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A WAY OF CALCULATING THE EFFECTS OF PLASMA-WALL INTERACTIONS BY A SELF CONSISTENT METHOD

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ABSTRACT

Previous attempts to model plasma-wall interactions using the well known plasma transport codes such as that due to Düchs, have assumed an arbitrary set of boundary conditions at the edge of the plasma. In the present work these assumptions have been replaced by others directly related to the physical processes acting in the plasma. The method can be used to study the evolution of the temperature and density profiles in a cylindrical plasma carrying a constant axial electric current, for different types of plasma-wall interaction, and can be used in the limiting case of no plasma-wall interaction where the Düchs code cannot be used.

Examples of its use are given using an accelerated time scale and a simplified plasma-wall interaction.

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1. INTRODUCTION

The presence of impurities in the plasmas of Tokamak machines has a significant effect on the discharges[1]. The impurities can arise from the interaction of particles diffusing from the plasma with the walls of the vacuum vessel, the limiters and divertor. The evolution of impurities from the wall can occur by several processes amongst which are the removal of neutral atoms of wall material either by evaporation or by sputtering and the desorption of adsorbed material from the walls. The desorbed materials typically include oxygen and neutral atoms of the parent type of the ions in the discharge.

The penetration of the evolved impurities into the discharge affects the plasma parameters such as electron and ion temperatures T_e, T_i and the plasma electron density N, all of which affect the plasma diffusion coefficient and hence the rate of evolution of material from the walls. Thus there is a strong feed back loop between the plasma parameters and the plasma-wall interaction and for many purposes the plasma cannot be considered as independent of the wall.

The trend of Tokamak development to bigger machines, with more energetic plasmas, implies that plasma-wall interactions will increase in importance, and it is desirable, and in the future will become essential, to calculate both the cooling effect of the wall on the plasma, and the power flow to the walls due to the plasma. Knowledge of these quantities will assist in the choice of suitable wall materials.

Attempts have been made to model the plasma wall interaction by using the computer codes developed to describe the transport processes in tokamak plasmas[2,3,4] which permit the degree of ionisation of the injected impurities to be calculated with considerable refinement. These codes have been developed in order that the changes of the temperature and density profiles with time which are observed experimentally can be compared with the predictions of theory and the diffusion coefficient thus determined.

These codes are a method of solving the coupled differential equations which describe the time dependence of the plasma parameters T_e, T_i, N and the electric current flowing in the plasma cylinder of circular cross-section. The electric current is either measured as the current density J or in terms of the azimuthal magnetic field B_θ associated with it. As with all problems involving differential equations it is necessary to specify a number of boundary conditions before a solution can be obtained. It is usual to specify the values of T_e, T_i, N and B_θ at or close to the edge of the plasma.

If the differential equations are a true description of the processes operating in the plasma, then the choice of a set of boundary conditions derived from experimental observations will automatically generate a solution in which the various constraints, such as conservation of energy, will be obeyed. On the other hand an arbitrary set of boundary conditions could result in an impossible solution.

Dangers such as this are present when attempts are made to model a plasma-wall interaction when there is no experimental guide as to the choice of boundary values of T_e, T_i and N. The rate of injection of material from the wall into the plasma will be strongly dependent on the boundary values. To use the codes in the conventional way to model the wall reaction it is necessary to compute the plasma profile and then check if the boundary values are consistent both with the plasma model and the model of the wall reaction.

An alternative approach followed here is to identify the processes which relate the boundary values of T_e, T_i and N and to construct a code which automatically takes the relationships into account as the equations are solved. A code has been written to test this concept and some of the results obtained are presented. Some arbitrary assumptions have been made to overcome gaps which exist in the present knowledge of plasma transport processes, but the solutions obtained are not very sensitive to these assumptions, so that it is possible to set up a model of a plasma-wall interaction which can be used for the purposes of sensitivity analysis.

2. THE GEOMETRY OF THE MODEL

The geometry is conventional, the plasma, Figure 1, is a straight cylinder of radius R cm, with axial symmetry having a magnetic field B_Z imposed on it. The radius is defined by a diaphragm limiter, and the plasma properties are evaluated at a sufficient distance from the limiter for there to be no axial variation in the plasma properties. Particles diffusing out of the plasma column enter a tenuous plasma which surrounds the main column. If the density of the tenuous plasma is less than 10^{12} particles cm^{-3} and if the particle energies are greater than 10eV or so, their mean free path in the axial direction, defined by Coulomb scattering on other charged particles, will be greater than 10m[5], thus in a reasonable size of tokamak the particles, once they have diffused from the plasma, can reach the limiter without making a collision with other particles and energy transport to the limiter is by convection rather than by thermal conduction. A few of the escaping particles may move radially from the plasma column and reach the walls of the tube surrounding the plasma; again convective transport is assumed. The particles incident upon these surfaces are assumed to be absorbed and a plasma wall interaction is simulated by injecting impurities across the boundary of the plasma in a flux of uniform density.

The plasma column carries an axial electric current of density J(kA cm^{-2}), the total current I_o (kA) being constant. This current generates an azimuthal magnetic field B_θ (kG) at radius r.

$$B_\theta = \frac{2}{10r} \int_o^r 2\pi\, J\, r\, dr . \qquad (1)$$

3. THE DIFFUSION COEFFICIENT D

The plasma is assumed to be hydrogenous with a diffusion coefficient D, which, following Düchs [(2)], is split into two components D_P, D_B as follows:

$$D^{-1} = D_P^{-1} + D_B^{-1} \qquad (2)$$

$$D_P = G_1 \frac{8.366 \times 10^{-6}\, N(T_e + T_i)\, \ln\Lambda}{B_\theta^2\, T_e^{3/2}} \qquad (3)$$

$$D_B = G_2 \frac{6.3 \times 10^{-3}\, (T_e + T_i)}{B_Z} \qquad (4)$$

where the plasma density N is measured in units of 10^{12} cm^{-3}, the electron and ion temperatures T_e, T_i are in eV, the magnetic field in kG, and time in microseconds.

