A NUMERICAL INVESTIGATION OF THE SCREENING OF THE ELECTRIC CHARGES IN A THUNDERCLOUD

by

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ABSTRACT

The potential distribution in and around a thundercloud in which the electric charges are maintained by convection currents is calculated using a numerical technique. Cylindrical symmetry has been assumed in the examples cited but a general three dimensional program is available. The program allows for arbitary distributions of convection currents and electrical conductivity.

It is found that typical electric charges in a thundercloud in order of ascending altitude are conservatively + 63, - 225, + 150 C instead of the few tens of coloumbs deduced from electrostatic theory. The surface charge of the cloud has little part in the shielding.

The convection currents assumed are consistent with a growth time of the charge which is less than 20 minutes.

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1. INTRODUCTION

The electric fields associated with thunderclouds have been studied by many workers and static distributions of electric charge consistent with the fields deduced. An alternative to this electrostatic model of the cloud, in which the currents associated with the cloud are considered has been proposed. Exploitation of this current driven model has been neglected, due no doubt to the extensive numerical calculations required to obtain quantitative results. However recently developed computer programs allow such calculations to be made with ease.

This note describes the results of a series of such calculations on the current driven model and shows that it predicts charges in the cloud which are larger by a factor of about five than does the electrostatic model. These charges are consistent with a detailed analysis of the electrification processes in clouds given by Mason (1971) and with measurements of the charge transferred in a lightning flash reported by Uman (1969).

2. EXPERIMENTAL EVIDENCE

The relevant experimental evidence can be summarised as follows.

(a) The electric field of English thunderclouds has been measured at ground level and as a function of altitude by means of a sounding balloon caught in the axial updraught of air inside the cloud (Simpson et al 1937, 1940). The use of electrostatic theory shows that the field of an average cloud is consistent with charges of 4.0, - 20, + 24C uniformly distributed in spheres of radii 0.5, 1.0 and 2.5 km centred on the axis of the cloud, see figure 1a. There is a considerable cloud to cloud variation in the magnitude of the charges, the lowest charge is sometimes absent, its presence being associated with precipitation.

(b) Cloud properties vary with geographical location. Measurements of the ground level field of South African clouds lead to a point charge electrostatic model, figure 1b. The lowest charge is variable but its estimated value rarely exceeds 10 C, (Malan 1963).

(c) Wait (1951) has measured the electric field above thunderclouds. One inactive cloud had a field which using electrostatic theory was consistent with point charges of 39 and - 39C located on its axis at altitudes of 6.1 and 3.05 km. Active clouds created fields of several hundred volts per cm at altitudes of about 13 km. Wait reported that the clouds did not affect the conductivity of the atmosphere above the clouds. The electric field above the clouds indicated that they drove currents of between 0.1A to 6.0A to the upper atmosphere.

(d) Measurements of the electric field inside a thundercloud do not exceed several kV per cm, (Mason 1971), well below the expected value of the breakdown field of about 10 kV/cm (Simpson et al 1940). It is thought that lightning flashes originate at small localized regions of high field missed by sounding apparatus.

(e) The occurrence of a lightning flash does not destroy the electric field of a thundercloud.

(f) The time interval between lightning flashes, is much less than the time required for the field to increase from zero until the first flash occurs (Mason 1971).

(g) The charge transferred in a lightning flash is typically 20 C (Uman 1969). Points e , f are usually interpreted as indicating that the main charge centres in a thundercloud contain at least ten times the charge transferred in a lightning flash (Mason, 1971), that is at least 200 C.

3. REVIEW OF THEORY

The review of experimental evidence shows there is a discrepancy between the magnitude of the charges in the electrostatic model and those it is reasonable to infer exist in thunderclouds. The electrostatic model neglects the electric currents which flow inside the cloud and the surrounding atmosphere because of their finite electrical conductivity. These conductivities vary with position and the passage of an electric current in such a medium creates a space charge. (Chalmers 1967, 1) which can screen the charges in the cloud. These conduction currents would cause the decay of the charges in the cloud if they were not maintained by the injection of charged particles carried by air currents or falling under the action of gravity, such movements of charge constitute electrical convection currents.

