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ASPECTS OF THE EQUILIBRIUM AND STABILITY OF COUNTERSTREAMING ION TOKAMAKS

J G CORDEY
F A HAAS

CULHAM LABORATORY
Abingdon Oxfordshire

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J G Cordey and F A Haas

Culham Laboratory, Abingdon, Oxon. OX14 3DB, UK
(Euratom/UKAEA Fusion Association)

ABSTRACT

An anisotropic high-β equilibrium is derived for the counterstreaming beam tokamak (CBT). The critical β of the CBT is found to be of comparable magnitude to that occurring in a similar model of a scalar pressure tokamak. It is shown that the toroidal current which is essential for equilibrium can be maintained by the counterstreaming ions. Finally, a brief discussion of the stability of the device is given.

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I. INTRODUCTION

The fusion reactor potential of counterstreaming beams of Deuterons and Tritons was first considered at the beginning of the C.T.R. programme, for example, by Hawkins [1]. The basic concept is to fuse Deuterons and Tritons by firing 60 keV beams of these ions at each other. The recent production of 50 amp ion sources has revived interest in this subject. However, it can be shown that the current density in each beam has to be of order 10^7 amps/cm^2 for a significant number of fusion reactions to occur in a pathlength of a few metres. Thus to obtain large current densities and increase the pathlength the ions are injected into a torus where they are stacked as counterstreaming distributions of Deuterons and Tritons.

Both species of ions lose their energy mainly through Coulomb collisions with the background electrons, and recent calculations [2] [3] have shown that during a slowing down time there are sufficient fusion reactions so that the Q of this system (Q is the ratio of the thermonuclear power to the injected beam power) can be considerably greater than unity. These systems have a further advantage that Q > 1 is achieved with quite modest values of nτ ($\sim 10^{12}$) [4], and hence small scale experiments can be conceived to demonstrate the reactor feasibility of this device.

In this paper we show that a high-β tokamak-like magnetic field configuration can be produced by counterstreaming beams alone, thus combining the above advantages of the CBT (counterstreaming beam torus) with the known advantages of a tokamak. The most significant difference between the CBT equilibrium and the conventional tokamak is that the pressure in the CBT is highly anisotropic, with $p_{\parallel} \gg p_{\perp}$. Typically, it is found from Fokker-Planck studies [4]

of the collisional interaction between the ions, that $p_{||}/p_{\perp} \sim 4$. Thus in section II of this paper, using an expansion in the inverse aspect ratio $\varepsilon (=r_o/R_o)$, we derive a model of a high-β equilibrium with the ordering $p_{\perp} \sim \varepsilon p_{||}$. This equilibrium exhibits all the typical properties of a scalar pressure model of a high-β tokamak, namely, a critical β for current reversal and an upper limit to the pressure that can be confined. Interestingly, it is found that this upper limit is of similar magnitude to that obtained in a scalar pressure model of tokamak with the same current distribution [5,6].

From the equilibrium model the toroidal current density is deduced, and then in section III it is shown that this current can be established by either using an electric field, as in the conventional tokamak, or by the beams themselves. In principle, using beams alone would remove the necessity for the usual transformer and its circuitry, and also, which is more important, allow for steady state operation of the device. It is shown in Section III that for a pure plasma ($Z_{eff} = 1$), the beams themselves lead to a zero net current. With impurities present ($Z_{eff} \neq 1$), however, a net current can be established. In order to control the toroidal current profile in the CBT it is suggested that helium should be injected and estimates are obtained for the injected current required both in a present day tokamak and a future reactor.

In Section IV we give a cursory discussion of the stability of the CBT. We would expect, that as well as the normal tokamak instabilities, there would be additional unstable modes in the CBT due to the pressure anisotropy, for example, the mirror and firehose instabilities. In fact, in the first part of Section IV, it is shown that these modes are stable. The authors are unaware of any further MHD modes driven by anisotropy, although some may exist. As far as the conventional tokamak modes are concerned, the localised interchange has been discussed [7], but only for low-β ($\beta \sim \varepsilon^2$). However, the indications are that for these β values, anisotropy with $p_{||} \gg p_{\perp}$ has a beneficial effect. The influence of anisotropy upon the kink instability has been studied for the

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skin current model [8], but the more relevant diffuse current profiles have yet
to be investigated. Recent work [9] on microinstabilities suggests that the
most dangerous modes of instability will be the parallel sound and ion cyclotron
modes. Conditions for the stabilisation of these modes are given.

Thus despite a fairly thorough appraisal of the known literature we have
been unable to find any mode which seriously influences the viability of the
CBT concept.

II. AN ANISOTROPIC HIGH-β TOKAMAK EQUILIBRIUM

Mondelli and Ott [10], and more recently, Connor and Hastie [7], have
investigated anisotropic equilibria in a large aspect ratio, low-β tokamak of
circular cross-section. These workers follow Shafranov [11] and develop the
equilibrium as an expansion in ε, the inverse aspect ratio. To order ε^2 the
flux-surfaces are circles with the centres of their cross-sections displaced
inward by a variable distance Δ from the magnetic axis, and where $\Delta \sim \varepsilon$. The
analysis is general and applies to any pressure and current profiles for which
$\beta \sim \varepsilon^2$.

