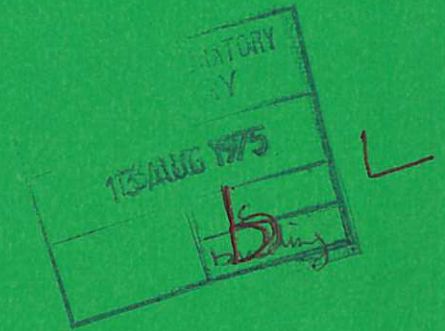


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EXPERIMENTS ON PLASMA TURBULENCE INDUCED BY STRONG, STEADY ELECTRIC FIELDS

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EXPERIMENTS ON PLASMA TURBULENCE INDUCED BY
STRONG, STEADY ELECTRIC FIELDS

by

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ABSTRACT

The consequences of applying a strong electric field ($E \geq m v_e \omega_{pi}/e$) to collisionless plasma based on (a) the results of computer simulations and (b) two quite different laboratory experiments, are compared. While the first suggests that the fluctuations produced by the induced instability do not prevent electron runaway, the effective electron collision frequency varying inversely with the time elapsed since E is applied, both laboratory experiments show that runaway does not occur, and that a sensibly constant frictional force (resistivity) acts for significantly long times.

May, 1975

In this paper I would like to discuss briefly the effect of applying a strong electric field to collisionless plasma. In particular I would like to compare what some simple ideas and prejudices lead us to expect to happen, what computer simulation experiments tell us ought to happen, and what actually does happen in two laboratory experiments which have been designed to allow the relevant instability and turbulent processes to occur unobstructed and which have been studied in sufficient detail.

First let me define what I mean by a 'strong' electric field. Just as a critical field in collisional plasma can be defined by the criterion that it be just sufficient to accelerate the average electron up to its thermal velocity, v_e , in one binary collision time, τ_c , i.e.

$$E_D = \frac{m v_e}{e \tau_c} = \frac{4 \pi m e^4 \ln \Lambda}{m v_e^2}$$

which is the well-known formula of Dreicer for electron runaway, so we can characterize a field in terms of whether or not it can do the same thing on a time scale characteristic of the growth of collective oscillations, say one ion plasma period ω_{pi}^{-1} , corresponding to a field

$$E_0 = \frac{m v_e \omega_{pi}}{e}$$

For our purpose we can regard the field as strong when $E \geq E_0$, although the concept is not very precise.

First, let us consider what we might expect to happen on the basis of elementary argument. Simply from the fact that electron-ion streaming electrostatic instabilities¹ acquire an enormously increased growth rate once the electron drift velocity, v_d , exceeds the thermal speed, v_e , and the strong field requirement ensures that this condition will be reached in

less than an ion plasma period, we can virtually disregard the slower growing kinetic or ion-acoustic form of the instability ($\gamma \ll \omega_{pi}$) and guess that once $v_d \gtrsim v_e^*$ the hydrodynamic two-stream instability, for which $\gamma \gtrsim \omega_{pi}$, will occur and the collective fields grow so rapidly that within a few growth times they reach such large amplitudes that some sort of highly nonlinear turbulent state is reached. This hypothetical turbulence, which is of course itself driven at the expense of the drift energy, will then not only act against the free acceleration but also, if sufficiently stochastic, heat the electrons until the threshold condition $v_d > v_e$ is violated; the fast-growing instability will then disappear and the fluctuations die away, thus once again allowing the driving field to provide free acceleration until instability recurs and the whole process then repeats. In other words, without worrying too much about the details of the processes involved, if the above argument is correct we should expect that, averaged over several ion plasma periods, the system should remain not very far from the marginal stability condition $v_d \approx v_e$. Schematically the time development would therefore look like that in Fig.1.

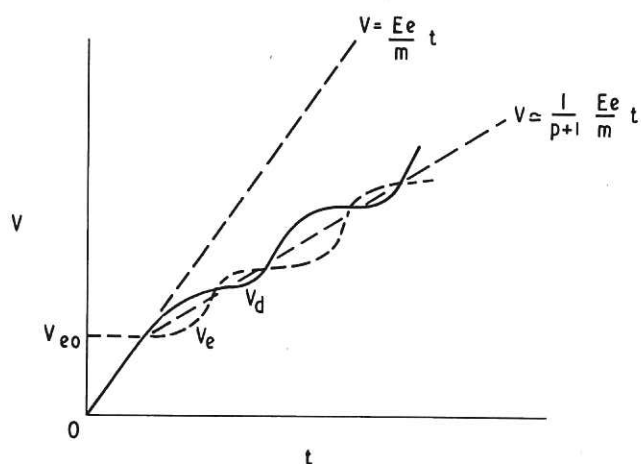


Fig.1

Temporal response of collisionless plasma to an applied electric field - based on conventional wisdom

Since this looks too simple to be true, let us turn to see what the computer, which, unlike plasma physicists, has no preconceived ideas about instabilities but obeys Newton's and Poisson's laws, says will happen in a simulated plasma when a constant electric field is applied.

