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ABSTRACT

The sufficient stability condition which follows directly from the energy principle is divided into the parts which depend respectively on $j_{\parallel}$ and $j_{\perp}$, which are the components of the equilibrium current parallel to, and perpendicular to, the magnetic field. The sufficient condition for $j_{\perp}$ not to cause instability is found to depend on the average normal curvature of the magnetic surfaces in the direction of $\underset{\sim}{B}$. The corresponding condition for $j_{\parallel}$ depends on the average geodesic torsion of the magnetic surfaces in the direction of $\underset{\sim}{B}$. For currents below approximately the Kruskal limit in a circular torus, the average normal curvature becomes negative and the $j_{\perp}$-modes are stable for negative pressure gradients. This confirms Mercier's earlier results for localised perturbations. On the other hand, to zero and first order in the aspect ratio of the torus (r/R_0) the average geodesic torsion is the same as for a straight cylinder so that there is no toroidal stabilising effect for $j_{\parallel}$-driven modes. Completely stable toroidal equilibria are possible which do not rely on magnetic shear. The relationship of these stability conditions with the $\int(dl/B)$ criterion is discussed.

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CONTENTS

I INTRODUCTION

It is well known that when a plasma is contained by a curved magnetic field such that ∇p has a component pointing in the direction of the centre of curvature, the plasma is prone to pressure driven instabilities. It is an inescapable fact that, if a confining magnetic field is to have the opposite (i.e. stable) curvature at all points, the lines of force cannot be closed within the system but must pass out through the wall of the containing vessel[1]. Loss of plasma along the lines of force can be minimised by the use of hydrid field geometries of the type first introduced by Ioffe et al[2] and considered theoretically by Taylor[3] and others, but particle collisions will cause these losses to be high in a final thermonuclear reactor. Because of this and the apparent ineffectiveness of magnetic shear to prevent pressure driven instabilities, renewed interest has been shown[4] in designing magnetic fields with closed lines of force whose curvature, although sometimes destabilising, has a net stabilising effect when averaged over the length of the lines.

In his study of toroidal stability, Mercier[5] has already shown that in certain circumstances toroidal magnetic fields can have a net stabilising curvature for certain modes of instability. Mercier has shown that for plasma perturbations which are localised in the vicinity of the magnetic surface for which $\underset{\sim}{k} \cdot \underset{\sim}{B} = 0$ (i.e. Suydam-type perturbations), there will be stability near the magnetic axis for certain ranges of the rotational transform depending on the curvature and torsion of the magnetic axis. ($\underset{\sim}{k}$ is the wave vector of the instability perturbation. In the particular case of a circular torus whose magnetic axis has radius R_0, the stability condition for such modes at a particular magnetic surface is given in the simple form[6]

$$\frac{dp}{dr}\left(\frac{B_{\theta 0}^2}{r^2} - \frac{B_{\varphi 0}^2}{R_0^2}\right) > 0 \qquad \ldots (1)$$

where the toroidal co-ordinates used (r,θ,φ) are related to cylindrical co-ordinates R, φ, z, relative to the symmetry axis (Oz) by $R = R_0 + r\cos\theta$,

$z = r \sin\theta$ and where $B_{\theta o}$ and $B_{\varphi o}$ are the field components on the magnetic surface at $\theta = \pm\frac{\pi}{2}$. Thus for φ-currents below the Kruskal limit where $B_{\theta o}/r = B_{\varphi o}/R_o$, negative pressure gradients are stable against such modes.

Here it is shown that because of the type of mode considered, Mercier's stability condition applies only to instabilities driven by ∇p or more accurately by $j_\perp$; it leaves undecided the question of instabilities due to $j_\parallel$. (The subscripts n, $\perp$, $\parallel$ are used to denote vector components in the directions of $\underset{\sim}{n}$, which is the unit vector normal to the magnetic surface in the outward direction, $\underset{\sim}{B} \wedge \underset{\sim}{n}$, and $\underset{\sim}{B}$ respectively.) General stability conditions are derived for the $j_\perp$ and $j_\parallel$ driven instabilities. The former confirms Mercier's result that the net effect of the φ curvature can be stabilising for $j_\perp$ driven modes, but the latter shows that the φ curvature has no effect to first order for $j_\parallel$ driven modes.

