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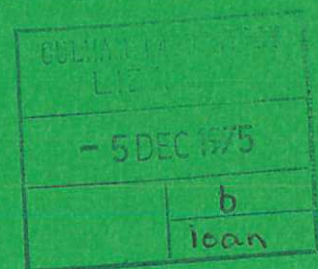


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# EMPIRICAL TOKAMAK SCALING

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## EMPIRICAL TOKAMAK SCALING

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### A B S T R A C T

Data from various experiments are fitted to three types of empirical scaling laws, for which  $\tau_E$  varies as  $a^2$ ,  $a$  or  $a^0$ . The results are used to predict the performance of future large tokamaks.

(Paper for presentation at the Seventh European Conference on Controlled Fusion and Plasma Physics, Lausanne, 1-5 Sept. 1975)

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Symbols  $R$  = major radius (m),  $a$  = limiter radius (m),  $I$  = plasma current (A),  $q_a, q_o$  = safety factors at limiter and magnetic axis,  $n_e, n_i$  = mean electron and ion densities ( $m^{-3}$ ),  $T_e, T_i$  = mean electron and ion temperatures (eV),  $V_R$  = resistive loop voltage (V),  $\beta_\theta$  = poloidal beta,  $\tau_E$  = plasma energy/input power (s),  $B_\phi$  = toroidal field (T),  $Z_{eff}$  = effective ion charge.

Introduction This paper attempts to predict the performance of future large tokamaks by using empirical expressions derived from the data of present experiments (Table I). Attention is restricted to hydrogen discharges, for which there was no large anomalous resistance due to turbulence. The implicit assumption is that the physics of the discharge will be unchanged in larger devices; a big assumption. However, no specific physical models are assumed, a priori.

The two basic parameters of interest are the plasma pressure which can be sustained by ohmic heating alone, and  $\tau_E$ . The latter allows an estimate of the additional heating required to reach a specified condition.

Effect of Varying  $q_a$   $q_a$  is an independent variable, but  $q_o$  tends to be near unity. The ratio determines the basic geometry of the discharge. It has been found on T3 and T4(1), and, more recently on TFR(16), that  $\tau_E$  increases as  $q_a$  increases up to

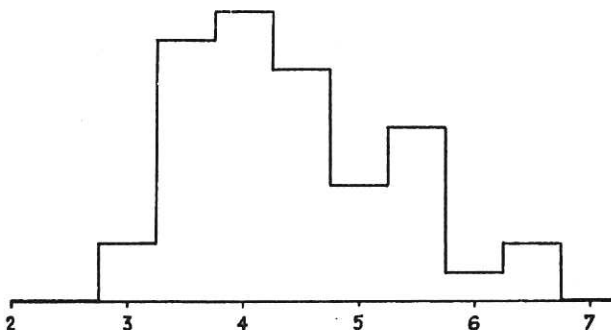


Fig. 1. Histogram of  $q_a$ .

$\sim 6$ , then remains constant. Conversely, when  $q_a \leq 2$ ,  $\tau_E$  decreases sharply and large disruptive instabilities are seen. For 90% of the data analysed here  $3 < q_a < 5.5$ , as shown in Fig. 1, a value which gives optimum plasma parameters for a given  $B_\phi$ . In this sense,  $q_a$  is not a proper independent parameter.

Effect of Varying  $n$  Many experiments of moderate size now show that  $\tau_E, \beta_\theta$  vary as  $n_e^\alpha$ , where  $\alpha \sim 0.4$  in the early Russian experiments (1), to  $\sim 0.8$  in ST (5), and 1.0 in Ormak (12) and Alcator (18). This implies the amount of plasma in the discharge does not have a big effect on the energy loss rate, or on  $T_e$ . It confirms the idea that particle balance and energy balance are controlled by nearly independent processes. In some experiments (14,16)  $n_e$  increases linearly with  $I$ , possibly due to plasma-wall interaction.

