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ABSTRACT

Unlike classical mechanics, the Copenhagen interpretation of quantum mechanics does not provide an objective spacetime picture of the actual history of a physical system. This paper suggests how the conceptual foundations of quantum mechanics can be reformulated, without changing the mathematical content of the theory or its detailed agreement with experiment and without introducing any hidden variables, in order to provide an objective, covariant, Lagrangian description of reality which is deterministic and time-symmetric on the microscopic scale. The basis of this description can be expressed either as an action functional or as a summation over Feynman diagrams or paths. The probability laws associated with the quantum-mechanical measurement process, and the asymmetry in time of the principles of macroscopic causality and of the laws of statistical mechanics, are interpreted as consequences of the particular boundary conditions that apply to the actual universe. The objective interpretation does not include the observer and the measurement process amongst the fundamental concepts of the theory, but it does entail a revision of our ideas of determinism and of time, since in a Lagrangian theory both initial and final boundary conditions on the action functional are required.

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1. INTRODUCTION

Classical mechanics provides an objective conceptual model to describe the time-evolution of any dynamical system; a model which may in principle be extended to include the complete history of the universe or any finite part of this history. Usually the description is expressed geometrically in terms of particle world-lines or field functions in space-time or by some combination of both, for example as in general relativity. According to the Cartesian mechanistic world-picture which has been extensively adopted in scientific practice, human "observers" may in the last resort be regarded as automata which are themselves just an incidental part of the total dynamical system (i.e. the "material world") and are subject to the same physical laws. Such a picture would for instance be normally used nowadays in brain research or in molecular biology. The measurement process can be treated classically in the same way as any other physical process so that it would be entirely consistent to imagine a history in which no observers happened to evolve.

By contrast, it has been generally concluded that the observer and the measurement process necessarily play a distinctive and essential role in quantum mechanics and that no consistent objective conceptual model for the history can be constructed. This lack of an objective description of reality was emphasized by Einstein and has been a topic of discussion and controversy for more than 50 years, (cf. the books and articles by Jammer 1966, 1974, Peierls 1967, Jauch 1968, Ballentine 1970, D'Espagnat 1971, 1976, Mehra 1973, Belinfante 1973, 1975, Scheibe 1973 from which most of the references may be traced).

The conceptual problem can be seen in its most elementary form in the case of a non-relativistic system of particles, described by a Hamiltonian H and a state-vector $|\psi(t)>$ in Hilbert space which satisfies a deterministic Schrödinger's equation

$$i\hbar \frac{d|\psi(t)>}{dt} = H|\psi(t)> \qquad (1)$$

so long as the system remains undisturbed, but which changes discontinuously each time a measurement is made by an observer who must be treated as external to the system itself (Von Neumann 1955, Dirac 1958). This "collapse of the state-vector" is not described within the framework of orthodox quantum mechanics (Wigner 1971a,b) and the standard Copenhagen interpretation must therefore be regarded as essentially subjective, although the division

$$\text{System + External Observer} \qquad (2)$$

is symmetrical between observers and can be made in an arbitrary number of ways, each observer being free to treat any other observer or measuring apparatus either as part of the system or on an equal footing with himself (Von Neumann 1955).

Quantum-mechanical emphasis on the essential role of the observer and of the measurement process appears to conflict with the continuous development of cosmological models from the time of Copernicus onwards, a development during which the earth and its inhabitants have been progressively displaced from their former central position in the universe. A subjective, anthropocentric or even solipsist reference to the individual observer has seemed to reverse this trend. This and other apparent conflicts have led to the formulation and extensive analysis of a number of well-known paradoxes such as those of EPR (Einstein, Podolsky and Rosen 1935), "Schrödinger's cat" (Schrödinger 1935, de Witt 1970, Ross-Bonney 1975) and "Wigner's friend" (Wigner 1963). On the other hand the Copenhagen interpretation has been considered by many people as having an epistemological advantage in concentrating on observable quantities, and it has been seen by some physicists (Wigner 1961, Von Weizsäcker 1973) and philosophers as having a possible relation to the metaphysical problems of mind and consciousness.

Of course, the great majority of physicists have accepted the Copenhagen interpretation for much more practical reasons: firstly because the predictive successes of quantum mechanics have been quite outstanding and no disagreement has been found between the results of experiment and the basic postulates of non-relativistic or of relativistic quantum theory, and secondly because of the conclusion referred to above, that no objective conceptual model for the history $\underline{h}$ can be constructed which does not involve some alteration in these postulates, e.g. by the introduction of "hidden variables".

The intention of this paper is to question this conclusion and to indicate, without as yet attempting a rigorous proof, that it may very well be incorrect. That is, it is suggested that one <u>can</u> construct an objective space-time description, entirely consistent with standard quantum mechanics (i.e. giving identical predictions for the results of experiment and involving no hidden variables in the accepted sense (Belinfante 1973)), which provides a conceptual model either for the complete history of the universe or for any finite part of this history. Observers are again treated as automata which are part of the total dynamical system and are subject to the same physical laws, so that just as in classical mechanics it would be consistent to imagine a history in which they had not evolved. The measurement process is treated in the same way as any other physical process and does not play a distinctive role. The probability laws of the Copenhagen interpretation are to be deduced from the basic postulates of the theory, leading to a reconciliation between this interpretation and a generalized Cartesian world-picture.

A hint that an objective description of quantum mechanical processes might be possible is provided by the EWG (Everett-Wheeler-Graham) relative-state or "many-worlds" interpretation put forward some 20 years ago (Everett 1957, Wheeler 1957, Graham 1970, de Witt 1970, 1971, de Witt and Graham 1971, 1973). The EWG interpretation discards the concept of the "collapse of the state-vector", treats observers as part of the system and derives the probability laws of the Copenhagen interpretation from the basic postulates of the theory, but instead of providing a model for a unique "actual" history $\underline{h}$ of the universe as in classical mechanics it describes a process in which the initial state $|i\rangle$ continually branches into an enormous number of parallel alternative histories $\underline{h}_\alpha$. In the objective interpretation put forward in this paper the history $\underline{h}$ will however be unique.

Although the proposed objective interpretation would enable many of the essential features of the Cartesian world-picture to be retained, it must be emphasized that the conceptual model which it uses is considerably more abstract that the geometrical description provided by classical mechanics, and that it involves a substantial revision of our usual concepts of determinism and of time. It is suggested that it is precisely this revision that is the significant new feature introduced by quantum theory rather than anything to do with the observer, and that it fits quite naturally with relativistic ideas and might provide a suitable framework for the quantization of general relativity. The mathematical formalism to be used is essentially that of Lagrangian quantum mechanics to be briefly discussed in §2.

Instead of the particle world-lines or field functions of the classical theory, the quantum description of the history $\underline{h}$ will be based on an action or "history" functional $\mathcal{H}$. The motivation for this approach may be seen from its classical analogue: if

$$\mathcal{H} \equiv \int_R \mathcal{L}(x)dx \qquad (3)$$

is the integral of the Lagrangian density $\mathcal{L}(x)$ over a 4-dimensional space-time region R with closed boundary S, then the values of physical quantities such as the charge-current vector $J^\mu(x)$ and the energy-momentum tensor $T^{\mu\nu}(x)$ at a point x in a specific history $\underline{h}$ may be represented as functional derivatives of $\mathcal{H}$, subject to prescribed boundary conditions $\underline{b}$ on S. For example the charge-current vector is

$$J^{\mu}(x) = \left(\frac{\delta \mathcal{H}}{\delta A_{\mu,e}(x)}\right)_{A_{\mu,e} = 0} \qquad (4)$$

where $A_{\mu,e}$ is an "external" variation of the electromagnetic potential function which modifies $\mathcal{L}(x)$ but not the boundary conditions on S. In a classical particle model $J^{\mu}(x)$ and $T^{\mu\nu}(x)$ would evidently reduce to sums of δ-functions along the particle world-lines. In this way the usual pictorial description of classical mechanics may be recovered. Other functional derivatives of arbitrary order may readily be constructed, and the action or "history" functional may thus be said to provide a complete description for the part of h that lies within R.