In conventional scalar pressure tokamak analysis a number of authors [5,6,12]
have considered high-β equilibria for which $\beta \sim \varepsilon$, and where $\Delta \sim 1$. For these
investigations it is necessary to prescribe simple forms for the pressure and
toroidal current density distributions. A feature common to all this work is
that for a given shape of plasma cross-section there is an upper limit to the
β which can be confined. A more practical limit is set by the onset of
toroidal current density reversal, which occurs at a somewhat lower value of β. In the
present section we investigate a simple model of a large aspect ratio, circular
cross-section, high-β tokamak, with anisotropic pressure ($p_{\parallel} \gg p_{\perp}$). It is of
particular interest to elucidate any β-limitations which may occur for this case.

In the CBT we envisage the resulting mass flow of the two ion species to be very small. Thus we consider the equations

$$\underline{j} \times \underline{B} = \text{div} \overset{\leftrightarrow}{p} \quad , \qquad \underline{j} = \nabla \times \underline{B} \qquad \Bigg\} \qquad (1)$$

and
$$\nabla \cdot \underline{B} = 0 \quad ,$$

where $\underline{B}$ is the magnetic field, $\underline{j}$ the current density, and the pressure tensor $\overset{\leftrightarrow}{p}$ is given by

$$\overset{\leftrightarrow}{p} = p_\perp (\overset{\leftrightarrow}{1} - \underline{e} : \underline{e}) + p_{\parallel} \, \underline{e} : \underline{e} \quad , \qquad (2)$$

where $\underline{e}$ is the unit vector along $\underline{B}$. For an axisymmetric torus

$$\underline{B} = \frac{T}{R} \underline{e}_\phi - \frac{\underline{e}_\phi \times \nabla \psi}{R} \quad ,$$

where R is the distance from the major axis, $\underline{e}_\phi$ is the unit vector in the toroidal direction, ψ the poloidal flux and T is an arbitrary function of position. Choosing an orthogonal coordinate system (ψ, ϕ, χ) and defining

$$\sigma_- \equiv \frac{p_{\parallel} - p_\perp}{B^2} \qquad\qquad \bar{p} \equiv \tfrac{1}{2}(p_{\parallel} + p_\perp) \quad , \qquad (3)$$

we obtain [13] from Eq. (1),

$$T(1-\sigma_-) = g(\psi) \qquad (4)$$

$$\frac{\partial \bar{p}}{\partial \chi} + \tfrac{1}{2} B^2 \frac{\partial \sigma_-}{\partial \chi} = 0 \qquad (5)$$

and
$$(1-\sigma_-) \left[\frac{T}{R^2} \frac{\partial T}{\partial \psi} - \frac{j_\phi}{R} \right] + \frac{\partial \bar{p}}{\partial \psi} - \frac{B^2}{2} \frac{\partial \sigma_-}{\partial \psi} = 0 \quad , \qquad (6)$$

where g is an arbitrary function of ψ. Following Mercier and Cotsaftis [13] we assume

$$\sigma_- = \sigma_-(\psi) \qquad T = T(\psi) \qquad \bar{p} = \bar{p}(\psi) \quad ,$$

and using Eq. (6) it is straightforward to derive the equation

$$H\psi = -R j_\phi \quad , \qquad (7)$$

where using cylindrical coordinates (R,ϕ,Z) based on the axis of symmetry, the operator H is given by

$$H \equiv \frac{\partial^2}{\partial R^2} + \frac{\partial^2}{\partial Z^2} - \frac{1}{R}\frac{\partial}{\partial R} \quad , \tag{8}$$

and

$$Rj_\phi = - \frac{\sigma_-'(\nabla\psi)^2}{2(1-\sigma_-)} + \left[TT'(1-\sigma_-) - \tfrac{1}{2}\sigma_-' T^2 + \bar{p}' R^2 \right]\frac{1}{1-\sigma_-} \quad . \tag{9}$$

We now apply Eq. (7) to a circular cross-section plasma of major and minor radii R_o, r_o respectively. Introducing local polar coordinates (r,θ,ϕ) based on the centre of the minor cross-section, H becomes [6]

$$H = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{\cos\theta \frac{\partial}{\partial r}}{R_o + r\cos\theta} + \frac{\sin\theta}{r}\frac{1}{R_o + r\cos\theta}\frac{\partial}{\partial\theta} . \tag{10}$$

The plasma is maintained in equilibrium by an externally applied vertical magnetic field. An approximate analytic method for determining this field for a conventional scalar pressure tokamak has been described by Haas [6]. The magnetic field at the plasma boundary $\psi = \psi_B$ is continuous, and since we shall consider a model in which $p_\perp = p_\parallel = 0$ at the interface, it follows that pressure balance is automatically satisfied. We choose forms for $T(\psi)$, $\bar{p}(\psi)$ and $\sigma_-(\psi)$, which ensure that $p_\parallel \gg p_\perp$. Specifically, we take

