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THE DISTORTION OF THE PLASMA ION DISTRIBUTION DURING HEATING BY ENERGETIC PARTICLES

by

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ABSTRACT

It is shown that the heating of a plasma by energetic ions can distort the background ion distribution. Examples of distortions caused by the neutral injection heating of tokamak plasmas and the alpha particle heating of reactors are given.

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I. INTRODUCTION

The understanding of the heating of a plasma by energetic ions is an essential step towards a fusion reactor since the temperature in a reactor will be maintained by the reaction product alpha particles, and one method of heating a plasma to ignition temperature is by injecting energetic ions. It has been tacitly assumed that during heating the background ion distribution remains Maxwellian. However, since the energetic ions interact more strongly with the large velocity ions in the tail of the ion distribution than the thermal ions, the distribution will be distorted at high energies.

This type of distortion is important for two reasons. First, in recent experiments[1,2] in which energetic neutral atoms were injected for plasma heating, the increase in temperature was determined by examining the charge-exchanged neutrals at energies from thermal up to ten or twenty times thermal. It is in this energy range that the distortion of the plasma ion distribution will be significant with this form of heating. Second, in a reactor heating of the D and T ions by the 3.5 MeV alpha particles will lead to enhanced high energy tails on the D and T ion distributions. The thermonuclear reaction rate of these distorted distributions is found to be slightly greater than the undistorted distribution.

Perhaps at this point it should be clearly stated that the energetic ions themselves (either injected or reaction products) also could be thought of as a distortion upon the total ion distribution; however in this paper the energetic ions will be regarded as a separate distribution, and the distortions of the background ion distribution considered here will be solely due to the collisional interaction of the background ions with the energetic ions. An experimental measurement of this distortion would almost certainly require that the injected energetic ion specie be different from the plasma ions, so that by using mass analysis the ions in the tail of the background distribution may be distinguished from the injected particles.

Another possible source of ion distribution distortion is due to the heating of the ions by the electrons. In present tokamak plasmas in which the background electron temperature is greater than the ion temperature, a small distortion in the ion distribution is produced.

The structure of the remainder of the paper is as follows. In Section II the Fokker-Planck equation for the background ions is solved. Then in Section III examples of distortions in present tokamaks and reactors are given.

II. THE ION DISTRIBUTION

Our starting point is the Fokker-Planck equation for the background ions and the form of this equation derived by Rosenbluth et al[3] will be used. In this equation there will be contributions to the Rosenbluth potentials g and h from the background ions themselves, the electrons and the energetic ions. The equation is considerably simplified by taking g and h to be isotropic, that is independent of angle; this assumption may be justified as follows: For the background ions and electrons it can be shown that the small departures from isotropy due to the geometry (trapped particle effects) or the electric field have only a negligible effect on g and h. For the energetic ions it has been demonstrated by several authors[4,5] that for velocities $v < v_c$ $(=(.75\,\pi^{\frac{1}{2}} m_e/m_i)^{1/3} v_e)$ the energetic ion distribution is isotropic, even if the injection is anisotropic; this is due to strong ion-ion scattering for $v < v_c$. Thus if the range of velocities is restricted to $v_i < v < v_c$ (v_i thermal velocity) all the distributions (plasma ions and electrons and energetic ions) may be assumed to be isotropic and hence the potentials g and h will be functions of v only.

The equilibrium Fokker-Planck equation for the background ion distribution f_i with isotropic potentials may be written in the form

$$(2v^2)^{-1} \frac{\partial^2}{\partial v^2} \left(v^2 \frac{\partial^2 g}{\partial v^2} f_i\right) - v^{-2} \frac{\partial}{\partial v} \left\{\left(v^2 \frac{\partial h}{\partial v} + \frac{\partial g}{\partial v}\right) f_i\right\} = 0 \tag{1}$$

where
$$v^2 \frac{\partial h}{\partial v} + \frac{\partial g}{\partial v} = 4\pi \sum_j Z_j^2 \left\{\frac{2}{3} v \int_v^\infty dv' \, v' f_j(v') - \frac{m_i}{m_j} \int_0^v dv' \, v'^2 f_j(v') - \frac{1}{3v^2} \int_0^v dv' \, v'^4 f_j(v')\right\} \tag{2}$$

$$v^2 \frac{\partial^2 g}{\partial v^2} = \frac{8\pi}{3} v^2 \sum Z_j^2 \left(v^{-3} \int_0^v dv' \, v'^4 f_j(v') + \int_v^\infty dv' \, v' f_j(v')\right) . \tag{3}$$

The first term of equation (1) is the velocity diffusion term and the second term is the friction term; the summation j in equations (2) and (3) is over the three species (plasma ions and electrons and energetic ions).