The natural generalization to quantum mechanics of the expression (3) is a history functional

$$\mathcal{H} = - i\hbar \; \ell n \; \langle f | i \rangle \qquad (5)$$

which (apart from the numerical factor) can be thought of as the logarithm of the S-matrix element $\langle f|i\rangle$ between an initial (ingoing) state $|i\rangle_{in}$ at some arbitrary time t_i in the remote past, and a final (outgoing) state $|f\rangle_{out}$ at some arbitrary time t_f in the remote future. A function such as the charge-current vector $J^{\mu}(x)$ may then be constructed in various equivalent ways, e.g. either as a functional derivative of the form (4) or as a normalized matrix product (T-product in general) in the Heisenberg picture:

$$J^{\mu}(x) = \frac{\langle f|j^{\mu}(x)|i\rangle}{\langle f|i\rangle} \qquad (6)$$

where $j^{\mu}(x)$ is the charge-current operator. (It is assumed here that re-normalization has been carried out so that expressions such as (5) and (6) are finite). Similarly for the energy-momentum tensor $T^{\mu\nu}(x)$ and for functional derivatives of higher order. The conservation laws $\partial J^{\mu}/\partial x^{\mu} = 0$, $\partial T^{\mu\nu}/\partial x^{\nu} = 0$ follow as a consequence of the corresponding operator relations, while the denominator in (5) ensures that the integral

$$\int_{\Sigma} J^{\mu} \, d\Sigma_{\mu} \qquad (7)$$

over a space-like surface Σ will be a real integral multiple of the elementary charge e if $|i\rangle$, $|f\rangle$ are eigenstates belonging to the same eigenvalue of the total charge.

In both classical and quantum theory "reality" is thus to be described by the functional derivatives of a history functional $\mathcal{H}$, the main difference between the two theories being how $\mathcal{H}$ is constructed mathematically. In a classical particle model, for example, $J^{\mu}(x)$ would reduce to δ-functions along the discrete particle world-lines as already described, while in the corresponding quantum model it might be represented as a sum over interfering Feynman paths. In both theories $\mathcal{H}$ is determined jointly by the form of the Lagrangian (or Hamiltonian) and by the boundary conditions $\underline{b}$.

The boundary conditions in classical mechanics may be represented in different ways. Instead of using a closed boundary S as in (3), it is more usual to adopt "1-point" boundary conditions in which Cauchy data on any space-like surface Σ_o determine the history $\underline{h}$ both in the future and also in the past of Σ_o. Classical mechanics is deterministic and time-symmetric on the dynamical micro-scale, but it appears from empirical evidence that these boundary data (which are left arbitrary by the dynamical theory) have been chosen by Nature so that the second law of thermodynamics is obeyed, leading to irreversibility on the macro-scale.

The option of using 1-point boundary conditions to define a unique history $\underline{h}$ is not available in quantum mechanics, and our objective interpretation will be said to employ <u>2-point boundary conditions</u> $\underline{b}\{|i>, |f>\}$ because the two states $|i>$, $|f>$ (together with the Hamiltonian or Lagrangian) jointly determine everything that happens during the history $\underline{h}$ between the times t_i, t_f, including the behaviour of observers and their measuring apparatus. The two states are treated entirely symmetrically as one might expect in a relativistic theory on the micro-scale. Just as in classical mechanics, all the macroscopic temporal asymmetry of the actual history $\underline{h}$ is to be attributed to the particular boundary conditions $\underline{b}$. Our interpretation requires a generalization of the concept of determinism because although $\underline{h}$ is <u>mathematically</u> determined by $\underline{b}$, internal observers whose sub-histories are part of $\underline{h}$ itself cannot obtain enough information to make definite predictions of future events on the quantum scale, and they must therefore rely on the probability calculus provided by the Copenhagen interpretation. The concepts of determinism and causality in classical and quantum mechanics will be discussed in §§3 & 4, while in §§5 & 6 we shall discuss the consistency of the proposed interpretation and the deduction of the quantum-mechanical probability laws.

2-point boundary conditions arise because one is now describing the history not by an evolving Schrödinger state-vector $|\psi(t)>$ but by a matrix element between prescribed initial and final states. That is, one is applying the standard formalism of S-matrix theory not just to a single collision as is usually done in particle physics, but, in principle, to the sequence of all the many collisions or other interactions occurring in a macroscopic time interval of order the duration of the universe $\sim 10^{10}$ years. It does not seem that there is anything in the standard derivation of the S-matrix from quantum field theory to say that this generalization cannot be made; in fact in quantum gravity one often does introduce a formal S-matrix between remote past and remote future states, but so far it does not appear that the logical conclusion has been drawn, namely that each element of such an S-matrix should correspond to a distinct history $\underline{h}$ of the complete universe and that its mathematical structure could usefully be analysed from this point of view. If the conjectures of §5 are correct, it should be possible to prove rigorously that, subject to appropriate boundary conditions $\underline{b}$, the relative frequencies of individual microscopic processes within $\underline{h}$ obey the quantum-mechanical probability laws irrespective of the existence of any observers.

Some generalization of the terminology used in this paper and of the expression (5) for the history functional would be entailed by the existence of "hidden" as well as initial and final surfaces in quantum gravity (Hawking 1976) but this generalization will not be attempted here.

2. LAGRANGIAN FORMALISM

The axiomatic formulation of non-relativistic quantum mechanics (Dirac 1958) with which the Copenhagen interpretation is associated is based on the Hamiltonian operator that appears in Schrödinger's equation (1). Since about 1949 it has however become increasingly clear that canonical Hamiltonian methods often turn out to be unduly complicated in relativistic quantum theory. Following the early work of Dirac (1933, 1945, 1958), a Lagrangian space-time approach to non-relativistic quantum mechanics which uses path integrals was explored by Feynman in 1948 (Feynman 1948, Feynman and Hibbs 1965), and led to the idea of Feynman diagrams in relativistic quantum electrodynamics (Feynman 1949a,b; Dyson 1949). Since that time, covariant Lagrangian space-time techniques of calculation have steadily been gaining ground from the earlier and more complicated canonical methods. Many by now familiar and interrelated concepts have been developed including path integrals, Feynman diagrams, Green functions, the S-matrix, action principles, source theory (Schwinger 1966, 1970, 1973a,b) and functional integration and differentiation (Rzewuski 1969). Comprehensive accounts of the formalism have been given by Schweber (1961) and de Witt (1965), while references to more recent applications of Feynman path integrals to field theory are quoted by Abers and Lee (1973). Although the canonical approach is still required for certain applications (Abers and Lee 1973), broadly speaking these concepts may be described as Lagrangian and covariant, and they provide an intuitive space-time picture for the structure of an individual element of the S-matrix in which the initial and final states are treated on an equal footing, i.e. the picture is symmetric between past and future.

In view of the elegance and simplicity of the covariant approach and the detailed agreement of the predictions of quantum electrodynamics with experiment, it is perhaps surprising that there has not hitherto been any axiomatic Lagrangian re-interpretation of the conceptual foundations of quantum mechanics. Probably the reason is that there has not yet been any true break with the older canonical formulation and the Copenhagen interpretation, since present-day theories continue to use local field operators and Lagrangians and can therefore still be put into Hamiltonian form if required even though this is rarely necessary in practical calculations. It is however logically awkward for the covariant space-time formalism to be conceptually underpinned by the much more complex canonical structure, especially since in classical relativistic field theory the dependence goes in the opposite direction.

Two further logical difficulties may be mentioned. The first concerns the quantization of general relativity (Isham, Penrose and Sciama 1975). This is a problem of considerable current interest because quantum effects are believed to be important both for the initial singularity of the "hot big bang" model and also for black holes (Hawking 1975a,b). Einstein space-time is a classical dynamical system that can be expressed in canonical Hamiltonian form (Ryan 1972) and quantized in a variety of ways, but it is difficult to give any operational meaning to a Schrödinger equation (1) for the state-vector of the entire universe, or to measurements of this state by an "external observer" with consequent "collapse of the state-vector".

