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# PARAMETRIC EXCITATION OF ION ACOUSTIC WAVES BY AN EXTERNAL CURRENT IN A MAGNETIZED PLASMA

C.N. Lashmore-Davies and R.S.B. Ong *
Euratom-UKAEA Association for Fusion Research
Culham Laboratory, Abingdon, Oxon, OX14 3DB, UK

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Abstract

We consider the parametric excitation of ion acoustic waves in a plasma by a low frequency spatially dependent external current source which acts as the pump field. We show that there is instability only when the pump resonates with the ion acoustic wave. The threshold and growth rate of the instability are determined for this case.

* also of Mathematical Institute, Oxford University, Oxford, UK

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## 1. Introduction

In a previous paper we studied the parametric excitation of ion acoustic waves in a plasma due to an external, low frequency modulation of the background magnetic field, (LASHMORE-DAVIES and ONG, 1974). The periodic changes in the magnetic field produce an oscillating equilibrium state of the plasma characterized by a modulation of the average density. This then couples to the ion acoustic waves. In that paper we restricted ourselves, for simplicity, to a one-dimensional analysis in which the pump was assumed to be spatially uniform. We now wish to extend this problem to the two-dimensional case including also the spatial dependence of the pump field.

As our model we consider an infinite plasma excited by an alternating external current source, which in turn induces an oscillating electromagnetic field - as well as a modulation of the plasma density, (LASHMORE-DAVIES, 1972). These low frequency oscillations then parametrically excite Alfvén and ion acoustic waves. We are mostly interested in the absorption of energy by the plasma. Hence we focus our attention on the excitation of ion acoustic waves with their relatively lower phase velocities. Further we simplify the problem by considering only waves propagating in a direction parallel to the background magnetic field.

## 2. The Oscillating Equilibrium

Consider an infinite uniform plasma with a stationary background magnetic field directed along the z-axis of a Cartesian coordinate system. An alternating external current source

$$\underset{\sim}{J}_o = J_o \hat{e}_x \cdot e^{i(k_{oy} y + k_{oz} z - \omega_o t)} \quad (1)$$

is assumed to flow in the x-direction. In equation (1) $\omega_o$ and $k_o$ are assumed given; hence we are dealing with a forced oscillation problem. First, we determine the 'oscillating equilibrium' state of the plasma in response to the alternating external current above. Since we shall always restrict ourselves to $\omega_o \ll \omega_{ci}$, the simple MHD equations with phenomenological damping coefficients are used:

$$\rho\left(\frac{\partial \underset{\sim}{v}}{\partial t} + \underset{\sim}{v}.\underset{\sim}{\nabla}\underset{\sim}{v}\right) = -\underset{\sim}{\nabla}p + \frac{1}{\mu_o}(\underset{\sim}{\nabla} \times \underset{\sim}{B}) \times \underset{\sim}{B} - \nu\rho\underset{\sim}{v} \tag{2}$$

$$\frac{\partial\rho}{\partial t} + \underset{\sim}{\nabla}.(\rho\underset{\sim}{v}) = 0 \tag{3}$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \underset{\sim}{\nabla} \times (\underset{\sim}{v} \times \underset{\sim}{B}) - \frac{\eta}{\mu_o}\underset{\sim}{\nabla} \times (\underset{\sim}{\nabla} \times \underset{\sim}{B}) \tag{4}$$

Furthermore,

$$\frac{1}{\mu_o}\underset{\sim}{\nabla} \times \underset{\sim}{B} = \underset{\sim}{J} + \underset{\sim}{J}_o \tag{5}$$

where

$\underset{\sim}{J}_o$ = the external (source) current

$\underset{\sim}{J}$ = the induced plasma current.