$$T(\psi) = \frac{b\psi_B}{r_o} = \text{constant} \tag{11}$$

$$\bar{p}(\psi) = \frac{a}{2R_o^2 r_o^2}(\psi_B^2 - \psi^2) \tag{12}$$

and

$$\sigma_-(\psi) = \frac{A}{\psi_B^2}(\psi_B^2 - \psi^2) \quad , \tag{13}$$

where a, b and A are dimensionless free parameters. Substituting the above forms into Eq. (7) and using Eq. (10), we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} - \frac{\varepsilon\cos\theta\frac{\partial\psi}{\partial r}}{1 + \varepsilon\, r\cos\theta} + \frac{\sin\theta}{r}\quad\frac{\varepsilon}{1 + \varepsilon\, r\cos\theta}\quad\frac{\partial\psi}{\partial\theta}$$

$$+ \frac{A}{\psi_B^2}\frac{\psi}{(1 - \sigma_-)}\left\{\left(\cos\theta\frac{\partial\psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial\psi}{\partial\theta}\right)^2 + \left(\sin\theta\frac{\partial\psi}{\partial r} + \frac{\cos\theta}{r}\frac{\partial\psi}{\partial\theta}\right)^2\right\}$$

$$+ \frac{b^2 A\,\psi}{1 - \sigma_-} - \frac{a\,\psi}{1 - \sigma_-}(1 + \varepsilon\, r\cos\theta)^2 = 0 \quad , \tag{14}$$

where r is now dimensionless, $r = 1$ representing the boundary and $\varepsilon = r_o/R_o$.
Taking $\varepsilon \ll 1$ we choose a,b and A such that $b^2 \sim a \sim A \sim 1$ and that $b^2 A - a \sim \varepsilon$.
We treat $\sigma_-(\psi)$ to be of order ε, and this is subsequently shown to be valid.
Expanding ψ in the form

$$\psi = \psi_o(r, \theta) + \psi_1(r, \theta) + \ldots \tag{15}$$

we can solve Eq. (14) order-by-order. Setting $\psi_o = \psi_B$ then the leading-order
equation is satisfied trivially. To first-order we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi_1}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi_1}{\partial\theta^2} + (b^2 A - a - 2\, a\,\varepsilon\, r\cos\theta)\psi_o = 0 \quad , \tag{16}$$

where $\psi_1 = 0$ at $r = 1$. This leads to the solution

$$\psi_1 = -\frac{1}{4}\psi_o(1 - r^2)(a - b^2 A + \varepsilon\, a\, r\cos\theta) \quad . \tag{17}$$

A typical plot of ψ_1 is shown in the figure. The flux-surfaces enclose an
outwardly displaced magnetic axis.

It follows that $p_{\parallel}$ and $p_{\perp}$ are given to leading-order by the expressions

$$p_{\parallel} = -\frac{\psi_B}{r_o^2 R_o^2}(a + A\, b^2)\psi_1 \tag{18}$$

and

$$p_{\perp} = -\frac{\psi_B}{r_o^2 R_o^2}(a - A\, b^2)\psi_1 \quad , \tag{19}$$

and thus confirming that our model describes a tokamak for which $p_{\parallel} \gg p_{\perp}$. Thus
to leading-order the pressure components are functions of ψ_1 only, and hence have
an approximately parabolic radial dependence.

As a consequence of Eq. (18) above

$$\frac{p_{||}}{B_\phi^2} = \frac{\psi_1}{\psi_B} \frac{1}{b^2} (a + A b^2) \sim \epsilon \quad ,$$

and hence as assumed earlier $\sigma_- \ll 1$. If we regard the model as consisting of anisotropic pressure in the presence of a scalar pressure background, p_o, then for consistency we must have $p_o \sim p_\perp \sim \epsilon \, p_{||}$.

We now discuss the current profiles. Since T = constant there is no poloidal current. By Eq. (9) the toroidal current density is given by

$$j_\phi = \frac{1}{R(1 - \sigma_-)} \left(\bar{p}' R^2 - \frac{R^2 B^2}{2} \sigma_-' \right) \quad ,$$

which for the present model gives

$$j_\phi = - \frac{\psi_B}{r_o^2 R_o} \left\{ a - b^2 A + 2 \, a \, \epsilon \, r \cos \theta \right\} \quad . \tag{20}$$

The total current, I, becomes

$$I = - \frac{\pi \psi_B}{R_o} (a - b^2 A) \quad . \tag{21}$$

We define the total-β to be $\beta = \dfrac{\langle p_\perp + p_{||}/2 \rangle}{\frac{1}{2} B_\phi^2}$, where the average is taken over the plasma minor cross-section. For an anisotropic pressure distribution there is no unique definition of β, and in fact, any combination of $p_{||}$ and $p_\perp$ in the above definition is allowable. The form that we assume here, however, is perhaps the most natural since it is the ratio of plasma energy density to magnetic energy density. (For a fusion reactor it is probably more appropriate to define β as $\beta = \langle p_{||} + \alpha p_\perp \rangle / B_\phi^2$, where α is a function of $\overline{\sigma v}$. In the vicinity of the maximum value of σ for the D-T reaction, α is close to unity. This definition exemplifies the fact that the thermonuclear reaction rate $\overline{\sigma v}$ per unit energy, is greater for a one dimensional Maxwellian with $p_{||}$-pressure than a two dimensional Maxwellian with $p_\perp$-pressure.)

