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AXIAL HEATING OF MAGNETICALLY CONFINED PLASMA WITH CO2 LASERS

by

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ABSTRACT

The coupled electron-ion heating equations, neglecting losses, in a CO_2 laser heated solenoid are solved for a laser intensity varying with time as $I = I_o t^{2/3}$. An analytical solution, without restriction on the ratio of electron-to-ion temperatures T_e/T_i is found, showing $T_{e,i} \sim t^{2/3}$. The heating wave which propagates along the solenoid is found to be supersonic having a velocity independent of time and varying as $I_o^{3/5}$. Low intensity heating is found to maximize T_i/T_e and minimize plasma length and laser energy requirements. The heating wave propagation is found to be consistent with an optical thickness of order unity in the heated plasma column. Considerations of electron-ion energy transfer, supersonic heating wave propagation and laser beam trapping lead to an optimum laser intensity parameter $I_o \simeq 2 \times 10^{13}$ watt cm^{-2} sec$^{-2/3}$.

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I. INTRODUCTION

Dawson et al (1971) first suggested the use of high power CO_2 lasers for axially heating long plasma columns, magnetically confined in the radial direction, to temperatures adequate to initiate thermonuclear fusion reactions. This scheme is of interest for two reasons: (a) experimentally, high power CO_2 lasers are available which can deliver very high energies in long pulses with high efficiency and (b) theoretically, a nearly fortuitous coincidence of parameters, including density n, temperature T, magnetic field B_o, wavelength λ and plasma column length L with radius R, combine to give efficient absorption of CO_2 laser energy leading to predictions of adequate confinement to exceed the Lawson criterion for D-T fusion.

Indeed, it may be possible that 10 μm radiation is near optimum for this scheme. Since absorption varies as $n^2\lambda^2$, required axial confinement, as $n\ell$ and radial confinement $B^2 \sim n$, for fixed T, a shorter wavelength would require a higher density to satisfy absorption length $\ell_{abs} \sim L$; conversely a longer wavelength would require a lower density. In the former situation, magnetic fields greater than reasonably practicable limits are necessary and in the latter, even longer plasma columns are required. Consequently, apart from efficient, high energy capabilities the CO_2 laser may be nearly ideally suited to solenoid heating.

The basic approach employs nearly uniform inverse bremsstrahlung absorption of the CO_2 laser beam energy which must necessarily propagate the entire column length in a density profile favourable for beam trapping (Steinhauer and Ahlstrom, 1971a; Humphries, 1974; Mani et al, 1975). Such profiles, requiring a density minimum on axis, can in fact be generated by the plasma heating resulting from absorbed laser energy (Burnett and Offenberger, 1974). Magnetic fields of several hundred kilogauss would be required to inhibit radial losses and the plasma length need be great enough to both absorb the laser energy and provide adequate confinement - determined in the simplest case by free axial streaming of plasma out the ends. A consistent set of parameters satisfying these requirements and the Lawson criterion are $T = 10$ keV, $n = 3 \times 10^{17}$ cm^{-3}, $B_o = 500$ kgauss, $L = 10^5$ cm, $R = 0.5$ cm. The confinement time and hence laser pulse time required is ~ 500 μsec.

Now for underdense plasma the electrons gain energy via inverse bremsstrahlung absorption of the laser energy and must then collisionally transfer it to the ions. Since the rate of ion heating varies as $(T_e-T_i)/T_e^{3/2}$, it is desirable to not heat electrons too quickly since for $T_e/T_i \gg 1$ coupling is poor and laser energy is needlessly wasted in heating electrons only. The major emphasis of this paper is, in fact, concerned with heating rates to minimize the electron temperature and laser energy requirement having regard to producing 10 keV ions in a time less than the confinement time.

In addition, since absorption varies as $T_e^{-3/2}$, as successive layers are heated up by the incident radiation they become transparent, allowing further penetration of the laser beam (Rehm, 1970; Steinhauer and Ahlstrom, 1971b). At high intensities a supersonic heating wave, called a 'bleaching wave', can propagate along the plasma column (Ahlborn and Strachan, 1973; Steinhauer, 1975a). It has been shown that the velocity of such a supersonic heating wave is governed by the requirement that the heated plasma maintain an optical thickness of order unity (Burnett and Offenberger, 1976). We shall show in the present work that a laser intensity profile having a $t^{2/3}$ time dependence satisfies this requirement and, moreover, leads to a constant velocity of the heating wave.

