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EFFECTS OF NEUTRAL INJECTION HEATING UPON TOROIDAL EQUILIBRIA

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ABSTRACT

Effects of neutral beam injection upon the equilibrium of a toroidal plasma are considered. The distribution function of energetic ions produced by the beam in the plasma is calculated for injection both parallel and perpendicular to the magnetic field taking account of effects due to the toroidal geometry. The effect of trapped particles on the current induced in the plasma by such a beam is calculated, together with the associated cross field diffusion. Loss mechanisms for the momentum deposited in the plasma by the neutral beam are considered. Ripples in the toroidal magnetic field strength are particularly efficient at destroying toroidal momentum and lead to flow velocities much less than the sound speed for typical injection parameters.

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1. INTRODUCTION

At the present time there is considerable interest in the use of high energy neutral beams for heating a toroidal plasma to ignition temperatures and preliminary experiments on this method of plasma heating in a Tokamak plasma have just been completed[1]. In this paper we discuss the changes to the normal toroidal equilibrium that are caused by the use of these large current high energy neutral particle sources. For example it has been demonstrated by Ohkawa[2] that large plasma currents can be produced by neutral beam injection and in fact this current can be made large enough to operate a Tokamak in a steady state configuration. Another possible consequence of injection with one beam parallel to the magnetic axis has been discussed by Callen and Clarke[3]; these authors show, from a consideration of the toroidal momentum balance equation, that the background plasma will begin to rotate in the toroidal direction with a very high velocity (many times greater than the sound speed).

The hot ions are injected as neutrals and then after ionization they slow down on the background plasma in a time τ_s say. During this time energy and momentum are being transferred to the background ions and electrons. The resulting equilibrium for the hot ion distribution function is derived in Section II by solving an appropriate Fokker Planck equation. This kinetic equation which includes the effects of toroidal geometry, such as particle trapping, is solved for the cases of injection parallel and perpendicular to the field lines. The results of this section are used in Section III where the plasma current produced by the injected ions is derived. The current carried by the injected ions is partly cancelled out by the motion of the plasma electrons which are accelerated by collisions with these hot ions. In previous calculations of the resulting current no account was taken of the trapped electrons which have here an effect analogous to their modification of the resistivity of a toroidal plasma. Using the standard techniques of neoclassical theory the effect of trapped particles is included in the calculation of the current in Section III. Finally in Section IV the build up of toroidal momentum is discussed. In particular the influence upon the plasma toroidal velocity of the small ripples in the toroidal magnetic field caused by the finite coil spacing is evaluated.

II. ENERGETIC ION DISTRIBUTION FUNCTION

The energetic ion distribution function is determined as a solution of the Fokker Planck equation. Starting with the collision operator in the Landau form we first make use of the inequality $v_i \ll v_h \ll v_e$ where v_i, v_e are the thermal velocities of the plasma ions and electrons and v_h is the mean velocity of the hot ions. This inequality will be satisfied in most injection heating schemes, apart from those in which the electron temperature is very low in comparison with the hot ion energy. The Fokker Planck equation in the guiding centre approximation can then be written in the following form for axisymmetric toroidal geometry with circular magnetic surfaces

$$\frac{\partial f_h}{\partial t} - \frac{\Theta v_\parallel}{r}\frac{\partial f_h}{\partial \theta} = \frac{1}{\tau_s}\left[\frac{2}{W^{\frac{1}{2}}}\frac{\partial}{\partial W}\left\{(W_c^{3/2} + W^{3/2})f_h\right\} + \frac{2\,\mu\,(W_c^{3/2} + W^{3/2})}{W^{3/2}}\frac{\partial f_h}{\partial \mu}\right.$$

$$\left. + \frac{m_i}{m_h}\left(\frac{W_c}{W}\right)^{3/2}\frac{v_\parallel}{B}\frac{\partial}{\partial \mu}\left(\mu v_\parallel \frac{\partial f}{\partial \mu}\right)\right] + S \qquad (1)$$

where W and μ are the usual constants of energy and magnetic moment given by $W = \frac{v^2}{2}$, $\mu = \frac{v_\perp^2}{2B}$, the parallel velocity $v_\parallel = \sqrt{2(W - \mu B)}$, where B is the magnetic field strength, f_h is the hot ion distribution function and S is the source of these ions. $\tau_s = \frac{3}{4\sqrt{2\pi}}\frac{m_h^{\frac{1}{2}}}{m_e}\frac{m_e T_e^{3/2}}{e^4 \ell n \Lambda Z_h^2 n}$ is the slowing down time and the critical energy