Thus it follows that

$$\beta = \frac{1}{4} \frac{a}{b^2} (a - b^2 A) .$$ (22)

Alternatively, we can define the poloidal-β to be $\beta_I = 8\pi I^{-2} \int p_{\parallel} dS$, which gives

$$\beta_I = \frac{a}{a - b^2 A} .$$ (23)

Defining the safety factor for an equivalent cylinder of radius r_o to be

$$q = \frac{2 \pi \epsilon B_\phi r_o}{I} ,$$

we obtain

$$q = \frac{2 \epsilon b}{a - b^2 A} \sim 1 .$$ (24)

We can now replace a, b and A by the physically meaningful parameters

β, q and I. Thus

$$j_\phi = \frac{I}{\pi r_o^2} (1 + 2\beta/\beta_c \ r \cos \theta)$$ (25)

and

$$\psi = \psi_B \left[1 + \frac{I}{4\pi I_c} (1 - r^2) \left(1 + \beta/\beta_c \ r \cos \theta \right) \right]$$ (26)

where $I_c = \psi_B R_o^{-1}$ and β_c is the so-called equilibrium limit, which in this case
is given by

$$\beta_c = \frac{\epsilon}{q^2} .$$ (27)

Note that from Eq.(25) the current density contours are vertical planes, the

magnitude of j_ϕ depending only on the distance from the centre of the minor

cross-section. We observe, as in the conventional model of tokamak, that as

β is increased j_ϕ decreases at the point A (see Figure) until at $\beta = \beta_c/2$

the toroidal current density vanishes. Any further increase in β leads to

a current density reversal. At $\beta = \beta_c$ the poloidal magnetic field vanishes

at point A thus providing an upper limit to the containment.

The $\beta \sim \varepsilon$ _anisotropic_ equilibrium derived in this section has an essentially parabolic pressure profile, with current density given by Eq.(25). Although these profiles can be established in a $\beta \sim \varepsilon$ _scalar_ pressure model [6], the manner in which they arise is different for the two problems. This can be seen most clearly from Eq. (6). For scalar pressure,

$$j_\phi = \frac{T}{R} \frac{dT}{d\psi} + R \frac{dp}{d\psi} \quad , \tag{28}$$

whereas for the CBT with a vacuum toroidal field (T = constant) and $\sigma_- \ll 1$,

$$j_\phi = - \frac{RB^2}{2} \frac{\partial \sigma_-}{\partial \psi} + R \frac{\partial \overline{p}}{\partial \psi} \quad . \tag{29}$$

Although the origins of j_ϕ are different in the two cases, for the particular models and orderings chosen, the right hand sides of Eqs. (28) and (29) cancel in leading-order to give the same j_ϕ. It is clear, of course, that the difference in form for T together with $p_\perp \sim \varepsilon p_\parallel$, means that the scalar pressure formulation cannot be recovered from the present work. We note that the magnitude of the equilibrium limit is similar to that for the scalar pressure model, the latter having the value $\beta_c = \frac{3}{2} \frac{\varepsilon}{q^2}$ on the definition adopted in this paper.

We now discuss the validity of the form for the pressure tensor adopted in Eq. (2). For reactor conditions the mean free path for collisions will be larger than any characteristic dimension of the system. Asserting that the Lorentz force plays a role analagous to that of the collision term, Chew, Goldberger and Low [14], expand the collisionless Boltzmann equation for the ions in terms of the Larmor radius (ion mass to charge ratio). In order to satisfy the leading-order equation for the zero-order distribution function, f_0, the electric field must be taken perpendicular to the magnetic field. This is a good approximation since the high mobility of the electrons ensures that $E_\parallel$ is insignificant. More precisely, we require the electron plasma frequency to be very large compared with the ion Larmor frequency - a condition easily satisfied in tokamaks. It follows from this approximation that f_0 is constrained

to have a particular functional form. On taking moments of the Boltzmann equation it is found by Chew et al that this constraint on f_o leads to the form of pressure tensor adopted in Eq. (2).

In this section we have established a high-β model of a tokamak with highly anisotropic pressure. The equilibrium β-limit is of a similar magnitude to that for the scalar pressure model, and could, presumably, be raised a further factor of two or three by considering the plasma to have an elliptic cross-section [15].

III. PRODUCTION OF THE TOROIDAL CURRENT

In this section it is shown that the toroidal current which is essential for the equilibrium can be provided either by the injected beams themselves, or by an electric field as in the conventional tokamak equilibrium.

It was first pointed out by Ohkawa [16] that the current in a tokamak could be supported by the injection of fast ions. The basic idea is that the transfer of momentum from the fast ions to the electrons by Coulomb collisions accelerates the electrons in a similar manner to an electric field and thereby generates a current. The circulating fast ions also constitute a current, which is in the opposite direction to the electron current, and only if the Z of the injected beam is different to the Z_{eff} of the plasma is there a net current.