The radial hydrodynamics of a laser heated solenoid have been solved numerically (Burnett and Offenberger, 1974) which show that a quasistatic approximation is valid for long heating times and magnetic fields of several hundred kilogauss. In this situation nearly constant axial density prevails through the heating period and radial losses are reduced to a small fraction of absorbed laser power except at the highest temperatures. Steinhauer and Ahlstrom (1975b) have constructed a simple model assuming single temperature ($T_e = T_i$), no thermal conduction losses radially or axially, trapped laser beam, slow heating and plasma tied to magnetic field. They then solved the "stacking" problem, determining axial temperature and position of the bleaching front. Yuen et al (1975) using similar approximations have analytically derived heating rates for electrons and ions and position of the bleaching front for constant laser intensity. They also point out that for optimum ion heating, when $T_e = 3T_i$ as first noted by Kidder (1971), a pulse proportional to $t^{2/3}$ is wanted. They note that a nearly constant propagation speed results, which however, they find independent of intensity and considerably larger than the value derived in the present work. Recently, Vagners et al (1975) have approached the same problem with the same assumptions using theory of optimal control to derive the optimum laser pulse shape; this can, however, only be done numerically.

In the present analysis, we use the same equations and approximations as the latter authors and show that the resulting equations for T_e and T_i admit analytic solutions for a $t^{2/3}$ intensity law. Temperature ratios are obtained for varying intensity and energy minimization deduced. Heating wave velocity is derived and restrictions on intensity deduced from considerations of adequate thermalization and absence of density shock formation. These conditions are shown to be equivalent to an assumption of optical thickness of order unity in the heated plasma column.

II. MODEL

We consider a uniform, stationary, fully ionized, deuterium-tritium plasma column along the z axis of a magnetic solenoid. The plasma is confined radially by a magnetic field B_o large enough to maintain constant density throughout the heating period. Heating occurs by inverse bremsstrahlung absorption of CO_2 laser radiation of intensity I (watt/cm^2) which will be taken to have a $t^{2/3}$ time dependence and constant beam diameter. Since the initial temperatures $T^o_{e,i}$ are much less than final values they will subsequently be ignored.

For concreteness, we shall take specific values of $n = 3 \times 10^{17}$ cm^{-3}, $\ell n\Lambda = 10$ and a plasma column $\sim$ 1 cm diameter confined radially by a magnetic field B_o = 500 kgauss which approximate reactor conditions. Since the plasma is well underdense, reflection and stimulated Brillouin backscattering losses are taken to be negligible (Offenberger et al, 1976). Plasma losses at z = 0 are ignored in this study though they are obviously important in a reactor.

With these approximations the electron and ion heating equations can be written as

[1] $$\frac{\partial T_e}{\partial t} = \frac{2}{3}\frac{k_\nu I}{nK_B} - \frac{(T_e - T_i)}{\tau_{ei}} + \frac{2}{3}\frac{\nabla .}{nK_B}(K_e \nabla T_e)$$

[2] $$\frac{\partial T_i}{\partial t} = \frac{(T_e - T_i)}{\tau_{ei}} + \frac{2}{3}\frac{\nabla .}{nK_B}(K_i^{\perp} \nabla T_i)$$

where the absorption constant (Dawson and Oberman, 1962) for CO_2 laser radiation $k_\nu \simeq 10^{-35}\, n^2 \ell n\Lambda / T_e^{3/2}$ cm^{-1}; the electron-ion energy equipartition time (for an average ion mass of 2.5 M_H) $\tau_{ei} \simeq 7.8 \times 10^8\, T_e^{3/2}/n\ell n\Lambda$ sec; the electron thermal conductivity (parallel to B_o) $K_e \simeq 310\, T_e^{5/2}/\ell n\Lambda$ watt cm^{-1} eV^{-1}; the ion thermal conductivity (perpendicular to B_o, for collision frequency << cyclotron frequency) $K_i^{\perp} \simeq 9.8 \times 10^{-22}\, n^2 \ell n\Lambda / B_o^2 T_i^{\frac{1}{2}}$ watt cm^{-1} eV^{-1}; Boltzmann's constant $K_B = 1.6 \times 10^{-19}$ J eV^{-1}; units for n, T and B_o are in cm^{-3}, eV and gauss. These transport coefficients can be found in Spitzer's monograph (1962).