The second difficulty is concerned with possible future generalizations of the Lagrangian formalism. One would like the freedom to be able to explore new concepts such as non-local Lagrangians and discrete space-time in an attempt to remove singularities (Blokhintsev 1973). There is no reason to expect that a theory based on such ideas would have a Hamiltonian analogue, and it would be desirable to have a physical interpretation ready for use should the need arise. It is argued in this paper that the mathematical framework for an objective space-time interpretation of Lagrangian quantum mechanics already exists and that its consequences deserve to be explored.

At the present time the choice must be largely a matter of personal taste between this new interpretation, the orthodox Copenhagen interpretation, and the other main contenders which are probably two in number: the EWG interpretation referred to in §1 and the statistical interpretation (Ballentine 1970). Apart from a possible generalization to be mentioned in §9, all four interpretations lead to identical predictions so that the choice can only be made on metaphysical grounds, e.g. Occam's razor and the conceptual advantage of having an underlying model for the "real" world. We shall briefly summarize the advantages and disadvantages of the four interpretations at the end of the paper. Our aim in the following sections will be to outline a set of arguments by which it should be possible to demonstrate that the objective interpretation is self-consistent and in agreement with everyday experience of causality and with the Copenhagen interpretation. The most obvious departure from previous ideas is the symmetrical appearance of the initial and final states $|i\rangle$, $|f\rangle$ in the boundary conditions $\underline{b}$ that are imposed on $\underline{h}$, and we therefore begin in §3 by examining the concepts of determinism and causality in classical mechanics.

3. MICROSCOPIC DETERMINISM AND MACROSCOPIC CAUSALITY

We wish to distinguish in this section between the exact microscopic determinism of the basic equations of classical mechanics, a concept which is fully symmetric between past and future, and macroscopic causality which appears to propagate only towards the future and also to involve an element of chance so that it is not completely deterministic.

Our familiarity with macroscopic causality has led to the well-known picture of the time-evolution of a deterministic Hamiltonian system as the "gradual unfolding of a canonical transformation". It should be recognized that this dynamical and time-directed point of view does not correspond to anything essential in the mathematical structure of Hamiltonian mechanics itself, which can just as well be thought of in a timeless and symmetric form. To emphasize this point we refer to the contrasting and equally well-known static picture introduced by general relativity in which the history of the universe is thought of as a "map laid out in curved 4-dimensional space-time". Although general relativity does treat space-time as a dynamical system it actually took several decades for the alternative evolutionary concept of "Hamiltonian cosmology" to develop, and then mainly for the purpose of quantization by the standard canonical techniques (Arnowitt et al 1962; Ryan 1972; Misner et al 1973). The microscopic determinism of classical relativity is in fact entirely symmetric between past and future and it allows influences to propagate on and within both halves of the light-cone. This is borne out by the symmetry of the covariant classical Poisson brackets (Peierls 1952) which become the commutators and anti-commutators of the Heisenberg picture in quantum mechanics.

In relativistic classical mechanics the history of a physical system or of the entire universe is usually described by a set of particle world-lines $x_i^\mu(s)$, a set of fields $\emptyset_\alpha(x)$ or a combination of both, satisfying underlying Lagrangian equations which are symmetric in time. Starting with this explicitly covariant and objective space-time picture, a canonical Hamiltonian formalism can be derived in which the complete history $\underline{h}$ is determined by specifying appropriate Cauchy data on a single arbitrarily chosen space-like surface Σ_o, from which the solution can in principle be computed both into the future and also into the past so preserving the original time symmetry. In practice of course the history of a sufficiently complex system cannot be computed very far either way since mathematically the problem is improperly posed and the solutions are unstable, small changes in the Cauchy data on Σ_o making arbitrarily large changes in $\underline{h}$. For example, a small variation in the data changes the impact parameters of particle collisions in the immediate future of Σ_o and so becomes amplified in the subsequent particle orbits. Similarly for the immediate past.

Our macroscopic experience seems to us to indicate that causality propagates only on and within the forward light-cone so that the past influences the future and not vice versa. Presumably for this reason, we conventionally choose the Cauchy surface Σ_o of the underlying <u>deterministic</u> theory to lie in the distant past, and imagine that the classical universe has been "started off" in some initial state on Σ_o and thereafter obeys the deterministic Hamiltonian equations of classical mechanics like a piece of machinery. Since the observed asymmetry in time is not contained in the underlying mathematical model it is assumed to be due to the particular choice of boundary conditions on Σ_o which leads to a history $\underline{h}$ in which entropy continually increases. A significant requirement of macroscopic relativity is that observers should not be able to send signals to one another outside the light cone or backwards in time. A signal must involve a message which contains a finite amount of information, and because of the connection between information and entropy which is exhibited in information theory (Brillouin 1962) it therefore comes within the framework of macroscopic causality. The need to use retarded potentials in classical radiation theory must also be attributed to the boundary conditions (Davies 1974).

Macroscopic causality does not appear to us to be truly deterministic since minute sub-microscopic details of the initial state are continually being amplified to produce macroscopic effects. It is a significant fact that as the total entropy continually increases, so paradoxically does the degree of apparent order in the universe. This is attributed (Davies 1974) to the existence of attractive forces and in particular to gravity, which break the underlying symmetry and ensure that the most probable state of maximum entropy is often not a random distribution of particles and field oscillations but an ordered structure. A typical example is a two-phase system of liquid + vapour below the critical point, contained within an insulating enclosure. Suppose that the system had started as a uniform supercooled gas, then the release of potential energy due to the condensation of the liquid phase appears as latent heat which raises the temperature and allows the entropy of the vapour phase to increase, giving a net increase of the total entropy as well as an increase in the apparent order.

According to the formal postulates of classical mechanics all this detailed structure is determined by the boundary conditions, being latent in the random choice of sub-microscopic state on Σ_0 or indeed on any other space-like surface. We conventionally interpret this to mean that once "blind chance" has selected the initial state, the subsequent history is determined by the workings of "inexorable fate". In real life, of course, this interpretation is too uncomfortable for practical use and we find it convenient to adopt a more common-sense point of view in which the workings of "blind chance" are distributed more uniformly throughout $\underline{h}$ and a "dice is thrown" at each collision much as in orthodox quantum mechanics; one of the standard approaches to statistical mechanics (Penrose 1970) is accordingly to replace the underlying deterministic equations of motion by a Markovian postulate.

The status of the "observer" in classical mechanics is somewhat ambivalent. The formal theory discussed in §1 simply regards each observer as an automaton who is part of the total dynamical system, so that his actions are fully determined by the boundary conditions on Σ_0, and physics takes no account of free will, mind or consciousness. Most of the universe has no observers, and although these do exist in our own limited region of space-time (possibly also elsewhere) they are formally subject to the same physical laws as the rest of the system. On the other hand we do in practice assume that the observer has a free choice as to which measurements to perform, he is often given a privileged status in axiomatic presentations of the theory of relativity, and he can also be regarded as "external" to a sub-system of classical particles on which he is making measurements, e.g. by optical means, since the disturbance caused by light of any chosen wavelength vanishes in the limit $\hbar \to 0$.

What we therefore appear to mean by "causality" is not the formal determinism of the underlying classical equations which is fully time-symmetric, but the experimental fact that influences due either to (apparently) random events occurring within h̲ or to (apparently) free decisions by human observers who are themselves part of the system seem to propagate only forwards in time, and we attribute this temporal asymmetry to the boundary conditions on Σ_o (Davies 1974).

Another illuminating paradox which may be mentioned here is that notwithstanding Heisenberg's principle of uncertainty, most of the deterministic behaviour of the observed universe appears to be either of quantum or of gravitational origin. All our permanent records including the genetic code ultimately depend on the existence of quantum-mechanical bound states (Schrödinger 1945) which in suitable circumstances can remain stable for billions of years. No comparable stability exists in classical mechanics which in many of its practical consequences is therefore less deterministic than the quantum theory.