II STABILITY OF A STRAIGHT CYLINDRICAL PLASMA

Considering first the simple case of a straight cylindrical plasma (coordinates r, θ, z), the well known sufficient condition for stability for a perfectly conducting plasma $j_z B_\theta/r \leqslant 0$, which follows directly from the energy principle, can be rewritten in the form

$$\frac{B_\theta^2}{rB^2}\frac{dp}{dr} - \frac{j_\parallel B_z B_\theta}{rB} \geqslant 0 \qquad \ldots (2)$$

Since $\frac{dp}{dr}$ and $j_\parallel$ are independent of each other, sufficient conditions for stability against modes driven by $\frac{dp}{dr}$ and $j_\parallel$ are respectively [8]

$$\frac{dp}{dr} \geqslant 0 \qquad \ldots (3)$$

and

$$j_\parallel B_\theta B_z/B \leqslant 0 \qquad \ldots (4)$$

When the magnetic field has no shear, condition (3) is also a necessary stability condition for pressure driven modes, since, after minimisation, all the other terms in the energy principle, including the $j_\parallel$ term reduce to expressions containing the factor $(\underset{\sim}{k} \cdot \underset{\sim}{B})$. Hence as $\underset{\sim}{k} \cdot \underset{\sim}{B} \rightarrow 0$, which is possible everywhere with no shear, only the $\frac{dp}{dr}$ term remains. When there is shear, the necessary stability condition for pressure driven modes is Suydam's well known condition[9]. It is pointed out here that Suydam's condition is also a sufficient condition for pressure driven modes, since if it is satisfied, only the $j_\parallel$ terms can make the energy integral negative.

The fact that the $j_\parallel$ term in (2) cancels with the stabilising terms as $\underset{\sim}{k} \cdot \underset{\sim}{B} \rightarrow 0$, shows that there is no net destabilising effect of $j_\parallel$ unless $\underset{\sim}{k} \cdot \underset{\sim}{B} \neq 0$. Thus it can already be seen that Mercier's restriction to modes such that $\underset{\sim}{k} \cdot \underset{\sim}{B} \simeq 0$ leads to the omission of the $j_\parallel$ destabilising terms. (The integral of the stabilising terms depends on the radial gradient of $\underset{\sim}{k} \cdot \underset{\sim}{B}$ and this in turn depends primarily on $j_\parallel$ and only weakly on $j_\perp$, if $\underset{\sim}{k} \cdot \underset{\sim}{B}$ is small. The necessary stability condition for $j_\parallel$ driven modes is obtained only by solving the Euler-Lagrange equation for the radial component of the plasma displacement[10].)

III TOROIDAL STABILITY

Turning to the case of a general toroidal plasma, the corresponding sufficient stability condition, as pointed out by Liley[11], is

$$\int \xi_n^2 \, \underset{\sim}{j} \wedge \underset{\sim}{n} \cdot (\underset{\sim}{B} \cdot \nabla \underset{\sim}{n}) \, d\tau \leqslant 0 \qquad \ldots (5)$$

where $d\tau$ is the element of volume. Resolving $\underset{\sim}{j}$ into its components as before, the sufficient conditions for, $j_\perp$ and $j_\parallel$ not to cause instability are

$$\int \frac{\xi_n^2 \, \nabla p}{\rho_\parallel} \, d\tau \geqslant 0 \qquad \ldots (6)$$

$$\int \frac{\xi_n^2 \, j_\parallel \, B}{\rho_T} \, d\tau \leqslant 0 \qquad \ldots (7)$$

$1/\rho_\parallel$ is the normal curvature of the magnetic surface in the direction of $\underset{\sim}{B}$, (which is the same as the curvature of $\underset{\sim}{B}$ in the plane of $\underset{\sim}{B}$ and $\underset{\sim}{n}$) and $1/\rho_T$

is the geodesic torsion of the magnetic surface in the direction of $\underset{\sim}{B}$.
[$\frac{1}{\rho_{\parallel}} \equiv \underset{\sim}{i}_{\parallel} \cdot (\underset{\sim}{i}_{\parallel} \cdot \nabla \underset{\sim}{n})$ and $\frac{1}{\rho_T} \equiv \underset{\sim}{i}_{\perp} \cdot (\underset{\sim}{i}_{\parallel} \cdot \nabla \underset{\sim}{n})$ where the $\underset{\sim}{i}$'s, like $\underset{\sim}{n}$, are unit vectors.] Since in toroidal equilibria, a ∇p requires some $j_{\parallel}$ as well as $j_{\perp}$, equation (3) is only the condition that ∇p should not cause an instability through $j_{\perp}$ as opposed to through $j_{\parallel}$.