The other equation in which $\underset{\sim}{J}$ appears is

$$\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B} = \eta\underset{\sim}{J}\ . \tag{6}$$

The equation of motion of the plasma can be written as

$$\rho\frac{\partial \underset{\sim}{v}}{\partial t} + \rho(\underset{\sim}{v}.\nabla)\underset{\sim}{v} = -\underset{\sim}{\nabla}p + \frac{1}{\mu_o}(\underset{\sim}{\nabla} \times \underset{\sim}{B}) \times \underset{\sim}{B} - \underset{\sim}{J}_o \times \underset{\sim}{B} - \nu\rho\underset{\sim}{v}\ . \tag{7}$$

We assume that $\underset{\sim}{J}_o$ is sufficiently small and calculate the linear response of the plasma to this source current. To do this we assume that all variables vary as $\exp i(k_{oy}\,y + k_{oz}\,z - \omega_o t)$. The linearized equations may then be written as

$$-i\omega_o\rho_o\underset{\sim}{v}_1 = -c_s^2 i\underset{\sim}{k}_o\rho_1 + \frac{1}{\mu_o}(i\underset{\sim}{k}_o \times \underset{\sim}{B}_1) \times \underset{\sim}{B}_o - \underset{\sim}{J}_o \times \underset{\sim}{B}_o - \nu\rho_o\underset{\sim}{v}_1 \tag{8}$$

$$-i\omega_o\underset{\sim}{B}_1 = i\,k_o \times (\underset{\sim}{v}_1 \times \underset{\sim}{B}_o) - \frac{\eta}{\mu_o}i\underset{\sim}{k}_o \times (i\underset{\sim}{k}_o \times \underset{\sim}{B}_1) \tag{9}$$

$$-i\omega_o\rho_1 + \rho_o i\underset{\sim}{k}_o.\underset{\sim}{v}_1 = 0 \tag{10}$$

where we take the equation of state to be $p = c_s^2 \rho$. Further, we also have

$$\underset{\sim}{k}_o \cdot \underset{\sim}{B}_1 = 0 . \tag{11}$$

As far as $\underset{\sim}{J}_o$ is concerned we shall always assume that damping effects are weak. Thus we neglect the resistivity and collisions in the determination of the response of the plasma to $\underset{\sim}{J}_o$, i.e. the 'oscillating equilibrium'.

Substitute equation (10) into equation (8) :

$$-i\omega_o \rho_o \underset{\sim}{v}_1 + i\underset{\sim}{k}_o c_s^2 \rho_o \frac{\underset{\sim}{k}_o \cdot \underset{\sim}{v}_1}{\omega_o} - i \frac{(\underset{\sim}{k}_o \times \underset{\sim}{B}_1)}{\mu_o} \times \underset{\sim}{B}_o = -\underset{\sim}{J}_o \times \underset{\sim}{B}_o$$

The components of this equation are

$$-i\omega_o \rho_o v_{1x} - i \frac{k_{oz} B_o B_{1x}}{\mu_o} = 0$$

$$-i\omega_o \rho_o v_{1y} + i k_{oy} \frac{c_s^2}{\omega_o} \rho_o (k_{oy} v_{1y} + k_{oz} v_{1z}) + \frac{i}{\mu_o}(k_{oy} B_{1z} - k_{oz} B_{1y}) B_o = J_o B_o$$

$$-i\omega_o \rho_o v_{1z} + i k_{oz} \frac{c_s^2}{\omega_o} \rho_o (k_{oy} v_{1y} + k_{oz} v_{1z}) = 0 .$$

We see immediately that $v_{1x}$, and hence $B_{1x}$, are independent of $v_{1y}$ and $v_{1z}$ (and $B_{1y}$ and $B_{1z}$). Thus as far as the calculation of the oscillating equilibrium is concerned we may take $v_{1x} = B_{1x} = 0$.

