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ABSTRACT

Although resistive diffusion is much slower than the growth of resistive instabilities, the conventional neglect of diffusion in tearing mode calculations is incorrect. The proper criterion for neglect of diffusion in resistive instability calculations, which is not satisfied for tearing modes, is $\omega \gg v/\delta$ where v is the resistive diffusion velocity and δ is the resistive layer thickness. The effect of diffusion is calculated in the limit of large and small $\omega\delta/v$ for the plane slab model, and new expressions for growth rate and stability boundary are obtained. The diffusion appears to have a stabilizing effect.

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I. INTRODUCTION

The resistive tearing instability of an incompressible plasma was first systematically investigated by Furth, Killeen, and Rosenbluth[1] (FKR). This calculation, as well as all subsequent analytic work[2,3], assumed a stationary equilibrium ($V_0 = 0$) ignoring magnetic field diffusion ($\partial B_0/\partial t = 0$). However, field diffusion is significant during the skin formation and current contraction phases of a tokamak discharge and instabilities occurring during these phases may be related to the tearing mode[4,5]. In a numerical simulation of the diffuse pinch Robinson[6] found that the diffusion velocity modified the growth of tearing modes at finite resistivity. In this paper, we investigate the effect of classical resistive diffusion on the tearing mode and show its effect to be significant even in the limit of vanishingly small resistivity.

FKR used the 'plane slab' model, in which the equilibrium depends only on y, the magnetic field is $\hat{x}\, B_{0x}(y) + \hat{z}\, B_{0z}$ and perturbations take the form $u(y) \exp i(k_x x + k_z z)$. They showed that in the high conductivity limit ($\eta \to 0$) resistivity is important only in the narrow layer around $\vec{k} \cdot \vec{B} = 0$; outside this layer the perturbation follows the ideal ($\eta = 0$) magnetohydrodynamic equations. By matching the solution within the resistive layer to the 'outer' magnetohydrodynamic solutions, FKR found resistive modes with growth rates much faster than normal resistive diffusion. However, they also showed that the most important resistive

modes--the tearing modes--are stable if $\Delta' < 0$, where Δ' is the change in logarithmic derivative of the ideal solution across the resistive layer.

FKR's assumption of a stationary equilibrium implies that the equilibrium satisfies

$$\nabla \times \eta \, (\nabla \times \vec{B}_0) = 0 \quad . \tag{1}$$

However, zero diffusive velocity was assumed for more general equilibria in their calculation because, for small η, the diffusive time scale $\tau_R = a^2/\eta \sim a/V_0$ is much longer than the resistive growth time of the instability, $\omega^{-1} \sim \eta^{-3/5}$, i.e., $\omega\tau_R >> 1$. Here a is the characteristic equilibrium scale length.

In this paper, we point out that the equilibrium diffusion velocity $\vec{V}_0$ (or its equivalent $\partial\vec{B}_0/\partial t$ if the equilibrium is nonstationary) <u>does</u> influence the growth of resistive instabilities even though these are rapid compared to diffusion times. We find that resistive layer motion is important if $\omega\tau_R = \mathcal{O}(a/\delta)$ where δ is the perturbation gradient scale length (or the resistive layer thickness). In other words, the importance of $\vec{V}_0$ is measured not by the normal resistive diffusion rate V_0/a but by the rate V_0/δ, i.e., the time of diffusion across the resistive layer. For tearing modes $\delta \sim \eta^{2/5}$ so that $V_0/\delta \sim \eta^{3/5}$ and is of the <u>same order</u> as the resistive instability growth rate (when $\partial B_0/\partial t \neq 0$, an equivalent statement is that field diffusion causes the singular surface to move a distance δ in the growth time ω^{-1}).

One consequence of including the diffusion velocity $\vec{V}_0$ in the calculation of resistive instabilities is that, even in the limit $\eta \to 0$, the stability criterion for tearing modes is no longer $\Delta' < 0$. The growth rates of the modes are also affected.

In Section II, the basic equations for the plane slab, resistive fluid model are described. Section III reviews the high conductivity matching procedure and the resistive layer in the high conductivity limit. Section IV discusses the solution in the outer and inner regions while Section V discusses the eigenvalue equation for the resistive modes in the presence of diffusion and gives some results including a new criterion for instability.