The form of the distribution functions which is substituted in equations (2) and (3) is as follows: for plasma ions and electrons, f_j is taken to be the Maxwellian $\left(f_j = \pi^{-\frac{3}{2}} n_j v_j^{-3} \exp(-v^2/v_j^2)\right)$ for the energetic ions (subscript h) $f_h = S\tau_s/\{4\pi(v^3+v_c^3)\}$, where S is the number of particles injected per unit volume per second, τ_s is the Spitzer slowing-down time. The latter expression for the energetic ion distribution was derived in references (4) and (5) and for simplicity charge exchange has been ignored ($\tau_{cx} \gg \tau_s$). For velocities v in the range $v_i < v < v_c$ equations (2) and (3) become

$$v^2 \frac{\partial h}{\partial v} + \frac{\partial g}{\partial v} = - n_i \sum_k Z_k^2 \frac{m_i n_k}{m_k n_i} - \frac{4 n_e}{3\pi^{\frac{1}{2}}} \frac{m_i}{m_e} \frac{v^3}{v_e^3} \tag{4}$$

$$v^2 \frac{\partial^2 g}{\partial v^2} = \frac{2}{3} S \tau_s \frac{v^2}{v_c} Z_h^2 \int_0^{v_o/v_c} \frac{u \, du}{1+u^3} + \frac{n_i v_i^2}{v} \sum_k Z_k^2 \frac{v_k^2}{v_i^2} \frac{n_k}{n_i} + \frac{4}{3\pi^{\frac{1}{2}}} \frac{n_e v^2}{v_e} . \tag{5}$$

Where the subscript i refers to the particular background ion species whose distortion is being calculated, the subscript h is for the energetic ions, the summation k is over the background ion species and enables impurities to be taken into account. Here it will be assumed that all the background ion species have the same temperature and then the two summations can be set equal to, say, Z_m as follows

$$\sum_k \frac{Z_k^2 n_k v_k^2}{n_i v_i^2} = \sum Z_k^2 \frac{m_i}{m_k} \frac{n_k}{n_i} = Z_m \quad . \tag{6}$$

The integral in the first term of equation (5) may be readily evaluated, and defining

$$K(v_o/v_c) = \int_0^{v_o/v_c} \frac{d\acute{u}\ \acute{u}}{\acute{u}^3+1} \tag{7}$$

a plot of $K(v_o/v_c)$ is given in Figure 1.

In equilibrium $(\frac{\partial}{\partial t} = 0)$ equation (1) may be trivially integrated to give

$$\frac{1}{2} \frac{\partial}{\partial v}\left(v^2 \frac{\partial^2 g}{\partial v^2} f_i\right) - \left(v^2 \frac{\partial h}{\partial v} + \frac{\partial g}{\partial v}\right) f_i = 0 \quad . \tag{8}$$

Substituting in the above equation the functions for g and h given by equations (4) and (5) and the subsidiary equations (6) and (7) and then integrating, gives

$$f_i = A\left(v^2 \frac{\partial^2 g}{\partial v^2}\right)^{-1} \exp\left[- \int_0^{v/v_i} \frac{2\,u\,du}{1+\rho u^3}\right] \quad . \tag{9}$$

with $\rho = 0.66'\, K\, S\, \tau_s\ Z_h^2\ v_i/(n_i\ v_c\ Z_m) + 1.33'\ n_e\ v_i\ (1 - T_i/T_e)/(\pi^{\frac{1}{2}}\ n_i\ Z_m\ v_e)$. (10)

Note that in equation (10) part of the second term was originally in the numerator, for $v < v_c$ the numerator may be expanded and the term incorporated in the denominator.

The first term on the righthand side of equation (10) is the contribution from the energetic ions and the remainder is the contribution from the electrons. The parameter ρ determines the extent of the distortion and in

Figure 2 the distribution function f_i is shown as a function of v^2 for different values of ρ .

III. EXAMPLES OF DISTORTIONS

(a) The heating of ions by the electrons

The first example we consider is the distortion of the ion distribution by electron heating alone. For this case ρ is small, the maximum value occurring for $T_e/T_i = 3$ (not untypical of present Tokamaks) with $Z_m = 1$ and $n_i = n_e$ the value of ρ_{max} is 0.008. As the reader can see from Figure 2, this is rather a small distortion.

(b) Neutral injection heating of Tokamaks

The next example we consider is the neutral injection heating of the Cleo tokamak, the parameters of the background hydrogen plasma are $T_i = 200$ eV, $T_e = 280$ eV, $n_e = 2 \times 10^{13}/cm^3$, $n_i = 8 \times 10^{12}/cm^3$, $Z_m = 1.56$ and $\tau_s = 0.009$ sec, the effective plasma volume occupied by the energetic ions is 10^5 cm^3. The injection of 1 amp of 22.5 keV hydrogen ions into this plasma gives $\rho = 0.01$. This is a fairly small distortion and did not affect the determination of the ion temperature increase.

For the injection of 26 keV helium ions (1 amp) into the same hydrogen plasma $\rho = 0.02$. Thus with helium injection the distortion is larger (due to $Z_h = 2$ in Equation (10)) and was taken into account when determining the temperature increase.

In future tokamak experiments designed to take large current sources (such as DITE) the distortion will be stronger since S (injected current/cm^3) will be increased by a factor of 4 and τ_s by a factor of 3 giving a ρ for hydrogen injection into a hydrogen plasma of 0.12.

