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A B S T R A C T

M.H.D. instabilities in Tokamaks are of the reactive type, for which a feedback loop with zero phase error can only provide marginal stability. However, by correctly choosing the frequency dependence of the feedback gain, this problem can be overcome, and the stability properties of the feedback loop improved.

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The lowest value of safety factor, q at which present tokamaks operate is usually set by the onset of the disruptive instability. This is preceded by a more or less sudden increase in the amplitude of M.H.D. instabilities with mode number, m = 2 or 3 (Mirnov and Semenov, 1971; Vlasenkov et al, 1973). Recently, a theoretical treatment of the feedback stabilisation of these instabilities has been given (Lowder & Thomassen, 1973), and this note deals with one of the practical problems presented by this work.

In what follows we assume the plasma displacement and related variables in the feedback loop vary with time as R exp λt, where λ is complex. The dispersion relation given by Lowder and Thomassen in the presence of feedback (equation (18)) can be written in the form

$$\lambda^2 - \Lambda^2 (1 - g) = 0 \qquad (1)$$

where Λ is the growth rate of the instability in the absence of feedback and g is the feedback gain, normalised to the gain required for stability, which is proportional to the ratio of the feedback current to the displacement of the plasma surface caused by the instability.

The form of equation (1) characterises the instability as being of the reactive type (Taylor and Lashmore-Davies, 1970). Notice that there is no term in λ in equation (1), which would correspond with damping; if there is any phase error in the feedback loop g becomes effectively complex, and equation (1) always has one unstable root.

Lashmore-Davies has shown how, in the case of flute instabilities in mirror geometry, by choosing the correct form of the frequency-dependence of the feedback gain, this difficulty can be overcome, and the sensitivity of the feedback loop to exact phase adjustment much reduced (Lashmore-Davies, 1971). A similar situation occurs in the present case, and the plasma-feedback network can be made stable by correctly choosing $g(\lambda)$. Consider the circuit shown in Fig. 1. It is easy to see that $g(\lambda)$ is of the form

$$g(\lambda) = g_o \frac{1 + \lambda\tau_D}{1 + \lambda\tau_F} \qquad (2)$$

where τ_D and τ_F are the time constants of the differentiating amplifier and the feedback control winding respectively. Substituting (2) into (1) gives

$$\lambda^3\tau_F + \lambda^2 + \Lambda^2(\tau_D g_o - \tau_F)\lambda + \Lambda^2(g_o - 1) = 0 \qquad (3)$$

and this equation will have no unstable roots provided the Routh-Hurwitz stability criteria are satisfied (Bellman, Glicksberg and Gross, 1958, Chapter 2). These give

$$\tau_D g_o > \tau_F,\ g_o > 1 \text{ and } (\tau_D g_o - \tau_F) > \tau_F(g_o - 1)$$

i.e.

$$\tau_D > \tau_F \text{ and } g_o > 1\ . \qquad (4)$$

Thus, the circuit shown in Fig. 1 provides the correct frequency dependence in the feedback loop for stable operation, provided equation (4) is satisfied. Notice, in contrast with equation (1), that all the coefficients of equation (3) are positive; the stability is no longer marginal and will not be affected by small errors in phase.

The above treatment, as also that of Lowder and Thomassen, takes no account of the real frequency of the instabilities, which may be due to plasma rotation. This will probably not make an important difference to the analysis provided Λ, τ_D^{-1}, $\tau_F^{-1} \gg \omega$, the instability frequency. Typically $\Lambda \sim 10^6\ s^{-1}$, τ_D^{-1}, τ_F^{-1} depend, of course, on the particular installation considered, and $\omega \sim 5 \times 10^4\ s^{-1}$.

Conclusion - Previous theoretical work on M.H.D. instabilities in Tokamaks has shown them to be of the reactive type, which are intrinsically more difficult to stabilise by feedback. However, it is shown that a proper choice of the frequency dependence of the feedback gain can overcome this difficulty in principle.

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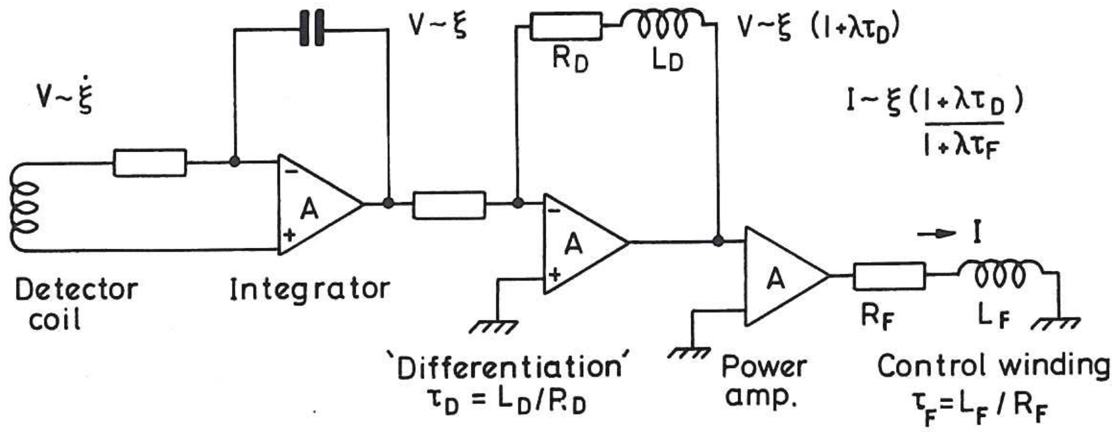

FIG. 1