II. BASIC EQUATIONS

We consider the usual 'plane-slab' or sheet pinch configuration in which equilibrium quantities are independent of x and z, the density gradient is $\rho(y)$, the magnetic field $\vec{B}_0 = \hat{z}\, B_{0z} + \hat{x}\, B_{0x}(y)$, and the flow velocity $\vec{V}_0 = V_0\hat{y}$.

Following FKR, we assume the fluid perturbations to be incompressible and to follow the simple scalar pressure equation

$$\rho \frac{d\vec{v}}{dt} = \vec{j} \times \vec{B} - \nabla p \quad .$$

The resistivity is taken to be uniform and the field equation, derived from the assumed Ohm's law, is

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}) - \frac{\eta}{4\pi} \nabla \times (\nabla \times \vec{B}) \quad .$$

For the moment, we assume that equilibrium is maintained in a steady state ($\partial/\partial t = 0$) against resistive diffusion by appropriate sources. (As we shall indicate later, a non-steady-state equilibrium, $\partial B/\partial t \neq 0$, can be dealt with by an appropriate reinterpretation of V_0.) Then the steady state is given by

$$\nabla \times (\vec{V}_0 \times \vec{B}_0) = \frac{-\eta}{4\pi} \nabla^2 \vec{B}_0 \tag{2}$$

or in the plane-slab by

$$V_0 \frac{\partial B_{0x}}{\partial y} = \frac{\eta}{4\pi} \frac{\partial^2 B_{0x}}{\partial y^2} \quad . \tag{3}$$

As described in the introduction, our main task is to determine the significance of V_0 for the theory of resistive tearing modes.

Introducing perturbations about the stationary equilibrium, writing $\vec{B} = \vec{B}_0 + \vec{B}_1$, $\vec{V} = \vec{V}_0 + \vec{V}_1$ with $V_1 \sim V(y) \exp [i (k_x x + k_z z) + \omega t]$ the linearized equations are

$$\omega \vec{B}_1 = \nabla \times (\vec{V}_1 \times \vec{B}_0) + \nabla \times (\vec{V}_0 \times \vec{B}_1) - \frac{\eta_0}{4\pi} \nabla \times (\nabla \times \vec{B}_1)$$

$$\rho_0 \nabla \times (\omega \vec{V}_1 + \vec{V}_0 \cdot \nabla V_1) = \frac{1}{4\pi} \nabla \times [\vec{B}_0 \cdot \nabla) \vec{B}_1 + (\vec{B}_1 \cdot \nabla) \vec{B}_0]$$

$$\nabla \cdot \vec{V}_1 = \nabla \cdot \vec{B}_1 = 0 \quad .$$

From this set, a pair of equations can be separated which involve only B_{y1} and V_{y1}. The remaining quantities are not needed for the stability analysis. Again, following FKR, we introduce dimensionless variables

$$\psi = B_{y1}/B \quad , \qquad W = - i\, kV_y\, \tau_R \quad , \qquad F = (\underline{k} \cdot \underline{B})/kB \quad ,$$

$$\alpha = ka \quad , \qquad p = \omega\tau_R \quad , \qquad C = V_0 \tau_R / a \quad ,$$

$$S = \tau_R/\tau_H \quad , \qquad y = a\mu \quad , \qquad \tau_R = 4\pi a^2/\langle\eta\rangle \quad ,$$

$$\tau_H = a(4\pi\langle\rho\rangle)^{1/2}/B \quad , \qquad \hat{\rho} = \rho_0/\langle\rho\rangle \quad , \qquad \hat{\eta} = \eta_0/\langle\eta\rangle \quad .$$