We shall further approximate these equations by neglecting axial electron thermal conduction losses and radial ion thermal conduction losses.

The validity of neglecting these terms has been discussed by previous authors (Steinhauer and Ahlstrom, 1975b; Yuen et al, 1975; Vagners et al, 1975) and we shall for the moment also ignore them. Further discussion of these approximations will be given later in this paper.

The resulting set of equations to be solved for T_e and T_i can therefore be written as

[3] $$\frac{\partial T_e}{\partial t} = \frac{aI}{T_e^{3/2}} - \frac{b(T_e - T_i)}{T_e^{3/2}}$$

[4] $$\frac{\partial T_i}{\partial t} = \frac{b(T_e - T_i)}{T_e^{3/2}}$$

where a = 126 cm^2 $joule^{-1}$ $eV^{5/2}$, b = 3.8 x 10^9 $eV^{3/2} \cdot sec^{-1}$, I = watt cm^{-2}. The auxiliary equation for the laser intensity

[5] $$\frac{\partial I}{\partial z} = - k_\nu I$$

has a solution

[6] $$I(z,t) = I(z=0,t)\, e^{-\int_0^z k_\nu \, dz}$$

Since it has been shown previously that the plasma self-regulates absorption with an optical thickness of $\sim$ unity (Burnett and Offenberger, 1976), propagation of the heating front up to z = L implies

[7] $$I(z,t) = I(z=0,t)\, e^{-k_\nu L} \simeq I(z=0,t)/e$$

is nearly independent of z and is a function of t only. In fact, the variation in intensity from z=0 to z=L is less than a factor of two and is subsequently ignored. This considerably simplifies the analysis and it will be shown that our solution is fully consistent with this approximation of a step heating wave.

III. SOLUTION FOR INTENSITY $\sim t^{2/3}$

We can eliminate T_i from Eqn's. [3],[4] to obtain an equation for T_e, which, in dropping spatial dependence is a function of time only

[8] $$\frac{d^2 T_e}{dt^2} + \frac{3}{2}\frac{1}{T_e}\left(\frac{dT_e}{dt}\right)^2 + \frac{2b}{T_e^{3/2}}\frac{dT_e}{dt} = \frac{a}{T_e^{3/2}}\frac{dI}{dT} + \frac{abI}{T_e^3} .$$

This equation, in fact,has a remarkable property. Even though it is a nonlinear equation for T_e, it possesses a unique solution for I varying as $t^{2/3}$. Thus, since in first approximation a $t^{2/3}$ dependence optimizes heating, we are in a position to vary the magnitude of I with a $t^{2/3}$ temporal characteristic and attempt to minimize energy requirements in heatings ions to 10 keV temperatures. We therefore take

[9] $$I(t) = I_o t^{2/3}$$

whereby it is trivial to show that T_e varies as $t^{2/3}$ as well. To calculate the constant of proportionality we take

[10] $$T_e = B^{2/3} t^{2/3}$$

or defining $y = B^{1/3}$ and substituting in [8] the electron temperature for a given I_o is determined by

[11] $$y^8 + 3by^5 - \frac{3}{2} aI_o y^3 - \frac{9}{4} abI_o = 0 \quad .$$

Solutions for y and B, along with other parameters to be discussed later, are given in Table I for various intensity values I_o.

With T_e solved, it is a simple matter to show that the general solution for T_i is given by

[12] $$T_i(t) = \frac{T_e(t)}{1 + \frac{2}{3}\frac{B}{b}} = \frac{T_e(t)}{\alpha}$$

i.e., no assumption on the ratio of T_e/T_i need be made. This turns out to be highly desirable from the point of view of heating ions with less energy requirements than for the case $T_e/T_i = 3$. The parameter α is a unique function of intensity I_o and can have values, which are also given in Table I, less than 3. Evidently, for low heating rates α can, in fact, approach unity.

