×10^{-4}	2.687×10^{-3}	2.176×10^{-11}	3.339	5.354×10^{-3}
2	3.747×10^{-2}	4.453×10^{-3}	5.813×10^{-3}	6.487×10^{-9}	22.28	2.382×10^{-1}
3	1.113×10^1	2.971×10^{-2}	1.197×10^{-2}	1.927×10^{-6}	148.7	1.060×10^1
4	3.307×10^3	1.983×10^{-1}	3.898×10^{-2}	5.726×10^{-4}	992.1	4.729×10^2
5	9.827×10^5	1.323×10^0	2.094×10^{-1}	1.701×10^{-1}	6620.	2.104×10^4

Table 1. Variation of energy, time, pressure and impulse with cycle number

($P_0 = 1$ bar, $r_1 = 10^6$ j/m²/s, $r_2 = 10^3$ j/m²/s/K, $s_m = 3 \times 10^{-7}$ m).

r_1 j/m ² /s	T (4) secs	E (4) j
10 ⁶	3.898x10 ⁻²	3.307x10 ³
10 ⁷	2.850x10 ⁻²	3.307x10 ³
10 ⁸	1.269x10 ⁻²	8.169x10 ¹
10 ⁹	1.467x10 ⁻³	3.390x10 ⁻²

Table 2. Variation of r_1 ($P_o = 1$ bar, $r_2 = 10^3$ j/m²/s/K, $s_m = 3 \times 10^{-7}$ m)

r_2 j/m ² /s/K	T (4) secs	E (4) j
10 ¹	3.163x10 ⁻²	3.307x10 ³
10 ²	3.181x10 ⁻²	3.307x10 ³
10 ³	3.898x10 ⁻²	3.307x10 ³

Table 3. Variation of r_2 ($P_o = 1$ bar, $r_1 = 10^6$ j/m²/s, $s_m = 3 \times 10^{-7}$ m)

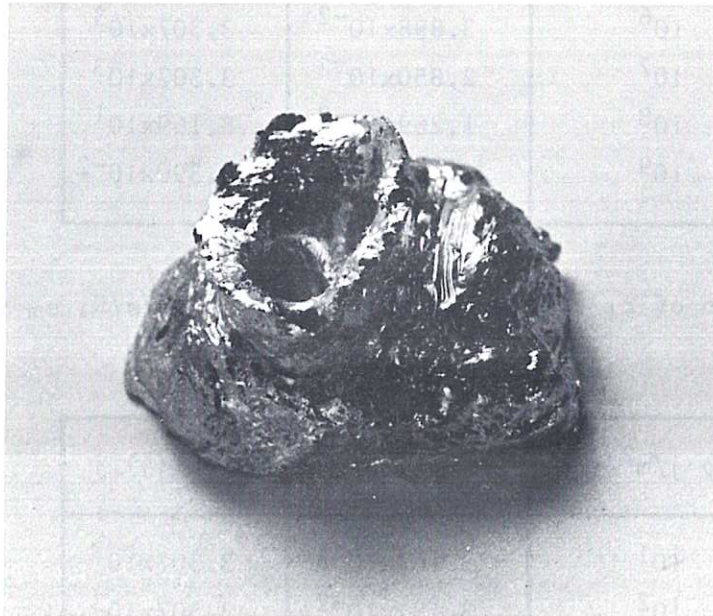


Fig.1 Photographic evidence showing explosions from within lumps of fuel.



Fig.2 Photographic evidence of jet penetration.

