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IV. Decay to Ion Cyclotron Waves

R N FRANKLIN
P D EDGLEY
S M HAMBERGER
R W MOTLEY

CULHAM LABORATORY
Abingdon Oxfordshire

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NONLINEAR BEHAVIOUR OF A FINITE AMPLITUDE ELECTRON PLASMA WAVE
IV. Decay to Ion Cyclotron Waves

R. N. Franklin,* P. D. Edgley,* S. M. Hamberger and R. W. Motley

UKAEA, Culham Laboratory, Abingdon, Oxfordshire.

* Department of Engineering Science, Oxford University)

ABSTRACT

Plasma in a magnetic field displays low frequency modes near the ion cyclotron frequency for waves propagating at an angle to the magnetic field. These modes are only slightly modified in a bounded plasma, and therefore can be excited by nonlinear decay of electron plasma waves which also propagate at an angle to the magnetic field.

The nonlinearly generated low frequency mode has been identified experimentally as an ion cyclotron wave by stimulating the decay. The resonant matching conditions have also been demonstrated.

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1. INTRODUCTION

In earlier papers experimental results and related theory were given for the fundamental nonlinear process involving longitudinal plasma waves and electron waves in particular (Franklin et al, 1975 (a,b), 1977). In this paper we describe results which depend upon the presence of a magnetic field in the plasma for their existence. However, the only requirement is that the ion cyclotron frequency ω_{ci} is less than, but comparable to, the ion plasma frequency ω_{pi}. Under these circumstances, for waves propagating at an angle to the magnetic field there are branches corresponding to a resonance at or below ω_{ci}, and a cut-off above or at ω_{ci} (see for example Franklin 1976). In a finite plasma the boundaries impose conditions on the wavenumber $\underline{k}$, such that propagation is always effectively at some angle to the axis of symmetry and therefore to any axial magnetic field.

The linear proagation characteristics of all modes involved in the nonlinear process are determined experimentally as in II.

For the specific case of ion cyclotron modes this has required the development of a method of exciting them without, at the same time, exciting ion acoustic modes. Stimulation of coupled wave instability below the natural threshold by introducing one of the product waves is shown to be a process which is specific to the particular wave types being excited nonlinearly, and therefore can be used as a means of identifying the decay product waves. Recent interest in the particular three-wave process described in this paper stems from a demonstration that, at high excitation power levels, it leads to significant heating of the plasma ions. We describe observations made using an incoherent driving field in which the spectrum of fluctuations excited is free from resonances, and thus the conversion to ion energy more direct.

2. THE DISPERSION RELATION FOR ION CYCLOTRON WAVES

The basic theory for waves propagating at an angle to a magnetic field in warm plasma at frequencies of the order of the ion cyclotron frequency have been known for some time (see for example, Stix, 1962).

However, there are certain complicating features of the experimental plasma which must be included in the theoretical model and therefore require some discussion before the experimental results are described. The two principal features are (i) the existence of a significant axial ion drift whose magnitude is, however, insufficient to excite instability, and (ii) the influence of the magnetic field on the ion energy distribution, allowing an inequality of the effective temperatures which describe the velocity components along and transverse to the magnetic field.

The axial drift of the ions relative to the laboratory frame arises from the fact that, in a single-ended Q-machine the hot plate conditions are such that its sheath is electron rich. The particle distribution functions in the plasma column for these conditions have been presented as Figure 2 in II. The ion distribution $f(v_i)$ is a displaced Maxwellian truncated at some velocity v_{ip} such at $\frac{1}{2}m_i v_{ip}^2 = eV_p$, where V_p is the potential difference between hot plate and plasma. The electron distribution $f(v_e)$ is an almost complete Maxwellian except that those electrons which can overcome the plasma-cold plate potential $V_c - V_p$ are collected by the cold plate, i.e. those with velocities $v < v_{ec}$ where $\frac{1}{2}m_e v_{ec}^2 = e(V_c - V_p)$, $f(v_e) = 0$.

Under conditions in which the cold plate is at floating potential the requirement that the ion and electron fluxes are equal, together with charge neutrality in the column, leads to the relation

$$\frac{\exp - (\eta_c - \eta_p)}{1 + \operatorname{erf}(\eta_c - \eta_p)^{\frac{1}{2}}} = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \frac{\exp - \eta_p}{\operatorname{erfc} \eta_p^{\frac{1}{2}}} \quad .. (1)$$

where $\eta_p = eV_p/k_B T_e$ and $\eta_c = eV_c/k_B T_e$.

This equation can be solved for any given value of cold to hot plate potential difference V_c to yield the corresponding values of v_{ip} and v_{ec}. In turn we can find both the mean ion drift velocity v_{Di} and an effective parallel ion temperature $T_{i\parallel}$ which can then be used in the dispersion relation discussed below, where the transverse ion temperature $T_{i\perp}$ is assumed to be the same as that of the hot plate.

The values of v_{Di}/c_s and $T_{i\parallel}/T_e$, where c_s is the ion acoustic speed and T_e is the electron temperature, are given in figure 1 as a function of the plasma potential relative to the hot plate $\eta_p = |eV_p/k_B Te|$. The case where the cold plate is maintained at a potential other than the floating potential η_f (for which no net current flows) can be shown to be given by

$$\exp\eta_p \operatorname{erfc}\eta_p^{\frac{1}{2}} = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \left[1 + \operatorname{erf}(\eta_c - \eta_p)^{\frac{1}{2}}\right] \exp(\eta_f - \eta_p) \quad ..(2)$$

and $j = n_s eV_{Ds} \left[1 - \exp(\eta_f - \eta_c)\right]$,
where n_s and v_{Ds} apply to $\eta_c = \eta_f$, and j is the current density. Figure 2 shows the relation between η_p and η_c for different values of η_f. For there to be a monotonic potential distribution (McIntyre, 1962) i.e. for the given solution to be valid, it is required that $\eta_c \geq \eta_p$. The points corresponding to zero net current ($\eta_f = \eta_c$) are also indicated. It is seen that for $\eta_c \geq \eta_f - 2$, η_p changes little with η_c: on the other hand both v_{Di} and $T_{i\parallel}$ depend quite strongly on η_p.