4. COMPARISON BETWEEN CLASSICAL AND QUANTUM MECHANICS

An alternative to the Cauchy or Hamiltonian concept of determinism in classical mechanics is the picture discussed in §1 in which the history h̲ within any finite connected region R of space-time is determined by data on its closed boundary S according to a variational principle $\delta\mathcal{H} = 0$, where $\mathcal{H}$ given by (3) is the action functional for the history. This is of course how the Lagrangian equations of motion are usually obtained.

The significant difference between classical and quantum mechanics may accordingly be stated in the following way, (using a non-relativistic particle model as an illustration):

A. In classical mechanics the history of a system can be determined by two alternative types of boundary condition*:

I. 1-point. Canonical coordinates and momenta (q̲,p̲) at any arbitrarily-chosen time t_o. (Any space-like surface Σ_o).

II. 2-point. Coordinates q̲ alone at any two times t_i, t_f enclosing the time-interval of interest. (Any surrounding closed surface S).

B. Quantum mechanics is mathematically equally deterministic, but only the 2-point boundary condition is now available.

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*There are some classical systems, e.g. the harmonic oscillator, for which Type II conditions give no history, or a history that is not unique.

In Lagrangian classical mechanics, for example, a particle world-line connecting two space-time points x_i and x_f is represented by the action integral

$$\mathcal{A} = \int_{x_i}^{x_f} L \, ds \tag{8}$$

and the corresponding charge-vector and energy-momentum tensor are δ-functions along the world-line. In Lagrangian quantum mechanics this definite world-line is replaced by a set of interfering Feynman paths connecting two sources $S_i(x_i)$, $S_f^*(x_f)$, whose contributions can be summed to give an action integral of the form

$$\mathcal{A} = -i\hbar \, \ln \iint S_f^*(x_f) \, K(x_f, x_i) \, S_i(x_i) \, dx_f \, dx_i \tag{9}$$

and the energy-momentum tensor is now calculated from the functional derivative of the Green's function or propagator K:

$$T^{\mu\nu}(x) = \frac{2\delta\mathcal{A}}{\delta g_{\mu\nu}(x)} = \frac{-i\hbar\iint S_f^*(x_f) \dfrac{\delta K(x_f,x_i)}{\delta g_{\mu\nu}(x)} S_i(x_i) \, dx_f \, dx_i}{\iint S_f^*(x_f) \, K(x_f,x_i) \, S_i(x_i) \, dx_f \, dx_i} \, . \tag{10}$$

This generalization is similar to the usual replacement of geometrical optics by physical optics but the picture is now fully symmetric between past and future: the function $T^{\mu\nu}(x)$ gradually spreads out from the "initial" source $S_i(x_i)$, reaches its maximum extent half-way, and then gradually contracts down to the "final" source $S_f^*(x_f)$. The picture uses Green functions rather than the wave-functions of classical optics or Schrödinger wave mechanics. Representation of the action integral as a sum over interfering paths (Dirac 1933, 1945, 1958; Feynman 1948: Feynman and Hibbs 1965) may be said to "explain" the principle of stationary action in classical mechanics in the same way that physical wave optics explains the principle of least time in geometrical optics.

Summarizing the arguments of §§3 & 4, we thus envisage two objective, time-symmetric and deterministic pictures of reality, classical and quantum, the first being the limit of the second as $\hbar \to 0$. Each picture assumes that the observed asymmetric behaviour of macroscopic causality and chance is to be simulated by an appropriate choice of boundary condition. In classical mechanics this boundary condition could be represented in many different ways, all mathematically equivalent to one another, and in order to avoid an apparent violation of the rules of commonsense the tacit convention was made that 1-point boundary conditions should be used with t_i or Σ_o lying in the distant past. Objective quantum mechanics no longer provides this degree of freedom for the way in which the boundary conditions b of §1 are imposed, but if these boundary conditions are properly chosen we still expect their consequences to conform to the macroscopic causality laws.

To avoid misunderstanding it should perhaps be remarked that the usual "causality condition" of dispersion theory and S-matrix theory (Hilgevoord 1962, Eden et al 1966, Nussenzveig 1972) is not connected with the macroscopic causality discussed in this paper. It is related to the Feynman $i\varepsilon$ prescription and to the covariant commutators and anti-commutators of the Heisenberg picture, both of which are symmetric in time except for a change of sign. This causality condition expresses the fact that real (i.e. not virtual) influences are constrained to propagate on and within the light-cone in a local Lagrangian field theory but may go in either direction. It cannot be connected with the observed 1-way propagation of macroscopic signals or classical retarded potentials since no asymmetry has been incorporated into the theory at this stage.

5. CONSISTENCY OF THE OBJECTIVE INTERPRETATION

The objective interpretation of quantum mechanics assumes that the actual history h of the universe (or any sub-history) is determined by:

A. The quantum-mechanical Lagrangian density $\mathcal{L}(x)$.

B. A set of rules by which $\mathcal{L}(x)$ is used to calculate the history functional $\mathcal{H}$.

C. The boundary conditions b, which in the Heisenberg picture are represented by the initial and final state-vectors $|i\rangle$, $|f\rangle$.

These assumptions are in 1-1 correspondence with their classical counterparts in §3. Since A and B have their equivalents in several alternative versions of "standard" Lagrangian relativistic quantum mechanics, consistency of the interpretation depends mainly on Assumption C.

To secure consistency and agreement with experiment it seems necessary to establish the truth of three conjectures which may be expressed broadly as follows:

I. Uniqueness. Definite physical processes occur within h, subject to limitations imposed by the Heisenberg uncertainty principle.

II. Frequency Distribution. The relative frequencies with which these processes occur in the actual history h are compatible with the quantum-mechanical probability laws.

III. Causality. The actual history h is compatible with the Second Law of Thermodynamics.

It appears that the validity of all three conjectures must depend on the actual boundary conditions b, and we note that so far as conjecture III is concerned this is also the situation in classical mechanics so that it need not be discussed explicitly here.

First it is necessary to explain what we mean by "definite physical processes" in Conjecture I. We do not of course mean definite particle world-lines as in the classical theory. In fact if there are N_i possible quantum states $|i\rangle$ that are compatible with the initial thermodynamic state of entropy $S_i = k \ln N_i$ and prescribed energy, angular momentum, charge, baryon number etc., and N_f possible quantum states $|f\rangle$ that are compatible with the final thermodynamic state of entropy $S_f = k \ln N_f$, (both numbers being large but finite), then the total number of alternative histories h is only $N_i N_f$. Equivalently we might say that the total amount of information (Brillouin 1962) that is available to define the actual history is only

$$S_i + S_f = k(\ln N_i + \ln N_f) \qquad (11)$$

and this is not enough for very much detail. The situation is quite different in classical mechanics where an infinite number of precisely-determined trajectories in configuration space evolve from any finite region in the initial phase space, however small.

At the same time we do not wish h to degenerate into what might be pictured as a random superposition of interfering Feynman paths or Feynman diagrams with no correlation between them, since this is not observed to happen in practice, and it seems plausible that Conjecture I will only be fulfilled if the boundary conditions are compatible with the formation within h of a sufficient number of macroscopic objects of substantial mass whose Feynman paths are localized in the immediate neighbourhood of the corresponding classical trajectories by interference effects. In other words, "geometrical optics" must be a good approximation for at least part of h.

Consider a body of N proton masses moving for a time T between sources at x_i and x_f in space-time. The radius of the central Fresnel diffraction zone at time $^T/2$ is

$$\delta = \sqrt{\frac{D\lambda}{2}} = \left(\frac{h}{2m_p}\right)^{\frac{1}{2}} \left(\frac{T}{N}\right)^{\frac{1}{2}}$$

i.e.

$$\delta = 4.4 \times 10^{-2} \left(\frac{T}{N}\right)^{\frac{1}{2}} \text{ cm} \qquad (12)$$

where $\lambda = h/Mv$ is the wavelength corresponding to the mass $M = Nm_p$ and $V = D/T$ is the velocity. The diffraction spreading away from the classical world-line is therefore very small for a macroscopic object with $N > 10^{24}$ even in a time $T \sim 10^{18}$ secs of order the duration of the universe (Peres and Rosen 1964).