$W_c = \left(\frac{3\sqrt{\pi}}{4} Z_i\right)^{2/3} \left(\frac{m_i}{m_e}\right)^{1/3} \frac{m_h}{m_i} T_e$ where Z_j and m_j are the charge number and mass of species j. Geometrical effects enter through the dependence of B on the poloidal angle θ while it is of course independent of the axisymmetric toroidal angle ϕ . Finally $\Theta(r)$ is defined as the ratio of poloidal and toroidal fields.

Eq. (1) is solved by expanding f_h as a series in τ_B/τ_s in the form $f_h = f_{ho} + \tau_B/\tau_s f_{hs} + \ldots$, where τ_B is the bounce period of a hot ion and τ_s is the slowing down time. The zeroth order equation is

$$\frac{\partial f_{ho}}{\partial \theta} = 0$$

which gives $f_{ho}(\mu, W, r)$. The function f_{ho} is then determined from the constraint on the next order solution which is

$$\tau_s \frac{\partial f_{ho}}{\partial t} = \frac{2}{W^{\frac{1}{2}}} \frac{\partial}{\partial W} \left\{ (W_c^{3/2} + W^{3/2}) f_{ho} \right\}$$

$$+ \frac{m_i}{4m_h} \left(\frac{W_c}{W}\right)^{3/2} \frac{B_o}{\eta W \left\langle \frac{1}{v_\parallel} \right\rangle} \frac{\partial}{\partial \eta} \left\{ \frac{(1 - \eta^2)\left\langle \frac{v_\parallel}{B} \right\rangle}{\eta} \frac{\partial f_{ho}}{\partial \eta} \right\} + \tau_s S \qquad (2)$$

where

$$\left\langle \frac{v_\parallel}{B} \right\rangle = \begin{cases} \oint \left\{ 2\,(W - \mu B) \right\}^{\frac{1}{2}} \frac{d\theta}{B} & \text{for passing ions} \\ \int_A^B \left\{ 2\,(W - \mu B) \right\}^{\frac{1}{2}} \frac{d\theta}{B} & \text{for trapped ions (A, B are turning points)} \end{cases}$$

and a corresponding definition holds for $\left\langle \frac{1}{v_\parallel} \right\rangle$, while $\eta = (1 - \mu B_o/W)^{\frac{1}{2}}$ and B_o is the field minimum on a magnetic surface.

In a Tokamak where $B = B_o(1 - \varepsilon \cos\theta)/(1 - \varepsilon)$ with $\varepsilon = r/R$, the functions $\left\langle \frac{v_\parallel}{B} \right\rangle$, $\left\langle \frac{1}{v_\parallel} \right\rangle$ may be expressed in terms of the elliptic functions E and K . in the neighbourhood of $\eta = 0$, i.e. the trapped region, the functions $\left\langle \frac{v_\parallel}{B} \right\rangle$ and $\left\langle \frac{1}{v_\parallel} \right\rangle$ may be approximated as

$$\left\langle \frac{v_\parallel}{B} \right\rangle \simeq \frac{\pi}{2 B_o} \sqrt{\frac{W}{\varepsilon}}\, \eta^2$$

and

$$\left\langle \frac{1}{v_\parallel} \right\rangle \simeq \frac{\pi}{\sqrt{W\varepsilon}}$$

In the opposite limit, $\eta \to 1$, i.e. in the region in which the hot ions are passing and parallel to the field lines, $\left\langle \frac{v_\parallel}{B} \right\rangle$ and $\left\langle \frac{1}{v_\parallel} \right\rangle$ are both independent of η. Eq.(2) has been solved in these two limiting cases; namely with all the particles trapped (perpendicular injection $\eta << 1$) or all passing (parallel injection $\eta \sim 1$).