CLM-P361

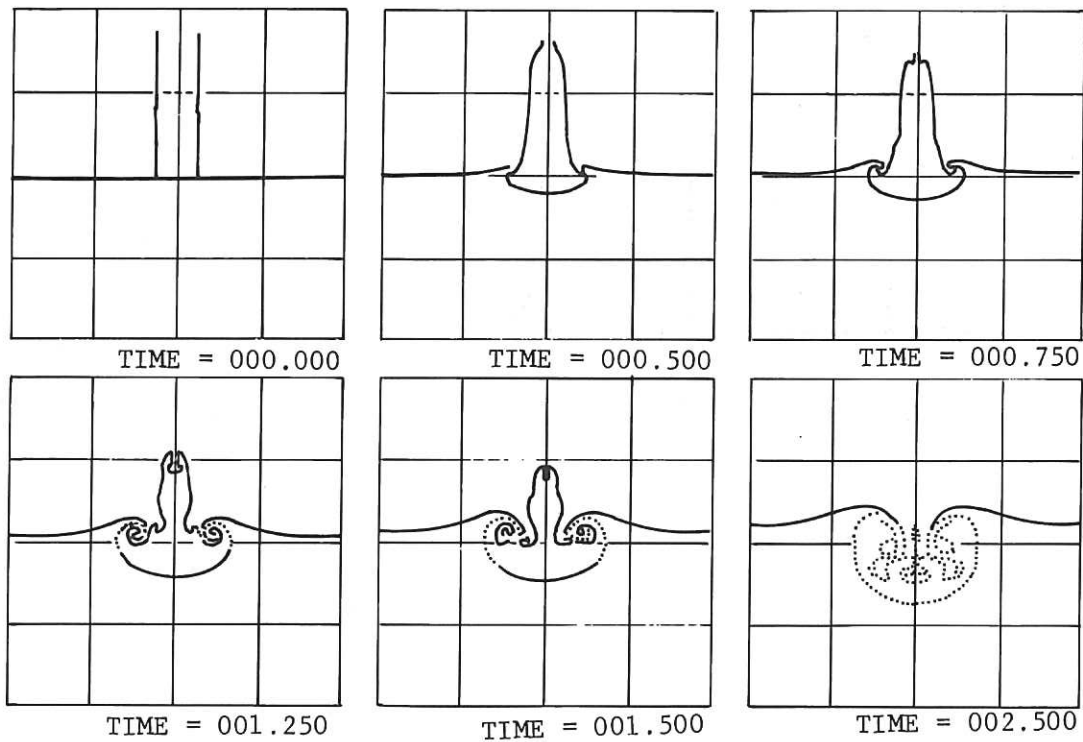


Fig.3 Computer simulation of penetration of a liquid by a liquid jet.

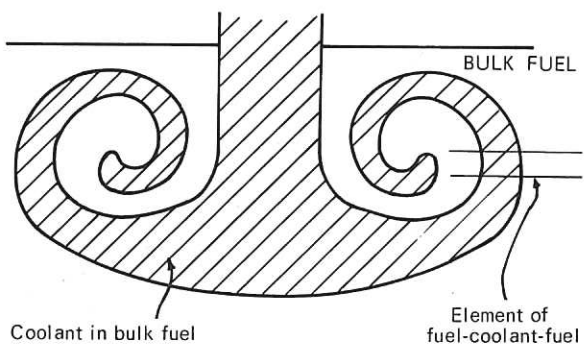


Fig.4 Schematic cross-section of a jet of coolant sometime after penetration.

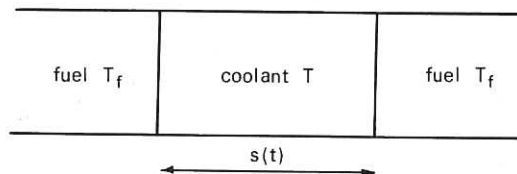


Fig.5 Element of fuel-coolant-fuel in which heat transfer occurs one-dimensionally.

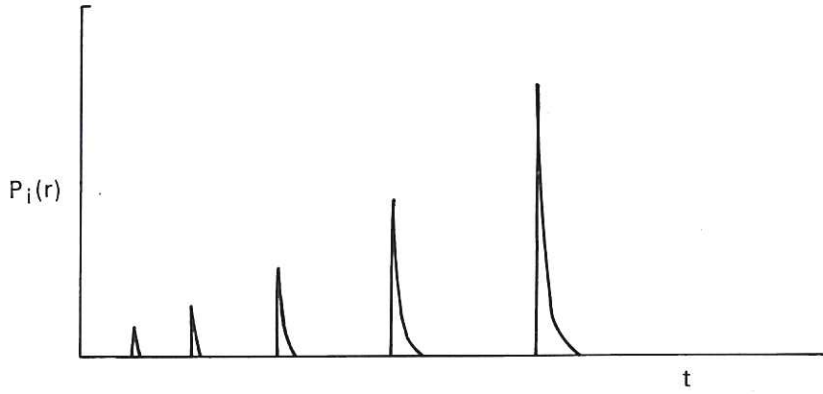


Fig.6 Pressure at the point r as a function of time.