The effective dielectric constant for electrostatic waves in a magnetised plasma propagating at an angle to the magnetic field and having wavenumber components $k_\parallel$ and $k_\perp$ parallel and perpendicular to the field respectively has been shown to be (Stix, 1962)

$$\varepsilon = 1 + \Sigma_{ji,e} \frac{\omega^2_{pj} m_j \exp-\lambda_j}{(k_\perp^2 + k_\parallel^2) k_B T_{j\perp}} \sum_{-\infty}^{\infty} I_n(\lambda_j) A_j^{(n)} \quad ..(3)$$

where

$$\lambda_j = \frac{k_\perp^2 k_B T_{j\perp}}{m_j \omega^2_{cj}} \equiv k_\perp^2 \rho j^2 \ , \quad \alpha_j = \frac{\omega - k_\parallel V_{Dj} + n\omega_{cj}}{k_\parallel \left(\frac{2k_B T_{j\parallel}}{m_j}\right)^{\frac{1}{2}}}$$

and

$$A_j^{(n)} = {}^{T_{j\perp}}/_{T_{j\parallel}} + Z(\alpha_j).\{(\omega - k_\parallel V_{Dj} + n\omega_{cj})^{T_{j\perp}/T_{j\parallel}} - n\omega_{cj}\}/k_\parallel \left(\frac{2k_B T_{j\parallel}}{mj}\right)^{\frac{1}{2}} ,$$

Z is the plasma dispersion function, ω_{cj} is the cyclotron frequency and ω_{pj} the plasma frequency of particles of type j. For cases such as ours in which $\omega_{ce} >> \omega_{pe} >> \omega_{ci}$, the electron susceptibility term in (3) can be approximated by

$$- \frac{1}{2k^2\lambda_D^2} Z' \left[\frac{\omega}{k_{\parallel} \frac{(2k_B T_e)^{\frac{1}{2}}}{m_e}} \right]$$

In the case of a finite, cylindrical plasma the dispersion relation is

$$D(\omega, k_{\parallel}, k_{\perp}) = k^2\varepsilon + k_{\perp}^2 = 0$$

where $k_{\perp}$ is the solution of the appropriate equation which ensures matching of the fields interior and exterior to the plasma at the plasma radius, a. For a plasma-filled waveguide this equation is $J_o(k_{\perp}a) = 0$, while for the present experimental situation with the metal walls removed to a radius $b(b > a)$ it is

$$\frac{J_o(k_{\perp}a)}{k_{\perp}aJ_o'(k_{\perp}a)} = \frac{I_o(kb)K_o(ka) - I_o(ka)K_o(kb)}{ka[I_o(kb)K_o'(ka) - I_o'(ka)K_o(kb)]} \quad ..(4)$$

The solution of these equations for the lowest radial eigenmode has been found numerically for parameters appropriate to the present experiment, the summation for ε being carried out over sufficient terms to achieve the desired accuracy. Typical results are shown in figure 3.

3. EXPERIMENTAL MEASUREMENT OF ION CYCLOTRON DISPERSION

Since, in general, at frequencies ω of the order of ω_{ci} it is possible for ion acoustic waves to propagate when $\omega \lesssim \omega_{pi}$ it is important to distinguish between the two types of wave in any experiment. A demonstration that both types can be excited simultaneously has been given by Sato et al (1974), who showed that while a wire probe immersed in the plasma excited only ion acoustic waves, a modulating magnetic coil outside the plasma excited both ion acoustic and ion cyclotron waves. Since the fields associated with each type of wave are different, it should be possible by excluding all electrostatic fields to excite only the ion cyclotron mode. Figure 4 shows an interferogram obtained using a magnetic field coil with grounded electrostatic screen to excite the wave. It can be seen (by comparison with Fig. 1 of Sato et al, 1974) that only one mode is excited, and by varying the magnetic field B the phase velocity of this mode can readily be shown to depend on B in the manner expected of an ion cyclotron mode.

The ion cyclotron waves are usually heavily damped, as can be seen from figure 4, so that only a relatively small portion of the dispersion diagram is experimentally accessible. A detailed comparison over this range is given in figure 5. Also shown is the ion acoustic mode at the same frequency as that for the ion cyclotron mode of figure 4 illustrating the characteristic difference in wave number.

Apart from the limitations imposed by damping, the magnitude of the magnetic field needed to confine the plasma (B = 0.2T) meant that to obtain a significant modulation of the field, relatively large alternating currents were required with consequently large ohmic dissipation. Therefore care had to be taken to ensure that the vacuum conditions were unaffected by outgassing from the modulating field coil.

4. EXCITATION OF THE HIGH-FREQUENCY-DRIVEN INSTABILITY

The experience of earlier workers (Chu, Bernabei and Motley, 1973) studying the instability was drawn on to the extent that the high frequency electric fields driving the instability were applied to a metallic ring entirely outside the plasma. For electron wave frequencies ω_o well below the electron plasma frequency ω_{pe}, the fundamental eigenmodes of the cylindrical plasma column have substantial fields outside the plasma, so that coupling to them by a ring electrode is stronger and the generation of higher modes less serious than for excitation by a fine wire probe as described in I at the same frequency. By this means the linear electron plasma wave dispersion could be measured in the frequency range from the plasma frequency ω_{pe} down to approximately 0.2 ω_{pe} with an accuracy better than 5%.