Equation (9) gives

$$-i\omega_o \underset{\sim}{B}_1 = i\underset{\sim}{k}_o \times (\underset{\sim}{v}_1 \times \underset{\sim}{B}_o)$$

$$\therefore \quad \underset{\sim}{B}_1 = -\frac{1}{\omega_o} \underset{\sim}{k}_o \times (\underset{\sim}{v}_1 \times \underset{\sim}{B}_o) \tag{12}$$

Substituting this into the y-component of the equation of motion gives:

$$\left(-i\omega_o\rho_o + i k_{oy}^2 c_s^2 \frac{\rho_o}{\omega_o} + i \frac{B_o^2 k_{oy}^2}{\mu_o \omega_o} + i \frac{B_o^2 k_{oz}^2}{\mu_o \omega_o}\right) v_{1y} + i k_{oy} k_{oz} \frac{c_s^2}{\omega_o} \rho_o v_{1z} = J_o B_o \tag{13}$$

The z-component of the equation of motion yields

$$v_{1y} = \left(1 - \frac{k_{oz}^2 c_s^2}{\omega_o^2}\right) \frac{\omega_o^2}{k_{oy} k_{oz} c_s^2} v_{1z} \,. \tag{14}$$

From (13) and (14) we can solve for $v_{1z}$ and obtain

$$v_{1z} = -i \frac{J_o B_o}{\omega_o \rho_o D} k_{oy} k_{oz} \frac{c_s^2}{\omega_o^2} e^{i(k_{oy} y + k_{oz} z - \omega_o t)} \tag{15}$$

where the spatial and temporal variations of $v_{1z}$ are written out explicitly, and

$$D(\omega_o, k_o) \equiv \frac{k_{oy}^2 c_s^2}{\omega_o^2} - \left(1 - \frac{k_o^2 C_A^2}{\omega_o^2}\right)\left(1 - \frac{k_{oz}^2 c_s^2}{\omega_o^2}\right) \tag{16}$$

$$k_o^2 \equiv k_{oy}^2 + k_{oz}^2 \,.$$

$$C_A \equiv (B_o^2/\mu_o \rho_o)^{\frac{1}{2}} = \text{Alfvén speed.}$$

The dispersion relation $D(\omega, k) = 0$ yields the compressional Alfvén and ion acoustic waves, which are the natural modes of the plasma described by the set of MHD equations (2) - (4). We can now solve for the other variables and obtain:

$$v_{1y} = -i \frac{J_o B_o}{\omega_o \rho_o D} \left(1 - \frac{k_{oz}^2 c_s^2}{\omega_o^2}\right) e^{i(k_{oy} y + k_{oz} z - \omega_o t)} \tag{17}$$

$$B_{1y} = i \frac{k_{oz}}{\omega_o} \frac{J_o B_o^2}{\omega_o \rho_o D} \left(1 - \frac{k_{oz}^2 c_s^2}{\omega_o^2}\right) e^{i(k_{oy} y + k_{oz} z - \omega_o t)} \tag{18}$$

$$B_{1z} = -i \frac{k_{oy}}{\omega_o} \frac{J_o B_o^2}{\omega_o \rho_o D} \left(1 - \frac{k_{oz}^2 c_s^2}{\omega_o^2}\right) e^{i(k_{oy} y + k_{oz} z - \omega_o t)} \tag{19}$$

It is observed that the condition expressed by equation (11) is satisfied by $B_{1y}$ and $B_{1z}$ above. Finally, from the equation of continuity we obtain

$$\rho_1 = -i \frac{J_o B_o}{\omega_o^2 D} k_{oy} e^{i(k_{oy} y + k_{oz} z - \omega_o t)} \tag{20}$$

In order to relate our analysis to the problem of transit time magnetic pumping we take the transverse wave number of the fields given in equations (15), (17) - (20) to be $k_{oy} = i\kappa$. This corresponds to a source current similar to equation (1) but which flows only in a thin sheet, i.e. the xz plane, at the origin. We let $k_{oy} = i\kappa$ where $\kappa$ is positive (negative) according to the case when y is positive (negative). The quantities (15), (17), (18), (19) and (20) which describe the 'oscillating equilibrium' state of the plasma are then essentially evanescent in the ±y-direction.