The existence of macroscopic objects seems to depend on gravitational and thermal condensation, i.e. on the expansion and adiabatic cooling of the universe from its initial "hot big bang" state, together with local gravitational condensation and radiative cooling which eventually lead to a hierarchy of structures extending from galactic clusters, galaxies, stellar systems, stars, planets, down to meteorites and dust grains. It appears in fact to be quite difficult to account for the formation of galaxies (Jones 1976) without assuming large-scale irregularities in the initial state, and conceivably for a different set of boundary conditions macroscopic objects would never be formed at all and h would simply represent a uniform expanding gas. However, the observed existence of massive objects does permit us to argue intuitively that h is likely to be unique on the macroscale, since any two alternative sets of macroscopic trajectories that are compatible with the same boundary conditions b can be expected to give contributions to $\langle f|i\rangle$ whose moduli are numerically so far apart in magnitude that the larger one will dominate the other and interference will not take place.

Uniqueness of the trajectories of the centre of mass and other macroscopic coordinates of recording apparatus, (to within the accuracy (12) that is consistent with the Heisenberg uncertainty principle), together with the quantum-mechanical stability of its internal bound states mentioned in §3, allows us to conclude that the records of measurements on individual microscopic processes will be well-defined. Interference effects will however occur in other circumstances, e.g. for "virtual" processes off the energy shell, and also for certain processes even on macroscopic length and time scales when quantized interaction with macroscopic bodies is not taking place. This applies for example to the usual interference experiment with 2 slits: here the Green function K is modified by the presence of the slits in such a way that $T^{\mu\nu}(x)$ splits into 2 parts which go through the slits and reunite again in the neighbourbood of x_f. Similarly for diffraction due to coherent scattering by a crystal lattice. This time-symmetric description using the Green function is different from the non-symmetric description familiar in classical physical optics or Schrödinger wave mechanics, which uses a wave-function propagating outwards from the source at x_i, and it will be mentioned again in §7. The predicted results are of course the same.

The term "probability amplitude" would not be appropriate in this objective interpretation, and interfering Feynman paths or diagrams are not to be regarded as alternatives. Their status is equivalent to that of the individual terms in the power series representation of a mathematical function or of the coefficients in a Fourier expansion. The meaning of concepts such as "electron", "photon", or even "atom" is not to be prejudged since it can in principle be rigorously deduced from the theory: "the mathematical formalism of the quantum theory is capable of yielding its own interpretation" (de Witt 1971). Thus in the slit experiment each electron should be thought of as neither a particle nor a wave but as a quantized pulse of charge, mass, energy, spin and momentum that spreads out from the source at x_i, splits into 2 parts while going through the slits, joins up again and collapses down to the source at x_f.

Probability concepts are not part of the conceptual description of objective reality: just as in classical statistical mechanics they are introduced by internal observers to predict the relative frequencies of the results of future measurements from the limited data available to them. In either case the individual results themselves are determined by the boundary conditions but cannot be predicted by an observer who is himself part of the system, the difference being that in classical mechanics the limitation is practical while in quantum mechanics it is fundamental. There are however no "hidden variables" in the usual sense and the inaccessible information is contained in the final state $|f\rangle$.

Wave functions are also not a fundamental part of the objective description and can best be thought of as auxiliary mathematical functions which are introduced for the purposes of calculation. This point will be examined further in §7.

6. QUANTUM-MECHANICAL PROBABILITY LAWS

To establish the plausibility of Conjecture II we rely on a theorem proved by Everett (1957) in connection with the EWG "relative-state" or "many-universes" interpretation of quantum mechanics, and subsequently examined further by Graham (1970, 1973), and by de Witt (1971) who used the Heisenberg picture which is also adopted here. Essentially these authors consider an ever-branching "tree" of parallel alternative histories, all growing from the same initial state $|i\rangle$ at time t_i. We shall assume that $|i\rangle$ is normalized:

$$\langle i|i\rangle = 1 \quad . \tag{13}$$

During each of these histories one or more internal observers make a sequence of quantum-mechanical observations, either of the same type or of different types, and they may if desired communicate their results to one another. All the observers together with their measuring and recording apparatus are regarded as part of the system and the state is not reduced in the usual way at each internal observation, so that every time a measurement is made there is a further fork or subdivision into a number of separate branch histories, one for each possible result. Interference is assumed not to occur between the separate superposed branches once an observation has taken place. By time t_f the state-vector (which is fixed in the Heisenberg picture) has been decomposed into a linear superposition of an enormous number of components:

$$|i\rangle = \sum_\alpha |f_\alpha\rangle\langle f_\alpha|i\rangle \qquad (14)$$

one for each branch α, where the $|f_\alpha\rangle$ are assumed to be orthogonal and normalized. A measure

$$m_\alpha = |\langle f_\alpha|i\rangle|^2 \qquad (15)$$

is associated with each branch, with sum

$$\sum_\alpha m_\alpha = 1 \quad . \qquad (16)$$

Everett, Graham and de Witt then show that for all but a subset of branches whose total measure tends to zero as the number of internal observations tends to infinity, the relative frequencies with which the results of the individual observations occur within any given branch are in accordance with the quantum-mechanical probability laws. Evidently the set $\{\langle f_\alpha|i\rangle\}$ is just the column of S-matrix elements for a fixed initial state $|i\rangle$ and a variable final state $|f_\alpha\rangle$, while (16) is the usual unitarity condition.

An equivalent point of view would be to retain the Copenhagen interpretation and to imagine a hypothetical statistical ensemble of universes which are all prepared in the same state $|i\rangle$ at the initial time t_i by a single external observer, who then allows them to satisfy Schrödinger's equation (1) undisturbed until the final time t_f, when he measures a complete set of commuting observables corresponding to the eigenstates of the orthogonal expansion (14). Then m_α is the relative frequency with which he finds the state $|f_\alpha\rangle$, and the results of Everett and de Witt show that if we associate one branch $|i\rangle \to |f_\alpha\rangle$ with each member of the ensemble according to its final state, almost all members will be associated with branches in which the results of internal observers obey the quantum-mechanical probability laws.

In the objective interpretation we discard the hypothetical concepts of branching, of an ensemble of universes and of external observation, and simply postulate that the unique history <u>h</u> of our single universe is determined by a boundary condition pair <u>b</u> {|i>,|f>}, coordinated in such a way that for a specified initial state |i>, the final state |f> is restricted to be such that the measure m_α defined by (15) is sufficiently large. The Everett theorem ensures that this postulate will then lead to the quantum-mechanical probability laws.

It must be admitted that physics at present provides no explanation for the particular choice of initial state |i>, nor for why the measure m_α should be appropriate. However, one can readily argue that the degree of arbitrariness in the choice of <u>b</u> is essentially the same as in classical mechanics. The low-entropy, highly-compressed initial state |i> of the big-bang expansion corresponds to specifying a small region Δ_o of the classical phase space associated with an initial surface Σ_o (§3), while the detailed trajectory which a classical universe would subsequently follow is defined by selecting a precise phase point P_o within Δ_o, and this is analogous to the choice of |f>. The weighting factor for the selection of P_o within Δ_o is provided by the invariant volume measure of phase space, and the second law of thermodynamics will be obeyed for all but a set of initial points P_o with negligible measure, but no convincing reason can yet be given for the particular choice of the region Δ_o, nor for why this weighting factor should be the appropriate one to use.