For the limit $\eta << 1$ the second term of Eq. (2) reduces to Bessel's operator and the resulting equation is solved by expressing f_{ho} as a Dini series

$$f_{ho} = \Sigma\, a_n(W)\, J_o(j_n\, \eta) \tag{3}$$

where the j_n are zeros of $J_o'(j_n)$. The differential equation for the $a_n(W)$ is

$$\frac{2}{W^{\frac{1}{2}}} \frac{d}{dW} \left\{ (W^{3/2} + W_c^{3/2})\, a_n \right\} - \frac{m_i}{4m_h} \left(\frac{W_c}{W} \right)^{3/2} j_n^2\, a_n$$
$$= \int_o^1 SJ_o(j_n \eta)\eta\, d\eta \Big/ \int_o^1 J_o^2(j_n\, \eta)\eta\, d\eta \tag{4}$$

The above equation can be solved by use of an integrating factor, and the solution is

$$a_n = \frac{W^{3/2\,\rho}}{2(W^{3/2} + W_c^{3/2})^{1+\rho}} \int_W^\infty dW\, W^{1/2 - 3/2\rho} \left(W^{3/2} + W_c^{3/2} \right)^{\rho} \frac{\int_o^1 S\eta\, J_o(j_n \eta) d\eta}{\int_o^1 \eta J_o^2\, (j_n \eta)\eta d\eta}$$

where $\rho = m_i j_n^2 / 12 m_h$

A typical solution for perpendicular injection is shown in Fig.1(a). where contours of f_{ho} are plotted as a function of $v_\parallel$ and v at the position of the magnetic surface where the magnetic field is a minimum ($\theta = 0$). From these contours one can see that there is a hump in the distribution function at high energies and that the distribution is also anisotropic in this region. The stability of such distributions and also of the corresponding ones for parallel injection has been discussed in a paper by Cordey and Houghton[5] and also by Stix[6].

For the limiting case $\eta \sim 1$ when the particles are passing the second term of Eq.(2) reduces to Legendre's operator and the equation is then solved by expressing f_{ho} as a series of Legendre polynomials

$$f_{ho} = \sum_{n=0}^{\infty} a_n(W)\, P_n(\eta) \ . \tag{5}$$

The equation for the $a_n(W)$ is similar to Eq.(4) and a typical solution is shown in Fig.1(b). Once again there is a hump in the distribution at high energies and the distribution is anisotropic. From the contours of Fig.1(b) one can calculate other parameters of interest such as the mean parallel velocity of the hot ions u_h. This is needed in the next section for the derivation of the plasma current.

III. THE PLASMA CURRENT

In this section the plasma current which is generated by the high energy ion beam will be calculated. Several possible models could be considered and to fix ideas we shall discuss the scheme proposed by Ohkawa[2]. In this scheme, to avoid increasing the toroidal momentum of the plasma, a low energy, high current beam is injected in the opposite direction to the high energy beam. (In general however, there will be a build up of toroidal momentum in neutral injection systems and the problem of its decay will be discussed in the next section.)

Assuming that the net ion momentum remains zero the mean velocity of the background ions u_i, is given by:-

$$u_i = m_h n_h u_h/(m_i n_i) \tag{6}$$

where n_h and u_h are the density and mean velocity of the energetic ions respectively. The total ion component of the current is then

$$j_i = e\, n_h\, u_h\, Z_h \left(1 - \frac{m_h Z_i}{m_i Z_h}\right) . \tag{7}$$

In order to obtain the electron contribution however, we must solve the electron Fokker Planck equation. This equation is expanded in the small larmor radius approximation, ordering time derivatives on the diffusion scale. In zero order we also ignore the effects of the energetic ions and conclude that the electron distribution function is a Maxwellian F_{me}. In first order we have the equation:-

$$\frac{v_{\parallel}}{r}\frac{\Theta\,\partial f_e^{(1)}}{\partial\theta} - \frac{m_e v_{\parallel}}{e\, r}\frac{\partial}{\partial\theta}\left(\frac{v_{\parallel}}{B}\right)\frac{\partial}{\partial r}F_{me} = C_{ei}(f_e^{(1)}) + C_{ee}(f_e^{(1)}) + C_{eh}(F_{me}) . \tag{8}$$