The calculation of the current in the CBT proceeds in a similar manner to that given by Ohkawa [16] but a kinetic approach is used so that the toroidal electric field may also be included. The starting point is the Fokker-Planck equation for the electrons which can be written in the form

$$\frac{\partial f_e}{\partial t} + \frac{eE}{m} \frac{\partial f_e}{\partial v_{\parallel}} = C(f_e, f_e) + \sum_i C(f_i, f_e) \tag{30}$$

where the collision operator C is in the form of Rosenbluth, Macdonald and Judd [17], for distributions which are axisymmetric about the magnetic field. To simplify the ion-electron collision operator in Eq. (30) the usual approximation $v_i \ll v_e$ is made; then to first order in v_i / v_e the operator becomes

$$\sum_i C(f_e, f_i) = \sum_i \overline{n_i v_{\parallel i}} \ Z_i^2 \ v^{-3} \left[2 \xi \frac{\partial f_e}{\partial v} - v^{-1} \frac{\partial}{\partial \xi} \left\{ (1 - \xi^2) f_e \right\} \right.$$

$$\left. - \frac{1}{2v} \frac{\partial^2}{\partial \xi^2} \left\{ \xi(1 - \xi^2) f_e \right\} - v \frac{\partial^2}{\partial \xi \partial v} (f_e \ v^{-1}) + v \xi \frac{\partial}{\partial v} (f_e \ v^{-1}) \right]$$

$$+ \frac{1}{2} \sum_i n_i \ Z_i^2 \ v^{-3} \frac{\partial}{\partial \xi} \left\{ (1 - \xi^2) \frac{\partial f_e}{\partial \xi} \right\} \tag{31}$$

where $\xi = v_{\parallel}/v$, $\overline{n v_{\parallel i}}$ is the mean flux of the ith species along the field lines and Z_i is the charge of the ith species.

In the velocity frame in which

$$\sum_{i}' \overline{n_i v_{\|i}} \, Z_i^2 = 0 \tag{32}$$

the above collision operator reduces to

$$\sum_{i}' C(f_e, f_i) = \frac{1}{2} \sum_{i} n_i \, Z_i^2 \, v^{-3} \frac{\partial}{\partial \xi} \left((1 - \xi^2) \frac{\partial f_e}{\partial \xi} \right) \quad . \tag{33}$$

This is now identical to the electron-ion collision operator for electrons colliding with cold stationary ions and is the operator used by Spitzer, Cohen and Routly in their calculation of the electrical conductivity of plasma. Thus using their results [18] the current may be written as follows,

$$j = E/\eta + \sum_{i}' e \, \overline{n_i v_{\|i}} \, Z_i \tag{34}$$

where the first term is the contribution from the electric field with η the Spitzer [18] resistivity. The second term is the contribution from the motion of the injected ions, background ions and any impurities.

Using Eq. (32) and defining $Z_{eff} = \sum_i n_i Z_i^2/n$ (where n is the electron density $n = \sum_i n_i Z_i$), Eq. (34) can be rewritten as

$$j = \frac{E}{\eta} + e \sum_{i}' \overline{n_i v_{\|i}} \, Z_i \, (1 - Z_i/Z_{eff}) \quad . \tag{35}$$

This form has the advantage that it is invariant under velocity transformations so $\overline{n v_{\|i}}$ can be specified in any reference frame. The above calculation is a uniform geometry one, and does not include the main effect of toroidal geometry which is the trapping of particles in the toroidal field gradient. This effect, however, has been calculated elsewhere [19], and the result may be trivially added to Eq.(35).

By examination of Eq. (35), one can see that if only Deuterons and Tritons are injected (Z = 1), then the beam contribution to Eq. (35) is zero, unless there are some impurity ions giving $Z_{eff} \neq 1$. Thus with no impurities i.e. $Z_{eff} = 1$ the current has to be generated by an electric field as in the conventional tokamak. With a radial distribution of impurities, giving $Z_{eff} > 1$, the injected

D-T beams alone can produce a current if the mean ion flux is non zero $(\overline{\Sigma n v_{\parallel i}} \neq 0)$, however the current profile and impurity profile will be related in the manner given by Eq. (35), and it may not be possible to obtain the current profile required to establish a stable equilibrium.

A better approach would be to inject a small density of ^{3}He or ^{4}He, at a high energy. Expanding in the small parameter $n_{3_{He}}/n_D$, the current density in the absence of any applied electric field, can be written in the form,

$$j = - e n_{3_{He}} \left[2 \overline{v}_{\parallel 3_{He}} - \frac{2}{n} \left(\overline{n_D v_{\parallel D}} + \overline{n_T v_{\parallel T}} \right) \right] \quad . \tag{36}$$

Since the D and T beams are injected in opposite directions the second term in the square brackets will be close to zero. The current profile in this case is dependent upon the deposition profile of the injected helium, which can be varied to some extent by changing the direction of injection.

We now give examples of typical CBT operation, based on the model of this paper. To demonstrate the existence of the CBT equilibrium and investigate its stability (which is discussed in the next section), we could for example inject 100 amps of 60 keV Deuterium or Hydrogen ions (50 amps in each direction) into a present generation tokamak of the size of say DITE (R = 110cm, r = 23cm B_ϕ = 30 kg). With an $n\tau_\epsilon \sim 5 \times 10^{11}$ cm^{-3} secs, the electron temperature would be 3 keV, and from the Q calculations [2] [3], this would give Q = 1 in an equivalent system with D-T injection. The equilibrium limit for q = 3, is β_c = 0.02 and this limit would be reached with the above injection power. The toroidal current required for the equilibrium is 200 K amps and this could be provided by injecting 3 amps of 80 keV ^{4}He.