Then τ_R and τ_H are the resistive and hydromagnetic time scales and the quantities $\langle\rho\rangle$ and $\langle\eta\rangle$ are measures of the mass density and resistivity. For our analysis we take $\langle\rho\rangle = \rho_0$ and $\langle\eta\rangle = \eta_0$. In these scaled variables, the system reduces to

$$\psi + \frac{FW}{p} + \frac{C\psi'}{p} = \frac{1}{p}(\psi'' - \alpha^2 \psi) \tag{4}$$

$$\psi'' - \alpha^2\psi - \frac{F''}{F}\psi = \frac{1}{S^2\alpha^2 F}\{C(W''' - \alpha^2 W') + p(W'' - \alpha^2 W)\} \tag{5}$$

and from Eq. (3), the equilibrium diffusion velocity becomes

$$C = F''/F' \quad . \tag{6}$$

As one would expect from the introductory remarks, the diffusion velocity V_0 is significant only in the thin resistive layer, over which it can be regarded as uniform. To avoid superfluous complications, we have therefore treated C as uniform in the above equations.

III. THE HIGH CONDUCTIVITY LIMIT

We are interested in the high conductivity limit $\eta \to 0$ or $S \to \infty$ (for thermonuclear plasma $S \sim 10^{10}$ and in current tokamak experiments $S \sim 10^{7}$). In this limit, FKR found resistive tearing modes with growth rates $p \sim S^{2/5}$ and we have implicitly confined our attention to these modes by ignoring gravitation and gradients of resistivity, which can lead to other resistive modes.

As $S \to \infty$, Eqs. (4) and (5) reduce to the ideal magnetohydrodynamic equations

$$(\psi + \frac{FW}{p}) = 0 \tag{7}$$

$$\psi'' - \alpha^2\psi - \frac{F''}{F}\psi = 0 \tag{8}$$

everywhere except in the neighborhood of $F = 0$ (i.e., $\vec{k} \cdot \vec{B} = 0$). Solutions of the ideal equations, satisfying the boundary condition $\psi = 0$ at the boundaries of the slab will exhibit a discontinuity in logarthmic derivative

$$\Delta' = [\psi_+'/\psi_+ - \psi_-'/\psi_-] \tag{9}$$

across the surface $\vec{k} \cdot \vec{B} = 0$. The behavior of resistive modes is determined by the requirement that the solution of the full resistive equations in the 'inner' resistive layer around $\vec{k} \cdot \vec{B} = 0$ should properly match

to this discontinuity Δ' in the "outer" solutions. Hence, the problem is reduced to calculating the change in logarithmic slope $\Delta_i'(p)$ of the resistive solution across the resistive layer. The growth rate is then determined by the dispersion relation $\Delta_i'(p) = \Delta'$.

As $\eta \to 0$, the width of the resistive layer $\delta \to 0$, the growth rate $p \to \infty$ and $W/\psi \to \infty$. One must therefore introduce appropriate scaled variables to calculate Δ_i' in this limit. The full tearing mode ordering written in terms of the gradient scale length δ requires that

$$p \sim \delta^{-1} \quad , \quad \psi \sim 1 \quad , \quad W \sim \delta^{-2} \quad , \quad \alpha \sim 1 \quad , \quad C \sim 1$$

$$F \sim \delta \quad , \quad F' \sim 1 \quad , \quad F'' \sim 1 \quad , \quad S^2 \sim \delta^{-5} \quad , \quad \frac{\partial}{\partial \mu} \sim \delta^{-1}.$$

Then the equations in the resistive layer become

$$\psi_0'' = 0 \tag{10}$$

$$\psi_1'' = p(\psi_0 + \frac{FW}{p}) \tag{11}$$

$$\frac{1}{S^2\alpha^2}(CW''' + pW'') - F^2W = pF\psi_0 - F''\psi_0 \quad . \tag{12}$$

The first important point of our work is already evident from these equations. In the tearing modes the effect of the diffusion velocity C is <u>not</u> of higher order in the δ (or η) expansion than other quantities, and so must be retained. In this connection it is important to note that

the order of the diffusion velocity is fixed by the equilibrium condition (3) and cannot be chosen arbitrarily. (One may also note that for the g-modes of FKR, the diffusion velocity <u>is</u> a higher order contribution in the η-expansion and for such modes V_0 may indeed be neglected.)