The electron-ion collision term in the above equation C_{ei} is well represented by the Lorentz form, corrected for the ion mass motion, while for the electron-electron collisions we use a model operator which possesses a pitch angle scattering term and a compensating momentum conserving contribution, given by Kovrizhnykh[7]. This form has been shown by Rosenbluth et al.[8] to give the same results for neoclassical transport theory as the full Fokker Planck form; it leads to approximately 20 per cent errors in calculating the Spitzer resistivity. The collision term C_{eh} between electrons with energetic ions is obtained by substituting the expression for f_h given by the Eqs.(5) or (6) into the Rosenbluth potentials. Then for energetic ion energies such that $W > W_c$ one finds that the transfer of momentum is predominantly to the electrons and the expression for C_{eh} may be written in the following form:-

$$C_{eh}(f_e) = -\frac{2\pi Z_h^2 e^4 \ell n\Lambda\, n_h u_h \cos\xi}{m_e^2 \quad W} \frac{\partial f_e}{\partial W} \tag{9}$$

where ξ is the angle between the injection line and the field lines and will be taken as zero for parallel or π for antiparallel injection in the following calculation.

Eq.(8) is linear in the effects of radial gradients and the ion beam, so we may ignore these gradient terms, merely adding at the end the results for the toroidal current and radial diffusion driven by the ion beam to the results of earlier authors for the transport induced by radial gradients[8]. Limiting our calculations to the banana regime of collision frequencies we solve Eq.(8) by conventional techniques. After lengthy calculation we obtain an electron current which when added to the ion current Eq.(7) yields a total current j

$$j = e\, n_h u_h Z_h \left\{1 - \frac{Z_h}{Z_i} + 1.46\, \varepsilon^{\frac{1}{2}} \left(\frac{Z_h}{Z_i} - \frac{Z_i}{Z_h}\frac{m_h}{m_i}\right) A(Z_i)\right\} \tag{10}$$

(Note u_h is defined as being positive when injection is parallel to the magnetic field.)

The numerical coefficient $A(Z_i)$ is given by

$$A(Z_i) = 1 + \frac{2.12}{3\sqrt{\pi} Z_i} \int_0^\infty \frac{hx^{3/2} e^{-x}}{h+Z_i} \Bigg/ \int_0^\infty \frac{he^{-x}}{h+Z_i} \tag{11}$$

where $h(x) = (1 - 1/2x)\,\eta(x) + d\eta(x)/dx$ with $\eta(x) = 2/\sqrt{\pi}\int_0^x e^{-t}\, t^{\frac{1}{2}} dt$ and arises from the energy dependence of the collision frequencies; it may be evaluated numerically and is tabulated in Table 1 for several values of Z_i.

The leading term in Eq.(10) is essentially that obtained by Ohkawa and, as he noted, vanishes if $Z_h = Z_i$. We have an additional contribution due to the electrons trapped in the toroidal magnetic field, which is significant for realistic tori ($\varepsilon \gtrsim 1/10$), and only vanishes if $\frac{Z_h^2}{Z_i^2} = \frac{m_h}{m_i}$. Thus we conclude that provided the mass or the charge of the ion beam differs from that of the plasma ions this current will not vanish.

If instead of taking u_i given by Eq.(6) we had assumed that the background plasma velocity was zero, and as will be shown in the next section there are good reasons for assuming that this is the case, then the expression for the current is the same as is given in Eq.(10), except that the second term in the inner bracket, $Z_i m_h / Z_h m_i$ is omitted. The correction due to the trapped electrons will give rise to a current for $Z_h = Z_i$ irrespective of the mass ratio in this case.