The methods of detecting and studying the unstable waves were essentially the same as those used in II except that it proved difficult to make accurate measurements of wavelength of the product waves in the presence of the large amplitude pump wave. For this reason it was necessary to use the device of stimulating the instability below its threshold. To do this a signal corresponding to one of the product waves is introduced into the plasma, when theory predicts that the well-defined threshold is modified as shown in figure 1 of II and reproduced as an inset to figure 6.

If such an effect is to be used to identify the type of product wave it is necessary to demonstrate that it is specific as to wave type, i.e. it can result in only one set of waves growing in the plasma. This was done in the following manner. Conditions were set up such that an ion acoustic decay instability could be produced in the plasma as described in II; signals were then introduced at the appropriate frequency both on the screened-coil exciter and the bare-wire probe. Figure 6(a) shows that there was interaction when the signal was applied to the bare-wire probe but not otherwise, showing that the stimulation is indeed specific. Figure 6(b) shows corresponding results at low excitation frequency when the low frequency product wave ω_1 is close to ω_{ci}. In this case there is interaction when the coil exciter is used, i.e. the stimulation is by an ion cyclotron wave but not when an ion acoustic wave is launched. This is the primary evidence for identifying the low frequency product as an ion cyclotron wave. The variation of frequency with pump wave frequency and with magnetic field will be described later.

The nature of the nonlinear process which underlies the instability can be revealed provided the wavenumbers and frequencies of all participating waves can be measured. In particular, it can be determined whether the process is the parametric excitation by a long wavelength (electromagnetic) pump wave of an electron plasma wave and an ion cyclotron wave, as envisaged by Porkolab (1974) or by a travelling electron plasma wave with an oppositely propagating product electron wave and companion ion cyclotron wave.

In order to achieve this a tuned receiver with bandwidth much less than the frequency of an ion cyclotron wave was used to accept signals at the higher product frequency and filter out those at the pump frequency. Then by beating together two signals, one from a fixed probe and the other from an axially traversible probe, the wavelength of the product high frequency wave could be determined interferometrically. It was found to lie on the linear dispersion for electron plasma waves at the appropriate frequency ($\sim$ 9.7 MHz) propagating in the fundamental mode of the column.

Thus it is clear that the pump and both product waves were all azimuthally symmetric. In order to determine whether the process is due to a three-wave resonance, measurements of the wave number were also required. Given that the pump frequency ∿ 10MHz is very much greater than the ion cyclotron frequency, 133 kHz for Na^+ at 0.2T, a resonant process with an electromagnetic pump wave would require $\underline{k}_{\omega_0-\omega_1} \simeq -\underline{k}_{\omega_1}$ while an electron plasma wave pump would require $\underline{k}_{\omega_1} \simeq 2\underline{k}_{\omega_1}$, $\underline{k}_{\omega_0} \simeq -\underline{k}_{\omega_0-\omega_1}$. Under typical conditions $\left(\frac{\omega_{pe}}{2\pi} = 50\text{ MHz}, \frac{\omega_0}{2\pi} = 11\text{ MHz}\right)$ it was found from the linear dispersion curves that $k_0 = 0.177\text{ cm}^{-1}$ and $k_1 = 0.35\text{ cm}^{-1}$. This shows clearly that the pump is an electron plasma wave, launched by the perturbing probe and not an electromagnetic wave associated with its field.

In order to vary the parameters of the experiment in a systematic manner, and provide further evidence for the above conclusion, the axial magnetic field was varied keeping the plasma frequency and the pump frequency constant. With a constant $\underline{k}\omega_0$, the value of $\underline{k}\omega_1$ would be expected to remain constant, but because the similarity variables for the ion cyclotron dispersion are ω/ω_{ci} and k_i/ω_{ci}, one would expect a variation of ω_1 with B. Figure 7 shows the measured values of $^{\omega}/\omega_{ci}$ as a function of B compared with the measured and theoretical dispersion shown as a solid curve, the scaling being chosen to give coincidence at B = 0.2T. It is seen that there is good agreement for variation of the magnetic field by a factor of 5.

A feature of the work of Chu, Bernabei and Motley was the excitation of the second ion cyclotron wave. In a plasma with stationary ions this branch is characterised by its frequency being below the ion cyclotron frequency; for propagation at an angle θ to the magnetic field, the resonance $(k \to \infty)$ occurs at $\omega_{ci}\cos\theta$. However, the dispersion is modified by ion drift in our plasma and therefore, except at very long wavelengths, there are two branches with frequencies above the ion cyclotron frequency and with different thresholds. The second branch can also be observed in a decay process; which of the two occurs under any given set of experimental conditions depends on the level of the pump and the relative values of the thresholds. Figure 8 illustrates the transition from one product mode to the other as the magnetic field is varied.

5. THE ACTION OF AN INCOHERENT PUMP

The practical interest in the process of ion cyclotron wave excitation by a radio-frequency pump wave, and in particular its relevance to thermonuclear plasma applications, stems from the heating observed rather than the underlying mechanism which this paper has so far been concerned to reveal. It is, therefore, of some value to discover what happens when one deliberately sets out to eliminate coherence from the pump, and especially to examine the resultant spectrum of low frequency fluctuations in the plasma in that case.

This is most conveniently done by taking the output of a source of white noise and amplifying and filtering it to give a broad-band spectrum in the appropriate frequency region and introducing such a signal to the plasma on a probe whose response is not frequency sensitive. When the injected power exceeds a threshold value we find that a broad band of low frequencies is excited over the frequency range corresponding to resonant decay from the range of driving frequencies. Under such circumstances a rapid and efficient transfer of energy to the ions raising the ion temperature is expected.

6. CONCLUSIONS

In a magnetised plasma a large amplitude electron plasma wave propagating at an angle to the magnetic field can decay nonlinearly producing an ion cyclotron wave and a back-scattered electron plasma wave satisfying the three-wave resonant matching conditions.