Holzer and Saxon (1952) have investigated a simple model of a cloud and have calculated the magnitude of the convection currents required to maintain a point charge. This model did not differentiate between the conductivity inside the cloud and that of the surrounding atmosphere. Kasemir (1963) considered a model in which the conductivity inside the cloud was one third that of the surrounding atmosphere at the same altitude. This investigation was qualitative in nature and it was deduced that if the electrostatic model predicted point charges of 5, - 40 C in order of ascending altitude, the true charges should be 60, - 340 and 50 C.

4. SCOPE OF THIS INVESTIGATION

The aims of this study are to investigate the properties of the current model of a thundercloud to try and verify Kasemir's hypothesis and so reconcile the size of the electric charges in the cloud with that transferred in a lightning flash. An additional objective is to use a more realistic geometry, including finite sized charge distributions, than in the previous investigations.

5. MATHEMATICAL FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

The differential equation governing the steady state distribution of currents of the thundercloud is obtained in a similar way to that of Holzer and Saxon (1952). The charge centres of the cloud are assumed to be maintained by a set of convection currents of density $\underline{J}_c$ which are independent of the electric field $\underline{E}$. The separation of charge due to the convection currents is balanced by the conduction current driven by the electric field of the cloud, the conduction current density is $\sigma\underline{E}$ where σ is the electrical conductivity. If there is no accumulation of charge at any point the divergence of the total current density $\underline{J}_c + \sigma E$ is zero, so that

$$\mathrm{div}\,(\sigma\underline{E}) = -\,\mathrm{div}\,\underline{J}_c \quad . \tag{1}$$

The electric field is related to the potential ϕ by

$$\underline{E} = -\,\mathrm{grad}\ \phi \quad . \tag{2}$$

The combination of these expressions results in the desired differential equation

$$\mathrm{div}\left(\frac{\sigma}{\sigma_o}\,\mathrm{grad}\ \phi\right) = \frac{1}{\sigma_o}\,\mathrm{div}\,\underline{J}_c \tag{3}$$

where σ_o is the conductivity at some reference point. Equation 1 is analgous to that governing the potential in a medium of dielectric constant Kr and charge density ρ , namely

$$\mathrm{div}\,(\mathrm{Kr}\ \mathrm{grad}\ \phi) = -\,\rho/\varepsilon_o \quad . \tag{4}$$

Comparison of equations 1 and 2 shows the charge density in the cloud is high in the regions where the convection current changes rapidly with position. The model of the cloud has been investigated by integrating equation 3 to obtain the field $\underline{E}$ of the cloud, using observed values of the relative conductivity σ/σ_o and a set of assumed values of $1/\sigma_o$ div $\underline{J}_c$ and adjusting the value of $\underline{J}_c$ until the calculated field is in fair agreement with the observed electric field of a cloud. When the fields are in agreement, the electric charge contained in any volume of the cloud is equal to the normal component of the electric flux passing through the surface of the volume. The solution is thus insensitive to the absolute value σ_o of the atmospheric conductivity. The convection current enters the equation by way of its divergence and because the divergence does not define the current uniquely, an incorrect choice of current does not invalidate the charge distribution, provided the calculated electric field is correct. Thomas (1973) has written a set of computer programs for solving equation 3 at a set of mesh points using a digital computer. To save the use of computer time, the two dimensional form of the programs were used.

6. COORDINATE SYSTEM AND BOUNDARY CONDITIONS

The region in which the solution is found is a cylinder 19 km high and 150 km radius, see figure 2. These dimensions are much larger than those of a typical thundercloud. The origin is at the centre of the lower plane of the cylinder with the z axis vertical. The conductivity of the atmosphere at altitude z km outside the cloud is assumed to be,

$$\sigma = \sigma_o \exp(0.2z) \quad . \tag{5}$$

This is the value assumed by Holzer and Saxon (1952). Its use may result in an underestimate of the shielding because it predicts a smaller variation of conductivity with altitude than has been found by some workers (Chalmers 1967,2). A value of $\sigma_o = 2.575 \times 10^{-14}$ mho/m is taken to link the convection currents to the electric field, this corresponds to a conductivity at 5 km altitude of 7×10^{-14} mho/m which is the value given by Mason (1971). The plane $z = 0$ is assumed to be the ground plane at which $\phi = 0$ and the vertical field at ground level at points distant from the cloud is assumed to equal the fair weather field of 150 V/m; this is met by setting the potential of the curved surface of the cylinder at altitude z km to

$$\phi = 750\,(1 - \exp(-\,0.2z))\ \text{kV} \quad . \tag{6}$$

The upper face of the cylinder at 19 km altitude is taken as an equipotential surface, these boundary conditions are discussed by Holzer and Saxon (1952).