A reactor CBT with parameters, R = 10 m, r = 1 m, B_ϕ = 55 kg, would have β_c = 0.05 for q = 2.5. For an $n\tau_\epsilon \sim 10^{13}$ cm^{-3} secs the electron temperature would be 10 keV giving Q = 5 [4], thus the injection of 166 Megawatts of D-T at 80 keV would give a thermal output of 1000 Megawatts. The toroidal

current of 1 Megamp which is required for equilibrium could be provided by the injection of 60 amps of helium at 80 keV. The plasma density would be $6 \times 10^{13} \text{cm}^{-3}$, and this allows the 80 keV fast neutral atoms to adequately penetrate the plasma.

So to summarise this section, to produce a toroidal current without an electric field requires the presence of impurities, and to obtain a given profile one may have to resort to the injection of a small number of high energy helium ions. The injection requirements for CBT operation in the present generation of tokamak experiments and future fusion reactors have been given, and these are well within the range of present technology.

IV. DISCUSSION OF STABILITY

In the present section we discuss the stability theory relevant to our equilibrium. We begin with the magnetohydrodynamic modes. Using the CGL Energy Principle [14], Mercier and Cotsaftis [13] have discussed the localised interchange perturbations in an axisymmetric toroidal plasma. They show that a necessary condition for stability is

$$- \sigma_{\perp} \left[\frac{1 + 5/3 \; \sigma_{\perp}}{1 + 2\sigma_{\perp}} \right] < \sigma_- < 1 \quad , \tag{37}$$

where $\sigma_{\perp} = p_{\perp}/B^2$ and $\sigma_- = \dfrac{p_{\parallel} - p_{\perp}}{B^2}$. The left-hand inequality is generally referred to as the mirror instability, whilst the right-hand inequality is referred to as the firehose instability. These criteria are required to be satisfied at all points throughout the plasma. It is clear that the equilibrium described in the first section of this paper satisfies both inequalities.

The validity of the CGL Energy Principle is uncertain. In particular, the heat flux along the magnetic field is neglected in order to close the moment equations, and it can be shown that this Energy Principle leads to optimistic results. Kruskal and Oberman [20], and Rosenbluth and Rostoker [21] have developed the so-called Kinetic Energy Principle. This is valid under conditions, and in situations, where the CGL principle is invalid. Further, it takes account of trapped particles. Recently, Connor and Hastie [7] have used this form to obtain

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a criterion for stability against localised interchanges in general axisymmetric toroidal equilibria. Their result contains an additional stabilising contribution due to trapped particles. In carrying through their programme of minimisations they require

$$\sigma_- < 1 \text{ and } 1 + \sigma_\perp > 0 \quad , \tag{38}$$

where $\sigma_\perp$ is now defined to be

$$\sigma_\perp = \frac{2p_\perp + C}{B^2} \quad ,$$

C being a 'pressure-like' moment of the form

$$C = \sum_i m_i \int \frac{B}{v_\parallel} \frac{\partial f_i}{\partial \varepsilon} (\mu B)^2 \, d\mu d\varepsilon \quad .$$

We observe that the first criterion in Eq. (38) above is identical with the firehose condition of Mercier and Cotsaftis [13], whereas the second condition is a modified version of the mirror criterion. Both conditions, however, are again satisfied by our high-β equilibrium. It has not proved possible to apply Connor and Hastie's general criterion for localised interchanges to the high-β equilibrium obtained in this paper. So we must content ourselves with reviewing the application of their criterion to low-β equilibria.

To simulate the effect of parallel injection, they assume

$$p_\perp = p_0(r) \quad , \quad C = -2p_0 \quad , \quad p_\parallel = p_0 + \hat{p}_\parallel$$

where p_0 is the scalar background pressure and $\hat{p}_\parallel$ is an additional longitudinal pressure introduced by the beam. They find that the beam produces a more stable situation than an isotropic plasma. In fact, ignoring the role of trapped particles which are only important near the magnetic axis, the safety factor on axis, q_0, may be reduced from $q_0 = 1$ to $q_0 \simeq 0.8$ for sufficiently large parallel pressures.

As regards the gross or kink instabilities, no calculations pertaining to anisotropic plasmas with diffuse currents in tokamak geometry are known to us. We expect that modes corresponding to those occurring in scalar pressure tokamaks should not be significantly affected by anisotropy. There is

the possibility of additional modes other than the firehose and mirror. Investigation, however, of the admittedly unrealistic model of a cylindrical anisotropic pinch, with a skin-current [8], shows no evidence of new modes. For the conventional tokamak, the form of current profile [22] and toroidicity [23] have important effects on the kink modes, so it would be necessary to repeat these calculations for anisotropic equilibria. Consideration of the vertical shift instability using the CGL principle shows, as anticipated, that the stability problem for this mode is identical with that for the equivalent scalar pressure model of tokamak [15].