Figure 2 shows results which are typical of such computer experiments appropriate to our strong field conditions. These have been carried out in many different laboratories², including Culham, using a variety of computing techniques, different dimensionality, ion mass, and so on, but the results differ only in details and they all show essentially the same general plasma

* Actually about $(1.3-1.4) v_e$, but this does not affect our argument significantly.

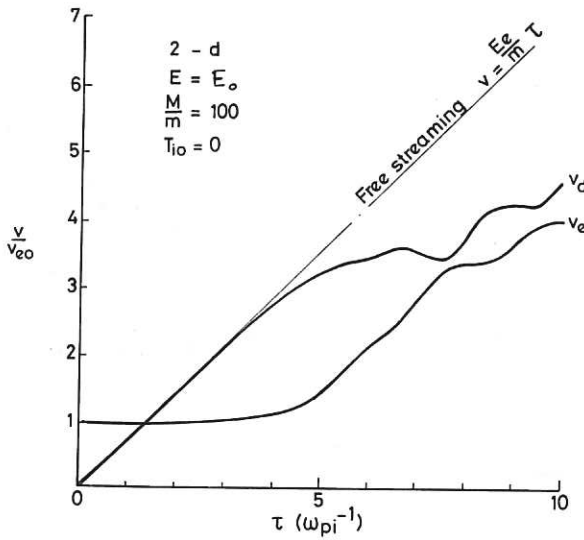


Fig.2

Temporal response of collisionless plasma to an applied electric field

$E = E_0 = \frac{m v_{e0} \omega_{pi}}{e}$ according to 2-dimensional simulation code.

experimenter, making only the additional assumptions that energy is conserved, and that the dissipated energy goes only into electron thermal motion (i.e. ignoring ion heating and stored collective fluctuation energy).

Power balance then gives for the averaged quantities \bar{v}_d and \bar{v}_e :

$$E e \bar{v}_d = \frac{d}{dt} \left(\frac{1}{2} m \bar{v}_d^2 + \frac{1}{2} m \bar{v}_e^2 \right)$$

(assuming for simplicity a one-dimensional system). Imposing the condition $\bar{v}_d = \bar{v}_e$ then results in

$$\bar{v}_d = \frac{1}{2} \frac{E e}{m} t \quad \dots (1)$$

(compared with $v_d = \frac{E e}{m} t$ for free-acceleration).

In other words the current increases linearly with time, but at half the rate it would have done had no collective interaction occurred. Had we allowed the thermal energy to be equally shared among p translational degrees of freedom, then equation (1) would have become

$$\bar{v}_d = \frac{1}{p+1} \frac{E e}{m} t \quad \dots (1a)$$

Similarly we can calculate the effective collision frequency ν and electrical conductivity σ (in esu) from the heating rates:

$$\nu = \frac{1}{\bar{v}_d^2} \frac{d}{dt} \left(\frac{1}{2} \bar{v}_e^2 \right) = \frac{1}{t} \quad \dots (2)$$

behaviour. In brief, they all more or less agree with our rather naive model of marginal stability. In other words, the electrons seem to go on heating indefinitely, while the mean drift velocity, that is the current, increases linearly and indefinitely with time. More simply, despite the spectacular effects of frictional heating via the current driven turbulence, the electrons still run away, albeit somewhat more slowly than they would have done had no collective interaction occurred.

Let us examine the consequences of keeping $\bar{v}_e \simeq \bar{v}_d$ on those macroscopic quantities of interest to the

$$[\text{or} \quad v = \frac{p}{t} \quad \dots (2a)]$$

and

$$\begin{aligned} \sigma &= \frac{1}{j^2} / \frac{d}{dt} \left(\frac{1}{2} m \bar{v}_d^2 \right) \\ &= \frac{n e^2}{m} t \end{aligned} \quad \dots (3)$$

$$[\text{or} \quad \sigma = \frac{n e^2 t}{m p} \quad \dots (3a)]$$

In terms of total current I and line density N the electrical 'resistance' per unit length becomes (for $p = 1$)

$$R = \frac{E}{I} = \frac{m}{N e^2} \frac{1}{I} \frac{dI}{dt} . \quad \dots (4)$$

This time dependent resistance looks, of course, like an inductance per unit length (emu) of

$$L = \frac{m c^2}{N e^2} \quad \dots (5)$$

This pseudo-inductance represents, of course, just an addition to the usual electron inertial inductance which is usually (though not always) small compared with the circuit inductance.