The screening term Λ has the value defined by Spitzer [(5)], being nine times the number of plasma electrons in a Debye sphere.

G_1, G_2 are numerical coefficients set at the start of each complete run. Yoshikawa [(6)] has suggested that the factor G_1 should be of the order of 140, but later computations [(4)] have used values in the region of 520. The term D_P is the pseudo-classical diffusion coefficient and is infinite on the axis of the plasma because $B_\theta = 0$ on the axis. The diffusion coefficient D must be finite and is unlikely to exceed the Bohm value[7] and to avoid any singularity on the axis, it has been assumed to be related to D_P by equation 2, thus by a suitable choice of G_1 and G_2, D can be made to vary from the value D_B on the axis, to a value approximately equal to D_P at the plasma boundary r = R. When $G_2 = 1$ and $T_e = T_i$, D_B is the usually accepted value of the Bohm diffusion coefficient[7]. In the examples given later $G_1 = 600$, $G_2 = 1$ and Bohm diffusion is predominant over the inner 10% of the plasma volume.

4. THE TRANSPORT EQUATIONS

The plasma parameters T_e, T_i, N and B_θ are assumed to diffuse with time t in the plasma column in a way which is described by the simplified transport equations 5,6,7,8; the numerical coefficients in the equations are for the system of units defined earlier.

The first of the differential equations is that for the diffusion of the magnetic field B in a quiescent plasma with the Spizter conductivity[8].

$$\frac{\partial B}{\partial t} = \frac{5.2}{4\pi} \frac{\partial}{\partial r} \left(\frac{\ln\Lambda}{T_e^{3/2}} \frac{1}{r} \frac{\partial (rB)}{\partial r} \right). \tag{5}$$

The plasma diffuses across the magnetic field with a diffusivity D so that,

$$\frac{\partial N}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(D r \frac{\partial N}{\partial r} \right) + S_N \tag{6}$$

where S_N is the rate of creation of electrons per unit volume due to the ionisation of impurities, in units of $10^{12} \text{cm}^{-3}\ \text{s}\ 10^{-6}$.

The equation for the transport of energy amongst the electrons is:

$$\frac{\partial}{\partial t} (NT_e) = \frac{1}{r} \frac{\partial}{\partial r} \left(rDT_e \frac{\partial N}{\partial r} + rND \frac{G_4 T_e}{T_e + G_5 T_i} \frac{\partial T_e}{\partial r} \right) - \frac{1.6012}{A} 10^{-3} \frac{T_e - T_i}{T_e^{3/2}} N^2 \ln\Lambda$$
$$+ \frac{1.7174}{4\pi} \times 10^5 \frac{\ln\Lambda}{T_e^{3/2}} \left[\frac{1}{r} \frac{\partial}{\partial r} (rB) \right]^2 - G_3\, 6.435 \cdot 10^{-8}\, T_e^{1/2} N^2 - S_E \tag{7}$$

where S_E is the power drain on the electrons due to the presence of impurities in units of 0.4151 erg cm^{-3} s . 10^{-6}. The symbols G_3, G_4, G_5 are numerical factors which are set at the beginning of each computer run.

The first two terms on the right hand side of equation 7 account for the transfer of energy by convection and conduction, the third term is that for the energy exchange between electrons and ions, the fourth represents the power generated by the passage of the electric current and the fifth is that for the power radiated by the plasma as free-free radiation so that G_3 is identified as the Gaunt factor (9).

The corresponding equation for the transfer of energy amongst the ions is:

$$\frac{\partial}{\partial t}(NT_i) = \frac{1}{r}\left(rDT_i\frac{\partial N}{\partial t} + C_1 r\frac{\partial T_i}{\partial r}\right) + \frac{1.6012 \times 10^{-3}}{A} \frac{T_e - T_i}{T_e^{3/2}} N^2 \ln\Lambda - S_I \quad (8)$$

where S_I is the power loss of the ions to the impurities by ionisation, excitation and charge exchange, in the same units as for S_E.

The value assumed for the ionic thermal conductivity is:

$$C_1 = G_6 \frac{6.614 \times 10^{-4} \ln\Lambda \, N^2 A^{\frac{1}{2}}}{B_Z^2 \, T_i^{\frac{1}{2}}} \quad (9)$$

where A is the mass of plasma ions relative to that of a proton, and G_6 is a numerical factor set at the beginning of each computer run. If $G_6 = 1$ then the expression is that for the classical value of ionic thermal conductivity (9).

5. THE PREVIOUSLY USED BOUNDARY CONDITIONS

The two previous sections have summarised the usual formulation of the transport codes and the transport coefficients appearing in them, but before a solution can be obtained a set of boundary conditions is required. The usually accepted sets of conditions are described below.

The symmetry of the model requires that in the axis of the plasma:

$$B_\theta = 0 \qquad \frac{\partial T_e}{\partial r} = 0 \qquad \frac{\partial T_i}{\partial r} = 0 \qquad \frac{\partial N}{\partial r} = 0 \quad (10)$$

at all times.

On the outer edge of the plasma at $r = R$, the dependent variables, B_θ T_e, T_i and N take the values $B_{\theta R}$, T_{eR}, T_{iR}, N_R. Because I_o, the total current carried by the plasma is constant then B_R is a constant and:

$$B_{\theta R} = \frac{2I_o}{10R} \; . \tag{11}$$

The codes previously used for investigating tokamak transport problems (1,2) and a newer code (10), either assume that T_{eR}, T_{iR}, N_R have predetermined values usually constant, or that the values of the dependent variables may be extrapolated across the boundary of the plasma to a value of $r = R'$ and that the values of T_e, T_i and N at $r = R'$ take predetermined values. This last assumption allows T_{eR}, T_{iR} and N_R to vary, but has the disadvantage of introducing a constraint on the values of the gradients of these quantities at the edge of the plasma. These assumptions are legitimate if enough experimental evidence is available to allow a suitable choice of boundary conditions to be made.