Mercier's treatment of modes for which $\underset{\sim}{k} \cdot \underset{\sim}{B} \to 0$ shows that (6) is a necessary as well as sufficient condition when there is no shear, as in the straight cylinder case. Condition (7) is not a necessary one since, as for a straight cylinder, part of the expression in (7) will be cancelled by one of the stabilising terms in the energy principle.

IV APPLICATION TO SIMPLE CIRCULAR TORUS

To illustrate these general results we consider the case of a circular torus with $R/r \gg 1$ and make use of Shafranov's result[12] that to first order in r/R the magnetic surfaces have circular cross sections in the $r\theta$ planes with centres displaced from the magnetic axis. For each magnetic surface its own co-ordinate system is chosen so that $r = 0$ at the centre of the surface's circular cross section. In this case the appropriate vectors on the magnetic surface are $\underset{\sim}{B} = (0, B_\theta, B_\varphi)$; $\underset{\sim}{n} = (1, 0, 0)$ whence

$$\frac{1}{\rho_{\parallel}} \simeq \frac{B_\theta^2}{rB^2} + \frac{B_\varphi^2 \cos\theta}{RB^2}$$

and

$$\frac{1}{\rho_T} \simeq \frac{B_\theta B_\varphi}{rB^2} - \frac{B_\theta B_\varphi \cos\theta}{RB^2}$$

$$d\tau = rR\, d\theta\, d\varphi \left(\frac{dr}{d\psi}\right) d\psi$$

where ψ is the integral of the flux within the major circle of radius R passing through the point. Considering firstly condition (6), after integrating with respect to φ, this becomes

$$\iint \xi_n^2 \frac{dp}{d\psi}\left(\frac{B_\theta^2}{rB^2} + \frac{B_\varphi^2 \cos\theta}{RB^2}\right) rRd\theta\, d\psi \geqslant 0 \qquad \ldots (8)$$

where ξ_n is now the amplitude of a normal mode displacement which is periodic in φ.

On a magnetic surface $B_\varphi R$ is a constant, so that to a first approximation $B_\varphi = B_{\varphi o}(1 - \frac{r}{R_o}\cos\theta)$. Similarly, from Shafranov[12], to a first approximation $B_\theta = B_{\theta o}(1 + \Lambda \frac{r}{R_o}\cos\theta)$, where Λ is an integral function of $B_{\theta o}$ and p_o within the magnetic surface. (The subscript o denotes the value at $\theta = \pm\frac{\pi}{2}$ which is the average value.) For low $8\pi p/B^2$ the displacement ξ_n must be proportional to $\frac{dr}{d\psi}$, namely $(1/2\pi R B_\theta)$, since this leads to minimum bending of the lines of force. This assumption is a vital one, since the stabilising effect of the φ curvature for $j_\perp$ driven modes depends directly on the θ variation of ξ_n. If ξ_n were independent of θ the net stabilisation effect would be zero to first order. [However, in that case the extra stabilising term due to the line bending would be of the order

$$\xi_{no}^2 \left(\frac{d\psi}{dr}\right)_o^2 \frac{B_\theta^2}{4\pi r^2}\left[\frac{d}{d\theta}\left(\frac{dr}{d\psi}\right)\right]^2 = \frac{\xi_{no}^2 B_\theta^2 \sin^2\theta}{4\pi R^2}(\Lambda + 1)^2$$

which will be large compared with the terms in (8) unless $\frac{dp}{dr}$ is of the same order as $rB^2/8\pi R^2$. Hence the condition for ξ_n to be proportional to $dr/d\psi$ is

$$\frac{B_\theta^2}{B^2}\left|\frac{dp}{dr}\right| \ll \frac{B_\theta^2 r}{8\pi R^2} \quad \text{or} \quad \left|\frac{dp}{dr}\right| \ll \frac{B^2 r}{4\pi R^2} \qquad \ldots (9)]$$