Kasemir (1963) assumed that the conductivity inside a thundercloud was one third that of the surrounding atmosphere at the same altitude, this assumption is used here and the conductivity inside the cloud is written as

$$\sigma = \frac{1}{3}\,\sigma_o \exp(0.2z) \quad . \tag{7}$$

The conductivity may be significantly different from this value, immediately before or after a lightning flash, which limits the analysis presented here.

7. THE CONVECTION CURRENTS

The position of the cylindrical thundercloud and of two cylindrical zones are shown in figure 2, which defines the dimensions. The zones are the regions in which the convective currents have finite divergence. The lower zone is associated with the lowest charge in the cloud, and the upper zone with the two uppermost charges. Towards the base of the upper zone some process produces pairs of charged particles of opposite signs, which are sorted by the updraught of air inside the cloud, this carries the positive charges upwards leaving the negative ones behind, creating the two upper and largest charges in the cloud, figure 1. The exact distribution of the electric currents which are driven by air currents or gravity are not known, but their general form can be deduced as follows. The experiments of Simpson and others (1937, 1943) show that the charge density on the axis of a cloud varies with altitude in a way similar to that shown in figure 3a. If the currents are assumed to be vertical of density J_c, then the charge density of figure 3a defines the divergence of the convection current, that is it defines dJ_c/dz. Figure 3b shows in a general way a variation of J_c with altitude consistent with the charge concentrations of figure 3a. In the calculations reported here, the density of the upper convection current has been assumed to be

$$J_c = C_1 e^{-\alpha z} \text{Sin}^3 \pi \left(\frac{z-z_B}{z_A-z_B} \right) \text{Cos} \left(\frac{\pi}{2} \frac{R^2}{R_o^2} \right) \tag{8}$$

which has the necessary property of being a single humped distribution with smooth transitions at its boundaries. In a similar way the convection current associated with the electric charge has been assumed to be given by

$$J_c = C_2 \left(1 - \text{Sin}^2 \frac{\pi}{2} \frac{z-z_C}{z_D-z_C} \right) \text{Cos} \left(\frac{\pi}{2} \frac{R^2}{R_1^2} \right) \qquad z_D > z > z_C \tag{9}$$

$$J_c = C_2 \qquad z_c > z > 0 \ . \tag{10}$$

A possible driving mechanism for the lower current is the production of pairs of charged particles in the region between z_D and z_C, the current consisting in the fall of negative charged particles from this region under the action of gravity, once the particles pass the level z_C they fall with

TABLE 1

Parameters used in expressions 8 and 9 for the convective currents in the clouds of figures 4 and 5

$\alpha = 0.6\ \text{km}^{-1}$								
figure	z_A km	z_B km	R_o km	z_D km	z_C km	R_1 km	C_1 A/km^2	C_2 A/km^2
4	9.8	3.0	1.5	2.99	2.0	2.0	1.75	- 0.0081
5	9.8	3.0	3.0	2.99	2.0	3.0	1.07	- 0.0088

TABLE 2

Convective currents strengths in the clouds of figures 4 and 5 as a function of altitude

	Altitude km	figure 4 A	figure 5 A
Upper Current	8.2	0.0177	0.0434
	5.3	0.0592	0.1449
	4.2	0.0923	0.2263
Lower Current		0.0636	0.1497

constant velocity and charge to the earth's surface.

The convection currents need not be vertical, it is easy to find a radial convection current inside the cloud which has the same divergence as that of equations 9 and 10.