To summarize, if the plasma responds to the applied field in the way suggested above, we should not expect to see the current determined by a well-defined resistance, nor could the quantities we call conductivity or collision frequency be said to be convenient parameters to describe the plasma.

Now let us compare these predictions to what is, in practice, seen in experiments. For this I would like to present results obtained using two very different kinds of apparatus which involve very different actual plasma parameters and which use quite different diagnostic methods. These two experiments are summarized in Table I. The first column refers to the now dismantled TWIST stellarator³, in which high voltage pulses, of about 100 kV per turn and lasting about half a microsecond, were electro-magnetically induced around a magnetically confined toroidal plasma column, isolated from the silica walls of the vacuum vessel, 10 cm in diameter and 2 m circumference (Fig.3). The working gas was usually hydrogen, and the plasma density about 10^{12} cm^{-3} . The induced currents were typically several kiloamperes, and quantities like the drift velocity, electrical conductivity and so on were obtained from the measured currents, density, and voltages in the usual way. The electron temperature, which rose extremely rapidly from a few eV

TABLE I
Parameters of the Two Laboratory Experiments

Configuration	TWIST	THESEUS
	Toroidal Stellarator	Linear Q-machine
Plasma density (cm^{-3})	10^{12}	10^9
Ion	H^+	K^+
Plasma dimensions (cm)	$r = 5$, $R = 32$	$r = 1.5$, $L = 80$
Voltage pulse (max)	100 kV	100 V
E (V/cm) (max)	500	1
Pulse duration (μs)	0.3	2
I_{max} (A)	$\leq 10^4$	< 1
T_{eo} (eV)	~ 3	0.2
T_e (max) eV	$> 10^4$	20 - 80
B_{\parallel} (kG)	3	3
Diagnostics	Magnetic, Microwaves, X-rays, etc.	Electron Wave dispersion
Repetition rate (sec^{-1})	10^{-2}	10^2

to some tens of keV in about 100ns , was derived from the X-ray spectrum produced by bremsstrahlung from electrons hitting a carbon target immersed in the plasma. This method is of course, notoriously insensitive to the shape of the electron distribution, but was the most direct available, and the results were consistent with estimates of the plasma energy density derived from other measurements. The electrical conductivity was unravelled from the voltage and current waveforms by making proper allowance for the circuit inductance, care being taken to include any skin current effects in a self-consistent manner. Electrostatic probes were used to monitor the fluctuations and so confirm the existence of two-stream instability in the cases analyzed. The ohmic power dissipated was also used to estimate the instantaneous electron temperature on the assumptions of sufficient energy confinement.