7. WAVE FUNCTIONS AND THE MEASUREMENT PROCESS

According to the objective interpretation, the history <u>h</u> of the universe or any subhistory within <u>h</u> is described by a history functional $\mathcal{H}$, itself determined by boundary conditions <u>b</u>. For the purpose of discussion we shall idealize the functional $\mathcal{H}$ into the logarithmic amplitude (5) of what may be thought of as a reduced Feynman diagram containing vertices (real collisions) connected by lines (real particles), so that <u>h</u> is pictured as a space-time network of events. Fig. 1 illustrates the simple case of binary collisions in which each vertex has two incoming and two outgoing lines. Actual situations will be much more complicated; for example processes involving real charged particles will radiate an arbitrary number of low-frequency photons. The network shown in Fig. 1 is therefore only schematic but it is enough to allow the corresponding infinite set of true Feynman diagrams associated with the perturbation-theory treatment of the same process to be specified.

It is useful to distinguish between the primary and secondary interpretations of the mathematical structures and concepts which appear in the theory. The primary interpretation is objective and relates to the history h as seen from a vantage point outside space-time; this is the "over-all space-time point of view" of Feynman (1949a). The secondary interpretation is concerned with statistical predictions about the future course of h which are made by internal observers from the limited amount of information that is available to them. The observers together with their measurements and predictions are part of h itself. The same formalism is used in both cases, but (for example) an element of the S-matrix describes an actual occurrence according to the primary interpretation, but is used to calculate the probability amplitude for this occurrence according to the secondary interpretation.

The space-time network of events illustrated in Fig. 1 is mathematically determined by the past and future boundary conditions b. This form of determinism is microscopic and time-symmetric, so that although influences are constrained to propagate on or within the light-cone they may go in either direction of time. The underlying deterministic network nevertheless supports a higher-level macroscopic history in which causality is unidirectional, entropy increases, and quantum processes appear indeterministic to internal observers.

Wave-functions are to be regarded as auxiliary concepts which are introduced in the following way (Feynman 1949a,b; Schwinger 1966, 1973a). With each line $\alpha\beta$ of the network joining two vertices α and β, $(t_\beta > t_\alpha)$ may be associated two distributed sources $S_\alpha(x_\alpha)$ and $S^*_\beta(x_\beta)$ and corresponding 1-particle wave functions, with ψ_α propagating forwards in time from region x_α, and ψ^*_β propagating backwards in time from region x_β. The sources and wave-functions are to be constructed in a self-consistent way from the diagram, and in the notation of Fig. 1 we may write symbolically formulae such as:

$$S_{C3} = \Lambda_{C3412}\, \psi^*_{E1}\, \psi_{A4}\, \psi_{B3} \tag{17}$$

$$S_{C4} = \Lambda_{C3412}\, \psi^*_{D2}\, \psi_{A4}\, \psi_{B3} \tag{18}$$

and

$$\psi_{C3} = G_a\, S_{C3} \tag{19}$$

$$\psi^*_{E1} = G^*_b\, S^*_{E1} \tag{20}$$

where Λ is the vertex part corresponding to collision C, G_a is the Green function for the propagation of particle a, and G_b^* is the conjugate of the Green function for the propagation of particle b. Corresponding 2-particle sources and wave-functions

$$S_{C34} = \Lambda_{C3412} \psi_{A4} \psi_{B3} \tag{21}$$

$$\psi_{C34} = G_{ab} S_{C34} \tag{22}$$

may also be constructed.

These wave-functions are not regarded as an essential component of reality in the objective description, except in the sense that (with appropriate normalization) the charge-current vector for the particle a propagating between C and D can be written as

$$J_a^\mu = e \psi_{D2}^* \gamma^\mu \psi_{C3} \quad . \tag{23}$$

Wave-functions can however be used by internal observers who are themselves part of h to calculate probabilities for the results of subsequent measurements. Suppose for example that in Fig. 1 two particles a, b with known wave-functions ψ_{A4}, ψ_{B3} collide at C, and that the state of a is observed at D by an observer 0_1 who uses measuring apparatus M_1 so that the wave-function ψ_{D2}^* is known, Then ψ_{C4} is known and may be used to predict the probability distribution for the results of an observation at E on particle b by an observer 0_2 who uses measuring apparatus M_2.

One might argue, as in the paradox of Einstein, Podolsky and Rosen (1935), that the measurements D, E could be very far apart and outside each others' light-cones; how can one measurement affect the other? It is a consequence of quantum mechanics that the type of measurement made at D (i.e. which set of complete commuting observables is measured) must affect the set of possible wave-functions ψ_{C4}; yet the observer 0_1 responsible for D may not decide which type of measurement to choose until long after collision C, "by which time ψ_{C4} has already started on its way".

Two answers may be made to this criticism:

(a) According to the objective interpretation, quantum mechanics is a mathematically deterministic theory on a similar footing to classical mechanics. Neither theory provides for any "free-will" by internal observers who are themselves part of h. The observers 0_1, 0_2 (who may or may not be the same person) need not have completed their decision processes (which are also part of h) until long after C, but the results of these decision processes and the actual measurements which are made are determined by the overall boundary conditions on $\underline{h}$.

(b) The argument that D cannot influence C is based on the idea of causality (§3) propagating only forwards in time. But it has been widely accepted in classical statistical mechanics since Boltzmann's H-theorem was first put forward that time-asymmetric causality should be regarded as a macroscopic concept (Davies 1974), which is imposed on the underlying time-symmetric dynamical equations by the particular choice of boundary conditions on Σ_o (§3). Virtually all experimental evidence indicates that individual microscopic processes are symmetric in time. The only restriction which relativity imposes on determinism in a microscopic theory ought therefore to be that it propagates on or within the light-cone; the direction is arbitrary. Thus ψ^*_{E1} helps to determine ψ_{C3}, while ψ^*_{D2} helps to determine ψ_{C4}, and no inconsistency between the measurements D and E can ever arise.

This argument resolves the paradox of Einstein, Podolsky and Rosen (EPR) but the picture is a remarkable one nevertheless. Observers 0_1, 0_2 may be light-years apart, yet their actions are partly correlated via a collision C which may have occurred billions of years ago. However in order to satisfy accepted ideas of macroscopic causality the only significant requirement, already mentioned in §3, appears to be that observers should not be able to send signals to one another outside the light-cone or backwards in time. A signal involves a macroscopic transfer of information and can only be transmitted forwards in time. The microscopic correlation between D and E cannot be used by observers 0_1 and 0_2 to send signals to one another, and the requirements of relativity are satisfied by the fact that the link goes via DCE and not directly across the space-like path DE. The EPR paradox is therefore to be resolved by making a careful distinction between symmetric, microscopic determinism and asymmetric, macroscopic causality.

It is clear from §6 that the Copenhagen interpretation remains valid for calculating probabilities, while Schrödinger's equation for an arbitrary number of particles can be used for the calculations of eigenstates in the usual way. However wave-functions do not "collapse"; they simply get discarded and replaced by new ones. Thus after the result of measurement D is known the appropriate wave-function to use for further predictions about particle a is no longer ψ_{C3} but ψ_{D4}.

A wave-function can be used to make predictions about the behaviour of any sub-system that is temporarily out of contact with its surroundings by cutting the lines that join it to the future. For example, a wave-function for particle a is obtained in Fig. 1 by cutting the line CD, while a 2-particle wave-function for the sub-system $a \otimes b$ is obtained by cutting both CD and CE. The usefulness of this procedure decreases as the sub-system becomes larger, and vanishes altogether in the limit when the sub-system is the entire universe and all the future-going lines have been cut. Then the state reduces to the initial state $|i\rangle$, and no apparatus is left with which to make a measurement. In this sense the Schrödinger state-vector for a complete set of interacting fields disappears from the theory in the objective interpretation.

8. CLASSICAL SOURCES AND FIELDS

The objective interpretation makes no sharp separation between classical and quantum systems. The entire history h obeys the laws of quantum mechanics but classical mechanics is an accurate numerical approximation for many processes whose wavelengths are sufficiently short and frequencies sufficiently high. Its status is equivalent to that of the WKB approximation in optics or wave-mechanics.