## 3. Derivation of the Coupled Mode Equations and the Dispersion Relation

We now derive the equations for the natural modes of oscillation of the plasma including the nonlinear interaction between these modes and the oscillating equilibrium. Again we start from equations (2) - (4) with the equation of state $p = c_s^2 \rho$. However, in the succeeding analysis for the natural modes we take into account the resistivity and the effective damping coefficient. The equation of continuity may now be written in the form:

$$\frac{\partial \rho}{\partial t} + \rho_o \underset{\sim}{\nabla} \cdot \underset{\sim}{v} = - \underset{\sim}{\nabla} \cdot (\rho_1 \underset{\sim}{v} + \rho \underset{\sim}{v}_1) \tag{21}$$

where the subscript zero denotes the stationary equilibrium, the subscript one denotes the 'oscillating equilibrium', and the wave quantities (or natural modes of oscillation) are indicated without any subscript. Similarly, the equation of motion becomes:

$$\frac{\partial \underset{\sim}{v}}{\partial t} + \frac{c_s^2}{\rho_o} \underset{\sim}{\nabla}\rho - \frac{1}{\mu_o \rho_o} (\underset{\sim}{\nabla} \times \underset{\sim}{B}) \times \underset{\sim}{B}_o = -\nu \underset{\sim}{v} - \frac{\rho_1}{\rho_o} \frac{\partial \underset{\sim}{v}}{\partial t} - \frac{\rho}{\rho_o} \frac{\partial \underset{\sim}{v}_1}{\partial t} - (\underset{\sim}{v}_1 . \underset{\sim}{\nabla})\underset{\sim}{v}$$

$$- (\underset{\sim}{v} . \underset{\sim}{\nabla})\underset{\sim}{v}_1 + \frac{1}{\mu_o \rho_o} (\underset{\sim}{\nabla} \times \underset{\sim}{B}_1) \times \underset{\sim}{B} + \frac{1}{\mu_o \rho_o} (\underset{\sim}{\nabla} \times \underset{\sim}{B}) \times \underset{\sim}{B}_1 . \qquad (22)$$

The equation for the magnetic field reduces to

$$\frac{\partial \underset{\sim}{B}}{\partial t} - \underset{\sim}{\nabla} \times (\underset{\sim}{v} \times \underset{\sim}{B}_o) = -\frac{\eta}{\mu_o} \underset{\sim}{\nabla} \times (\underset{\sim}{\nabla} \times \underset{\sim}{B}) + \underset{\sim}{\nabla} \times (\underset{\sim}{v} \times \underset{\sim}{B}_1) + \underset{\sim}{\nabla} \times (\underset{\sim}{v}_1 \times \underset{\sim}{B}) \qquad (23)$$

In equations (21) - (23) we kept only those terms which can contribute to a wave response of the form $\exp i\,(\underset{\sim}{k} . \underset{\sim}{r} - \omega t)$. Furthermore, we have neglected terms involving products of the oscillating equilibrium fields, because these fields are the pump fields and have therefore the highest frequency. These terms can then only give a response at $2\omega_o$ or zero frequency.

We now consider the parametric excitation of ion acoustic waves by the modulation of the stationary equilibrium state by the external source current $\underset{\sim}{J}_o$. It is the modulation of the plasma density which couples to the acoustic waves. For simplicity we consider only acoustic waves propagating along the uniform background magnetic field directed along the z-axis. For these waves $v_y$, $B_y$ and $B_z$ are zero and only $\rho$ and $v_z$ are of interest. The relevant equations are the continuity equation (21) and the z-component of the equation of motion:

$$\frac{\partial v_z}{\partial t} + \frac{c_s^2}{\rho_o} \frac{\partial \rho}{\partial z} = -\nu v_z - \frac{\rho_1}{\rho_o} \frac{\partial v_z}{\partial t} - \frac{\rho}{\rho_o} \frac{\partial v_{1z}}{\partial t} - (\underset{\sim}{v}_1 . \underset{\sim}{\nabla})\, v_z - (\underset{\sim}{v} . \underset{\sim}{\nabla})\, v_{1z} . \qquad (24)$$