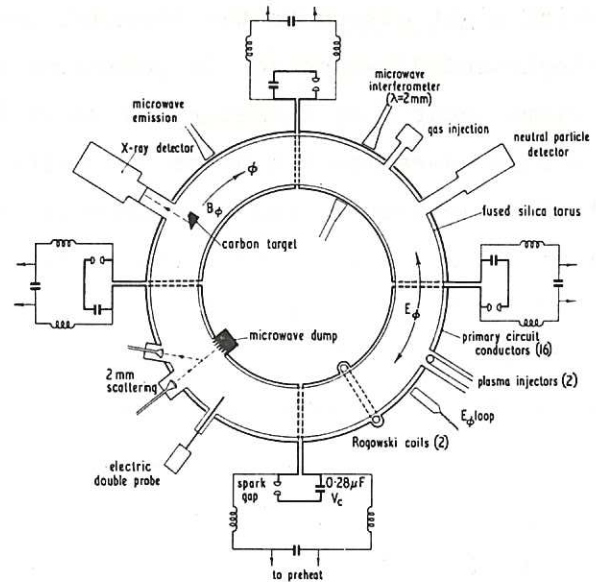


Fig.3

Schematic arrangement of TWIST
turbulent heating experiment

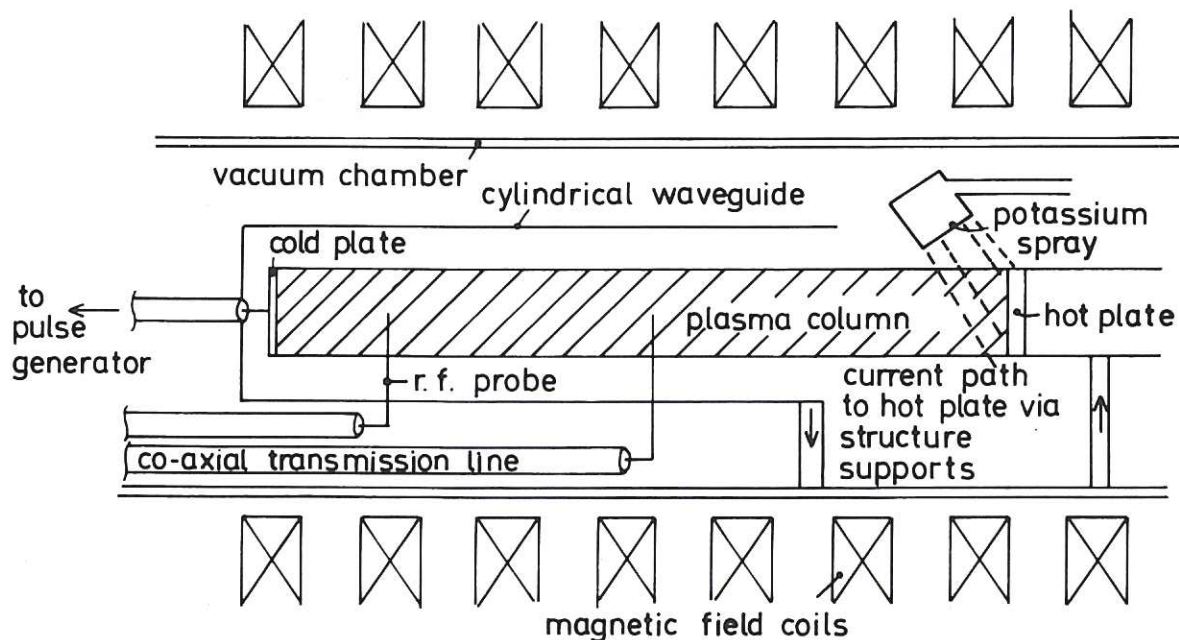


Fig.4

Schematic arrangement of the THESEUS experiment

The other experiment, which uses a much simpler apparatus, has been designed to avoid many of the uncertainties, e.g. about initial conditions, which might attach to the toroidal one. It is conducted in a conventional single-ended Q-machine, in potassium plasma of density a few 10^9 cm^{-3} , in a column about 3 cm diameter and 80 cm long (Fig.4). A voltage pulse of 1-2 μs duration and up to 100 volts is applied via a 50Ω series resistor to the electrodes, and the currents drawn are less than one ampere. This time the initial electron temperature is only 0.2 eV, and rises to around 20 eV. These parameters have been chosen so that while there is sufficient time for the turbulence to develop there should be no significant energy loss from the plasma region examined.

The details of this experiment will be reported separately in a contributed paper⁴. One unique feature lies in its diagnostics: by a very careful study of the dispersion of long wavelength, propagating electron plasma waves made using a fast sampling technique, it is possible to follow in great detail the time and space development of, e.g. the density (and to some extent its profile); the drift velocity (instead of relying on current measurements); and the r.m.s. velocity (averaged over many Debye lengths).

Table II shows the important plasma parameters again, but this time normalized in such a way to show that, although they look very different, the essential physics studied in both experiments should be comparable.

A typical set of results from TWIST are summarized in Fig.5 and shows the variation with time of the various quantities of interest.

	Stellarator	Q-Machine
$\frac{E_e}{m v_e \omega_{pi}}$	4	2 - 7
$\omega_{pi} \tau$	~ 200	~ 80
$\frac{\omega_{ce}}{\omega_{pe}}$	~ 1	~ 10
$\frac{\omega_{ci}}{\omega_{pi}}$	3×10^{-2}	3×10^{-2}

It shows that the drift velocity first starts to increase at a rate determined by the circuit inductance until it passes the initial thermal velocity, $v_e(0) \sim 10^8$ cm/sec. This is the moment when a strong instability, which from its frequency we have earlier⁵ at least tentatively identified with two-stream, occurs. The thermal speed then rises very rapidly, but the drift velocity far less so, partly of course due to the circuit inductance. However, at quite an early time the circuit becomes resistive, the current increases much more slowly and becomes almost in phase with the electric field. If we examine the calculated conductivity, we find that it is more or less constant, and most certainly does not increase $\propto t$, in contrast to equation (3). In fact it stays fairly close to a value consistent with an empirical formula derived earlier⁶ for such conditions

$$\sigma = 0.5 \left(\frac{M}{m} \right)^{\frac{1}{3}} \omega_{pe} \quad \dots (6)$$

which we had previously found from the resistance at peak current.