Stimulation of a decay process by injecting a product wave when the pump wave is below threshold intensity is specific as to wave type and allows the decay to be identified.

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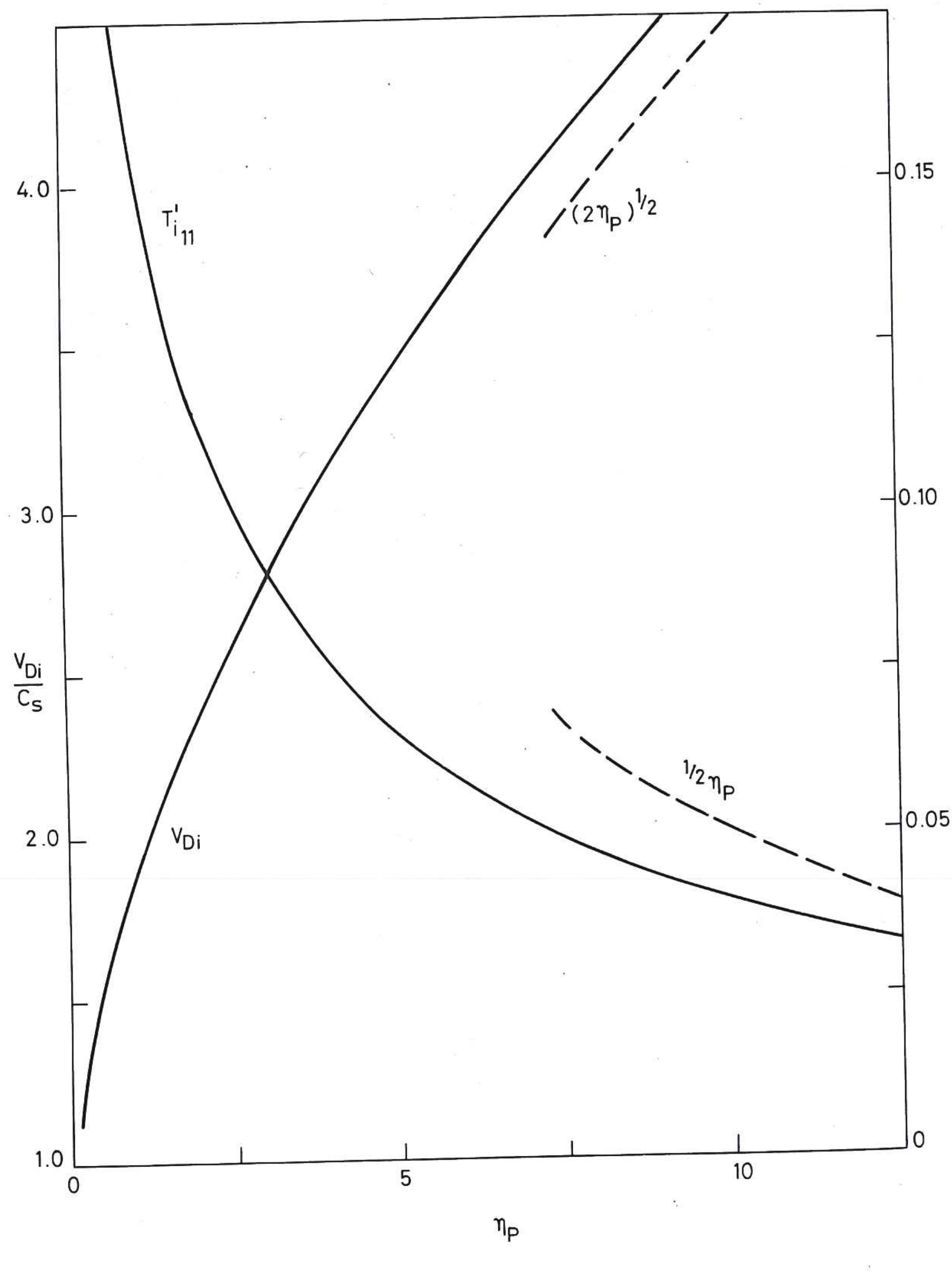

Fig.1 The drift velocity v_{Di} normalized to the ion acoustic speed c_s and the effective ion temperature parallel to the electron (hot plate) temperature for a single ended Q-machine as a function of the normalized plasma potential relative to the hot plate η_p.