IV. FUNCTIONAL BEHAVIOR

A. THE OUTER REGION

In the outer region the solutions are given by the ideal magnetohydrodynamic equation (8). All we need note here is that as one approaches the resistive layer the ideal magnetohydrodynamic solution behaves like

$$\psi \sim A\mu + B\,[1 + C(\mu \log \mu - \mu)]$$

so that $(\psi'/\psi) \sim A/B + C \log \mu$. The inner solution will later be shown to match this behavior. Although (ψ'/ψ) is divergent as $\mu \to 0$, the difference Δ' between (ψ'/ψ) on the two sides of the resistive layer, remains finite and is given by the coefficients A, B, of the magnetohydrodynamic solution.

B. THE INNER REGION

In the inner region, the lowest order solution is $\psi_0 = 1$ (the ordering ensures that matching will only be possible if $\psi_0' \equiv 0$). Then the quantity we need, Δ_i', is given by

$$\Delta_i' = \int_{-\infty}^{\infty} (p + FW)\, d\mu \quad . \tag{13}$$

The problem is to calculate W and hence Δ_i'. Before attempting this, we note from (12) that as $\mu \to \infty$

$$FW \sim -p + \frac{C}{\mu}$$

so that ψ'/ψ varies as $C \log \mu$ + constant. This is exactly the behavior needed to match the outer solution and the divergent logarithmic terms may be ignored provided matching is carried out at points equidistant from $\mu = 0$. That is, we define

$$\Delta_i' \equiv \lim_{x \to \infty} \int_{-x}^{x} (p + FW)\, d\mu \quad . \tag{14}$$

We now consider the resistive layer equation in more detail. Introducing new variables $\mu \equiv \delta\theta$, and

$$W \equiv p^{3/4} (\alpha^2 S^2/F'^2)^{1/4}\, h \quad ,$$

with

$$\delta^4 = (p/\alpha^2 F'^2 S^2)$$

we obtain

$$\lambda \frac{d^3h}{d\theta^3} + \frac{d^2h}{d\theta^2} - \theta^2 h = \theta - \lambda \tag{15}$$

$$\Delta_i'(p) = |\alpha F' S|^{-1/2}\, p^{5/4} \int_{-\infty}^{\infty} (1 + \theta h)\, d\theta \tag{16}$$

so

$$\Delta' = \Delta_i'(p) = |\alpha F'S|^{-1/2}\, p^{5/4}\, G(\lambda) = \delta p\, G(\lambda) \tag{17}$$

where from Eq. (6) we have defined

$$\lambda \equiv C/\delta p$$

$$= \frac{|(\alpha F'S)|^{1/2}}{p^{5/4}} \left|\frac{F''}{F'}\right| \quad .$$

Note from our ordering that $p\delta = \mathcal{O}(1)$ and $C = \mathcal{O}(1)$ so that λ is also $\mathcal{O}(1)$. Then since $p \sim S^{2/5}$, the growth rate given by (17) is of the same order as that of the usual tearing modes.

V. THE FUNCTION $G(\lambda)$, GROWTH RATES, AND CRITICAL Δ'

The calculation of Δ_i', or equivalently of $G(\lambda)$, when $\lambda = 0$ was carried out by FKR[1] and other authors[3]. (In fact, in these calculations λ was retained in the term on the right side of Eq. (15), but $\lambda h'''$ was omitted from the left side. Since only the antisymmetric part of $h(\theta)$ contributes to Δ', this omission has the erroneous effect of making Δ' appear independent of λ!) We shall not repeat these calculations for $\lambda = 0$, but simply quote the result[1,2,7]. This is equivalent to

$$G(0) = 2\pi \frac{\Gamma(3/4)}{\Gamma(1/4)} \tag{18}$$

which, in conjunction with (17) leads to the well-known results that tearing modes are stable when $\Delta' < 0$ and given by

$$p^{5/4} = [\Gamma(1/4)/2\pi\Gamma(3/4)]\ (\alpha F'S)^{1/2}\ \Delta'$$

when $\Delta' > 0$.

The calculation of $G(\lambda)$ when $\lambda \neq 0$ is more difficult and we consider only the limits of small and large λ. The latter will provide a new criterion for instability and the former a modification to the growth rates of unstable modes.