8. APPLICATION TO MALAN TYPE CLOUDS

The theory discussed earlier is now applied to a cylindrical cloud of 5 km radius extending from 2.0 km to 10.5 km altitude and approximating to the South African clouds studies by Malan (1963). The ground level field of these clouds can be represented by the point charge model of figure 1b. Measurements of the field distribution inside the clouds are not available, and comparison of the results of the calculations with the charge distribution of figure 1a is not permissible because of the puny character of English thunderclouds relative to those of South Africa.

The sizes and strengths of the convective currents used in the calculations are listed in tables 1, 2. Mason (1971) gives the mean radius of the upper convective current to be in the range 1.0 to 5.0 km.

Figure 4a shows the first of the models considered, and defines the linear dimensions of the convective current zones. The electric field of this cloud has been calculated using the convective current strengths listed in tables 1, 2. The vertical component of electric field of this cloud at ground level, on the axis of the cloud and above the cloud at an altitude of 13 km are shown in figures 4b, c, d. The field at ground level resembles that shown in figure 1b with the lowest charge present. If the lower convective current is omitted the field resembles that shown in figure 1b with the lowest charge absent. Figure 4e shows the charges contained in various zones of the cloud.

Figure 5a shows a cloud with larger convective current zones than that of figure 4. The electric field of this cloud has been calculated using the convective current strengths listed in tables 1 and 2. Figures 5b, c and d show the vertical field of the cloud at ground level, on the axis of the cloud, and above it at an altitude of 13 km. For the purpose of comparison the vertical field at ground level for a point charge electrostatic model of 6, - 40, 40C at 2, 5, 10 km altitude has been shown in figure 5b, and figure 5d shows the vertical field at an altitude of 13 km for an electrostatic model of point charges of 39, - 39C at altitudes of 3.05 and 6.05 km, (Wait 1951). Figure 5e, shows the electric charges contained in the cloud and figure 5f the surface charge on the cloud. Figure 6 shows the potential contours around the cloud.

9. DISCUSSION

Figure 7 shows the normals to the equipotentials of figure 6 and illustrates the pattern of the electric field of the cloud.

The maximum field strength inside the cloud of 5.6 kV/cm is below the probable value of the breakdown strength of the atmosphere, see section 2 above, and greater than the maximum observed strength in clouds, but there is no evidence that the field strength at the point of origin of a lightning flash has been measured. If the lightning flashes are triggered by a small local perturbation which either increases the local field strength or lowers the breakdown strength then the calculated value of 5.6 kV/cm appears to be reasonable. Mason has proposed a detailed theory of thundercloud electrification which gives a limiting value on the large scale electric field in a cloud of about 5 kV/cm. The fields above the cloud are within the limits of Wait's observations and correspond to currents of 0.026 A flowing to the upper atmosphere.

TABLE 3

Charges contained in the different models considered in this investigation

Model	Size of Cloud	Figure	Upper Charge	Central Charge	Lowest Charge
Electrostatic	Average	1b	+ 40 C	- 40 C	+ 10 C
Current Driven	"	5	+ 186	- 224	+ 63
Current Driven	Small	4	+ 58	- 86	+ 25

The agreement between the fields predicted by the electrostatic models and the current driven model of a thundercloud used in figure 5 is good. The worse agreement is near the axis for the vertical field at ground level, where the electrostatic appears to be at its weakest in representing the field of a real cloud. The charges in the cloud are greater by a factor of about five, see table 3, than those in the electrostatic model. The surface charges on the cloud are negligible compared with those inside the cloud.

The current driven model of the cloud proposed here appears to be a satisfactory representation of a thundercloud.

10. VARIATION OF CONDUCTIVITY OF THE CLOUD

The fields of two clouds of similar dimensions have been calculated, in one the conductivity at altitude z was that used previously,

$$\sigma = \frac{1}{3}\sigma_o \exp(0.2z) \quad . \tag{11}$$

In the other it was less

$$\sigma = \frac{1}{10}\sigma_o \exp(0.2z) \quad . \tag{12}$$

The convection currents in the lower conductivity cloud were about 0.3 times the strengths of those in the higher conductivity cloud, to give similar axial and ground level fields, the charges contained inside the clouds were the same to an accuracy of 5%.