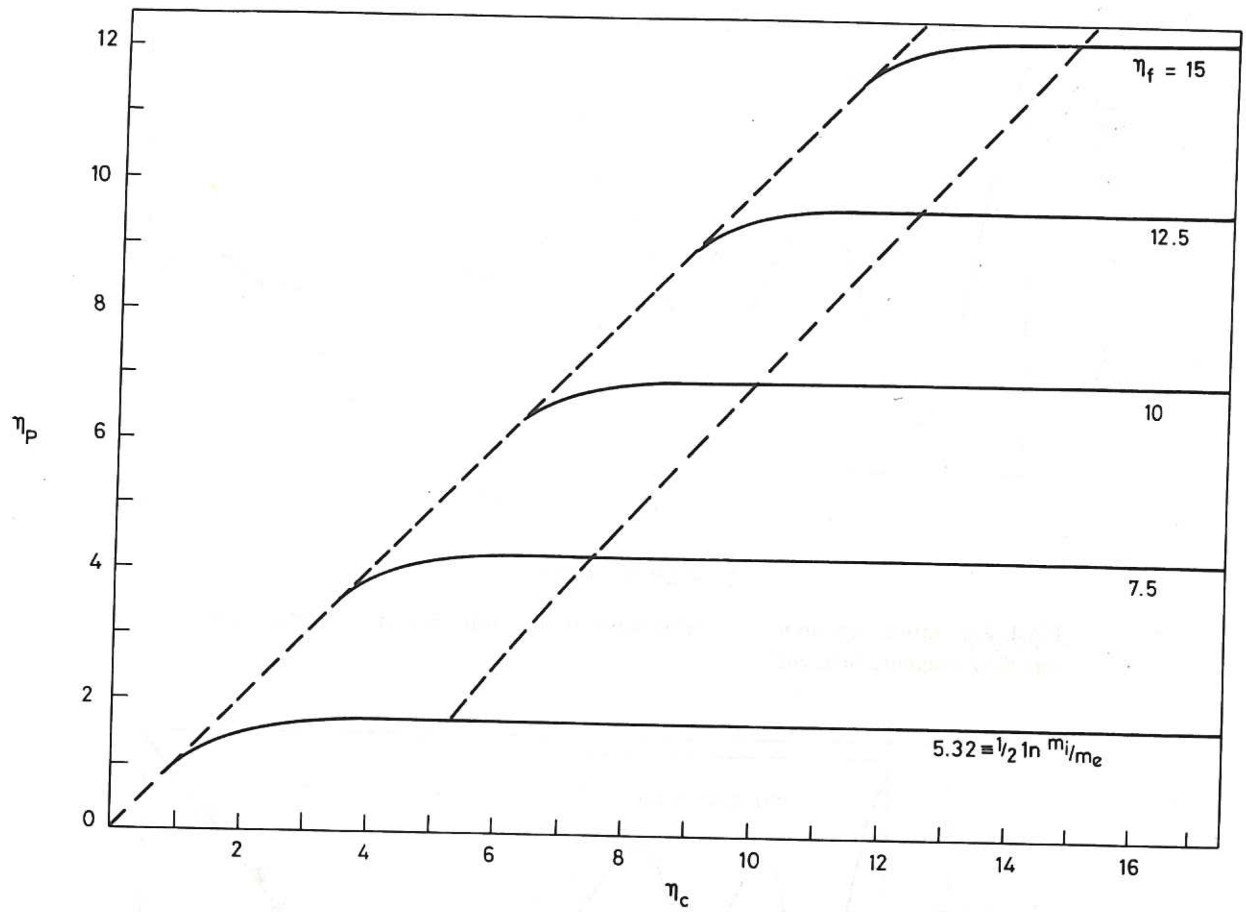

Fig.2 The relation between η_p and η_c the normalized cold plate potential for different values of the floating potential η_f for a sodium vapour plasma.

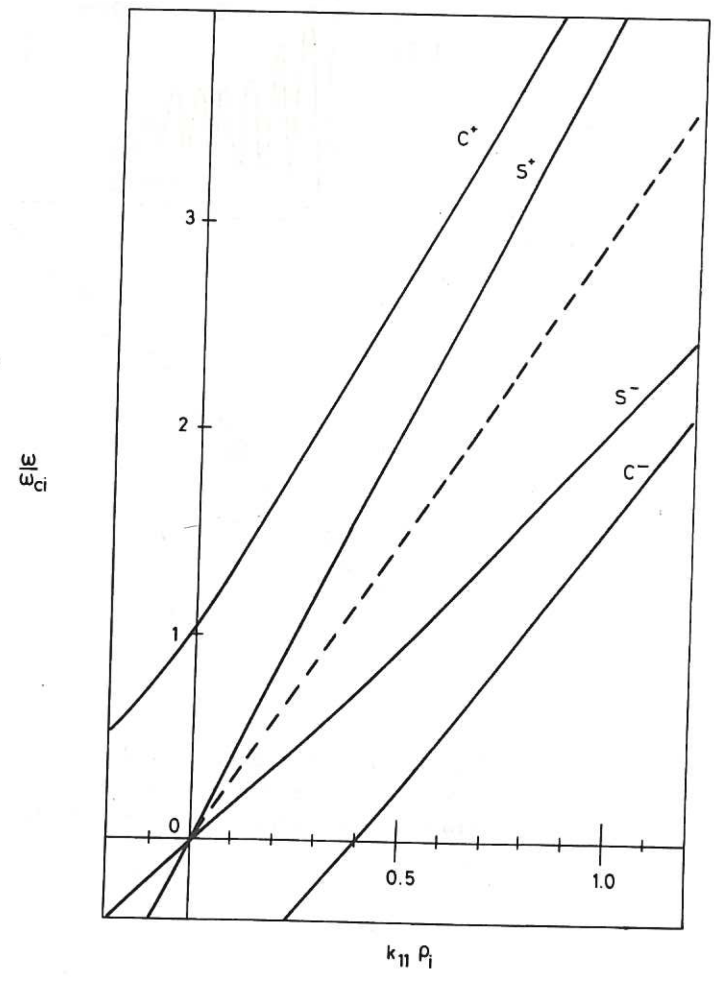

Fig.3 Dispersion curves for the ion cyclotron and ion acoustic waves in a bounded drifting plasma with the following parameters: $v_{Di}/c_s = 3$, $T_{i\|}/Te = 0.1$, $\rho_i/\lambda_{De} \equiv \omega_{pi}/\omega_{ci} = 2$, $\rho_i/a = 0.1$, $k_\perp a = 2.405$.

