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DIFFUSE MODE GENESIS

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ABSTRACT

An exact condition for parasitic oscillation in laser elements with isotropic scattering surfaces is given in the form of an integral equation for the surface brightness. Analytic solutions valid for large gain are given for spherical, cylindrical and rectangular symmetry.

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Consider an amplifying medium with one or more scattering surfaces adjacent to it. Let each surface element be illuminated by scattered light, amplified according to the length of path. If the scattering is isotropic, the flux incident at any point is[1]

$$\Phi' = \int_S \frac{B' \cos\theta \cos\psi \, dS}{r^2}$$

where θ and ψ are the inclinations of the surface normals to the light path r joining an element of area dS on the scattering surface to the illuminated element, B' is the perceived brightness of the source with surface brightness B,

$$B' = B \exp(\alpha r)$$

and α is the gain coefficient of the medium. If ρ is the diffuse reflectivity, the scattered flux $\Phi'' = \rho\Phi'$. As a result, the illuminated element has a local brightness B'' given by $\Phi'' = \pi B''$, from Lambert's law for a uniform source. Therefore

$$\pi B'' = \rho \int_S B e^{\alpha r} \frac{\cos\theta \cos\psi \, dS}{r^2}$$

an integral equation for the surface brightness.[2] Because it assumes equilibrium between the gain of the medium and the distribution of surface flux, this relation constitutes the threshold condition for parasitic oscillation generated via surface scattering.[3] Solutions to the integral equation relate the physical size of the system to the coefficients of gain and scattering, as in the analogous critical problem.[4] Whispering gallery modes have been excluded from the analysis because they are generated by propagation over a smooth surface[5] rather than by scattering from a diffuse one.

For certain symmetries the brightness function B(r) is virtually constant and the integral equation reduces to an identity for the threshold gain. In other cases this is not so. We consider a few examples:

1. The sphere, a totally symmetric body for which the assumption of constant surface brightness is exact. With $\mu = \cos\theta$ and $A = 2\alpha R$, where R is the radius, we find

$$(2\rho)^{-1} = \int_0^1 \exp(A\mu)\mu \, d\mu = A^{-1}[(1-A^{-1})e^A + A^{-1}]$$

in agreement with the published result.[6] In the limiting cases of high and low reflectivity this reduces to

$$A = \frac{3}{2}(\rho^{-1} - 1) \text{ and } e^A = (2\rho)^{-1}A$$

for $\rho \to 1$ and $\rho \ll 1$ respectively.

2. The circular disc, for which the assumption is less accurate, but adequate if the disc is thin. The analysis is complicated by total internal reflection (TIR), which requires the evaluation of a double integral; therefore we simplify the solution by considering the index matched case. Let the disc be of height h and diameter 2R. For diffuse reflection at the curved surface,

$$\rho \int_{-\pi/2}^{\pi/2} \exp(A \cos \theta) d\theta = \pi D,$$

where $D \equiv \frac{2R}{h} \gg 1$. For large gain, $e^A \gg 1$, we expand $\cos \theta = 1 - \frac{1}{2}\theta^2$ to second order, and integrate the resulting gaussian between infinite limits, whence

$$e^A \simeq \left(\frac{\pi A}{2}\right)^{\frac{1}{2}} \rho^{-1} D.$$

For a gain $e^3 \simeq 20$ across a disc whose shape factor $D = 5$, we have $A = 3$ and therefore $\rho \simeq 54\%$. In the case of a similar disc with TIR at the plane faces the threshold gain parameter $A \simeq 0.9$ for a refractive index of 1.56.[3,7,8] Such a drastic reduction underlines the importance of index matching for increasing the permissible stored energy in disc lasers.[9] To a first approximation the same expression can be used to estimate the threshold of an elliptical disc[7,8] (major axis = 2R), because of the gain narrowing effect on the divergence of the scattered radiation.[10]

3. Parallel plates. The integral equation for rectangular plates with a uniform gain medium between them can be written as follows

$$\pi B(x',y') = \rho \iint B(x,y) \frac{e^{\alpha r}}{r^4} h^2 \, dxdy$$

where $r^2 = (x-x')^2 + (y-y')^2 + h^2$, (x,y) and (x',y') are the Cartesian co-ordinates of points on each surface, and h is the plate separation. Clearly $B(x,y)$ has rectangular symmetry, and this fact can be used to find an analytic expression for the threshold in the large gain case. If $L^2 \gg b^2$ and $b \sim h$, where L is the length and b is the breadth of either plate, then in the limit of large gain ($e^{\alpha L} \gg 1$) an asymptotic expansion of the integral yields the following result to first order:

$$e^{\alpha L} \simeq \pi \alpha L \rho^{-1} \left[\frac{L^3}{bh^2}\right]$$

where we have assumed $B(0,y) \equiv B(L,y')$ for $|y-y'| \leq b$. Therefore if $b = h$ and $L = nb$, we have

$$n < \left[\frac{e^{\alpha L}}{\pi \alpha L}\right]^{\frac{1}{3}}$$

as a limiting condition on parasitic oscillation in a rectangular system (because $\rho \leq 1$). For example let $\alpha L = 10$. Then $n < 9$ approximately for self-oscillation to be possible e.g. in a system with $L = 2$m and $\alpha = 0.05$ cm^{-1} - typical of a large TEA CO_2 laser - it is sufficient to choose $b = h < 22$ cm to prevent the generation of diffuse modes via scattering from the plates.

The integral equation method outlined here constitutes an exact treatment of the problem which should find general application, including numerical methods of solution in any given case. The asymptotic expansions derived from it are useful approximations for large gain and complement those previously given for low to medium gain.[6] In addition to defining a generation theshold, solution of the surface brightness distribution allows all the parameters of the radiation field to be determined throughout the active volume e.g. flux, intensity distribution, density.

Finally, we note that in the large gain approximation it is the reflectance of the scattering surface in the direction of the maximum gain length which is the determining factor in parasitic oscillation, whereas for low gain it is the angular distribution which is concerned.

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