11. TIME CONSTANTS OF THUNDERCLOUDS

The convective current strengths used are those required to maintain the charge distribution in the cloud and not those required to create it. A partial check on the accuracy of the model used, is to calculate the time required for the currents to separate charges of the size of those found in the cloud and thought to be carried in a lightning flash. The resulting times should be comparable to or somewhat greater than the observed time constants. For the cloud of figure 4b the upper convective current would create the upper charge of 147 C in 17 min, which is in good agreement with the observed growth time of 10-20 min. If the charge transferred in a lightning flash is 10 C, the calculated time between flashes is approximately 45 s, the measured value for a cloud with a modest cell is 20-30 sec, the agreement is regarded as satisfactory.

12. CONCLUSIONS

(a) The electric charges inside a thundercloud deduced from a model which includes the effects of convective currents and the conductivity of cloud and surrounding atmosphere are greater than those deduced from an electrostatic model by a factor of five. The charges inside the cloud are shielded by the space charge created when an electric current flows in a medium of varying conductivity.

(b) It follows from (a) that the charge transferred in a lightning flash based on the electrostatic model and measurements of the electric field outside the cloud are suspect and reliance should be given to those based on current measurements (Uman 9).

(c) The simple convective model of the cloud gives a realistic description of the fields associated with a thundercloud, the values of the convective currents are consistent with the observed time constants of typical clouds.

(d) Contrary to Kasemir's hypothesis the surface charge residing on the cloud appears to have little effect on the screening of the cloud charges. The screening appears to be insensitive to the variation in conductivity across the surface of the cloud.

(e) The cloud charges deduced from the convective model are concentrated towards the axis of the cloud.

(f) The cloud charges predicted by the conductivity model are consistent with theory of thundercloud electrification advanced by Mason, which is based on the internal dynamics of the cloud.

(g) When more accurate measurements of the values and spatial distribution of the conductivity inside a thundercloud become available some further computation would be desirable using the convective model.

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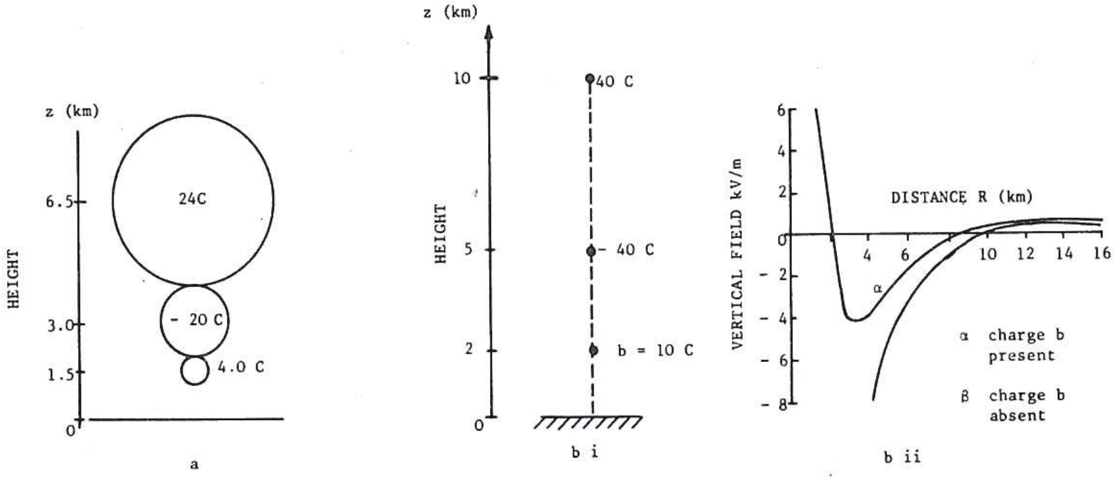

Fig. 1(a). Electrostatic model of a thundercloud proposed by Simpson and Robinson on the basis of field measurements at ground level and inside English clouds.

(b). Electrostatic model of a thundercloud proposed by Malan on the basis of field measurements at ground level below South African clouds. (i) Charge distribution; in about half the clouds investigated the lowest and smallest charge was absent. (ii) Vertical component of electric field at ground level with and without lowest charge present.