6. INTERDEPENDENCE OF THE BOUNDARY CONDITIONS

An examination is now made of the constraints which apply to the boundary conditions and their interdependence is discussed. As before, the total current carried by the plasma is regarded as constant so that B_R is known. The first effect to be examined is the conservation of electrical energy.

The electric current flowing through the plasma supplies energy to the system in two ways. Firstly, it heats the electrons in the plasma by ohmic heating and secondly, it changes the magnetic energy stored inside the plasma boundary. The rate at which electrical power is supplied to a unit length of the plasma column can be expressed in terms of the flux of the Poynting vector, and can be written symbolically, See Appendix 1, as:

$$\frac{1}{8\pi}\left[\frac{5.2\ \ln\Lambda}{T_e^{3/2}} \, B_\theta \, \frac{\partial (rB_\theta)}{\partial r}\right]_{r=R} = \frac{1}{4}\frac{\partial}{\partial t}\int_o^R B_\theta^{\,2} r \, dr + \frac{1}{8\pi}\int_o^R \frac{5.2}{T_e^{3/2}} \ln\Lambda\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)\right]^2 r \, dr \, . \tag{12}$$

The first term on the righthand side of equation 12 is the inductive power supplied to the plasma, and the second term is the ohmic power. The $\ln\Lambda$ term is only weakly dependent on the plasma parameters, and equation 5 shows that $\frac{\partial B_\theta}{\partial t}$ is strongly dependent on the electron temperature and only weakly dependent on the other parameters; equation 12 thus shows that there is a strong independence between T_e and B_θ.

The ohmic power transmitted to the electrons per unit volume of the plasma, is balanced by the power sinks, W_1 to W_5, (see equation 7), where:

W_1 = the rate of increase of the thermal energy of the electrons.
W_2 = the power transferred to the ions by collisions.
W_3 = the power radiated as free-free radiation.
W_4 = the power transferred to the impurity ions by collisions.
W_5 = the power transferred by convective and conductive effects,
so that equation 12 can be re-written as:

$$\frac{1}{8\pi}\left[\frac{5.2\ \ln\Lambda}{T_e{}^{3/2}}\ B_\theta\ \frac{\partial(rB_\theta)}{\partial r}\right]_{r=R} = \frac{1}{4}\frac{\partial}{\partial t}\int_o^R B_\theta{}^2 r\ dr + \int_o^R 2\pi\ (W_1+W_2+W_3+W_4)r\ dr + W_6 + W_7 \qquad (13)$$

where the terms W_1-W_4 are in units of 10^{12} erg cm^{-3} s^{-1}, W_6 is the power flowing to the plasma boundary due to the convection of the electrons and W_7 is the power flowing up to the plasma boundary due to the electronic component of the thermal conduction.

If the tokamak heating system is of reasonable efficiency the energy conducted to the plasma boundary by the electrons, represented by the term W_7 in equation 13, should only be a small fraction of the ohmic dissipation in the plasma. The left hand side of the equation is only weakly dependent upon the value of W_7 which is the only term on the right hand side directly dependent upon $(\partial Te/\partial r)_R$. The proposed method of solution uses equation 13 to define a value of T_{eR} which is self consistent with the spatial distribution of B_θ, assuming that W_7 is negligibly small. This is equivalent to setting $(\partial Te/\partial r)_R = 0$; experimental evidence as to the values of the temperature gradient will be given later. The numerical method employed to determine T_{eR} is to assign values of B_θ, T_e, T_i and N to a set of mesh points spaced across a radius of the plasma and also at the boundary of the plasma at time t_1. The updated values of the variables at the interior mesh points at time $t_1 + \Delta t$ are now found by any of the standard procedures for the numerical solution of diffusion equations. This gives enough information to obtain numerical approximations to the integrals on the right hand sides of equations 12 and 13. Since B_θ is known at the boundary for all values of time it follows that the mean value of $B_\theta\{\partial(rB_\theta/\partial r\}_R$ over the time interval is known which then gives the mean and updated values of T_{eR}.

The neglect of the term W_7 in equation 13 has been justified on theoretical grounds. It is also in accord with the general trend of experimental results.

Experimentally determined profiles of electron density and temperature such as those given in [1] and also the results of computer simulations using the pseudo-classical diffusion coefficient [10] give results which show that near the plasma boundary

$$T_e \ \partial N/\partial r \ < \ N \ \partial T_e/\partial r,$$

thus implying that $W_7 < W_6$ in equation 13. The method used to update T_{eR} can also accommodate small values of conductive power loss if it is possible to find a suitable approximation for W_7.

The value for T_{iR} is found by considering the power balance of the ions. The ions absorb power from the electrons, some of which increases the thermal energy of the ions, some is passed to the impurities in the plasma by methods which are included in the term S_I in equation 8, and the remainder is transported through the plasma and convected to the walls. The power carried to the plasma boundary by the ions per unit length of the column is I_i, where:

$$I_i\left[\text{erg cm}^{-1}\ \text{s}\cdot 10^{-6}\right] = (4.806\pi)\int_0^R \left[\frac{1.6012\times10^{-3}}{A}\ \frac{T_e - T_i}{T_e^{3/2}}\,N^2 \ln\Lambda - S_I - \frac{\partial}{\partial t}\,(NT_i)\right] r\,dr. \quad (14)$$

The power is carried across the plasma boundary by the escaping ions, which are assumed to have a radial drift velocity V_B (cm s 10^{-6}) so that we also have:

$$I_i \ = \ 4.806\ \pi\ N_R\ V_B\ T_{iR}\ R\ . \quad (15)$$

The value of V_B is open to debate and depends on the detailed model which is set up to describe the edge of the plasma. For the illustrative example it is assumed that:

$$V_B \ = \ G_7 \left(\frac{T_{iR}}{A}\right)^{\frac{1}{2}} \quad (16)$$

where G_7 is a parameter which is set at the beginning of each computer run. If $G_7 = 1$ then V_B is of the order of the thermal velocity of the ions, an assumption sometimes made in reactor design studies [11]. Because electrons and ions must escape from the plasma at equal rates the electrons carry energy across the plasma boundary at a rate equal to $4.806\ \pi\ N_R\ V_B\ T_{eR}$. This power flow is equal to the term W_6 in equation (13), and it is assumed to be small compared to the ohmic power dissipated in the plasma.