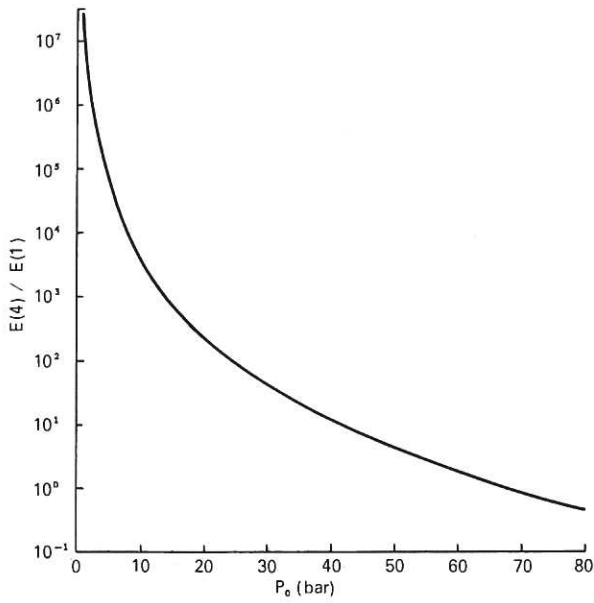


Fig.7 The effect of external pressure on the energy released: $E(4)/E(1) \text{ v } P_o$. (For Figs.3.11-3.15, $r_1 = 10^6 \text{ J m}^{-2} \text{ s}^{-1}$, $r_2 = 10^3 \text{ J m}^{-2} \text{ s}^{-1} \text{ K}^{-1}$ $\text{sm} = 3 \times 10^{-7} \text{ m}$).

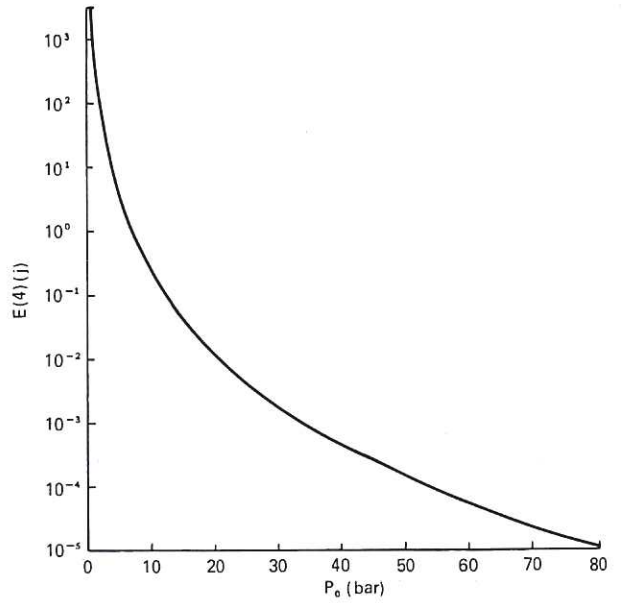


Fig.8 The effect of external pressure on energy released: $E(4) \text{ v } P_o$.