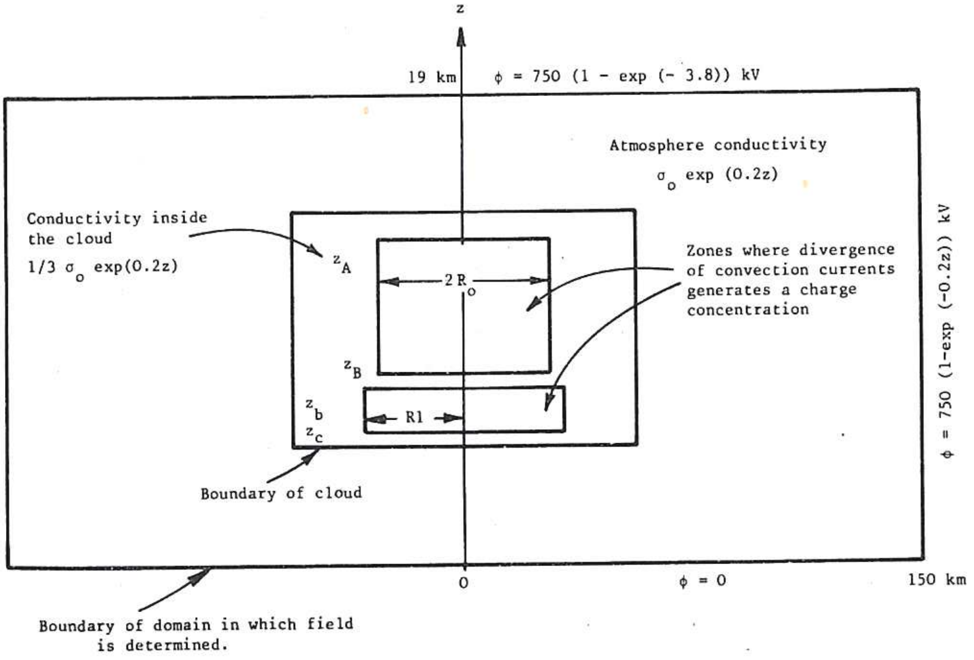

Fig. 2. Theory diagram, illustrating the cross-section of the cylinder in which the differential equation governing the current flow has been solved, and the boundary conditions used. This cylinder contains a cylindrical cloud which in its turn contains two cylindrical zones where the convection of charge has finite divergence and produces large charge concentration.

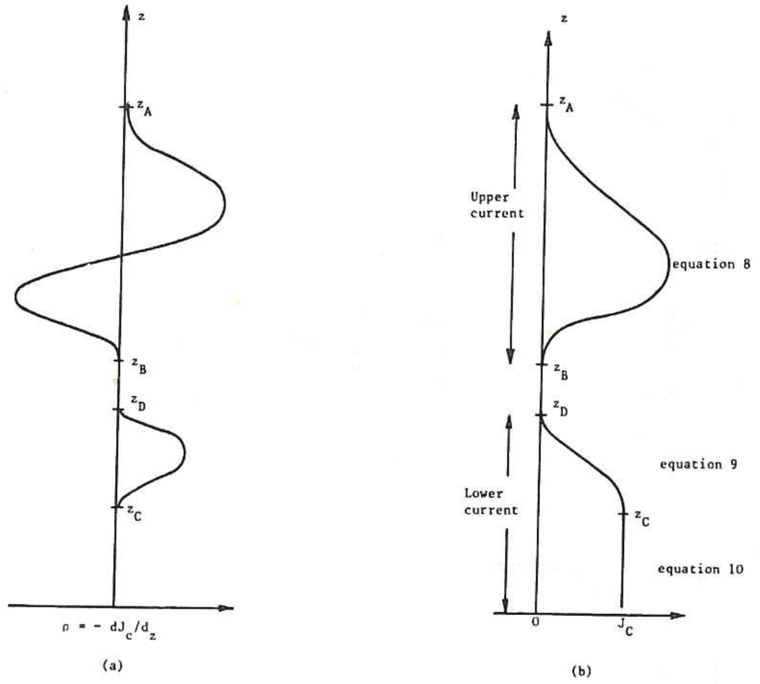

Fig. 3(a) Schematic arrangement of the charge distribution on the axis of a cloud. (b) Vertically directed convection currents consistent with the charge distribution in a, the forms assumed for the various parts of the current are indicated.