Had the behaviour been as predicted by the simulations, the current would have remained inductive (as shown in Fig.5) and the drift velocity reached values many times larger than those observed. Needless to say, the fact that the drift velocity is significantly below v_e for most of the time, while the plasma is still apparently subject to an instability which generates turbulence sufficiently strong to limit the current, is still not explained.

Let us now turn to the results from the much simpler linear experiment, which in many ways more nearly approaches the ideal conditions of the simulations. For example, the radius is much less than the smallest possible current penetration depth, $c/\omega_{pe} \sim 10$ cm, so that we should not expect to encounter any complicated current distributions. In any case, the important parameters, such as v_d and v_e , are measured in an unambiguous way.

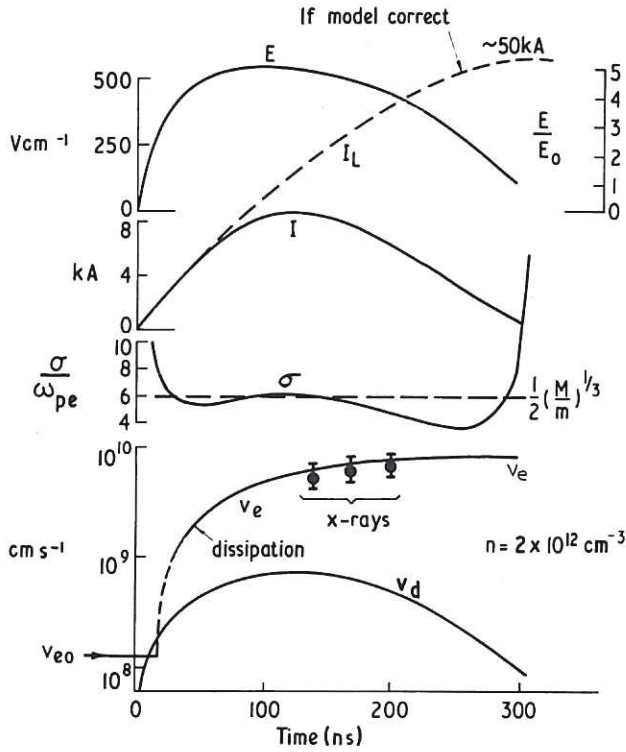


Fig.5

Typical results from TWIST, showing time dependence of various quantities of interest

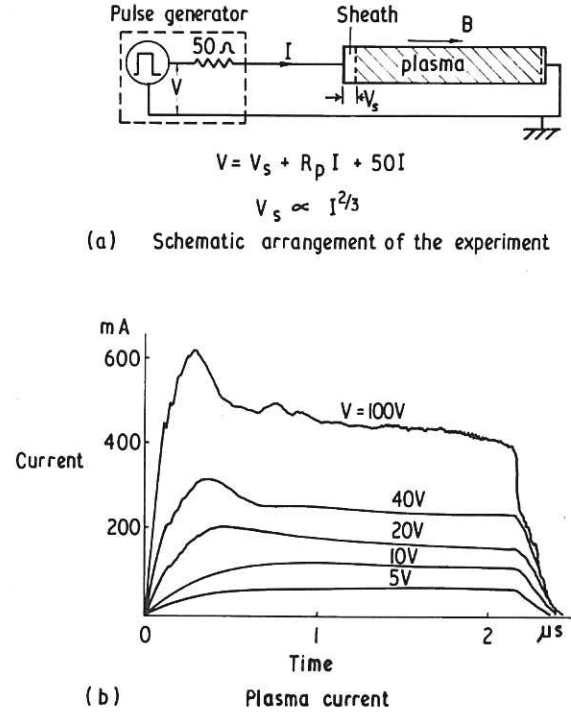


Fig.6

THESEUS (a) Simplified circuit arrangement, (b) plasma current waveforms for various pulse generator voltages

Figure 6(a) shows a simplified arrangement, with the plasma column in series with the 50Ω source impedance, together with (Fig.6(b)) current waveforms for various pulse generator voltages: for low generator voltages the current waveform is determined entirely by the inductance (which happens in this case to be mostly inertial), the hotplate sheath condition, and the source impedance, the plasma resistance being negligibly small. At higher voltages this changes, and for $V \geq 30$ volts, the plasma acquires a significant resistance. Since the hot-plate electron emission is always kept space-charge (and not temperature) limited, it is possible to allow for the effective sheath impedance if we assume that its perveance remains constant, and therefore to deduce the electric field in the plasma column itself and hence derive a mean plasma conductivity. This is shown in Fig.7 for a fixed time $1\mu s$ after the pulse is applied, plotted in normalized units σ/ω_{pe} against the ratio of drift to initial thermal velocity. As you can see, the conductivity falls rapidly when $v_d/v_e \geq 1.3$, which linear theory predicts as the appropriate threshold for two-stream instability, and reaches