Finally, assuming B_θ/B_φ to be of the same order as r/R, the integration with respect to θ yields to a first approximation

$$\int \xi_{no}^2 \frac{dp}{d\psi}\left[\frac{B_{\theta o}^2}{B_o^2} - (1 + \Lambda)\frac{r^2}{R_o^2}\frac{B_{\varphi o}^2}{B_o^2}\right] d\psi \geqslant 0 \qquad \ldots (10)$$

and hence a sufficient stability condition for $j_\perp$ driven modes is

$$\frac{dp}{d\psi}\left[\frac{B_{\theta o}^2}{B_o^2} - (1 + \Lambda)\frac{r^2}{R_o^2}\frac{B_{\varphi o}^2}{B_o^2}\right] \geqslant 0 \ . \qquad \ldots (11)$$

Thus, provided the factor $(1 + \Lambda)$ is positive, the net effect of the φ

curvature is stabilising for $j_\perp$ driven instabilities. For a pinch system with no hardcore $(1+\Lambda)$ is always positive[12] and example values are (a) for $B_{\theta o}$ proportional to r and $B_{\varphi o}$ constant, $(\Lambda + 1) = 1.25$ and (b) for $B_{\theta o}$ proportional to r and $B_{\varphi o}$ decreasing with r so that dp/dr is small compared with $B_{\theta o}^2/8\pi r$, $(\Lambda + 1) = \frac{1}{4}$. For a Levitron $(\Lambda + 1)$ can have positive or negative values. (It is not clear why Mercier's formula (1) does not contain Λ. His general stability condition[5] contains a term identical with (5), but the detailed derivation of (1) has not been published.)

Turning to the $j_{\parallel}$ driven modes for a circular torus, condition (7) becomes

$$\iint \xi_n^2 \, j_{\parallel} \frac{B_\varphi B_\theta}{B} \left[1 - \frac{r \cos \theta}{R} \right] \frac{R \, d\theta \, d\psi}{\left(\frac{d\psi}{dr}\right)} \leqslant 0 \qquad \ldots (12)$$

and hence to zero and first order in r/R the θ integral gives

$$\int \xi_{no}^2 \, j_{\parallel o} \frac{B_{\varphi o} B_{\theta o}}{B_o \left(\frac{d\psi}{dr}\right)_o} \, d\psi \leqslant 0 \qquad \ldots (13)$$

which is the same condition as for a straight cylindrical plasma. This result is due to the fact that the contribution to the geodesic torsion from the φ-curvature is $(r \cos \theta)/R$ times the θ-curvature contribution. That is, it is of one higher order in r/R_o and secondly, even to this order its integral is zero.

In addition the stabilising terms in the energy principle will be unaltered to first order. Hence, for a toroidal plasma in which $j_\perp \ll j_\parallel$, so that the dp/dr terms can be neglected, the stability conditions will be the same as for the corresponding magnetic field configuration in a straight cylinder. In fact, in experiments on the B-1 Stellerator, good agreement was obtained[13] between the experimentally observed current ranges for $m = 2,3,4$ and 5 instability modes and the ranges predicted by theory[14] for $j_\parallel$ driven $(p = 0)$ modes in a straight tube. Since these results are for currents well below the Kruskal limit, the good agreement confirms that the toroidal (φ) curvature has little effect on $j_\parallel$ driven modes.

Recombining the conditions (10) and (13) yields the sufficient condition for stability against all modes

$$\left(\frac{dp}{dr}\right)_o \left[\frac{B_{\theta o}^2}{B_o^2} - (1 + \Lambda)\frac{r^2\, B_{\varphi o}^2}{R_o^2\, B_o^2}\right] - \frac{j_{\parallel o}\, B_{\varphi o}\, B_{\theta o}}{B_o} \geqslant 0 \qquad \ldots (14)$$

V RELATIONSHIP WITH THE $\int(dl/B)$ CRITERION

In many theoretical treatments of systems with variable magnetic curvature, it has been the practice to assume both $\beta(\equiv 8\pi p/B^2)$ and $j_\parallel$ are infinitesimally small in which case the only instability modes which are energetically possible are the pure interchanges which satisfy $\delta B = 0$. The stability of the plasma against such modes is related in a simple manner to the variation with position of the volume occupied by a tube of force with given flux. The sufficient stability condition obtained is [17]

$$\frac{dp}{d\psi}\,\frac{d}{d\psi}\left(\oint \frac{dl}{B}\right) \geqslant 0 \qquad \ldots (15)$$

where the integration is along a line of force.