Combining equations (15) and (16) gives,

$$T_{iR}^{3/2} = \frac{\sqrt{A}\, I_i}{4.806\, \pi\, N_R\, R\, G_7} \quad . \qquad (17)$$

This equation is used to define T_{iR}.

The value of N_R is found from the particle balance in the plasma. The rate at which particles diffuse from the plasma column per unit length I_N (10^{12} cm^{-1} s 10^{-6}) is:

$$I_N = 2\pi \int_0^R r \left(\frac{\partial N}{\partial t} - S_N\right) dr . \qquad (18)$$

This rate of loss must equal $2\pi\, R\, N_R\, V_B$, so that:

$$N_R = \frac{I_N}{2\pi R V_B} \qquad (19)$$

thus defining N_R.

The values of T_{iR} and N_R thus depend on the assumptions made concerning V_B which in turn depends on the value chosen for G_7. Some of the experimental evidence for the value of G_7 is examined in Section 9, and it is shown that if G_7 is chosen to be in the range consistent with results of present experiments, that is, $G_7 = 10^{-2} - 10^{-3}$, then the calculations show that the predictions of the model, especially the rate at which particles leave the plasma, are not sensitive to the value chosen for G_7.

7. ORGANISATION OF THE CODE

The numerical method used to integrate the equations 5,6,7,8, is to establish a set of n mesh points equispaced across the radius of the plasma with the first mesh point on the axis of the plasma, and the plasma boundary bisecting the segment between the last and the penultimate point, Figure 2. The four differential equations are replaced by a set of difference equations in the usual way[12,13]. The Crank-Nicholson method[13,14] has been used for updating the values of the dependent variables.

At the beginning of the first time step it is necessary to assume an initial distribution of the variables which is an approximate solution of the transport equations and which also satisfies the constraints identified in Section 6.

If the code is run for a succession of time steps using such an approximation, then the values of T_{eR}, T_{iR} and N_R oscillate with decreasing amplitude, the oscillation becoming negligible after a few tens of steps, showing that the approximation improves as the steps proceed. Because of this it is usual to disregard the results of the first time steps and to take the time zero from the end of the m^{th} time step where m is in the region of 20 to 40, which is equivalent to neglecting at most the first 60 microseconds of the life of the plasma.

8. RESULTS FOR THE CASE OF NO PLASMA-WALL INTERACTION

The code has been run for the limiting case of no plasma-wall interaction for which the previous codes are inapplicable. In this case the terms S_E, S_I and S_N in equations 6,7, and 8 are all zero. The dimensions chosen are typical of the present generation of tokamaks and are:

$$R = 25\text{cm}, \quad B_Z = 28\text{kG}, \quad I_o = 270\text{kA}, \quad A = 1.0 .$$

The numerical coefficients G_1, G_2 governing the diffusion coefficient were set at 600 and 1 as previously discussed, the Gaunt factor G_3 was unity.

The coefficients G_4, G_5, G_6 appearing in equations 7,8, and 9 which define the thermal conductivity of the plasma were set equal to:

$$G_4 = 1 \quad G_5 = o \quad G_6 = 200.$$

These values make the electron thermal conductivity in equation 7 equal to ND and the ionic thermal conductivity in equation 9 200 times the classical value.

The particles diffusing from the plasma were assumed to cross the plasma boundary with a velocity V_B, (equation 16), which was 0.001 times the ion speed. No special significance is attached to this assumption; its advantage is that it generates a solution which changes significantly in a conveniently short computer run, making it a good standard condition for the production of illustrative examples.

Figures 3,4,5,6 show the calculated radial distributions of the axial current density J, the temperatures T_e, T_i and the density N at times t = 0 and 6.10^{-3} s where:

$$J = \frac{2.5}{\pi} \frac{1}{r} \frac{\partial}{\partial r} (rB_\theta) \quad \{\text{kA cm}^{-2}\}. \tag{20}$$

The initial distribution of J, T_e, T_i and N with r were parabolic. The striking feature of the results is the rapid rise in the temperatures at the edge of the plasma relative to the central values. Figure 7 shows the change in the boundary values T_{eR}, T_{iR} and N_R and the average particle containment time τ_p with time, where:

$$\tau_p = \frac{\int_0^R 2\pi \; r \; N \; dr}{I_N} \; . \tag{21}$$

Calculations with the Düchs code[4] give results that are insensitive to the value of the ionic thermal conductivity. This result has been verified by re-running the code with the values of G_6 set equal to unity so that the ionic thermal conductivity was equal to the classical value. The results are shown in Figures 3,4,5,6.

The results were obtained with the assumption that the power conducted to the plasma boundary by the electrons was negligibly small, so that $(\partial T_e/\partial r)_R$ should be zero. Because the numerical solution has been obtained at a set of mesh points and because of the method of defining T_{eR}, the calculated gradient is finite at the boundary. A check on the accuracy of the results can be made by estimating the conducted power and comparing it with the ohmic dissipation. In the examples given the conducted power did not exceed 2.10^{-3} of the ohmic power, so that the accuracy of the determination of T_{eR} is of the order of 0.15%. In view of the coarse spacing of the mesh points, (17 were used), and the accelerated time scale which enhanced the losses at the edge whilst keeping the ohmic dissipation constant, this is satisfactory.

The effect of a finite conduction of power to the boundary by the electrons was simulated by putting $W_7 = 0.25 \; W_6$ in equation 13, thus making the conducted power one quarter of the convected power. This lowered the value of T_{eR} by less than 9%, and had a negligible effect upon T_{iR} and N_R, the principal parameters governing the plasma wall interaction. Thus the neglect of W_7 in equation 13 is justified.