Source functions such as $J^{\mu}(x)$ and $T^{\mu\nu}(x)$ may be constructed from the history functional as in equation (4). The question is, what do they mean? In particular, what meaning can be given to using J^{μ} in the classical Maxwell's equation to construct a vector potential $A^{\mu}(x)$ and electromagnetic field $F^{\mu\nu}(x)$, to using $T^{\mu\nu}(x)$ in Einstein's equations to construct a metric tensor $g^{\mu\nu}(x)$, or to deriving sets of macroscopic hydrodynamic and thermodynamic equations?

The first point to notice is that these functions always exist as part of objective reality in the sense that they can be calculated from $\mathcal{H}$, although they cannot be measured with complete accuracy by apparatus that is itself within $\mathcal{H}$. They are not to be regarded as probabilities or ensemble averages. They are relatively smooth functions of space and time, and although they are in principle complex, their imaginary parts are negligibly small when averaged over a large number of particles, and the real parts of J^{μ} and $T^{\mu\nu}$ separately satisfy the conservation laws.

Broadly speaking the question of what these functions mean is answered by what physicists always do in practice. The electrons in a plasma or in a radio antenna obey the laws of quantum mechanics, but we are accustomed to calculate classical potentials A^{μ} from their total charge-current vector J^{μ}. These potentials can be used to describe processes involving very large numbers of long wavelength photons. It is also meaningful to use Schrödinger's equation to calculate the motion of an individual charged particle in a classical field. One can say that the classical field picture is valid if the sum of the amplitudes of Feynman diagrams calculated in this way is a good approximation to that calculated from the exact quantum-mechanical theory.

There is a logical difficulty in general relativity in using the Heisenberg picture as in §1, because the operators in this picture are meant to be given once-for-all, and to be independent of the state-vector $|\psi\rangle$ in the Copenhagen interpretation, or of the vectors $|i\rangle$, $|f\rangle$ or the history $\underline{h}$ in the objective interpretation. They satisfy Lagrangian equations of motion and their commutators depend on the light-cone structure of space-time. This formalism relies on the metric $g^{\mu\nu}$ being a c-number and will not work if it is an operator.

A possible solution for most regions of space-time might be a self-consistent, iterative approach in which a c-number background metric $\eta^{\mu\nu}$ used in the Heisenberg picture is obtained by solving Einstein's equation using as source the real part of the $T^{\mu\nu}$ constructed from $\mathscr{H}$ itself. Quantization by expansion about a WKB solution has been discussed in the general case by Rajaraman (1975), and for quantum gravity expansion about a classical background metric has been pioneered by de Witt (1965, 1967, 1975). Quantization about flat space-time has been reviewed by Salam (1975). Provided that the difficulties of renormalization can be overcome, the fact that one can now avoid the external observer and the state-vector for the complete system and adopt a more space-time point of view may make it easier to combine quantum mechanics and general relativity.

9. CONCLUDING REMARKS

This paper has outlined an alternative interpretation of quantum mechanics which mathematically is identical to the Copenhagen interpretation but conceptually is quite different. Each interpretation represents a departure from the ideas of classical mechanics but the departures are in different directions.

The Copenhagen interpretation is based on the measurement concept: it deals with the calculation of probabilities for the results of future measurements by an external observer and with the verification of the results of these calculations by experiment. It is a formal mathematical calculus which works with an abstract Hilbert space and it does not provide a conceptual model for an objective real world. A similar remark applies to the statistical interpretation (Ballentine 1970) which does not describe an individual system but only an ensemble of systems. Some people have welcomed this epistemological approach and the agreement with experiment has of course been outstanding. Nevertheless the lack of an objective description of reality is awkward in practice and it has been difficult to combine the ideas of quantum mechanics with those of general relativity. The canonical Hamiltonian formalism which provides the main foundation of the Copenhagen interpretation of non-relativistic quantum mechanics has in recent decades been more and more replaced by a number of alternative but equivalent Lagrangian formalisms, which are fully covariant and emphasize an underlying symmetry between past and future. These mathematical developments seem to provide a hint for a reorganisation of the conceptual foundations of the theory.

The objective interpretation employs a real space-time in which actual processes happen quite independently of the observer except for those processes in which he himself plays an integral part. According to this interpretation the measurement concept no longer has any connection with the foundations of quantum mechanics, so that the human observer is once again removed from a privileged central position which he lost at the time of Copernicus but appeared to regain in 1925. The objective description is more abstract than is commonly used in classical mechanics since it is based on a history functional $\mathcal{H}$ rather than on particle world-lines or field functions, but the most fundamental departure from the ideas of classical mechanics is that 2-point initial and final boundary conditions are now mandatory, whereas in the classical theory either 2-point (Lagrangian) or 1-point (Hamiltonian) boundary conditions could be used. This is analogous to the generalization from geometrical to wave optics, and is represented by the replacement of a definite classical trajectory between two points ($\underline{q}_i$, t_i) and $\underline{q}_f$, t_f) by a set of interfering Feynman paths.

The new interpretation makes a clear distinction between two levels of description of reality. The underlying microscopic level is deterministic and symmetrical between past and future, and is concerned with relative frequencies of actual events in $\underline{h}$ rather than with probabilities. Superimposed on this is a macroscopic level in which the asymmetric laws of one-way causality and chance apply, laws whose origin must be sought in the particular boundary conditions $\underline{b}$ imposed on the history $\underline{h}$. This distinction between microscopic and macroscopic levels is similar to that of classical statistical mechanics.

There are no hidden variables in the theory and the Copenhagen interpretation remains valid, as indeed one must expect in view of its precise agreement with experiment. However the probability laws associated with this interpretation are no longer to be regarded as a fundamental postulate but as a deduction from the theory together with the boundary conditions that have been imposed. While the underlying description of the history h depends on functional derivatives or Green functions, internal observers who are themselves part of h use wave-functions to make predictions of the results of future experiments from the limited information that is available to them. These wave-functions may be constructed in a variety of different ways in relativistic quantum mechanics, and their meaning is to be determined from the mathematical formalism. No a priori meaning is given to concepts such as "particle" or "wave", and the extent to which they are useful as part of a description of objective reality is to be deduced from the theory itself. Classical mechanics is an approximation to the exact laws of quantum mechanics in the limit $\hbar \to 0$ rather than an independent theory. Insofar as the actions of observers are regarded as physical processes they are determined by the boundary conditions, and the theory makes no statement about the problems of free-will, mind or consciousness.

The objective interpretation has the advantage of providing a more complete, straightforward and intuitive picture of objective reality than the Copenhagen interpretation, and it also includes this interpretation at the secondary level (§§6 & 7). It is conceptually simpler than the EWG "relative state" or "many-universes" interpretation with which it has several points in common, notably the concept of the internal observer and the deduction of the quantum-mechanical probability laws. The EWG interpretation is however asymmetric in time because a single initial state expands continually into a many-branched tree of alternative histories, while the objective interpretation is symmetrical in time and provides a history which is unique.

From a pedagogical point of view it is suggested that the objective interpretation is to be preferred to the others, since it offers the attractive future prospect of being able to derive all the formalism and the concepts of relativistic and non-relativistic quantum theory, general relativity, statistical mechanics and classical dynamics from a single consistent set of basic axioms. It is also closely related to the Feynman diagram and path representations which have many advantages both intuitively and for purposes of practical calculation.

The distinctive new feature, to which some may take exception, is the symmetrical use of initial and final boundary conditions on the history h, involving a reassessment of our present ideas of causality and determinism. Such a generalization has been latent in the Lagrangian variational approach since the days of Maupertuis (Yourgrau and Mandelstam 1968), and although attractive to some physicists (Wheeler and Feynman 1945, Planck 1950 p.179) it has usually been rejected because of the progressive and apparently quite successful elimination from modern science of the earlier teleological concept of a "Final Cause", a noteworthy example being the introduction and development of the theory of evolution by natural selection, and later of molecular biology, so providing at least a qualitative explanation for the existence of highly complex biological structures in terms of random processes which conform to the second law of thermodynamics.