Scaling with  $I$  and  $a$  It is difficult to disentangle the effects of increasing  $I$  and  $a$  independently, since larger experiments tend to run higher current discharges. In any case, the range of  $a$  is limited to a factor  $\sim 3$ . Therefore, three possible types of scaling are tested, for which  $\tau_E$  varies approximately as  $a^2(A)$ ,  $a(B)$  or  $a^0(C)$ . We always have  $\tau_E = 3 \mu_o IR \beta_\theta / 8V_R$ , so a knowledge of  $V_R$ , or the plasma conductivity, is sufficient to relate  $\tau_E$  and  $\beta_\theta$ . It is remarkable that 95% of the data points considered have  $2 < V_R < 4$ , but it is not possible to distinguish the constancy of  $V_R$  from that of  $V_R/R$ .

Type A was first suggested by workers on TM3 and T3 in the form

$$\tau_E \sim a^2 n_e^{0.3} B_\theta \sim a n_e^{0.3} I.$$

A test of this scaling is shown in Fig. 2. It does not look very convincing, and the corresponding scaling for  $\beta_\theta \sim n_e^{0.4} I^{-0.3}$  predicts a plasma resistivity lower than classical for large devices. The mean value of  $\tau_E/aI$  is  $\sim 5 \times 10^{-7} (sm^{-1} A^{-1})$ .

Type B is illustrated in Fig. 3. 85% of the data is within a factor of 2 of the expression  $\tau_E = 8 \times 10^{-25} n_e I^2$ , though a higher exponent for  $I$ , up to about 1, would fit the data equally well.

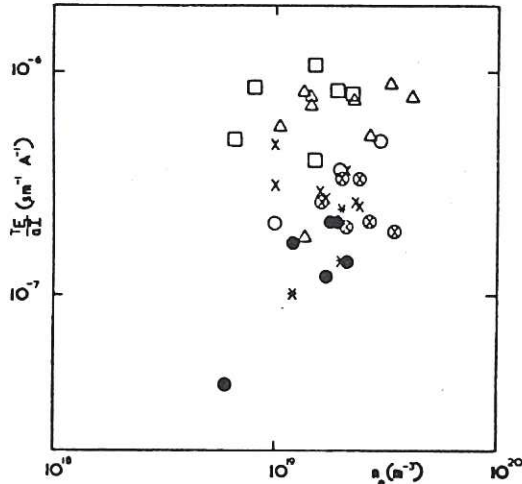


Fig. 2. Type A scaling.

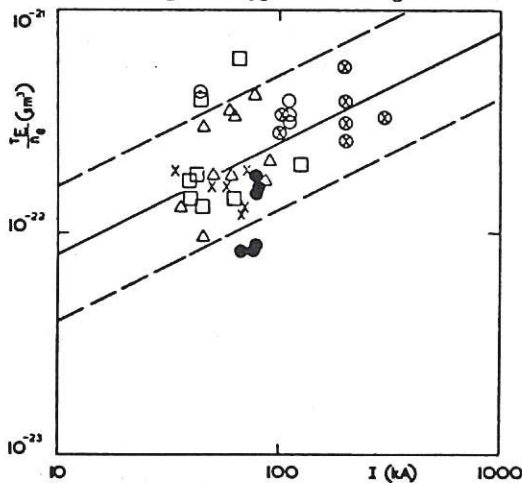


Fig. 3. Type B scaling.

Type C is illustrated in Fig. 4. The data fits the expression  $T = T_e + (n_i/n_e) T_i = 1700 [\text{mean ohmic power density (MW/m}^2)]^{1/2}$  which leads to

$$\tau_E = 10^{-23} n_e T^{1/2}$$

This expression is similar to that suggested by recent results in Alcator (18), and also agrees with the scaling found on ATC (11), in the form  $\beta_\theta/n_e Z_{\text{eff}}^{1/2} = 5.8 \times 10^{-18}/I$ , if  $Z_{\text{eff}} = (T/T_\sigma)^{3/2}$ .