9. INJECTION OF NEUTRAL GAS

It remains to be shown that the code can be used with a power sink, due to the presence of impurities in the plasma of the order of magnitude to be expected in an experimental situation. The transport of impurities in a plasma has been subjected to intensive numerical investigation[2,15,16], and because these treatments can be grafted on the present code, the test to be described has been made with an over-simplified plasma wall interaction, and transport theory.

The impurities are assumed to be neutral atoms of the parent type of the ions in the plasma, which are desorbed from the walls and enter the plasma at a rate proportional to that of ions diffusing out of the plasma. When the stream of neutrals enters the plasma it is attenuated as the neutral particles undergo ionising and charge exchange collisions, so that the neutral density is greatest near the edge of the plasma. Düchs[16] has used the approximation that the density H of the neutrals a distance ℓ inside the surface of the plasma is:

$$H = H_R \exp\left(- \int_o^{\ell} (S+C) \frac{N d\ell}{v_D} \right) \qquad (22)$$

where H_R is the density of the stream at the plasma boundary, v_D is the drift velocity of the neutrals and S,C are rate coefficients such that rate of ionisation per unit volume is SHN and the charge exchange rate per unit volume is C H N.

The hot neutral particles produced by the charge exchange reactions may escape from the plasma or they may penetrate the plasma and undergo additional charge exchange or ionising collisions. In the present examples, energy transport inside the plasma by the hot neutrals is neglected as is the possibility that those escaping from the plasma may rebound from the walls and also cause desorption of gas from the wall. The neglect of the subsequent collisions of the charge exchanged particles is not very important in the present case, because their mean free path is large enough to ensure that they have a large probability of escaping from the plasma.

The neutral hydrogen atoms are assumed to have the classical ionisation cross-section by electron impact[17]. Graphs of the variation of S with T_e have been computed[18] and up to electron temperatures in the region of 1keV are reasonably well fitted by:

$$S = 2.3 \cdot 10^{-1} \left(1 - \exp\left(-\frac{T_e}{27}\right)\right)^2 T_e^{-0.25} \text{ cm}^2 \text{ s } 10^{-6} \qquad (23)$$

where N is measured in units of 10^{12} cm^{-3}.

Riviere[19] has reviewed the experimental measurements of the charge exchange cross-section $\delta(E_i)$ for protons and hydrogen atoms at proton energies E_i in excess of 100eV and has suggested the empirical relation:

$$\delta(E_i) = \frac{6.937 \times 10^{-15} (1 - 0.0673 \ln E_i)^2 \text{ cm}^2}{1 + 1.1112 \times 10^{-15} E_i^{\,3.3}} \; . \qquad (24)$$

This expression gives values for $\delta(E_i)$ which are approximately 20% greater than the theoretical values of Dalgarno and Yadev [20]. Experimental values for the cross-section at lower proton energies have not been found in the literature but extrapolation of Riviere's formula to leV gives a cross-section of 6.9×10^{-15} cm^2 which compares well with the theoretical value of 4.7×10^{-15} cm^2. For this reason equation 24 has been used for all ion energies in excess of leV. Interpretation of this cross-section over a Maxwellian distribution of ion energies gives an expression for C:

$$C \sim 9 \times 10^{11} \; \delta \; (T_i) \; \sqrt{T_i} \quad cm^3 \; s \; 10^{-6} \; . \tag{23}$$

The neutral term S_N appearing in equation 6 is obtained directly from the expression for S_H. For the power sink term S_E in equation 7, allowance must be made for the electron atom collisions which do not ionise the atoms but which produce radiation. It is assumed that for hydrogen that on average each ionising collision corresponds to an energy sink which is a factor G_8 times the ionisation energy, where G_8 is in the region of 2-3, for typical Tokamak plasmas[21]. The ion energy sink term S_I is obtained by multiplying the charge exchange rate by the difference of the ion temperatures and the temperature T_N of the neutrals.

The code has been run assuming the ion thermal conductivity is 200 times the classical value, with the other disposable coefficients as before, and the injection of neutral particles starting at time t = o. The particles are assumed to be injected across the plasma boundary at a rate Q per unit area proportional to the proton loss rate

$$H_R = \frac{Q}{v_D} \; . \tag{24}$$

The mean drift velocity of the neutral atoms was assumed to be 2/3 of their thermal velocity, which corresponds to a cosine distribution of velocities about a radius. Some results are shown in figures 3,4,5, and 6, the rate of injection of neutrals being chosen to keep the number of electrons per unit length of the plasma constant, the neutral atom temperature was assumed to be 2.0 eV and G_8 was taken to be 2.0[21].

10. THE VALUE OF THE DRIFT VELOCITY V_B

The analysis has assumed that the particles escape from the plasma by drifting across the plasma boundary with a drift velocity V_B which was proportional to the thermal velocity of the ions at the boundary:

$$V_B = G_7 \left(\frac{T_{iR}}{A}\right)^{\frac{1}{2}} \quad \text{cm s } 10^{-6} .$$

Although G_7 is sometimes assumed to be unity, so that V_B would be approximately equal to the thermal velocity of the ions, no experimental confirmation of this has been found. The properties of tokamak plasmas at radii approximately equal to the limiter radius have not been intensively investigated, although values of N_R of 2.10^{12} cm^{-3} have been reported for the JFT-2 and ST machines(22,23). These values are only consistent with the published values of mean density and confinement time if $V_B = 3.10^4$ cm s^{-1} (JFT-2) and 10^4 cm s^{-1} (ST). The values of T_{iR} are not known but are probably at least several eV so that V_B is at most 0.01 times the thermal velocity of the ions and is probably less. The effect of the variations of G_7 on the computer simulations has been assessed by comparing the results of two sets of calculations, in one of which $G_7 = 0.01$ and in the other $G_7 = 0.001$, the other parameters being unchanged. The results at a time of 6 msec are summarised in Table I. The table includes the values of N calculated at r = 24.2 cm as well as the values of N_R which correspond to r = 25 cm, the rate at which the charged particles strike the wall is proportional to $G_7\, N_R\, T_{iR}^{\frac{1}{2}}$.