As far as microinstabilities are concerned there would appear to be two possible driving mechanisms that could result in instability. These are the anisotropy $p_{\parallel} \gg p_{\perp}$ and the positive slope of the ion distribution function ($\frac{\partial f}{\partial v_{\parallel}} > 0$) at low energies. Since the injection velocity of both the D and T beams is much less than the Alfvén speed, all of the Alfvén wave modes of instability discussed by Berk et al [9] are stable. However, the parallel sound instability [9], driven by $\frac{\partial f}{\partial v_{\parallel}} > 0$, and the ion cyclotron instability [24], driven by anisotropy ($p_{\parallel} \gg p_{\perp}$), may be unstable. The condition for stability of the parallel sound mode has been given by Berk et al [9], and may be written for the CBT in the form $m_e v_e^3 < 2m_b v_b \Delta v_{b\parallel}^2$, where the subscript b refers to either of the beam species and $\Delta v_{b\parallel} = \left[\frac{\partial f}{\partial v_{\parallel}} \Big|_{v_{\parallel} = v_{b\parallel}} \right]^{-\frac{1}{2}}$. The above stability condition is not usually satisfied for CBT plasmas; however the introduction of a small density of thermal ions can stabilise this mode.

The ion cyclotron mode [24] has a somewhat more stringent condition for stability, this is, $m_e v_e^3 < 2m_b v_{b\perp}^2 \Delta v_{b\parallel}$. Once again this condition is weakened by the addition of a thermal distribution of ions [24]. For both of these modes magnetic shear, which should be stabilising, has yet to be included in the model.

V. CONCLUSIONS

An anisotropic high-β equilibrium has been derived for a colliding beam tokamak device. This equilibrium is similar to the conventional tokamak, and the critical-β that can be contained is of comparable magnitude. The toroidal current necessary for equilibrium can be provided by the injected D-T beams alone in the presence of impurities; however, to obtain a greater control over the current profile it is suggested that high energy ^{4}He should be injected.

We have shown that the firehouse and mirror instabilities are stable for $\beta \sim \varepsilon$ in CBT devices; and we have identified the most serious microinstabilities as being the parallel sound mode and the ion cyclotron mode. However, despite a fairly extensive survey of the literature, we have been unable to find any serious instability. Examples are given of CBT operation under typical tokamak and reactor conditions.