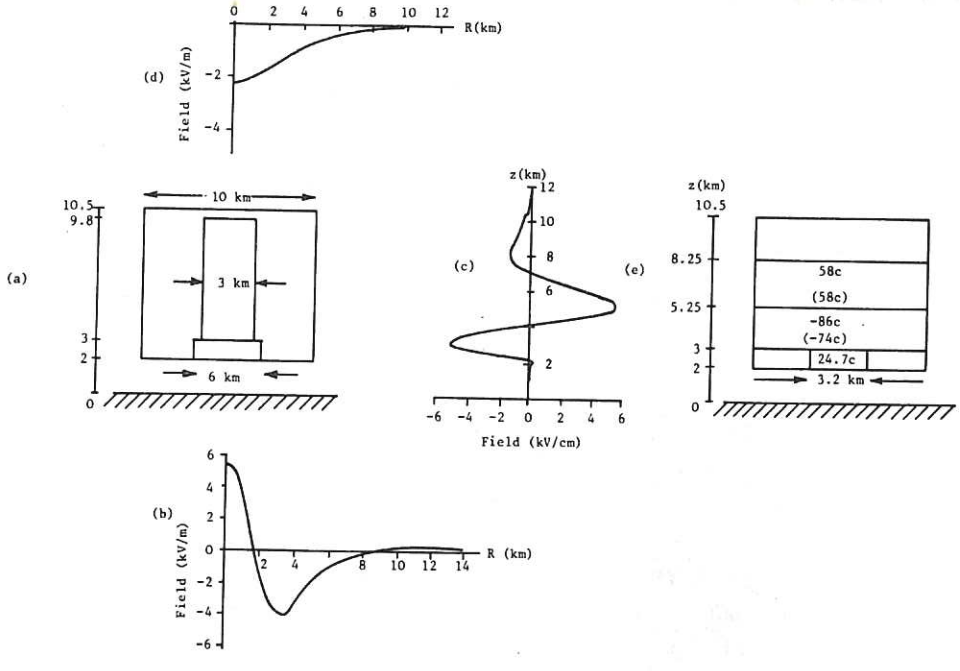

Fig. 4. First cloud configuration investigated. (a) Geometry of cloud, for dimensions and convective current strengths see tables 1 and 2. (b) Vertical component of electric field at ground level. (c) Vertical component of electric field on axis of cloud as a function of altitude. (d) Vertical component of electric field above cloud at an altitude of 18 km. (e) Electric charge contained in various zones of the cloud, the values in brackets indicates the charge contained inside the boundaries of the convection currents.