The table shows that changing G_7 by a factor 10 changes the values of T_{iR}, T_{eR}, and $G_7\, N_e\, T_{iR}^{\frac{1}{2}}$ by a factor of 1.8. N_R is the only parameter which changes by a factor of about 10, but the changes in the density profile of the plasma are concentrated towards its boundary. The values of N calculated inside the plasma boundary at r = 24.2 cm change by a factor of less than two and the effect of changing G_7 upon the calculated density profile becomes less marked towards the centre of the plasma.

To summarise, the results of the computations are not very sensitive to variations in the value assumed for G_7 if this is confined to the range which is compatible with the results of current experiments. The only parameter which has a proportional change comparable with that in G_7 is N_R but the severe changes in the density profile are limited to a thin layer near the surface of the plasma which is thin enough to have a high transmission for injected neutral particles. Thus, although the value of V_B requires refinement the results obtained with the present assumption are sufficiently insensitive to the value of G_7 to allow a model of the plasma-wall interaction to assess the relative merits of different wall materials with an arbitrarily chosen value of G_7 in the range 0.01 - 0.001.

11. DISCUSSION

The treatment of the boundary conditions of the plasma transport codes given here has the advantage over the earlier methods in that it requires fewer arbitrary assumptions and is not unduly sensitive to the assumptions for values covering the range relevant to tokamak experiments.

The first of the two assumptions made in the present treatment is that the power flow to the plasma boundary by the thermal conduction of electrons is much smaller than the ohmic dissipation in the plasma. It has been shown that the values of T_{ie}, T_{iR} and N_R and the rate of injection of impurities into the plasma are insensitive to the conducted power provided that it is small.

The second assumption is related to the velocity with which the escaping particles drift out of the plasma column. The need to make such an assumption emphasises a gap in our present knowledge of plasma properties. However it was shown in Section 9 that the rate of injection of atoms into the plasma is not critically dependent upon the value of the drift velocity provided it is chosen to be within the range consistent with present experiments. The effects on the plama profiles of changing this assumption are mainly confined to a narrow region near the plasma edge, so that the bulk of the plasma is insensitive to the value of the drift velocity.

The values of T_{iR}, τ_P, T_{eR} and N_R are strongly dependent upon the existence of the plasma wall interactions. In the examples given the interaction dominates the plasma properties in the outer half of the plasma ($r > 12$ cm), where over 60% of the charged particles in the plasma are to be found. It is necessary to include the wall effects in any model of the plasma.

The earlier methods of solution assumed that the values of T_i, T_e, and N at or near the plasma boundary were predetermined, thus implying that the plasma wall interaction was known. The present method allows the boundary values to vary over a wide range according to the plasma-wall interaction and is suitable for investigating such interactions.

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TABLE I

Plasma Parameters Calculated at t=6ms for Different Values of G_7

Plasma-Wall Interaction	G_7	$N(10^{12}cm^{-3})$ r = 24.2cm	$N_R(10^{12}cm^{-3})$	T_{iR} eV	T_{eR} eV	τ_p ms	Flux of particles to wall (arbitrary units) $G_7 N_R T_{iR}^{\frac{1}{2}}$
No	0.001	6.0	0.66	77	478	24.7	5.8
No	0.01	10.7	0.044	53	426	52.3	3.2
Yes	0.001	10.8	7.6	25	132	4.0	38
Yes	0.01	19.7	1.53	19	111	2.6	67

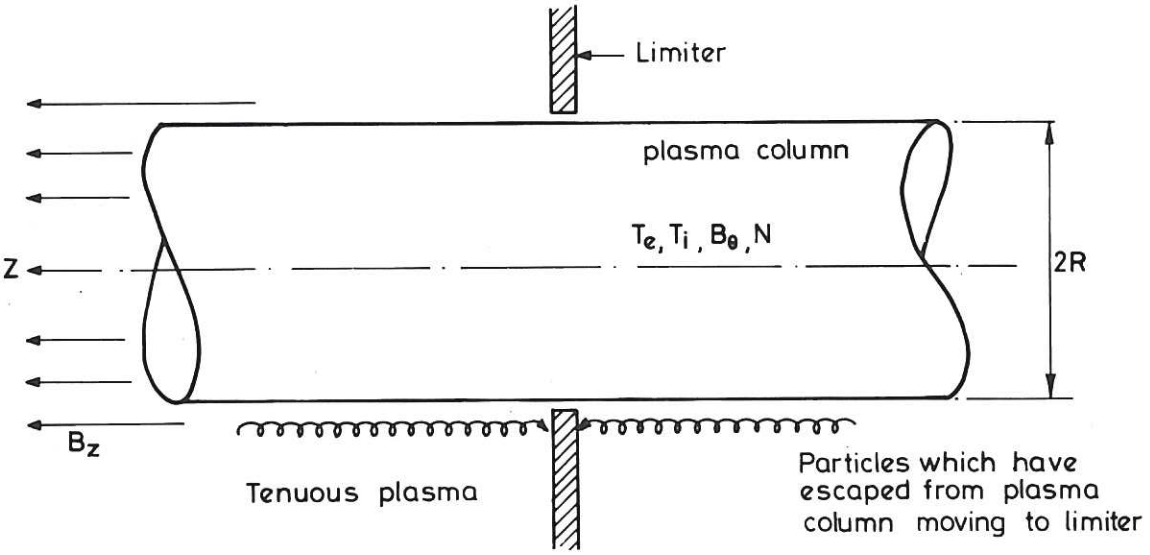

Fig.1 Illustrating the geometry of the calculations.