It can be shown that this is a special case of the general condition derived above for $j_\perp$-driven modes, namely equation (6). Thus applying the integral in (6) to the volume contained by a tube of force ($d\tau \sim dl/B$), it follows from (6) that there will be stability against $j_\perp$-driven modes if for each line of force

$$\oint \frac{\xi_n^2\, \nabla p}{\rho_\parallel}\left(\frac{dl}{B}\right) \geqslant 0 \qquad \ldots (16)$$

For low β, ξ_n must be proportional to $1/\nabla\psi$ and in addition, from the equilibrium pressure balance,

$$\underset{\sim}{n} \cdot \nabla\left(p + \frac{B^2}{8\pi}\right) = \underset{\sim}{n} \cdot \left(\frac{\underset{\sim}{B} \cdot \nabla \underset{\sim}{B}}{4\pi}\right) = \frac{B^2}{4\pi}\,\underset{\sim}{n} \cdot (\underset{\sim}{i}_\parallel \cdot \nabla \underset{\sim}{i}_\parallel)$$

$$= -\frac{B^2}{4\pi}\,\underset{\sim}{i}_\parallel \cdot (\underset{\sim}{i}_\parallel \cdot \nabla \underset{\sim}{n}) = -\frac{B^2}{4\pi\rho_\parallel} \qquad \ldots (17)$$

so that provided

$$|\nabla p| \ll |B^2/4\pi\rho_{\parallel}| \qquad \ldots (18)$$

$$\frac{1}{\rho_{\parallel}} = -\frac{1}{2B^2}\frac{dB^2}{d\psi}\nabla\psi$$

and condition (16) becomes

$$-\frac{dp}{d\psi}\oint\frac{1}{B^4}\frac{dB^2}{d\psi}(Bdl) \geqslant 0$$

which is the same as (15) provided

$$4\pi|j| \ll |\nabla B| \;, \qquad \ldots (19)$$

since elements of magnetic potential Bdl can then be chosen which are independent of ψ and the number of such elements is also independent of ψ.

Thus β and j must be small enough to satisfy conditions (18) and (19) for the $\int(dl/B)$ criterion to be valid. In addition the assumption $\xi_n \sim 1/\nabla\psi$ requires

$$|\nabla p/\rho_{\parallel}| \ll B^2(\nabla\psi)_0^2\left[\frac{d}{dl}\left(\frac{1}{\nabla\psi}\right)\right]^2 \qquad \ldots (20)$$

On the other hand, the stability conditions (5), (6) and (7) are valid for all β and j. However, the application to a circular torus in section IV makes the same assumption about ξ_n and therefore requires conditions (20), which takes the form given in (9).

A further point which should be noted is that when $\beta \neq 0$, the most unstable modes are often not the pure interchanges ($\underset{\sim}{k} \cdot \underset{\sim}{B} = 0$) but rather the quasi-interchanges[18] for which $\underset{\sim}{k} \cdot \underset{\sim}{B}$ is small but not zero. In such modes, a tube of force moves outwards for parts of length and inwards in between. The net change in volume of a tube is zero, and, at marginal stability, $\nabla \cdot \underset{\sim}{\xi} = 0$ and the dominant component of $\underset{\sim}{\xi}$ is $\xi_{\parallel}$. It is these modes which cause the necessary stability condition to be $dp/d\psi \geqslant 0$ when the mean normal curvature is positive and when there is no shear, whereas the corresponding stability condition for a pure interchange mode in such a case is

$$\frac{1}{p}\frac{dp}{d\psi} \geqslant -\frac{\Upsilon}{V}\frac{dV}{d\psi}$$

where $V = \int (dl/B)$.