In order to study the interaction between the 'oscillating equilibrium' and ion acoustic waves we write equation (21) and (24) into coupled mode form (LASHMORE-DAVIES, 1973). We make use of the frequency and wave number selection rules

$$\omega_o \simeq \omega_{s1} + \omega_{s2}$$
$$k_o = k_{s1} + k_{s2} \tag{25}$$

where $(\omega_{s1}, k_{s1})$ and $(\omega_{s2}, k_{s2})$ are the frequency and wave numbers of the ion acoustic waves. First we eliminate $\rho$ from equations (21) and (24), and obtain

$$-i(\omega - kc_s)\,(\omega + kc_s)v_z = -\omega\nu v_z - \frac{k c_s^2}{\rho_o}\left\{\rho_1 \frac{\partial v_z}{\partial z} + v_z \frac{\partial \rho_1}{\partial z} + \rho \underset{\sim}{\nabla}\cdot\underset{\sim}{v}_1 + \underset{\sim}{v}_1\cdot\underset{\sim}{\nabla}\rho\right\}$$
$$-\omega\left\{\frac{\rho_1}{\rho_o}\frac{\partial v_z}{\partial t} - \frac{\rho}{\rho_o}\frac{\partial v_{1z}}{\partial t} - v_{1z}\frac{\partial v_z}{\partial z} - v_z\frac{\partial v_{1z}}{\partial z} - v_{1y}\frac{\partial v_z}{\partial y}\right\}.$$

Now we put $\omega = \omega_1$, where $\omega_1 = k_1 c_s$, except in the factor $\omega - k c_s$, where we put $\omega \to i\dfrac{\partial}{\partial t}$. We obtain

$$\left(\frac{\partial}{\partial t} + i\omega_{s1}\right) v_{z1} = -\frac{1}{2}\nu v_{z1} - \frac{c_s}{2\rho_o}\left\{\rho_1\frac{\partial}{\partial z} v_{z2}^* + v_{z2}^*\frac{\partial \rho_1}{\partial z} + \rho_2^*\,\underset{\sim}{\nabla}\cdot\underset{\sim}{v}_1 + \underset{\sim}{v}_1\cdot\underset{\sim}{\nabla}\rho_2^*\right\}$$
$$-\frac{1}{2}\left\{\frac{\rho_1}{\rho_o}\frac{\partial v_{z2}^*}{\partial t} - \frac{\rho_2^*}{\rho_o}\frac{\partial v_{1z}}{\partial t} - v_{1z}\frac{\partial v_{z2}^*}{\partial z} - v_{z2}^*\frac{\partial v_{1z}}{\partial z} - v_{1y}\frac{\partial v_{z2}^*}{\partial y}\right\} \tag{26}$$

Next we evaluate the terms on the right hand side. First we distinguish between the following two cases:

a. $k_{s1}/k_{s2} > 0$

b. $k_{s1}/k_{s2} < 0$ .

Case a. In this case we have

$$k_{oz}\, c_s = (k_{s1} + k_{s2})\, c_s$$
$$= \omega_{s1} + \omega_{s2} = \omega_o\,. \tag{27}$$

Using these relations the equation above becomes

$$\left(\frac{\partial}{\partial t} + i\omega_{s1}\right) v_{z1} = -\frac{1}{2}\nu v_{z1} - \frac{i}{2}\frac{J_o B_o \omega_{s1}}{\omega_o^2 \rho_o D}\kappa v_{z2}^* e^{i(k_{oz}z + i\kappa y - \omega_o t)}$$