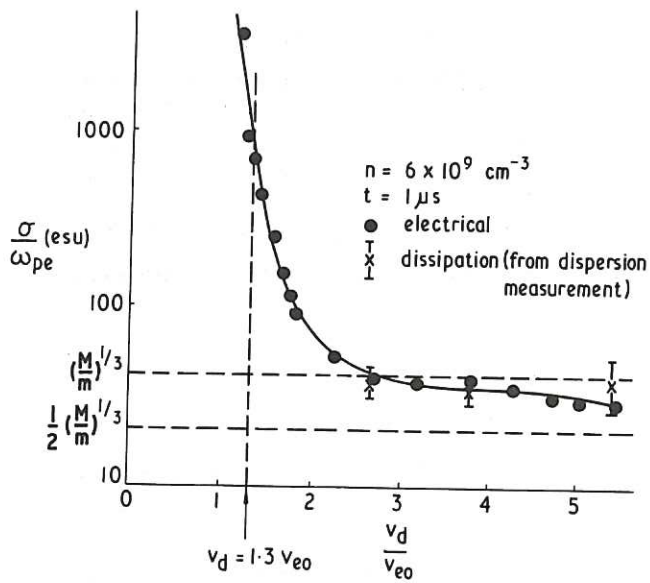


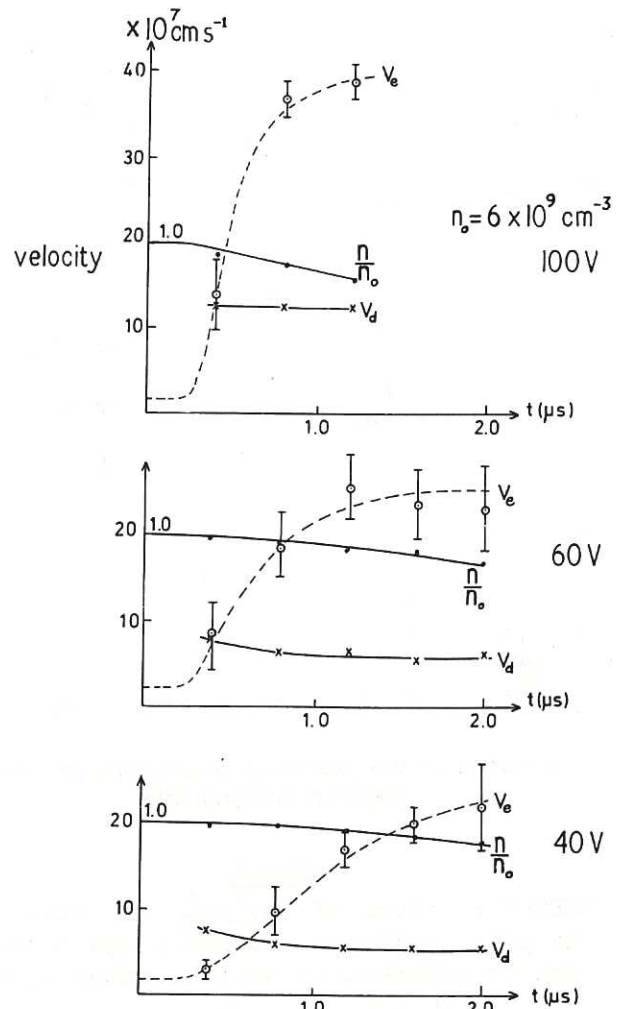
Fig.7

THESEUS: Plasma conductivity measured at a time $1 \mu s$ after pulse is applied. The experimental points \bullet are obtained from electrical measurements, those Φ are derived indirectly from measurements of velocity moments

a kind of saturation or plateau value for large v_d ; the actual value agrees within a factor two of that given by the empirical formula (6), i.e. essentially the same as in the toroidal experiment.