The modification to the current from trapped particles is reminiscent of the corrections to the conductivity of a toroidal plasma arising from trapped particles[4]. Since the beam plays a similar role to an applied toroidal electric field one expects the analogue of the Ware pinch effect. Calculating the diffusion flux from the solution of Eq.(8) we find the plasma diffusion caused by the beam is

$$\Gamma = 1.46 \frac{\varepsilon^{\frac{1}{2}} m_e}{e B_\theta} n_h u_h \nu \left(1 + \frac{0.53}{Z_i}\right)\left(1 - \frac{m_h}{m_i}\frac{Z_i^2}{Z_h^2}\right) \qquad (12)$$

where

$$\nu = \frac{4}{3} \frac{\sqrt{2\pi}\, e^4 \ln\Lambda\, Z_h^2 n_e}{m_e^{1/2} T_e^{3/2}} .$$

As an example of the above effects we consider the one used by Ohkawa[2], $Z_i = 1$, $Z_h = Z$ and $m_h/m_i = Z$; then after evaluating $A(1)$ from Eq.(11) we find that the total plasma current given by Eq.(10) may be written in the form

$$j = e\, n_h u_h Z(1 - Z)\,(1 - 2.54\, \varepsilon^{1/2}) . \qquad (13)$$

One can see from this equation that the toroidal correction term (the '$\varepsilon^{1/2}$' term) is very large indeed for small aspect ratio tori and could reduce the current to a very small value. The diffusion flux given by Eq.(12) may be written in the form

$$\Gamma = \frac{2.24\ \varepsilon^{1/2}\ m_e\ n_h\ u_h\ \nu}{eB_\theta}\left(1 - \frac{1}{Z}\right) \qquad (14)$$

for the above values of Z_i, Z_h etc. If we compare this with the flux due to the density gradient[8] we find that they are roughly of the same order of magnitude and so the diffusion of the plasma may be significantly enhanced or reduced depending upon whether the beam is directed parallel or antiparallel to the field lines. Of course the expressions for j and Γ are local ones and if the beam can be concentrated near the axis of the torus the toroidal effects are small as $\varepsilon = r/R \rightarrow 0$, but in practice the beam will exist over a considerable portion of the minor cross-section and the toroidal modifications will be important.

IV. SLOWING DOWN OF THE TOROIDAL FLOW BY RIPPLE VISCOSITY

In general, as well as heating the plasma, the injected ions increase the toroidal momentum of the background plasma. This can result in large toroidal plasma flows, particularly if there is only one beam whose direction of injection is parallel to the magnetic axis. With opposing beam injection (one beam parallel and one anti-parallel to the field line), the injected momentum can be reduced but not cancelled everywhere, since the injection process is necessarily asymmetric. There are several possible mechanisms by which toroidal momentum can be lost in a torus. Callen and Clarke[3] have shown that if the only momentum loss is provided by perpendicular viscosity then the toroidal velocity builds up to a value much larger than the sound speed. Another possible momentum loss is by convection of toroidal momentum and an expression for this loss has been given by Kovrizhnykh[9]. This latter mechanism gives a faster loss rate of momentum than perpendicular viscosity, but the equilibrium toroidal velocity is still larger than the plasma sound speed for the parameters of most heating schemes.

In this section we show how ripples due to the finite spacing of the toroidal field coils can slow down the background plasma. We first discuss a model magnetic field consisting of a uniform magnetic field B in the z direction on which is superimposed a small ripple field with amplitude $\frac{\delta}{2}$ and wavelength L, the total magnetic field being given by

$$B = B_o \; (1 + \tfrac{1}{2} \; \delta \cos z/L). \qquad (15)$$

The equilibrium ion velocity arises through the competition between the momentum input and the friction between passing particles and the trapped ions which cannot be accelerated, and may be calculated using the techniques of neoclassical theory. The rate of loss of momentum of a plasma moving through a ripple field has been indirectly derived previously in connection with magnetic pumping calculations[10] where the heating of the plasma appears as an increase of momentum in the wave frame. Rather than reproduce these calculations we present the following simple argument. Consider a plasma with number density n moving with velocity u along a magnetic field given by Eq.(15). For a particle to be trapped in a ripple well $\frac{v_{\parallel}}{v} < \delta^{\frac{1}{2}}$ at the minimum of the well. The number of particles scattered into the well per second equals $n\delta^{\frac{1}{2}} \; (\nu_{ii}/\delta)$. The average velocity of these particles is δ u, so that the rate of loss of momentum by the plasma due to ripple trapping is:-

$$\frac{dp}{dt} = -1.46 \ mn\nu_{ii} \, \delta^{\frac{1}{2}} \, u \quad . \tag{16}$$

The constant 1.46 which has been inserted in Eq.(16) arises from geometrical effects associated with the magnetic field profile[8] when one uses a more complete theory based on the analogy with magnetic pumping. Eq.(16) is only valid in the "weak collisional" regime, and by this we mean that a particle must stay trapped in the well for at least one bounce period; this condition can be written in the form $\nu_{ii} < \delta^{3/2} \, v_i/L$.