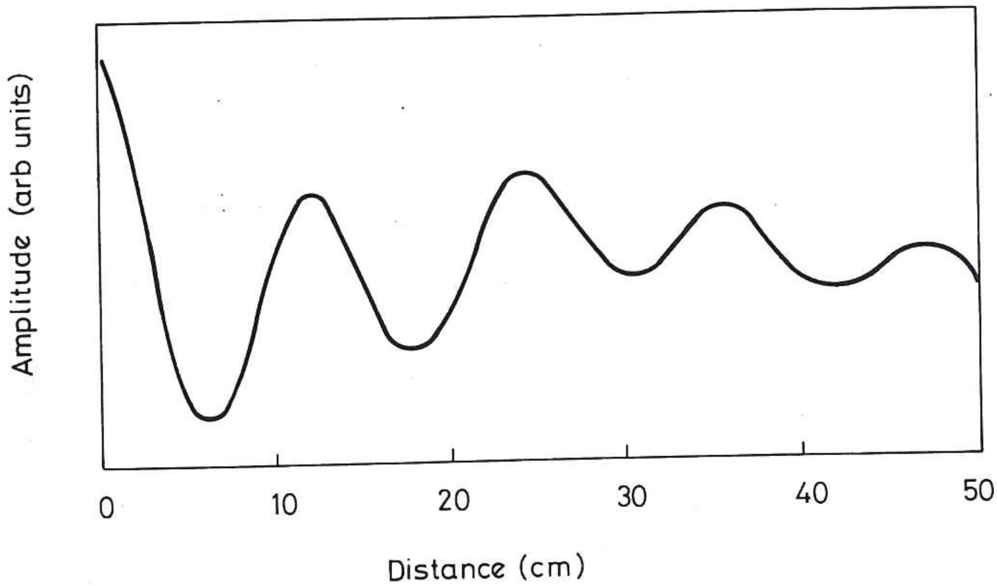

Fig.4 Raw interferogram data showing single mode excitation of an electrostatically screened magnetic field coil.

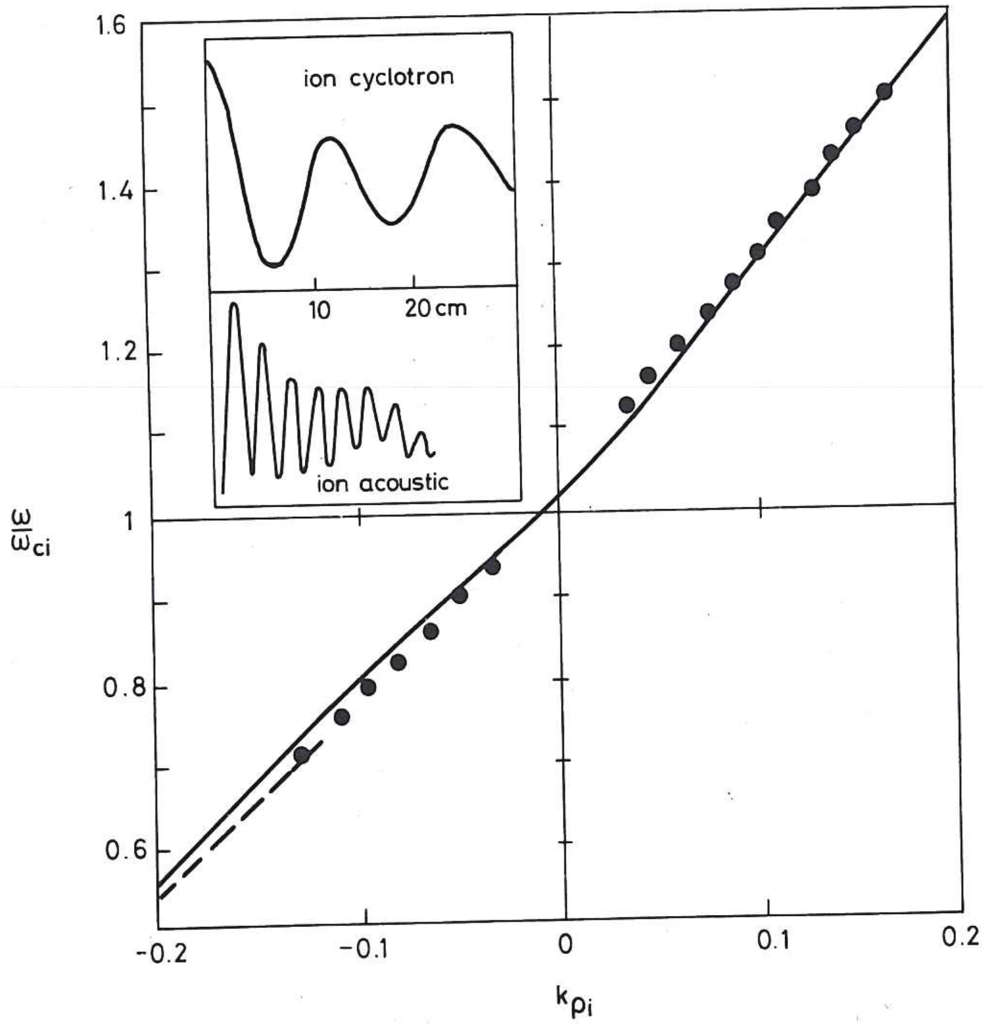

Fig.5 A comparison of theoretical and experimental dispersion for the electrostatic ion cyclotron wave. The inset shows a comparison of ion cyclotron and ion acoustic interferograms at the same frequency of 170 kHz (ω/ω_{ci} = 1.28). The ion drift v_{Di} is taken as 3.0 c_s.