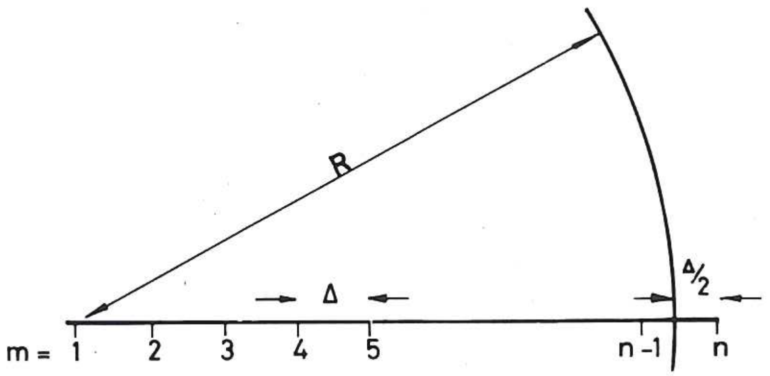

Fig.2 Illustrating the disposition of n equispaced mesh points across the plasma radius.

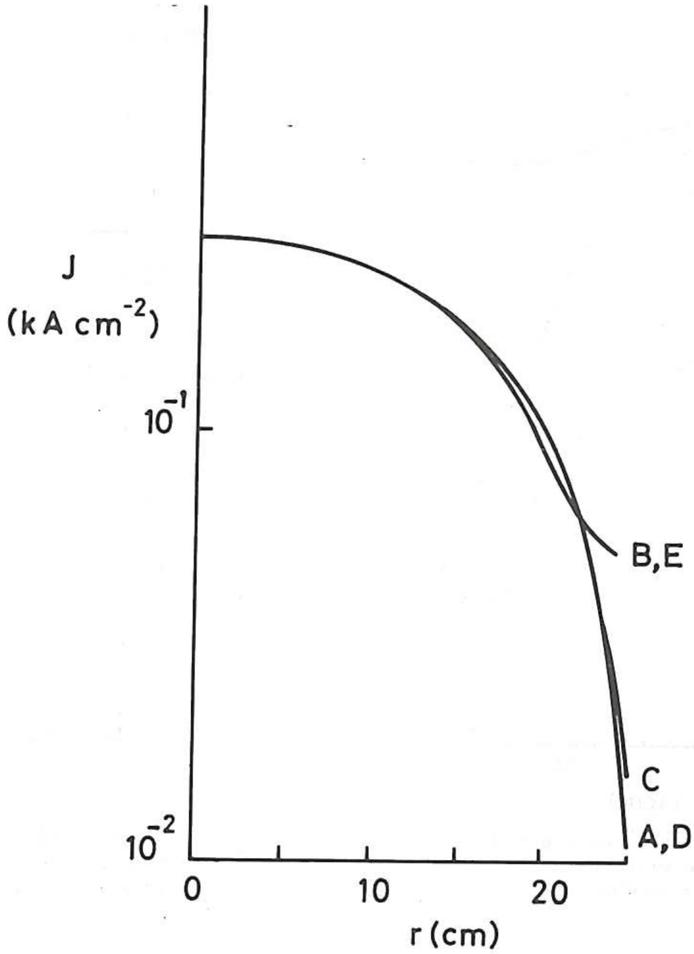

Fig.3 Current density as a function of radius

A	Pure plasma t = 0
B	Pure plasma t = 6 msec
C	As for B but with neutral injection
D	Pure plasma t = 0
E	Pure plasma t = 6 msec.

For A,B,C the ionic thermal conductivity is 200 x classical.
For D,E the ionic thermal conductivity is classical.

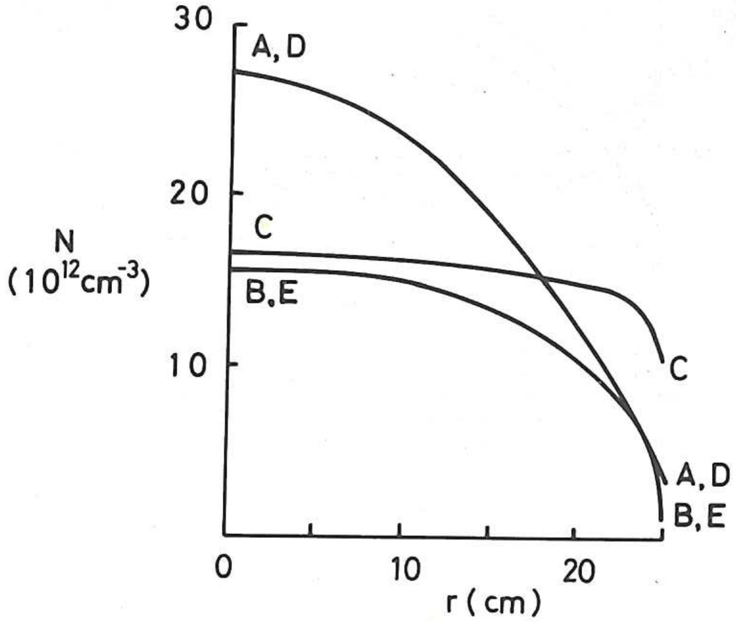

Fig.4 Variation of electron density with radius
Identifiers as for Figure 3.