It is now straightforward to calculate the energy flux ϕ required to heat the ions to any desired temperature. Integration of [9] gives

[13] $$\Phi = \frac{3}{5} I_o t^{5/3}$$

which for a specific value of $T_i = T_e/\alpha$ defines a time $t = \alpha^{3/2}\ T_i^{3/2}/B$ and therefore flux

[14] $$\Phi = \frac{3}{5}\ \frac{I_o \alpha^{5/2} T_i^{5/2}}{B^{5/3}}$$

Flux magnitudes and corresponding times to heat ions to temperatures of 1 and 10 keV are also given in Table I for various intensity values.

Evidently there is a considerable premium on heating electrons too rapidly which inhibits efficient collisional transfer of energy to the ions. In raising the incident intensity a factor of 50, the time required to achieve a given ion temperature is reduced a factor of less than 3, but the energy requirement is increased nearly a factor of 10. Thus low intensity heating is highly desirable. The intensity cannot be reduced indefinitely, however, since there is a lower limit set by the requirement that no shock waves be generated, i.e. the heating wave be supersonic (Burnett and Offenberger, 1976). This will be discussed further in section V.

IV. VELOCITY OF HEATING FRONT

Since the absorption length for the CO_2 laser radiation is very short at the head of the propagating heat wave ($\sim$ 1 mm for $n = 3 \times 10^{17}$ cm^{-3} and $T = 1$ eV), the velocity of expansion into the unheated gas is given by energy conservation $\rho V \varepsilon = I$, where ρV = mass flux, ε = internal energy per unit mass and I = intensity of laser radiation (since all is absorbed in the heat front). Thus we can write

[15] $$V = \frac{dL}{dt} = \frac{(\gamma - 1)\ I}{nK_B\ T_e\ (1 + \frac{1}{\alpha})}$$

where γ = ratio of specific heats, L = plasma column length and other parameters are as defined before.

For an intensity given by [9] and temperature by [10], the velocity is constant in time

[16] $$V = \frac{(\gamma-1)\, I_o}{nK_B\, (1 + \frac{1}{\alpha})\, B^{2/3}}$$

and the plasma length L = Vt. Values for V, L_{max} and I_{max} are given in Table II for times appropriate to heating ions to 1 and 10 keV temperatures The vacuum intensity is ∿ e times greater (a more correct value will be determined shortly). These results show that length requirements are substantially reduced for lower intensity heating. Though stimulated Brillouin backscatter may not be important for long pulse, multimode laser beams, clearly lower intensity is desirable as insurance. The advantage to be had in using reasonably low intensity radiation for minimizing plasma length, laser energy required and potential backscatter is evident.

We can derive an alternative expression for the velocity of the front using the self-regulating characteristic of the heated plasma to maintain an optical depth ∿ 1. Thus we take

[17] $$k_\nu L = \beta$$

where $\beta \leqslant 1$ and is to be determined by comparing velocities obtained from Eqn's. [15] and [17] using the same temperature profile (as given by Eqn. [10]). We obtain from

[18] $$\frac{10^{-35} n^2 \ell n\Lambda\, L}{T_e^{3/2}} = \beta$$

a velocity $U = B\beta/9$ for the given n and $\ell n\Lambda$. Values of β required for U = V are given in Table II. Evidently β ≃ constant for all laser intensities with a mean value of 0.67. That this number is essentially constant substantiates analytically the previous numerical evidence for the plasma to maintain a constant optical thickness ∿ unity. This in turn implies that the vacuum laser intensity should be approximately $2I_o$.

We also find that the velocity depends on intensity; empirically from

Table II, a sixth-tenths power law on intensity is inferred. That this is essentially correct can be seen by combining Eqn's. [15] and [18] to obtain

[19] $$V = \frac{dL}{dT} = \frac{(\gamma-1)\ \beta^{2/3}\ I_o\ t^{2/3}}{(10^{-35}\ell n\Lambda)^{2/3} K_B (1 + \frac{1}{\alpha})\ n^{7/3}\ L^{2/3}}$$

or substituting values

[20] $$\frac{dL}{dT} = \frac{2.5\ I_o\ t^{2/3}}{(1 + \frac{1}{\alpha})\ L^{2/3}}$$

which when integrated gives

[21] $$L = Vt = \left[\frac{2.5\ I_o}{(1 + \frac{1}{\alpha})} \right]^{3/5} t \quad .$$

Thus the velocity is constant for a given intensity and varies as $I_o^{3/5}$. Numerical values of velocity from [21] are the same as in Table II, as indeed they must be for parameter $\beta = 0.67$. This intensity variation is in contradistinction to Yuen et al (1975) who find a constant heating front velocity independent of intensity for $I \sim t^{2/3}$.