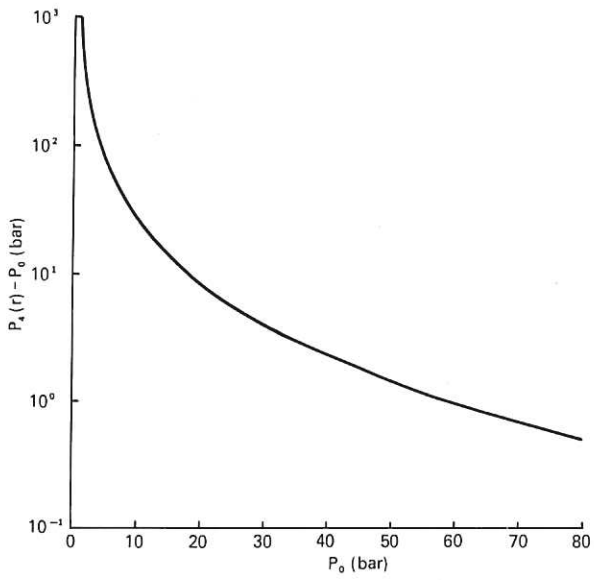


Fig.9 The effect of external pressure on the pressure measured at r: $P_4(r) - P_0 \propto P_0$.

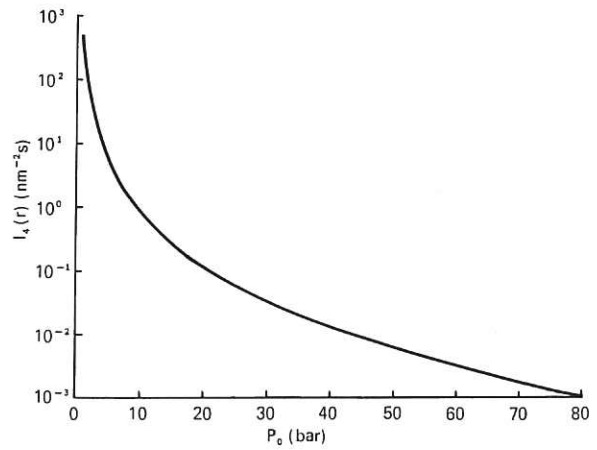


Fig.10 The effect of external pressure on the impulse felt at r: $I_4(r) \propto P_0$.

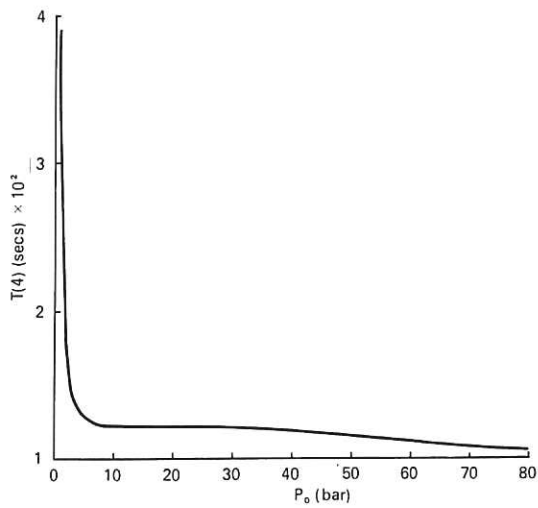


Fig.11 The effect of external pressure on the elapsed time: $T(4) \propto P_0$.

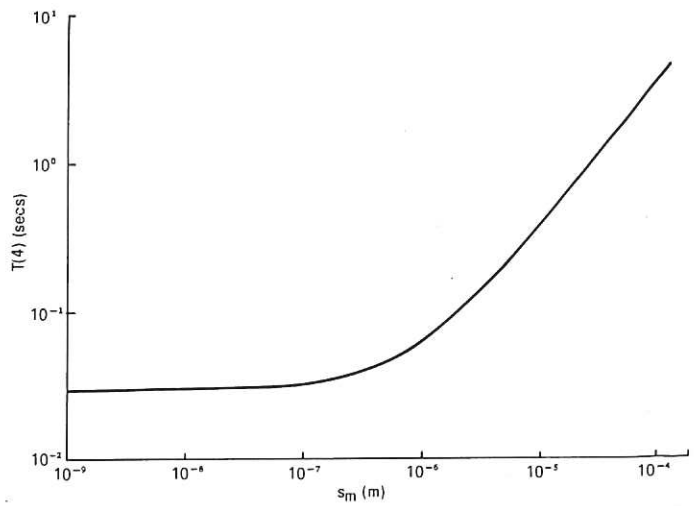


Fig.12 The effect of the minimum particle size on the elapsed time: $T(4) \propto s_m$.