Let us now comment on the relation of the simple model magnetic field structure used above, to a more realistic field possessing transform ι. The plasma is now pumped through the more important poloidal variation in the magnetic field strength[11] $\sim \varepsilon B_o$, $\varepsilon >> \delta$. This has the result that an equilibrium parallel velocity $u_{\parallel}$ almost balancing any poloidal rotation due to $E \wedge B$ drifts arising from a radial electric field Φ', will be set up

$$u_{\parallel} \simeq \frac{\Phi'}{B_\theta} \quad . \tag{17}$$

Thus if we know the radial electric field we know the $E \wedge B$ drifts and $u_{\parallel}$ and hence the toroidal flow velocity u. The radial electric field is determined by the ambipolar condition. The force $m\dot{u}$ on the plasma induces a non-ambipolar contribution to the radial flux

$$\Gamma_i = \frac{m_i n\dot{u}}{eB_\theta} \quad , \quad \Gamma_e \simeq 0 \quad . \tag{18}$$

In the presence of the non-axisymmetric ripple the conventional diffusion due to the density gradient is no longer ambipolar and one can obtain the radial electric field by equating this non-ambipolar contribution to that due to the beam. In an axisymmetric situation the ion banana diffusion flux would be estimated by:-

$$\Gamma_i = \varepsilon^{\frac{1}{2}} \nu_{ie} \frac{m_i T_i}{e^2 B_\theta^2} \frac{e\Phi'}{T_i} , \tag{19}$$

where Φ' is the radial electric field, and would equal the electron flux. In the presence of the ripple, ion-ion collisions can contribute to the ion banana diffusion since momentum is no longer conserved, the effect being proportional to $\delta^{\frac{1}{2}}$, the region of velocity space affected by the ripple, as shown by a detailed analysis. Thus we replace $\varepsilon^{\frac{1}{2}} \nu_{ie}$ by $\delta^{\frac{1}{2}} \nu_{ii}$ in Eq.(19). This non-ambipolar contribution yields, on equating to the part induced by $m\dot{u}$,

$$\frac{\Phi'}{B_\theta} \simeq \frac{\dot{u}}{\nu_{ii} \delta^{\frac{1}{2}}} . \tag{20}$$

Hence

$$u \simeq u_{\parallel} \simeq \frac{\Phi'}{B_\theta} \simeq \frac{\dot{u}}{\nu_{ii} \delta^{\frac{1}{2}}} \tag{21}$$

agreeing with the model result Eq.(16). This argument has supposed we can neglect localised particle diffusion[12] with respect to the contribution considered.

The complete toroidal momentum balance equation for the plasma has the form

$$\frac{\partial u}{\partial t} = -1.46 \nu_{ii} \delta^{\frac{1}{2}} u + n_h u_o/(n \tau_s) \tag{22}$$

where the first term on the r.h.s. is the ripple viscosity term and the second term is the source of momentum from the hot ions, in which u_o is the component of the injected hot ion velocity along the field lines. The solution of the above equation is

$$u = \frac{0.69 n_h u_o}{n \tau_s \nu_{ii} \delta^{\frac{1}{2}}} \left\{ 1 - \exp(-1.46 \nu_{ii} \delta^{\frac{1}{2}} t) \right\} . \tag{23}$$

As mentioned previously the ripple amplitude δ is a function of r ; an appropriate form for a Tokamak is $\delta = \alpha + \beta r^2/a^2$, where typical values for α and β are $\alpha \sim 3 \times 10^{-3}$, $\beta \sim 2 \times 10^{-2}$ (these particular values are for the DITE Tokamak experiment). From Eq.(23) we see that the toroidal velocity is largest at $r = 0$ on the magnetic axis and that this velocity reaches its maximum value $u_{max} = n_h u_o/(n \tau_s \nu_{ii} \alpha^{\frac{1}{2}})$ in equilibrium when $t \gg 1/(1.46 \nu_{ii} \delta^{\frac{1}{2}})$. For the parameters of DITE (injection current 8 amp, $n \sim 3 \times 10^{13}$ and $T_e \sim 1$ keV) $u_{max} = 0.025 u_o$. This is of the order of a fifth of the sound speed which is very small indeed.