We let $\gamma_s = \frac{1}{2}\nu$, and write the equation for $v_{z1}$ as

$$\left(\frac{\partial}{\partial t} + i\omega_{s1} + \gamma_s\right) v_{z1} = -\frac{i}{2}\frac{J_o B_o \omega_{s1}}{\omega_o^2 \rho_o D}\kappa v_{z2}^* e^{i(k_{oz}z + i\kappa y - \omega_o t)}$$

Similarly the equation for $v_{z2}$ may be written as

$$\left(\frac{\partial}{\partial t} + i\omega_{s2} + \gamma_s\right) v_{z2} = -\frac{i}{2}\frac{J_o B_o \omega_{s2}}{\omega_o^2 \rho_o D}\kappa v_{z1}^* e^{i(k_{oz}z + i\kappa y - \omega_o t)}$$

We put $v_{z1,2} = V_{1,2}(t)\, e^{i(k_{s1,2}z - \omega_{s1,2}t)}$, where the slow time variation of the wave amplitude is denoted by $V_{1,2}(t)$. We use the matching condition $k_{oz} = k_{s1} + k_{s2}$ and obtain

$$\frac{\partial V_1}{\partial t} + \gamma_s V_1(t) = -\frac{i}{2}\frac{J_o B_o \omega_{s1}}{\omega_o^2 \rho_o D}\kappa V_2^* e^{-\kappa y} e^{-i\varphi t},$$

where $\varphi = \omega_o - (\omega_{s1} + \omega_{s2})$ = the frequency mismatch.

We let $\Gamma \equiv \frac{1}{2}\frac{J_o B_o}{\omega_o^2 \rho_o D}\kappa e^{-\kappa y}$, and write the above equation as

$$\left(\frac{\partial}{\partial t} + \gamma_s\right) V_1(t) = -i\Gamma\omega_{s1} V_2^* e^{-i\varphi t}. \tag{28}$$

Similarly,

$$\left(\frac{\partial}{\partial t} + \gamma_s\right) V_2^*(t) = i\Gamma\omega_{s2} V_1 e^{i\varphi t}. \tag{29}$$

We now look for a slow time scale variation of the form $e^{-i\omega t}$. The equations for $V_1$ and $V_2$ can then be written as

$$(-i\omega+\gamma_s)\,V_1 + i\,\Gamma\omega_{s1}(V_2^* e^{-i\varphi t}) = 0$$

$$-i\,\Gamma\,\omega_{s2}\,V_1 + (-i\omega+\gamma_s+i\varphi)(V_2^* e^{-i\varphi t}) = 0$$

This implies that

$$(\omega+i\gamma_s)(\omega+i\gamma_s-\varphi) = -\,\Gamma^2\omega_{s1}\omega_{s2}\;.$$

This leads to a threshold value for the excitation of the ion acoustic wave given by

$$\Gamma = \left\{\frac{[\gamma_s^2 + (\frac{\varphi}{2})^2]}{\omega_{s_1}\,\omega_{s2}}\right\}^{\frac{1}{2}}\;. \tag{30}$$

The growth rate of the instability is given by

$$\gamma = -\,\gamma_s + \Gamma\left\{\omega_{s1}\omega_{s2} - \frac{\varphi^2}{4\Gamma^2}\right\}^{\frac{1}{2}}\;. \tag{31}$$

<u>Case b</u> : Let $k_{s_1} > 0$ and $k_{s2} < 0$

In this case we have $v_{z_1} = V_1(t)e^{i(k_{s1}z-\omega_{s1}t)}$

and $v_{z2}^* = V_2^*(t)e^{-i(-|k_{s2}|z-\omega_{s2}t)}$

$$= V_2^*(t)e^{i(|k_{s2}|z+\omega_{s2}t)}$$

Again we evaluate the terms on the right hand side of equation (26). We use the relations

$$\omega_o \simeq \omega_{s1} + \omega_{s2}$$
$$k_{oz} = k_{s_1} - |k_{s2}| \tag{32}$$

and obtain

$$\left(\frac{\partial}{\partial t} + i\omega_{s_1}\right) v_{z1} = \frac{1}{2}\nu v_{z1} - \frac{i}{2}\frac{J_o B_o}{\omega_o^2 \rho_o D} \kappa e^{-\kappa y} k_{oz} c_s v_{z2}^* e^{i(k_{oz} z - \omega_o t)}$$