The standpoint taken in this paper is a pragmatic one. There is at present an apparent serious incompatibility between the almost universal use of the Cartesian picture of an objective material world in ordinary life and throughout most of science, and the formal Copenhagen interpretation of quantum mechanics which seems to deny the validity of such a picture. In this situation, it may well be preferable to have a teleological picture of the history of the universe than no consistent objective Cartesian picture at all. The evidence from the development of relativistic quantum theory during the last 30 years is that a Lagrangian description is likely to be mathematically simpler (Occam's razor), and according to present indications one of the principal effects of the final boundary condition should merely be to impose the degree of randomness needed for a deduction of the quantum mechanical probability laws, the second law of thermodynamics, retarded electromagnetic radiation and other irreversible processes (Davies 1974). It has been explained in §6 that no violation of macroscopic causality will result if $|f_\alpha\rangle$ has been chosen in accordance with the measure m_α of equation (15), since the relative frequencies of individual events will be identical to those predicted by the Copenhagen interpretation.

The initial boundary condition $|i\rangle$ should presumably correspond to a gravitational singularity prior to the big-bang expansion. The nature of the final boundary condition $|f\rangle$ is not yet known: depending on the mean mass density of the universe it might correspond either to a final collapse or to a large number of individual black-hole singularities in an otherwise expanding system. The boundary conditions on the universe are in fact far from being understood at the present time (Jones 1976): it is difficult to explain the origin of the galaxies and of other non-uniformities such as rotation and large-scale magnetic fields without postulating some corresponding lack of uniformity in the initial state $|i\rangle$ prior to the big-bang expansion, since theory indicates that such irregularities develop too slowly to have grown from random $\sqrt{N}$ fluctuations. Making arbitrary asymmetries in the initial state responsible for all the observed large-scale structure seems unsatisfactory to some cosmologists especially since this state must also account for the observed high degree of symmetry of the microwave background radiation. A model in which the initial state $|i\rangle$ is symmetrical and it is the final state $|f\rangle$ that is responsible for the cosmological structure could therefore be an interesting alternative which in principle might be subjected to observational test, although it would involve some departure from the measure condition of §6 on the cosmological scale.

Although, apart from this possibility the objective interpretation leads to identical predictions to the established Copenhagen interpretation and therefore has the disadvantage that it cannot be experimentally tested, it does suggest a number of purely mathematical investigations, analogous to ergodic studies in classical mechanics, from which definite answers should be obtainable. As a typical example, consider a gas consisting of a large number of non-relativistic quantum-mechanical particles enclosed in a box with rigid walls during a time interval (t_i, t_f). Let the initial boundary condition $|i\rangle$ at time t_i be a pure state corresponding to a product of approximately localized single particle wave-packets, appropriately symmetrized or antisymmetrized, and similarly for the final boundary condition $|f\rangle$ at time t_f. Does the history $\underline{h}$ represented by the functional (5) correspond to one particular topological network of the type shown in Fig.1 whose amplitude far outweighs the rest, so that negligible interference takes place and individual collisions approximately localized in space-time in accordance with the Heisenberg uncertainty principle can be considered actually to occur, or do different networks with distinct topologies have comparable amplitudes so that interference takes place between them and $\underline{h}$ must be regarded as a superposition? Suppose that the walls are adiabatically moved during the time interval (t_i, t_f) so that macroscopic droplets condense out and subsequently evaporate again. Is there any circumstance in which alternative macroscopic histories might interfere? Questions of this kind are objective since they are not related to the existence of observers and they can in principle be answered by examining the structure of the S-matrix element $\langle f|i\rangle$.

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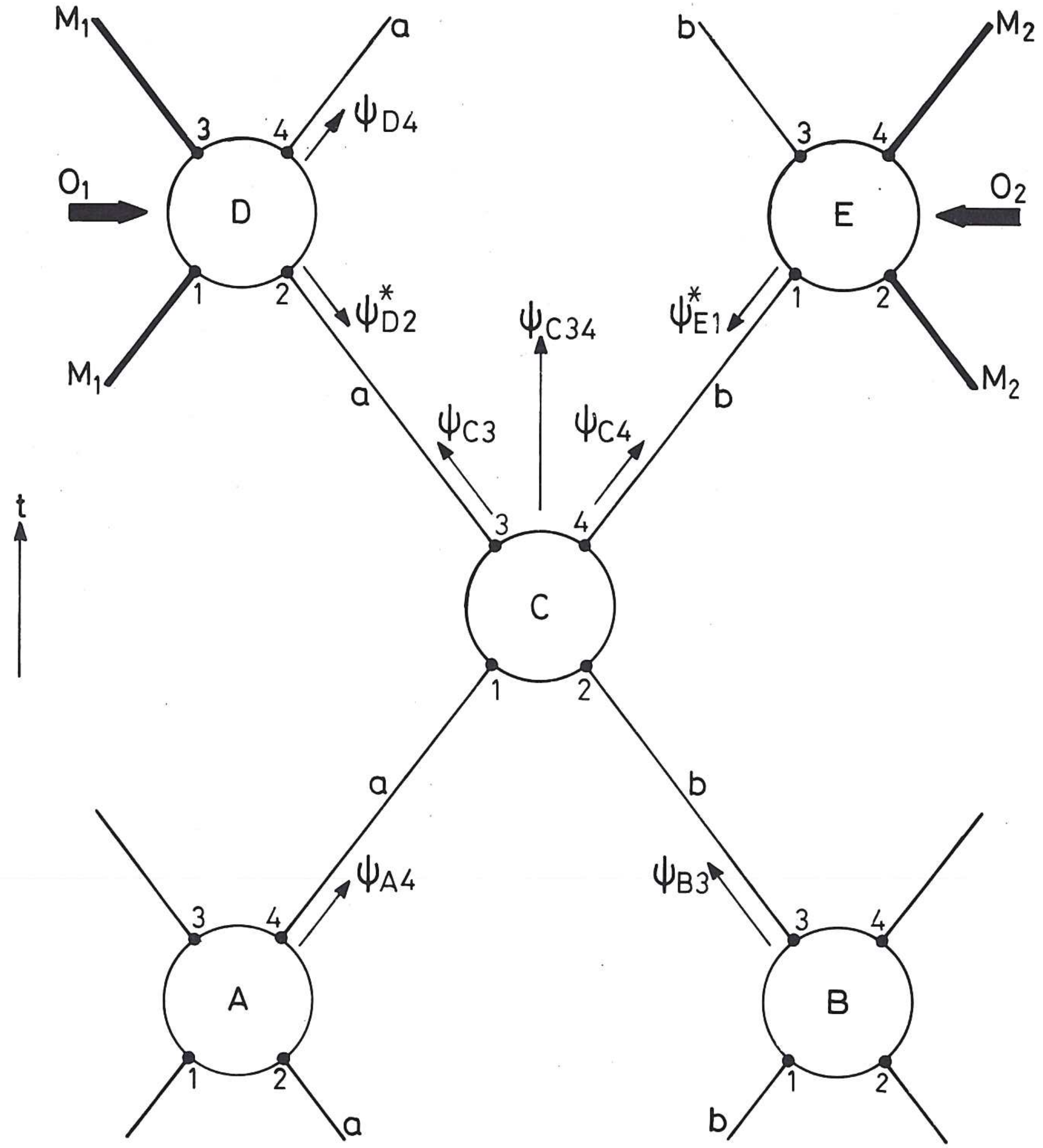

Fig.1 Network of binary collisions forming part of the history h.
A collision occurs at C between particles a,b. Measurements are made at D by observer 0_1 with measuring apparatus M_1, and at E by observer 0_2 with apparatus M_2. Two 1-particle sources S,S* and wave-functions ψ, ψ^* may be constructed by breaking any single Feynman line, 2-particle sources and wave-functions by breaking a pair of lines, and so on. These wave-functions are used for making statistical predictions and are discarded when no longer required. The measurements at D and E are correlated since ψ^*_{E1} helps to determine ψ_{C3}, and ψ^*_{D2} helps to determine ψ_{C4}.

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