(c) Alpha particle heating of a DT reactor

In the following the distortion of the deuterium distribution by collisions with the 3.5 MeV alpha particles is calculated. The production rate of alphas $S = \frac{1}{4} n_e^2 \overline{\sigma v}$ and since τ_s is inversely proportional to n_e the parameter ρ is independent of n_e. It does however depend on the temperature T through $\overline{\sigma v}$ and two cases $T = 20$ and 40 keV are considered (for $T = 20$ keV, $\rho = 2.36 \times 10^{-3}$ with $S = 8.75 \times 10^{-17}$ $n^2/cm^3/sec$, $Z_m = 1.66'$, $v_o/v_c = 2.089$, $K = .75$ and for $T = 40$ keV, $\rho = 2.7 \times 10^{-2}$ with $S = 2 \times 10^{-16}$ $n^2/cm^3/sec$, $v_o/v_c = 1.477$, $K = 0.6$). The resulting distorted distributions are shown in Figure 3 along with the undistorted distribution for comparison.

The distortion at 20 keV is small; however, for the 40 keV case there is a more significant distortion which results in a small increase in the thermonuclear reaction rate.

Acknowledgement

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References

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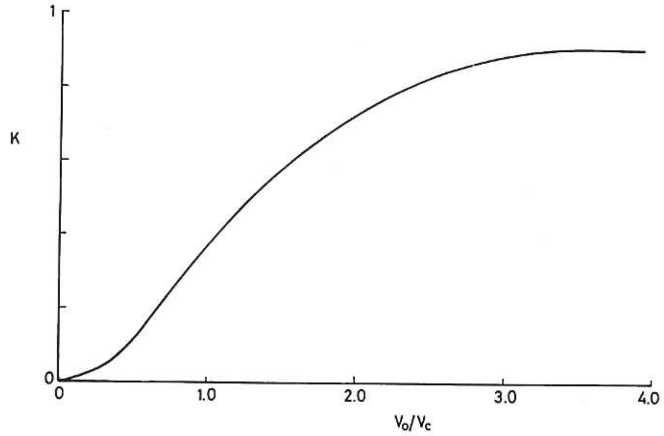

Fig 1. The constant K against v_o/v_c .

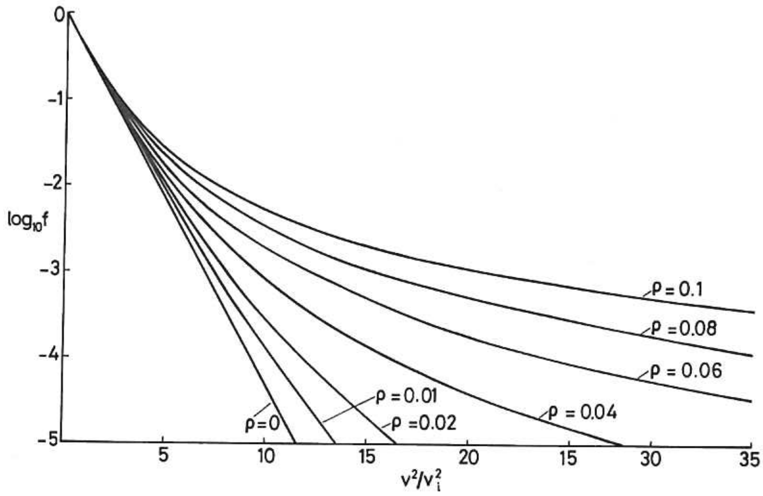

Fig 2. The ion distribution function f against v^2/v^2 for different values of the distortion parameter ρ .

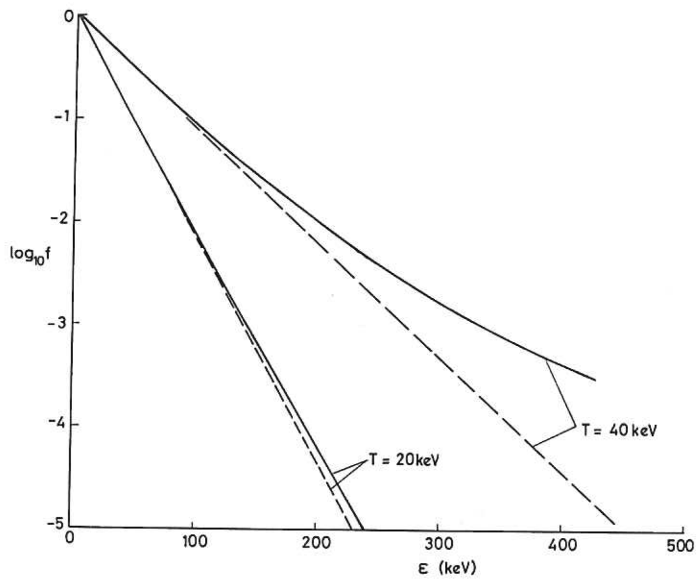

Fig 3. The distorted deuterium distribution (continuous line) as a function of energy for reactor temperatures of 20 and 40 keV. The broken lines are the corresponding undistorted distributions which are shown for comparison.