V. HYDRODYNAMIC LIMITS ON INTENSITY

Limits to the laser intensity which can be employed have been discussed in an earlier work (Burnett and Offenberger, 1976), wherein numerical simulations demonstrate the validity of limiting the intensity from above by $\tau_{ei} < L/V$ for adequate thermalization between ions and electrons and from below by $V > V_s$, where V_s = ion acoustic speed, to prevent shock waves from being formed. We shall show that these criteria can be satisfied for a reasonable but restricted range of intensities. Moreover, it is straightforward to show that the same limits obtain with $k_\nu L = \beta$ used explicitly, as are otherwise found, thus again demonstrating the remarkable self-regulating absorption property a laser heated plasma exhibits.

Using $\tau_{ei} = T_e^{3/2}/b = Bt/b < L/V = t$, the thermalization requirement reduces to $B < b$. From [11], the upper limit on I_o is found to be 7.8×10^{13} watt cm^{-2} sec$^{-2/3}$. Alternatively, if [18] is used, it is simple to show that $\tau_{ei} < L/V$ reduces to $I_o < 8 \times 10^{-3}\ \beta(1 + \frac{1}{\alpha})\ b^{5/3}$. This implies for our parameters $I_o < 8.9 \times 10^{13}$ watt cm^{-2} sec$^{-2/3}$, a result very close to the one above.

Likewise, using our results for temperature and velocity the requirement for supersonic propagation $V > V_s = [\gamma K_B T_e(1 + \frac{1}{\alpha})/M]^{\frac{1}{2}}$, reduces to

[22] $$I_o > 1925\ (1 + \frac{1}{\alpha})^{11/8}\ B^{11/12}\ L^{1/4} .$$

Numerically, we find $I_o \gtrsim 2.1 \times 10^{13}$ watt cm^{-2} sec$^{-2/3}$ for the maximum length of $\sim$ 1 km. On the other hand, using [18] one finds

[23] $$I_o > 7.27 \times 10^9 (1 + \frac{1}{\alpha})^{11/6} \frac{L^{5/9}}{\beta^{1/3}}$$

or numerically $I_o > 1.5 \times 10^{13}$ watt cm^{-2} sec$^{-2/3}$, in good agreement with the other value.

It has been suggested that the optimum heating wave velocity is of the order of the magnetoacoustic speed (which for most of the heating period is simply the Alfven speed, V_A) since the laser beam and column radius would self-regulate by diffraction and self-focusing effects (Burnett and Offenberger, 1976). For higher velocities, $V > V_A$, a region near the front would exist with no density minimum leading to diffraction losses, thereby reducing the intensity and slowing the heating wave. For lower velocities, $V < V_A$, self-focusing would increase the intensity, accelerating the front. From our present results for $t^{2/3}$ heating, we can determine the intensity for which $V \leq V_A$ $[= B_o/(4\pi nM)^{\frac{1}{2}}]$. With parameters as given before, $V_A = 1.3 \times 10^8$ cm sec^{-1} and

$$I_o \leq 9.1 \times 10^6 (1 + \frac{1}{\alpha}) \; B^{2/3} \qquad [24]$$

which implies $I_o \lesssim 2.2 \times 10^{13}$ watt cm^{-2} eV$^{-2/3}$. It is seen that this value is even more restrictive than that required for adequate thermalization and moreover, is essentially the same as that required for supersonic propagation. In fact, $V = V_A$ specifies a unique intensity value for optimum propagation and heating which is seen to be satisfactory from other considerations.

Summarizing then, an intensity varying as $I = I_o \; t^{2/3}$ with $I_o \simeq 2 \times 10^{13}$ may be near optimum for propagating a supersonic heating wave along a magnetically confined plasma column.