Now let us examine the detailed behaviour during the pulse. Fig.8

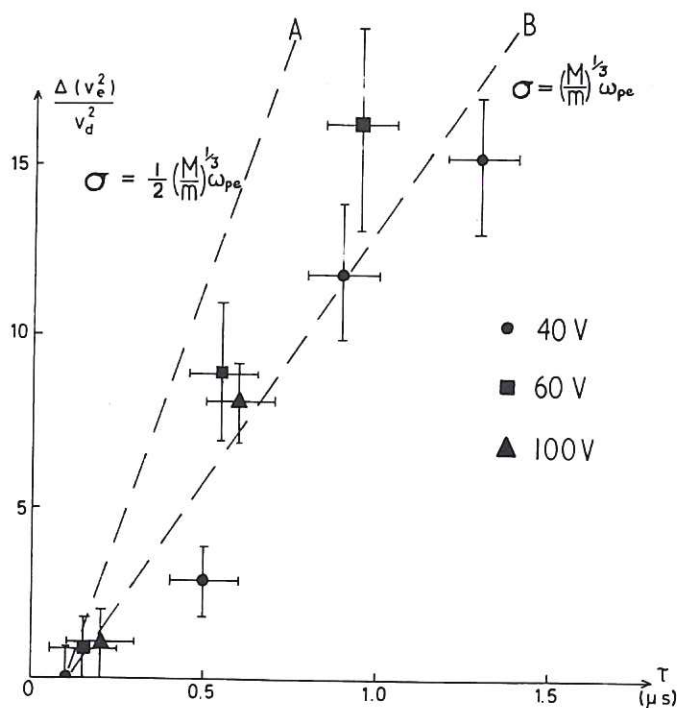
shows the variation of drift velocity, thermal velocity, and density for three different generator voltages, all large enough to produce instability. In this case all the quantities plotted have been derived from electron wave dispersion, and not from electrical measurements. In all three examples you can see that the density and drift velocity change very little with time, while the electron thermal velocity increases at a rate depending on the pulse voltage and, for most of the time, exceeds v_d (cf. Fig.3). Apart from the very early times (not shown), when free-acceleration occurs and electron inertia is important, there is no indication of electron runaway, although the electric field is strong according to our definition.



Evolution of electron density, n , drift velocity, v_d , and r.m.s. velocity v_e , (in the electron drift frame) for three different voltage pulses.

Fig.8

Time dependence of various quantities of interest in THESEUS



Estimation of the electrical conductivity, σ , from the electron 'heating' rate

Fig. 9

THESEUS: plots of $\Delta v_e^2/v_d^2$ vs time: for the pulse voltages. Lines A and B correspond to theoretical heating rates given by

$$\frac{d}{dt}(v_e^2) = 2\nu v_d^2 \quad \text{where}$$

$$\nu = \frac{1}{4\pi} \left(\frac{m}{M}\right)^{\frac{1}{3}} \omega_{pe} \quad \text{for A}$$

$$\nu = \frac{1}{8\pi} \left(\frac{m}{M}\right)^{\frac{1}{3}} \omega_{pe} \quad \text{for B}$$

Let us now summarize the main conclusions from the two laboratory experiments, in both of which $E > E_0 = \frac{m v_e \omega_{pi}}{e}$:

- (1) In both cases we have an approximately constant and well defined conductivity, which for large drift velocities has values consistent with the empirical scaling law

$$\sigma \approx 0.5 \left(\frac{M}{m}\right)^{\frac{1}{3}} \omega_{pe}$$

in complete contrast to the prediction that

$$\sigma \propto t.$$

- (2) In both cases a necessary condition for this behaviour is that the drift velocity v_d exceeds at some time the initial thermal velocity. Once that has happened, however, the thermal velocity soon afterwards exceeds the drift speed, i.e. the condition $v_e > v_d$ holds for most of the

We can, of course, derive an effective collision frequency or conductivity from the rate of apparent electron heating. Fig. 9 shows the increase in mean square velocity normalized to the drift velocity squared as a function of time for several cases. The dotted lines A and B represent the theoretical rate of heating corresponding to the constant effective collision frequencies shown. The appropriate values of electrical conductivity to fit data such as that in Fig. 9 are also shown in Fig. 7, where it can be seen that they agree well with the directly measured values. This agreement can be regarded as experimental certification of the assumption that the energy is conserved.

the subsequent time, while the turbulent condition is still maintained. This is in contrast to the prediction that $v_d \approx v_e$.

- (3) In each experiment there appears to be sufficiently good evidence of energy conservation (i.e. from agreement between the rate of electron heating and the electrically measured ohmic dissipation) that the original assumptions should apply.

The reason for such a large discrepancy between experiment and computer simulation is still not known. We have conducted a number of simulations in which various effects have been introduced in an attempt to make the model more realistic and in the hope of finding better agreement, e.g. in producing true resistive behaviour. For example, the effects of skin current distributions; a high initial fluctuation level (e.g. for an earlier stage of ion-sound turbulence); initial density modulation, external circuit effects, etc.