A. EXISTENCE OF CRITICAL Δ'

That tearing modes must be stable even in the presence of the diffusion velocity, if Δ' is less than some critical value Δ'_c can be shown as follows. Consider the homogeneous equation [corresponding to (12)],

$$LW \equiv \frac{1}{S^2\alpha^2}(cW''' + pW'') - F^2W = 0 \quad . \tag{19}$$

We multiply Eq. (19) by W^* and integrate over all μ to obtain

$$\frac{c}{2S^2\alpha^2}\int_{-\infty}^{\infty}(W''\,W^{*\prime} - W'\,W^{*\prime\prime})\,d\mu + \frac{p}{S^2\alpha^2}\int_{-\infty}^{\infty}|W'|^2\,d\mu$$
$$+\int_{-\infty}^{\infty}F^2|W|^2\,d\mu = \frac{1}{S^2\alpha^2}\left(CW''\,W^* - \frac{C}{2}|W'|^2 + pW'W^*\right)\Bigg|_{-\infty}^{\infty} \quad . \tag{20}$$

We see by taking the real part of Eq. (20), that there are no solutions to Eq. (19) which decay as $|\mu| \to \infty$ when Re $p > 0$. Hence the operator L has an inverse, and bounded solutions exist for Eq. (12). It has been previously shown that for $|\mu| \to \infty$,

$$p + FW \sim \frac{c}{\mu} + O\left(\frac{1}{\mu^2}\right) \quad ,$$

so that since Eq. (14) defines symmetric limits the function $\Delta_i(p)$ must be bounded for all Re $p > 0$. (Except possibly when $|p| \to \infty$; however in this limit we know that $G(\lambda)$ is given by Eq. (18).) Then the least of these bounds represents Δ'_c with $\Delta'_i(p) > \Delta'_c$ for all unstable modes.

B. SOLUTION FOR SMALL λ

We now turn to the evaluation of the central function $G(\lambda)$ and consider first the case when λ is small (i.e., C small). In this case we can expand h in powers of λ,

$$h = h_0 + \lambda h_1 + \lambda^2 h_2 + \ldots$$

and obtain from Eq. (15)

$$h_0'' - \theta^2 h_0 = \theta$$

$$h_1'' - \theta^2 h_1 = - h_0''' - 1$$

$$h_2'' - \theta^2 h_2 = - h_1''' \quad .$$

We note that h_0 and h_2 are odd functions of θ and that h_1 is even. Hence the latter does not contribute to Δ' or the growth rate. However it does contribute to the perturbed field structure and produces an asymmetric shift in the perturbed field across the singular surface, somewhat similar to that of the rippling mode[1]. The resulting small λ expansion for $G(\lambda)$ becomes,

$$G(\lambda) = 2\pi \frac{\Gamma(3/4)}{\Gamma(1/4)} + \lambda^2 \int_{-\infty}^{\infty} \theta \, h_2 \, d\theta + O(\lambda^4) \quad . \tag{21}$$

The hierarchy equations have been integrated numerically. In each integration an intrinsic numberical instability, proportional to exp $(\theta^2/2)$, is eliminated by methods developed by Wang and Dobrott[8]. The functions h_0, h_1, and h_2 are shown in Fig. 1. A further integration yields $G(\lambda)$ and hence, finally,

$$\Delta_i'(p) = 2.12\ p\delta + 0.28\ \frac{C^2}{p\delta}$$

where implicitly, $C^2/p\delta << 1$. This result can be written

$$\Delta_i'(p) = 2.12\ \frac{p^{5/4}}{|\alpha F'S|^{1/2}} + 0.28\ \frac{(\alpha F'S)^{1/2}}{p^{5/4}}\ C^2 \quad . \tag{22}$$

The growth rate, obtained by matching $\Delta_i'(p)$ to the outer solution, is seen to be smaller than that of the $C = 0$ case.