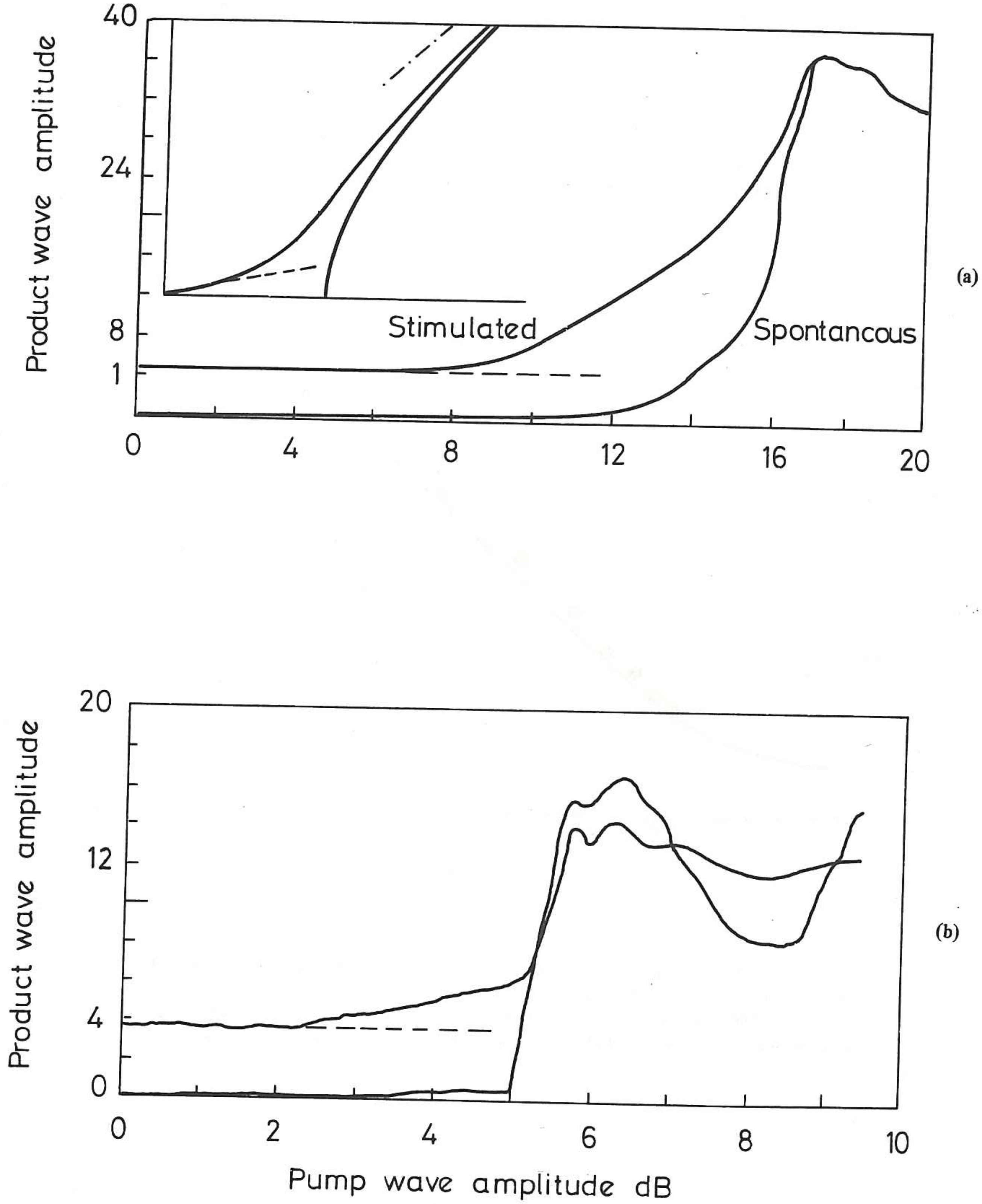

Fig.6 (a) the effect of stimulation on the ion acoustic decay instability showing the removal of the hard onset of the decay as predicted theoretically in the inset. (b) A similar stimulation of the ion cyclotron decay by a small amplitude in cyclotron wave. In both (a) and (b) there was no effect when stimulation was attempted with a wave of the other type.