CONCLUSION

In this paper we have calculated the energetic ion distribution in a toroidal plasma produced as a result of high energy neutral injection. The full toroidal geometry was taken into account and results for parallel and perpendicular injection were given. The plasma current produced by the injected ions was then deduced; the trapping of electrons was found to change the value of the current. Finally a new toroidal momentum balance equation was derived in which the slowing down of the plasma by ripple viscosity was included. It was found that even with large current injection schemes the equilibrium toroidal velocity of the background plasma was quite small.

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TABLE 1

Z_i	$A(Z_i)$
1	1.68
2	1.36
3	1.24
4	1.18
5	1.15
6	1.12

Variation of Coefficient $A(Z_i)$ with plasma ion charge number Z_i.

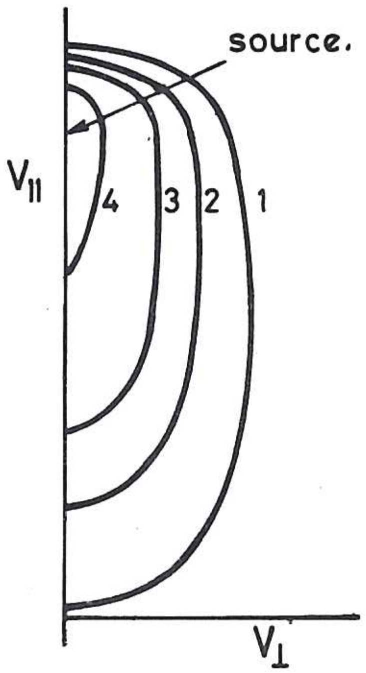

Fig. 1(a) Parallel injection, the source is at $V_{\parallel} = 2V_c$ ($W_{\parallel} = 4W_c$).

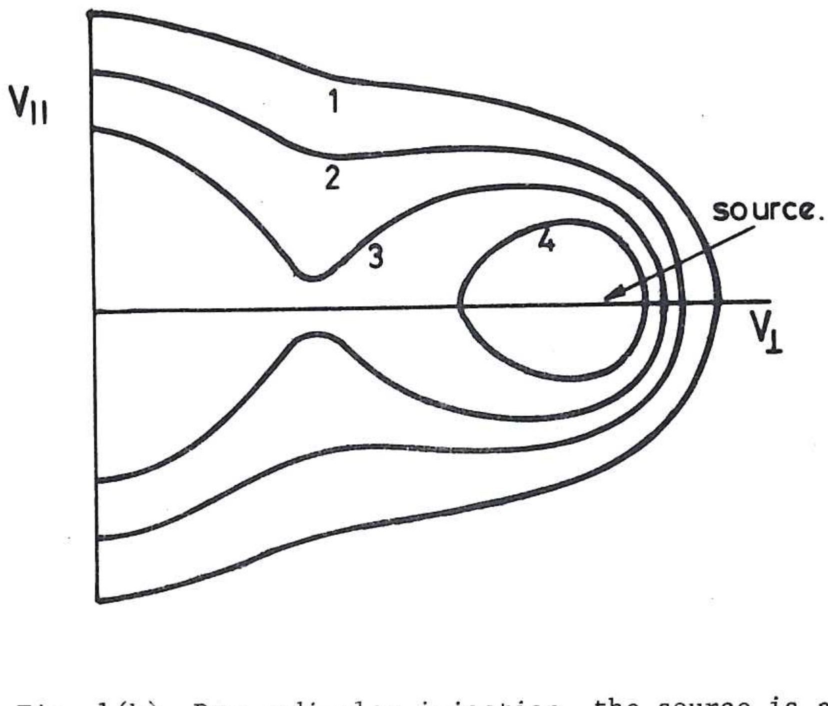

Fig. 1(b) Perpendicular injection, the source is at $V_{\perp} = 2V_c$ ($W_{\perp} = 4W_c$).