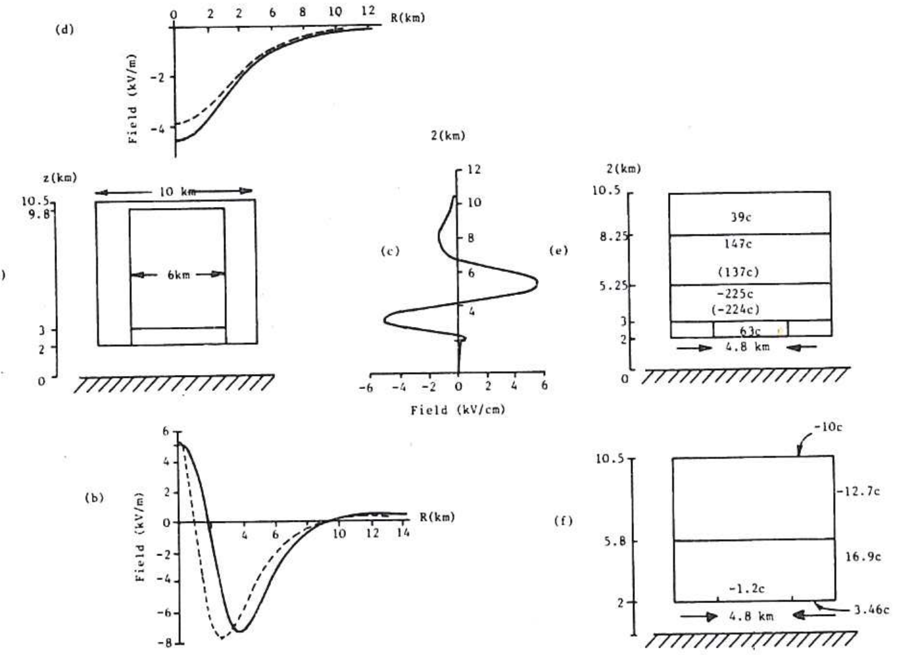

Fig. 5. Second cloud configuration investigated. (a) Geometry of cloud, for dimensions and convective current strengths see tables 1 and 2. (b) Vertical component of electric field at ground level. The broken line is the field for an electrostatic model of charges of 6 C, - 40 C and 4.0 C at altitudes of 2, 5 and 10 km. (c) Vertical component of electric field on axis of cloud as a function of altitude. (d) Vertical component of electric field above cloud at an altitude of 13 km. The broken line is the field for an electrostatic model with charges of - 39 C and 39 C at altitudes of 3.05 and 6.05 km see Wait (4). (e) Electric charge contained in various zones of the cloud, the values in brackets indicate the charge contained inside the boundaries of the convection currents. (f) Surface charge in cloud, the bottom face of the cloud has a charge distribution of - 1.2 C inside a circle of radius 2.4 km surrounded by an annulus containing 3.46 C.

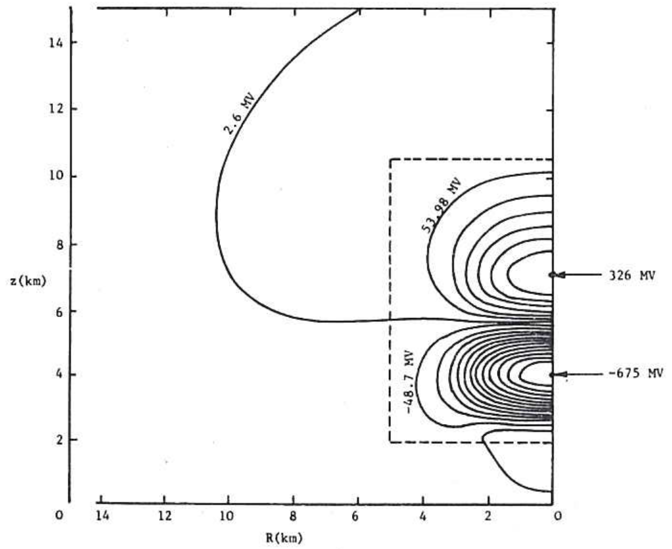

Fig.6 The potential distribution around the cloud of Fig.5 ; the broken line shows the boundary of the cloud. The contours are plotted at 51.4 MV intervals.

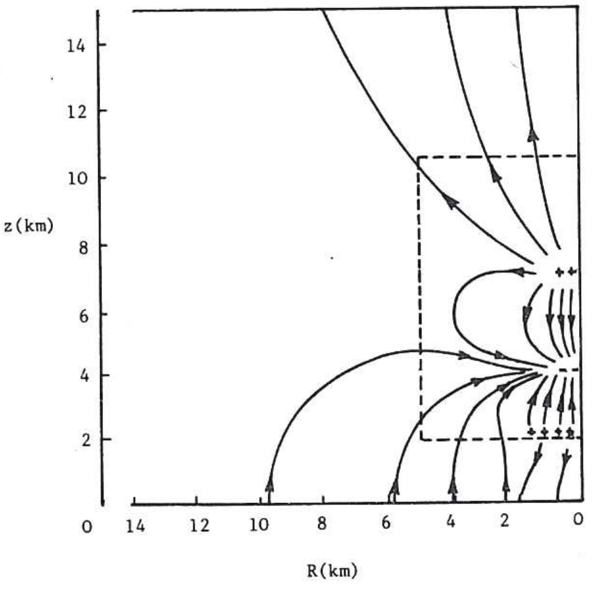

Fig.7 The normals to the potential distribution of Fig.6 showing the field distribution of the cloud.