C. SOLUTION FOR LARGE λ

The case of large λ arises when $p \to 0$ and so determines an instability boundary, replacing the $\Delta' > 0$ condition of earlier work. One cannot immediately set $\lambda \to \infty$ in Eq. (15) as the resulting function $h(\theta)$ would not have the correct behavior as $|\theta| \to \infty$. To obtain the correct behavior, we again introduce scaled variables

$$\theta = |\lambda|^{1/5}\ z \quad , \qquad h = |\lambda|^{3/5}\ f \quad .$$

Then we find

$$f''' - z^2 f = -1 \quad . \tag{23}$$

Incidentally, it should be noted that this is exactly the equation which one obtains by setting $p \to 0$ (or by treating p as higher order in δ), in the original Eq. (12). In the same limit of large λ, or small p, $G(\lambda)$ is given by

$$G_0 = \lim_{\lambda\to\infty} G(\lambda)/|\lambda| = \lim_{\lambda\to\infty} \int_{-x}^{x} z\, f(z) \tag{24}$$

and so is proportional to $|\lambda|$. Note that the next term is $O(|\lambda|^{1/5})$. Then Δ' is given as

$$\Delta' = p\delta\, |\lambda| \lim_{x\to\infty} \int_{-x}^{x} z\, f(z)\, dz + O\left(|\lambda|^{-4/5}\right)$$

or

$$\lim_{\lambda\to\infty} \Delta' = \Delta'_\infty = \left|\frac{F''}{F'}\right| \lim_{x\to\infty} \int_{-x}^{x} z\, f(z)\, dz \quad . \tag{25}$$

Although the large λ behavior for Δ' is not expressible as an even power series in λ, we see that it depends upon the magnitude of λ only.

The value Δ'_∞ may be found by the application of Fourier transforms. Taking the transform of Eq. (23), we obtain

$$\frac{d^2 g}{dk^2} + ik^3 g = -2\pi\,\delta(k) \quad .$$

Here

$$g(k) \equiv \int_{-\infty}^{\infty} e^{ikz}\, f(z)\, dz$$

is a continuous function, but has a discontinuous derivative at $k = 0$. It can be written

$$g(k) = \begin{cases} A\, U(k)/U(0) \quad , & k > 0 \\ A\, U^*(-k)/U(0) \quad , & k < 0 \end{cases} \tag{26}$$

with $U(k)$ expressed in terms of the first Hankel function,

$$U(k) = k^{1/2}\, H^{(1)}_{1/5}\left(\frac{2}{5}\, e^{i\pi/4}\, k^{5/2}\right) \quad .$$

The coefficient A is determined by

$$2A \cdot \mathrm{Re}\left(\frac{1}{U}\frac{dU}{dk}\right)_{k=0_+} = -2\pi \quad . \tag{27}$$

Then Δ'_∞ can be expressed in terms of Eq. (26) by noting that

$$\Delta'_\infty = \left|\frac{F''}{F'}\right| \lim_{x\to\infty} \frac{1}{2\pi}\int_{-x}^{x} dz \int_{-\infty}^{\infty} dk\ zg(k)\ e^{-ikz}$$

or

$$\Delta'_\infty = \frac{i}{\pi}\left|\frac{F''}{F'}\right| \lim_{x\to\infty} \int_{-\infty}^{\infty} dk\ g(k)\ \frac{\partial}{\partial k}\ \frac{\sin kx}{k} \quad .$$

We recall that $\partial g/\partial k$ is discontinuous at $k = 0$. Then integration by parts yields

$$\Delta'_\infty = \left|\frac{F''}{F'}\right| A\ I_m \left(\frac{1}{U}\frac{dU}{dk}\right)_{k=0_+}$$

where from Eq. (27) we find

$$\Delta'_\infty = -\ \pi \left|\frac{F''}{F'}\right| \tan\left[\arg\left(\frac{1}{U}\frac{dU}{dk}\right)_{k=0_+}\right]$$

or

$$\Delta'_\infty = \pi \left|\frac{F''}{F'}\right| \tan\ (\pi/10) \quad . \tag{28}$$

Recalling that $\lambda \to \infty$ corresponds to $p \to 0$, we conclude that Eq. (28) defines a new threshold for unstable tearing modes - replacing the previous threshold $\Delta' = 0$. Note that the diffusion velocity again has a stabilizing influence, as it did in the case of small λ in Eq. (22).