As $\beta \to 0$, for the quasi-interchanges to remain unstable, $\underset{\sim}{k} \cdot \underset{\sim}{B}$ must tend to zero so as to keep the stability terms due to line bending less than the destabilising term. For low β this requires a long wavelength along $\underset{\sim}{B}$. In a finite straight system the boundary conditions at the ends will set an upper limit to this wavelength, so that there will be a critical β below which the quasi-interchanges are no longer more unstable than the pure interchanges.

For toroidal systems the boundary condition is $2\pi/k_\varphi = L/n$ where L is the torus circumference and n is an integer. If there is magnetic shear, magnetic surfaces can always be found so that for a suitable choice of m and n the value of $\underset{\sim}{k} \cdot \underset{\sim}{B}$ is zero[5], (m is the wave number in the θ direction). On either side of these surfaces the required regions of arbitrarily small $\underset{\sim}{k} \cdot \underset{\sim}{B}$ will exist. (The quasi-interchange is a Suydam type instability in this case.) If, however, there is no magnetic shear the minimum value of $\underset{\sim}{k} \cdot \underset{\sim}{B}$ depends on the particular value of the rotational transform.

VI CONCLUSIONS

The results presented here confirm Mercier's result that the extra curvature, which a torus has over a straight cylinder, can have a net stabilising effect for $j_\perp$ driven modes. For a circular torus this effect will dominate the destabilising effect of the θ-curvature for currents below $(1 + \Lambda)^{1/2}$ times the Kruskal limit current (I_K).

Whereas the $j_\perp$-driven modes depend on the average value of $1/\rho_{\parallel}$, the normal curvature of the magnetic surface parallel to $\underset{\sim}{B}$, the $j_{\parallel}$-driven modes depend on $1/\rho_T$ the geodesic torsion of the magnetic surface parallel to $\underset{\sim}{B}$. In a circular torus, the φ-curvature makes a large contribution to $1/\rho_{\parallel}$ and can change the sign of the average value of $1/\rho_{\parallel}$ for φ-currents below $I_K(1 + \Lambda)^{1/2}$, but the contribution to $1/\rho_T$ is always small compared with the θ-curvature contribution. Hence, for small plasma pressure, the stability condition for $j_{\parallel}$-driven modes will be the same as for a straight tube. When dp/dr is finite and the current is less than $I_k(1 + \Lambda)^{\frac{1}{2}}$ the plasma will be more stable than the straight tube, because the pressure gradient term is then stabilising, but dp/dr must not be made too large (condition (9)), otherwise the instability will tend to become localised in those regions where the net curvature is destabilising for $j_{\perp}$-modes.

The general stability conditions (5),(6) and (7) are valid for all values of β and $j_{\parallel}$, but the application to a circular torus requires $\left|\frac{dp}{dr}\right| \ll rB^2/4\pi R^2$. It has been shown that the $\int(dl/B)$ criterion is a special case of the stability condition for $j_{\perp}$-driven modes which holds when conditions (18), (19) and (20) are satisfied.

A toroidal plasma will be stable without recourse to the effects of magnetic shear if it satisfies the three conditions (9),(11) and (14). For a toroidal pinch discharge, condition (14) cannot be satisfied for zero and small r, since there, $j_{\parallel o}$ is of necessity non-zero and in the destabilising direction, whereas dp/dr is either zero or small. Over the outer regions however it should be possible to satisfy all three conditions and since the volume involved can be greater, overall stability should be possible.

For a Levitron plasma,(i.e. a toroidal plasma containing a hardcore ring) the normal direction for $j_{\parallel o}$ is in the stabilising direction and hence (14) should be easily satisfied. For small r where dp/dr is positive,

$$\left[\frac{B_{\theta o}^2}{r^2} - (1 + \Lambda)\frac{B_{\varphi o}^2}{R_o^2}\right]$$

must be positive, which can easily be satisfied, since not only is B_θ^2 usually larger than B_φ^2 in this region but $(1 + \Lambda)$ will be small and may be negative. Where dp/dr is negative,

$$\left[\frac{B_{\theta o}^2}{r^2} - (1 + \Lambda)\frac{B_{\varphi o}^2}{R_o^2}\right]$$

must be negative. This requires a positive $(1 + \Lambda)$ and this can be achieved by making the major radius of the hardcore larger than the mean major radius of the containing torus.

(It should be noted that the above results are based on the assumption of perfect conductivity.)

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