CLM-P 387

OPTICAL SATURATION

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(To be submitted for publication in Physics Letters)

ABSTRACT

A general criterion is given for an amplifying medium to be self saturating. The concept of self-saturation gain has application to laser physics e.g. energy storage, and astrophysics (interstellar masers). Wide variations are shown to exist between different media.

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March 1974

An amplifying medium reaches saturation when the gain coefficient is significantly decreased by stimulated emission. If the gain is sufficiently large this can take place via amplified spontaneous emission[1] and the medium then becomes self-saturating. This has important consequences for laser amplifiers[2] and astrophysical masers[3].

The self-saturation gain G_s is most simply defined as the ratio of stimulated emission S at saturation to the spontaneous emission ε in the medium. Now $S = B\Delta n\ I_s/c = \alpha\ I_s$, where B is the Einstein coefficient for the stimulated process, Δn is the inverted population density, I_s is the saturation intensity, c the velocity of light and α the small signal gain coefficient.

Therefore $$G_s^{-1} = \frac{\varepsilon}{\alpha I_s}$$

Because the inversion density and the gain coefficient are reduced by a factor 0.5 - 0.7 in the saturated case, G_s may be defined more precisely as the ratio of the saturation intensity I_s to the emissivity ε/α of the medium. It should be noted that these expressions exclude solid angle because they represent an intrinsic property of the medium; geometric effects can be introduced later in solving specific problems via the equations of radiative transfer. The definition of G_s given above is perfectly general; as an example of its application a relation for G_s involving the rate constants of a two level system will now be derived.

For a radiative transition between two energy states with homogeneous broadening of the emission line we have[4] $\varepsilon = A_{21}\, n_2\, h\nu_{21}$, where A_{21} is the Einstein A coefficient connecting level 2 with level 1, n_2 is the population of level 2, ν_{21} is the radiation frequency; $\alpha = A_{21}\, ƛ_{21}^2\, \Delta n_{21}/\Delta\nu_{21}$, where $ƛ_{21} = \lambda_{21}/2\pi$, λ_{21} is the wavelength of the radiation, $\Delta\nu_{21}$ is the emission linewidth, the inversion density $\Delta n_{21} = n_2 - \gamma_2/\gamma_1\, n_1$, γ_2 and γ_1 are the statistical weights of levels 2 and 1, n_1 is the population of level 1 ; finally $I_s = 2\Gamma_{21}\, h\nu_{21}\, \Delta\nu_{21}/A_{21}\, ƛ_{21}^2$, where $\Gamma_{21}^{-1} = \Gamma_2^{-1} + (\gamma_2/\gamma_1)\, \Gamma_1^{-1}\, (1 - A_{21}\Gamma_2^{-1})$, and Γ_1 , Γ_2 are the total decay rates - radiative plus non-radiative - of levels 1 and 2 connected to all other states, including the transition $2 \rightarrow 1$.

Using these relationships we find

$$G_s^{-1} = A_{21}\, n_2/2\Delta n_{21}\, \Gamma_{21} .$$

Solution of the rate equations[4] for levels 1 and 2 gives

$$\frac{n_2}{2\Delta n_{21}} = \frac{1}{2}\left[1 - \frac{\gamma_2}{\gamma_1}\left(\frac{A_{21}}{\Gamma_1} + \frac{W_1}{W_2}\,\frac{\Gamma_2}{\Gamma_1}\right)\right]^{-1}$$

where W_1 , W_2 are the excitation rates. Let $q_{21} = n_2/2\Delta n_{21} = q\,(A,W,\Gamma)|_{2,1}$,

therefore

$$G_s^{-1} = q_{21}\, A_{21}/\Gamma_{21} \qquad (q_{21} \geq \tfrac{1}{2}).$$

A source of solid angle $\Delta\Omega$ has a component of spontaneous emission ε_1 of magnitude $\varepsilon\Delta\Omega/4\pi$, if the spontaneous process is isotropic. An analogous gain saturation parameter can be defined in this case[3]

$$g_s^{-1} = \frac{\varepsilon_1}{\alpha I_s} = q_{21} (A_{21}/\Gamma_{21})(\Delta\Omega/4\pi) .$$

The following examples show that the magnitude of G_s can vary over a wide range.

For astrophysical masers values of $G_s \sim 10^7$ to 10^9 were estimated previously[3], whereas an ideal 4 - level laser has $\Gamma_1 \gg \Gamma_2 = A_{21}$, whence $q = \frac{1}{2}$ and $G_s = 2$. Thus the gain in this case is essentially limited to $\sim g_s \simeq 8\pi/\Delta\Omega$. The minimum condition for an inversion $\Delta n > 0$ is $\gamma_1 \Gamma_1 > \gamma_2 A_{21}$; if $W_1 = 0$, $\Delta n_{21} \ll n_2$ and $G_s \ll 1$ ($\Gamma_2 \simeq A_{21}$). For nearly equal excitation rates $W_2 - W_1 = \Delta W$ in a gas where $\Gamma_2 = \Gamma_1 \gg A_{21}$ and $\gamma_2 = \gamma_1$, $G_s = (\Delta W/W_1)(\Gamma_2/A_{21})$[5]. An infrared gas laser with $W_2 \gg W_1$ and $\Gamma_{21} \gg A_{21}$ has $q_{21} \simeq \frac{1}{2}$ and $G_s \simeq 2\Gamma_{21}/A_{21} \gg 1$; hence the high gain, high energy characteristic of this device[6]. For a 3 - level laser $W_1 = -W_2$ and $G_s = 2\Gamma_2/A_{21} \geq 2$.

In the case of an inhomogeneously broadened emission line or a multi-line source, competition between stimulated emission on the different homogeneous components and cross-relaxation between them will increase the saturation gain G_s by a factor which is a function of the spectroscopic constants. This argument can be used to show that $G_s > 55$ dB for the TEA C02 laser[7]; for Nd: glass $G_s > 3$ dB.

Steady state conditions have been assumed throughout this analysis; in the transient case 'over-pumping' can produce temporary enhancement of pulse amplification[8]. In the absence of an external signal, the value of g_s gives a condition for the generation of saturated incoherent emission[1, 9]. The proposed 'super-radiant' X-ray laser[10] exceeds this requirement by several orders of magnitude.

There is also an important conclusion here for astrophysical masers; the self-saturation gain is always less than the threshold gain for maser oscillation via resonance scattering [11]. In these circumstances there would seem to be no meaningful distinction between the oscillator and partially saturated amplifier models of these sources [3, 11, 12, 13].

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