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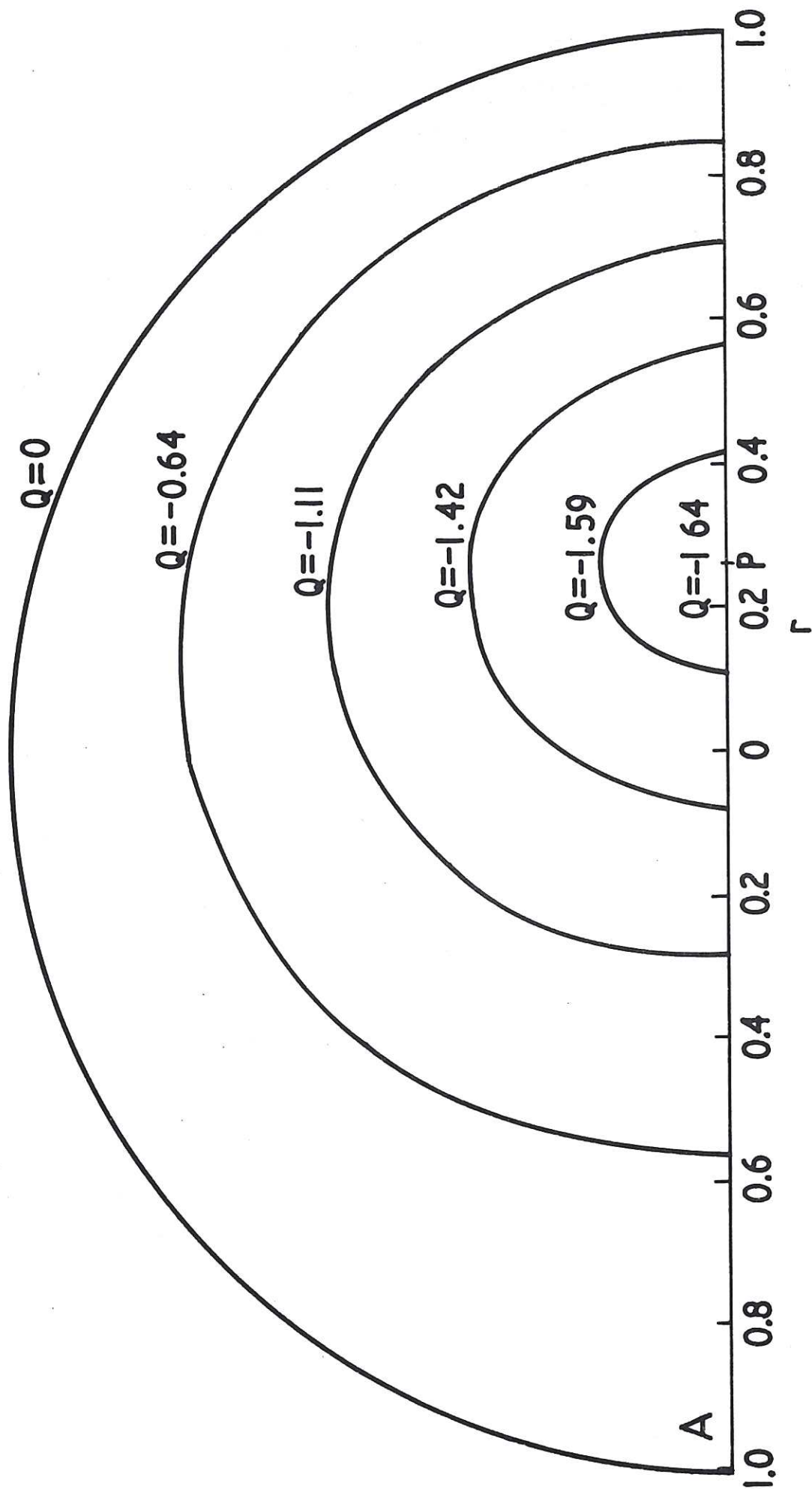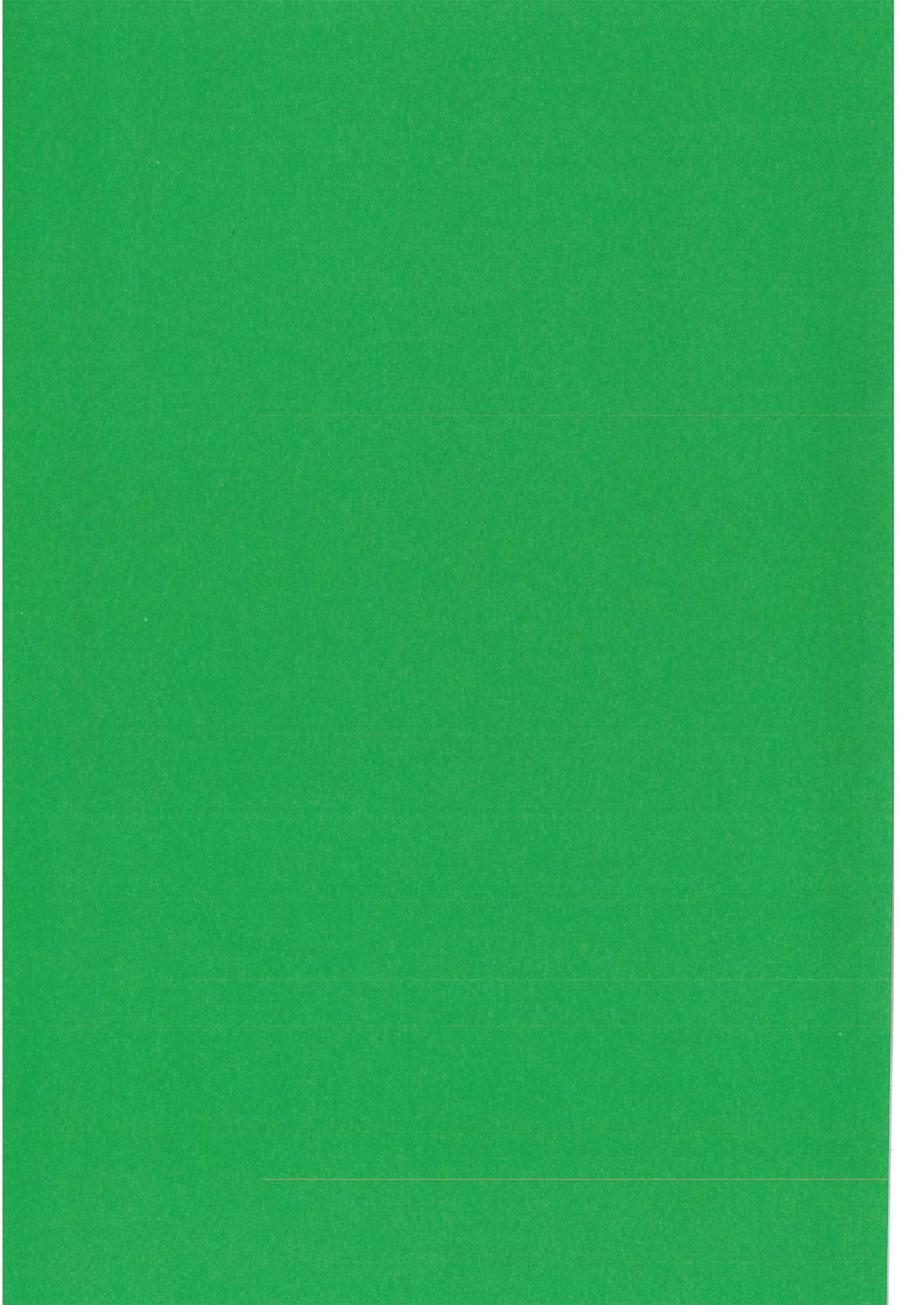[24] Mai L P and Horton W Jr. Preprint submitted to the Physics of Fluids.

Fig.1 Flux surfaces in the plasma for $\frac{1}{\epsilon a}(a - b^2 A) = 1.5$. The figure shows a plot of the function $Q = -(1 - r^2)\{\frac{1}{\epsilon a}(a - b^2 A) + r \cos \theta\}$. The magnetic axis P is displaced $\Delta = 0.25$ from the centre 0 of the plasma, the axis of symmetry being on the left of the figure.

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