VI. CONCLUSIONS

Although it apparently has a much longer time scale than that for resistive instabilities, the equilibrium diffusion velocity enters the calculation of tearing mode growth rates in the same order as other quantities. Consequently, even in the limit $\eta \to 0$, tearing modes are affected by the fact that the equilibrium of a resistive fluid is nonstatic.

When this effect is taken into account, we find that the growth rates of tearing modes, in the plane slab model, are less than those given by FKR and others. Also, the threshold of instability at $p = 0$ becomes $\Delta' > \Delta'_\infty$ (where Δ'_∞ is given by Eq. 28) instead of $\Delta' > 0$. (Unfortunately, although we can conclude that unstable modes arise when $\Delta' > \Delta'_\infty$ we can no longer conclude that all tearing modes are stable when $\Delta' < \Delta_\infty$. The argument[1] that there are no overstable tearing modes is no longer valid when $F'' \neq 0$.)

At this point, we should remark on the situation when the equilibrium is not stationary and $\partial B_0/\partial t \neq 0$. Since the influence of this change, like that of V_0, is important only in the resistive layer, it can be incorporated by carrying out the resistive layer calculation in a moving reference frame in which locally $\partial B/\partial t = 0$. The appropriate reference frame has velocity $-V_0$, and the results obtained above are, therefore, equally valid for this case.

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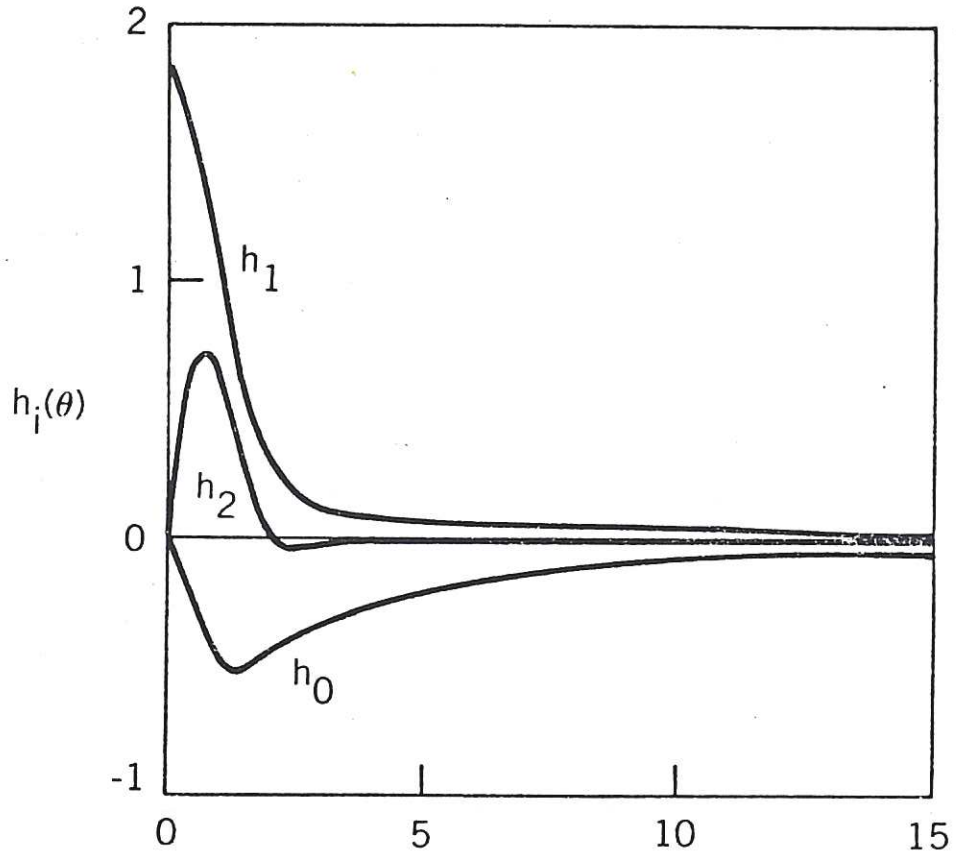

FIG. 1. $h_i(\theta)$ versus θ for $i = 1, 2, 3$. h_0 and h_2 are even with respect to θ, and h_1 is odd.