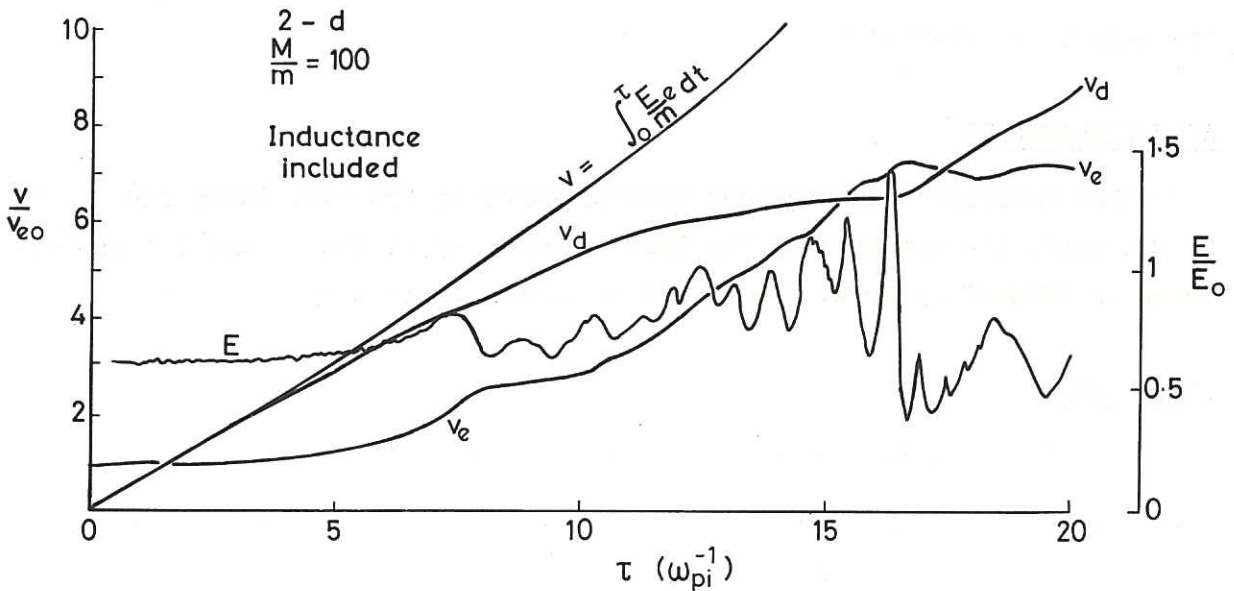


Fig. 10

Two-dimensional computer simulation, as Fig. 2 but with the effect of an external inductance included to reduce the free-streaming velocity. E is the mean instantaneous electric field in the plasma (now no longer constant)

An example of a 2-d calculation in which the effect of the circuit inductance has been simulated (for the toroidal case) is shown in Fig. 10. Despite the expenditure of much computer time, it appears that once again we have v_d and v_e keeping roughly in step, both increasing with time.

Now, why does this sort of discrepancy occur? One clue comes from various other observations made on TWIST in which we have tried to get some

idea of the fluctuation spectrum. The electrostatic probes showed that the initial burst of frequencies at around the Buneman frequency was followed by a surprisingly wide frequency spread. When we tried to measure the density fluctuation spectra using microwave scattering we found very large fluctuations with very wide frequency and wavelength spreads which are difficult to reconcile with any form of electrostatic modes. In fact, there appeared to be pronounced irregularities in the direction perpendicular to the electron drift with frequency spectra up to the electron frequencies. At the same time, we find very wide electromagnetic emission spectra which again are hard to explain as arising from electrostatic plasma fluctuations. We have thus been led to speculate whether there is in fact some sort of electromagnetic instability occurring, which of course cannot appear in the computer simulations because they do not include the magnetic field. Such an instability may take the form of a break-up of the current channel into unstable filaments. If this is indeed the case, then perhaps this will be seen in the Q-machine experiment.

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REFERENCES

- ¹ STRINGER, T.E., Plasma Physics, 6, 267 (1964).
- ² e.g. BORIS, J.P., DAWSON, J.M. and ROBERTS, K.V., Phys. Rev. Letts., 25, 706 (1970).
MORSE, R.L. and NIELSON, C.W., Phys. Rev. Letts., 26, 3 (1971).
- ³ HAMBERGER, S.M., JANCARIK, J., SHARP, L.E. and ALDCROFT, D.A. In 'Plasma Physics and Controlled Nuclear Fusion Research' (Madison, 1971) vol.II.
- ⁴ CLARK, W. and HAMBERGER, S.M. Second International Congress on Waves and Instabilities in Plasma, Innsbruck 1975.
- ⁵ HAMBERGER, S.M. and JANCARIK, J., Phys. Fluids, 15, 825 (1972).
- ⁶ HAMBERGER, S.M. and FRIEDMAN, M., Phys. Rev. Letts., 21, 674 (1968).