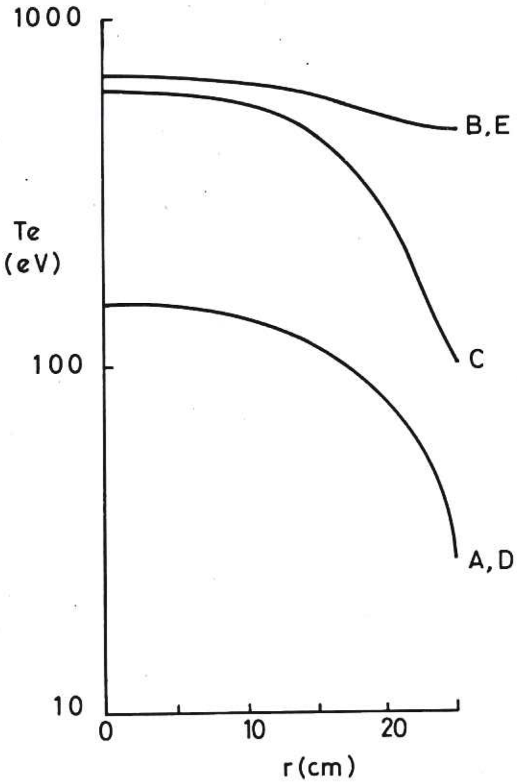

Fig.5 Variation of electron temperature with radius. Identifiers as for Fig.3

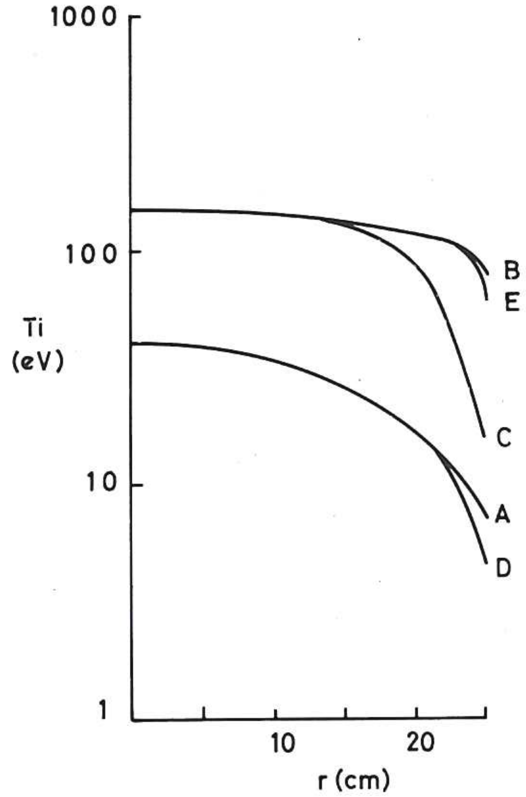

Fig.6 Variation of ion temperature with radius. Identifiers as for Fig.3

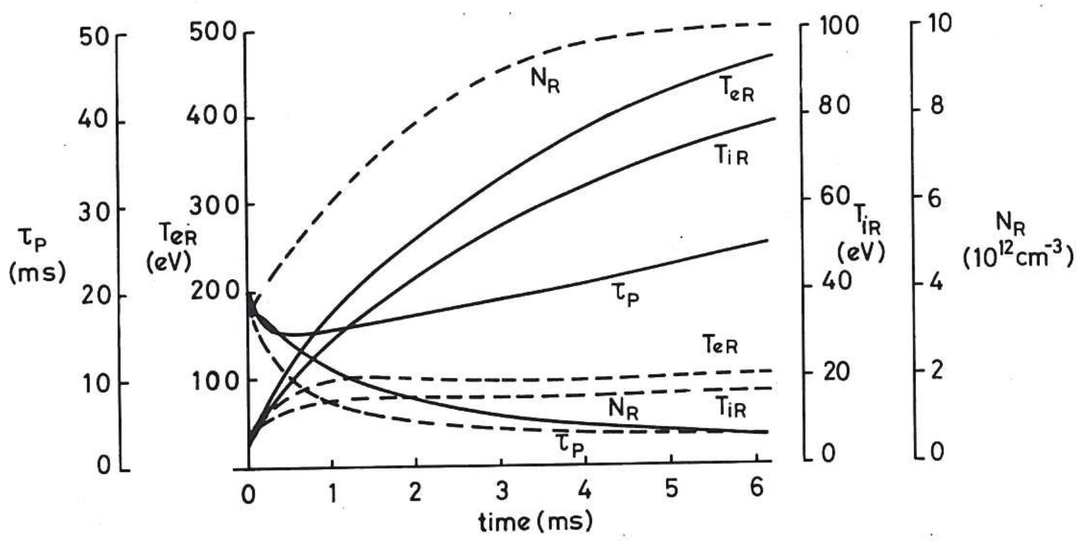

Fig.7 The variation of the boundary values N, T_e, T_i and the electron containment time with time, for a plasma whose ionic thermal conductivity is 200 times the classical value.
Full lines for no neutral injection
Broken lines for neutral injection

APPENDIX I

Inside the plasma, Maxwell's equations in the units used in the main text take the form:

$$\text{Curl } \underline{B} = 4\pi \frac{T_e^{3/2}}{\eta} \underline{E} \qquad \text{Curl } \underline{E} = \frac{\partial B}{\partial t}$$

where $\underline{B}$ is the magnetic field in kG due to the current flowing in the plasma and $\underline{E}$ is the associated electric field in e.m.u. times 10^{-9} and $\eta = 5.2 \ln\Lambda$.

Expanding the vector identity for $\frac{1}{4\pi}$ Div $(\underline{B} \times \underline{E})$

$$\frac{1}{4\pi} \text{Div } (\underline{B} \times \underline{E}) = \frac{1}{4\pi} \underline{E} \cdot \text{Curl } \underline{B} - \frac{1}{4\pi} \underline{B} \cdot \text{Curl } \underline{E}$$

$$= \frac{T_e^{3/2}}{\eta} \underline{E}^2 - \frac{1}{8\pi} \frac{\partial}{\partial t} \underline{B}^2 .$$

Integrating the identity over the column contained in a unit length of the plasma column and transforming the left hand side into a surface integral by Gauss' Theorem gives:

$$\left[\frac{r}{8\pi} \frac{\eta}{T_e^{3/2}} \underline{B} \cdot \underline{E} \right]_{r=R} = 2\pi \int_0^R \frac{T_e^{3/2}}{\eta} \underline{E}^2 \, r \, dr + \frac{1}{4} \frac{\partial}{\partial t} \int_0^R \underline{B}^2 \, r \, dr .$$

As the current is in the z direction, $\underline{E}$ has one component E_z, and $\underline{B}$ one component B_θ, and substituting into the above equation gives equation 12 after a little manipulation:

$$\frac{1}{8\pi} \left[\frac{\eta}{T_e^{3/2}} B_\theta \frac{\partial}{\partial r} (rB_\theta) \right]_{r=R} = \frac{1}{8\pi} \int_0^R \frac{\eta}{T_e^{3/2}} \left(\frac{1}{r} \frac{\partial}{\partial r} (rB_\theta) \right)^2 r \, dr + \frac{1}{4} \frac{\partial}{\partial t} \int_0^R B_\theta^2 \, r \, dr$$

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