We let $\gamma_s = \frac{1}{2}\nu$, and rewrite the equation for $v_{z1}$ as

$$\left(\frac{\partial}{\partial t} + i\omega_{s1} + \gamma_s\right) v_{z1} = -\frac{i}{2}\frac{J_o B_o}{\omega_o^2 \rho_o D} \kappa e^{-\kappa y} k_{oz} c_s \frac{\omega_{s1}}{\omega_o} v_{z2}^* e^{i(k_{oz} z - \omega_o t)}$$

Similarly the equation for $v_{z_2}$ may be written as

$$\left(\frac{\partial}{\partial t} + i\omega_{s2} + \gamma_s\right) v_{z_2} = +\frac{i}{2}\frac{J_o B_o}{\omega_o^2 \rho_o D} \kappa e^{-\kappa y} k_{oz} c_s \frac{\omega_{s2}}{\omega_o} v_{z1}^* e^{i(k_{oz} z - \omega_o t)}$$

We now proceed as in case (a) and obtain the equations for the wave amplitudes $V_1(t)$ and $V_2^*(t)$ :

$$\left(\frac{\partial}{\partial t} + \gamma_s\right) V_1(t) = -i\Gamma k_{oz} c_s \frac{\omega_{s1}}{\omega_o} V_2^*(t) e^{-i\varphi t} \qquad (33)$$

$$\left(\frac{\partial}{\partial t} + \gamma_s\right) V_2^*(t) = -i\Gamma k_{oz} c_s \frac{\omega_{s2}}{\omega_o} V_1(t) e^{i\varphi t} \quad . \qquad (34)$$

Solving equations (33) and (34) as before we obtain the dispersion relation

$$(\omega + i\gamma_s)(\omega + i\gamma_s - \varphi) = \Gamma^2 k_{oz}^2 c_s^2 \frac{\omega_{s1}\omega_{s2}}{\omega_o^2} \quad . \qquad (35)$$

It can easily be seen that there are <u>no</u> growing solutions for this equation and there is no parametric excitation of ion acoustic waves for case (b). The reason for this is evidently due to the fact that momentum cannot be conserved for this case. When $\omega_o \neq k_{oz} c_s$, the oscillating current source produces a transverse momentum through $v_{1y}$ (see equation (14)). This momentum cannot be balanced by the excitation of a pair of ion acoustic waves since they do not carry any transverse momentum even when the waves propagate obliquely to $\underline{B}_o$ !

## 4. Conclusions

We have considered a pump field of the type used in transit time magnetic pumping experiments and have shown that ion acoustic waves will only be excited parametrically when the pump field itself resonates with the ion acoustic wave i.e. when $\omega_o \approx k_{oz} c_s$. Otherwise there is no parametric excitation of ion acoustic waves and the pump wave only modifies the real part of the frequency. As already mentioned, the reason that the pump wave does not excite the ion acoustic waves, except at resonance, is evidently due to the fact that the acoustic waves cannot balance the transverse momentum of the pump wave. However, we have only considered a very simple model of the plasma. For a more realistic model which takes account of the confinement of the plasma there may be other low frequency waves, e.g. drift waves, which can balance the transverse momentum of the pump. The parametric excitation of such waves could be detrimental to confinement.

## REFERENCES


LASHMORE-DAVIES C.N. and ONG R.S.B. (1974) Phys. Rev. Lett. 32, 1172.

LASHMORE-DAVIES C.N. (1972) Plasma Phys. 14, 357.

LASHMORE-DAVIES C.N. (1973) Culham Lab. Rep. CLM-P 342.