VI DISCUSSION

With our values for temperature $T_{e,i}(t)$ and length $L(t)$ we can now estimate how serious the neglect of electron and ion thermal loss terms are. We shall consider the case for $I_o = 2 \times 10^{13}$ for which the absorption term $aI \; T_e^{-3/2}$ can be expressed as $1.6 \times 10^6 \; t^{-1/3}$ eV sec. Electron-ion energy exchange $b(T_e - T_i)T_e^{-3/2}$ can be written as $b(1 - \frac{1}{\alpha}) \; B^{-1/3} \; t^{-1/3} = 7 \times 10^5 \; t^{-1/3}$ eV sec^{-1} which is roughly half the electron absorption rate. Using the result that $k_\nu L = \beta$, the axial electron thermal conduction losses can be approximated by $7.8 \times 10^4 \; T_e^{\frac{1}{2}} = 9 \times 10^7 \; t^{1/3}$ eV sec^{-1}. This implies that thermal conduction losses are less than electron-ion energy exchange for $t < 700$ μsec; moreover, the effect is to smooth out axial temperature gradients rather than lead to a genuine loss of energy out of the column. Particle flow out the ends is a more important loss; the corresponding timescale is $L/2V_{Th} = 500$ μsec at maximum temperature. Any means of stoppering the end losses would be highly desirable.

Ion thermal conduction losses radially are more difficult to approximate analytically since the detailed radial temperature profile is required. An upper bound is given by $2K_i^{\perp} T_i/3nK_B R^2$ which for an effective $R^2 \sim 0.5\ cm^2$ can be expressed as $10^8\ t^{1/3}$ eV sec^{-1}. This loss is exceeded by electron-ion energy transfer for $t < 600$ μsec. In practice, losses will be somewhat below this with correspondingly longer confinement times.

With these limitations on heating times (or the alternative of providing better confinement), we have obtained an exact solution to the coupled electron and ion heating equations for a laser beam intensity varying as $t^{2/3}$. The temperatures are found to have the same dependence on time which is that required for optimizing ion heating. It has been shown that, in general, low intensity heating maximizes T_1/T_e, minimizes the plasma length and minimizes the laser energy required to achieve a given T_i.

From considerations of electron-ion energy relaxation, supersonic heating wave propagation and beam trapping we have found an optimum laser intensity $\simeq 2 \times 10^{13}\ t^{2/3}$ watt cm^{-2}. The electron temperature in this case is only 28% greater than that of the ions. In general, for $I \sim t^{2/3}$, the heating wave velocity is independent of time and varies as $I_o^{3/5}$.

Finally, we have seen that the heating wave propagation is consistent with a model in which the heated plasma behind the front maintains an optical thickness of 0.67.

It is perhaps worth remarking that these calculations show heating of a D-T plasma to kilovolt temperatures can be achieved with near state-of-the-art long pulse CO_2 lasers. A 35 m long plasma confined radially by less than 200 kgauss fields, when heated by a 300 kjoule, 30 μsec CO_2 laser pulse, would provide an attractive radiation source of neutrons and X-rays.

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TABLE I

I_o(watt cm^{-2} $sec^{-2/3}$)	y	B(10^9 $eV^{3/2}$ sec^{-1})	$\alpha = T_e/T_i$	t(μsec)		Φ(10^6 J cm^{-2})	
				1 keV	10 keV	1 keV	10 keV
10^{13}	1005	1.01	1.18	40	1264	0.28	89
2×10^{13}	1162	1.57	1.28	29	917	0.33	104
5×10^{13}	1415	2.83	1.50	20.5	647	0.46	145
10^{14}	1645	4.45	1.78	16.9	534	0.67	211
2×10^{14}	1920	7.08	2.24	15	474	1.1	346
5×10^{14}	2355	13.1	3.30	14.5	457	2.6	814

TABLE II

I_o(watt cm^{-2} $sec^{-2/3}$)	V(10^8 cm sec^{-1})	β	L_{max}		I_{max}(10^{11}watt cm^{-2})	
			1 keV(m)	10 keV(km)	1 keV	10 keV
10^{13}	0.75	0.67	30	0.95	0.12	1.2
2×10^{13}	1.2	0.69	35	1.1	0.19	1.9
5×10^{13}	2.1	0.67	43	1.4	0.37	3.7
10^{14}	3.3	0.67	56	1.8	0.66.	6.6
2×10^{14}	5.2	0.66	78	2.5	1.2	12
5×10^{14}	9.6	0.66	139	4.4	3.0	30