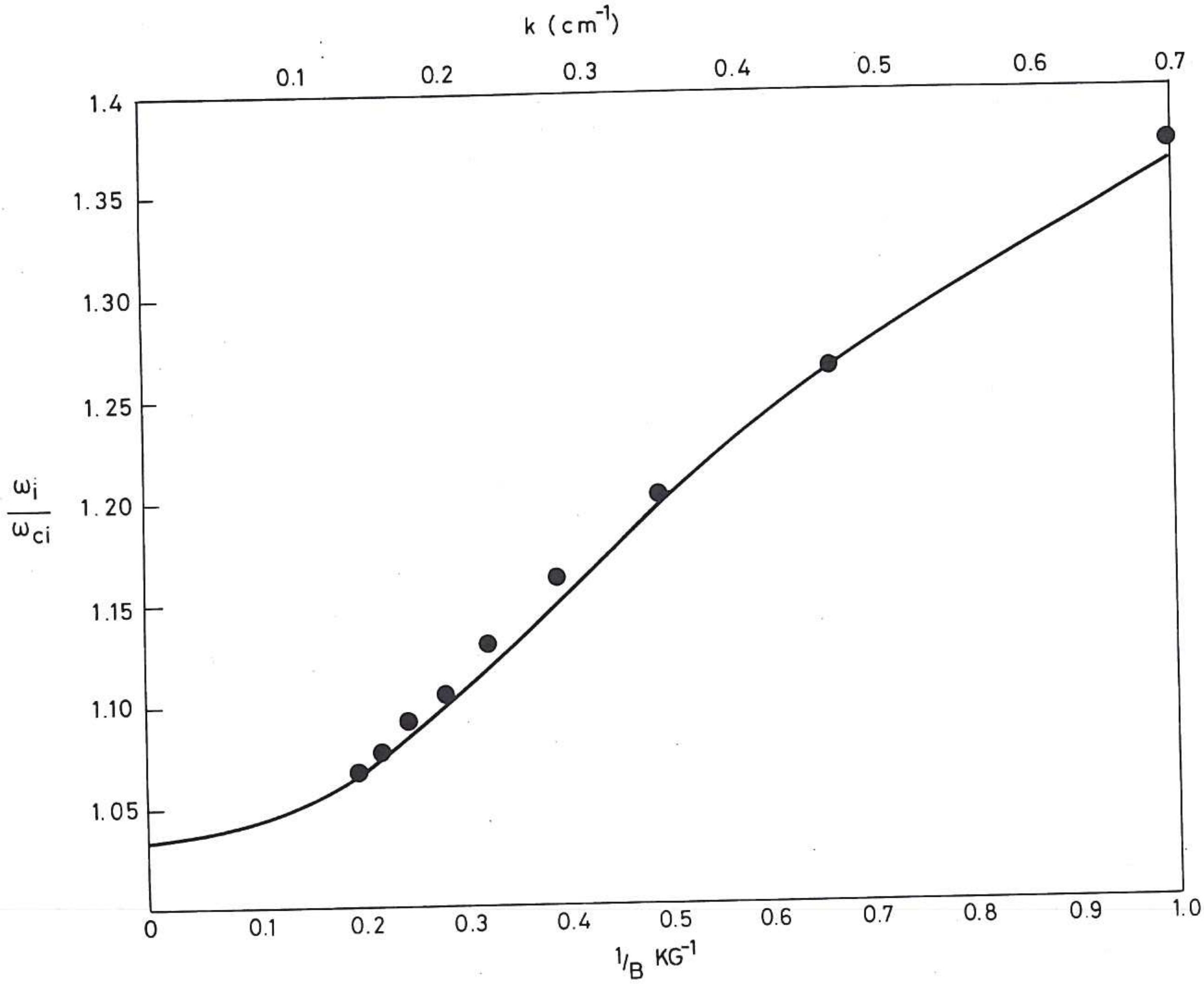

Fig.7 A comparison between theory and experiment for the product of the decay being an electrostatic ion cyclotron wave with wave-number $2k_o$ for a fixed frequency (i.e. wave-number k_o) driving wave and a variable magnetic field. The experimental points give the normalised product frequency ω/ω_{ci} as a function of B the theoretical curve is fitted at 0.2 T to give the scale for the upper wave-number scale.

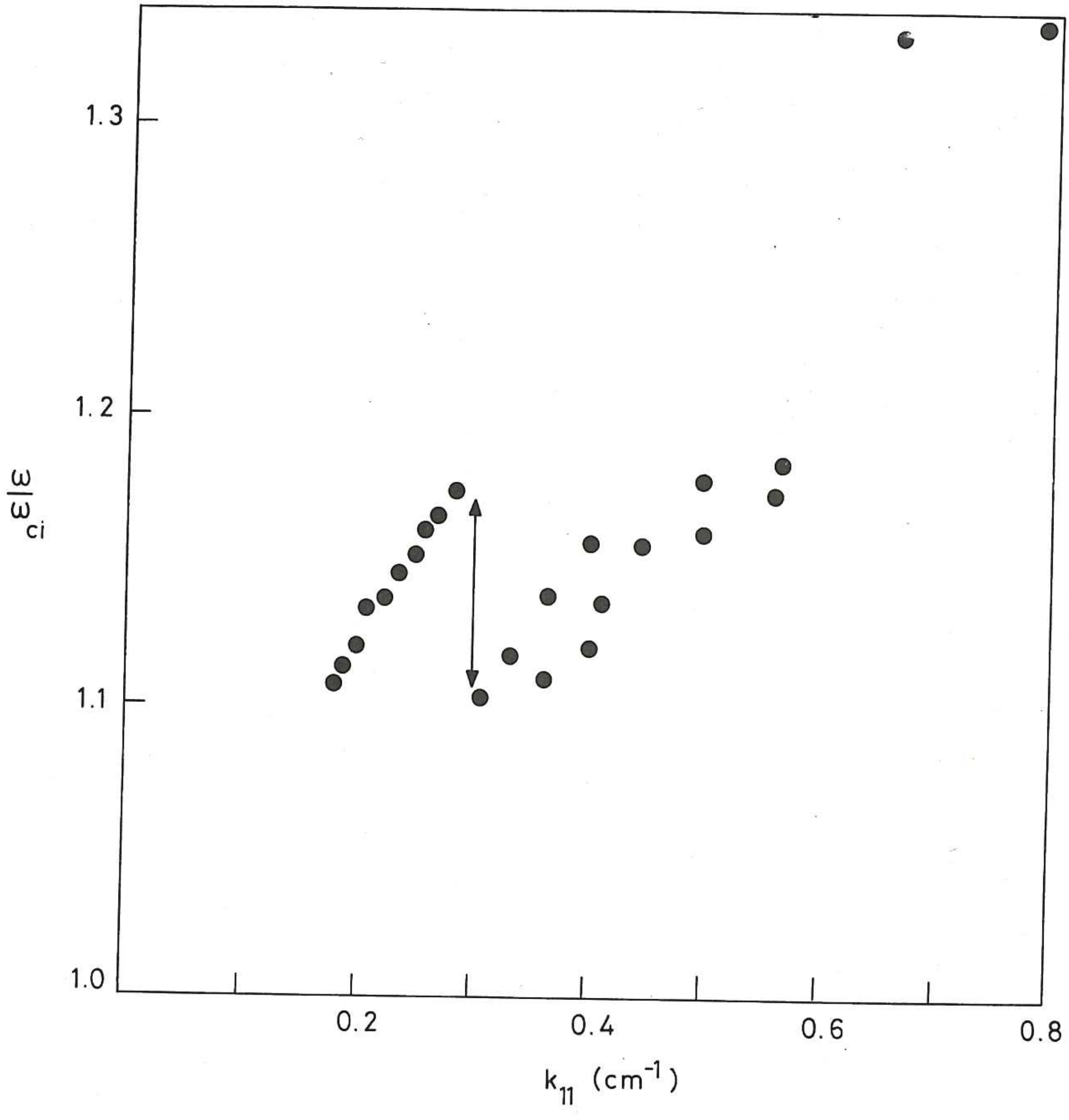

Fig.8 Low frequencies observed in the decay of long wavelength electron plasma waves indicating a transition from one ion cyclotron mode to another as the damping of one mode becomes heavy and the corresponding threshold is raised.