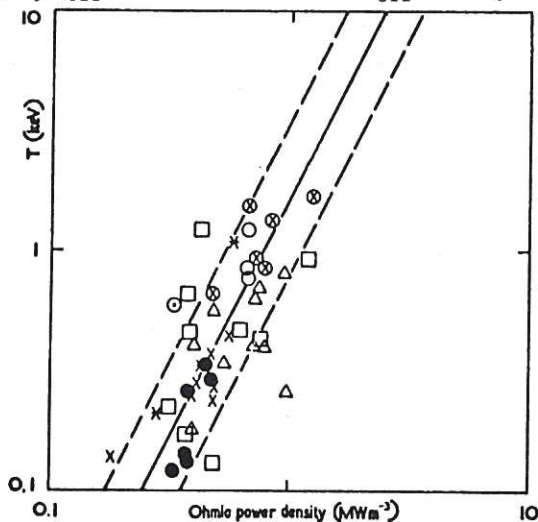


Fig. 4. Type C scaling.

Table II shows the results of applying the three types of scaling to PLT (Princeton Large Torus) and JET (Joint European Torus), assuming  $Z_{\text{eff}} = 4$ .

**Conclusions** There is presently insufficient data to choose between scalings for type B or C, but type A looks improbable. Type C scaling favours the high field, high current density type of experiment.

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The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry, no matter how small, should be recorded to ensure the integrity of the financial statements. This includes not only sales and purchases but also expenses and income. The document also highlights the need for regular reconciliation of bank statements and the company's records to identify any discrepancies early on.

In addition, the document provides a detailed breakdown of the accounting cycle, from identifying the accounting entity to preparing financial statements. It explains how each step contributes to the overall accuracy and reliability of the financial data. The document also includes a section on the importance of internal controls, which are designed to prevent errors and fraud within the organization.

The second part of the document focuses on the practical application of these principles. It provides a series of examples and exercises that illustrate how to record and classify transactions in the general ledger. These examples cover a wide range of business activities, from the purchase of inventory to the payment of salaries. The document also includes a section on the preparation of trial balances, which are used to verify the accuracy of the ledger accounts.

Finally, the document concludes with a summary of the key points discussed throughout the text. It reiterates the importance of accuracy, consistency, and transparency in financial reporting. The document also provides a list of references and resources for further study on accounting principles and practices.

The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry should be supported by a valid receipt or invoice. This ensures transparency and allows for easy verification of the data.

In addition, the document highlights the need for regular audits. By conducting periodic reviews, any discrepancies can be identified and corrected promptly. This proactive approach helps in maintaining the integrity of the financial data.

Furthermore, it is advised to use standardized accounting practices. This includes following established guidelines for recording income, expenses, and assets. Consistency in reporting is crucial for providing a clear and reliable picture of the organization's financial health.

**Financial Reporting and Analysis**

The second section focuses on the process of generating financial reports. It outlines the steps involved in compiling data from various sources and presenting it in a clear, concise format. Key reports mentioned include the Income Statement, Balance Sheet, and Cash Flow Statement.

Each report provides a different perspective on the organization's performance. The Income Statement shows the profit or loss over a specific period, while the Balance Sheet provides a snapshot of the organization's financial position at a given time. The Cash Flow Statement tracks the inflows and outflows of cash, which is essential for understanding the organization's liquidity.

Beyond just reporting, the document also discusses the importance of analyzing these reports. Management should look for trends, identify areas of concern, and make informed decisions based on the data. Regular analysis helps in forecasting future performance and adjusting strategies accordingly.

**Conclusion and Recommendations**

In conclusion, effective financial management is the backbone of any successful organization. By implementing robust record-keeping, regular audits, and thorough financial reporting, organizations can ensure their financial health and long-term sustainability.

The recommendations provided in this document are intended to guide organizations in adopting best practices. It is crucial to tailor these suggestions to the specific needs and scale of the organization. Regular communication and collaboration between departments are also key to successful financial management.