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A THEORY OF PARTICLE AND ENERGY FLUX FROM THE MAGNETIC FLUTTER OF DRIFT WAVES

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Abstract

A quasi-linear theory of particle and energy flux by magnetic flutter associated with drift waves is presented. It is shown that magnetic flutter can enhance the energy flux. However, particle diffusion is ambipolar and runaway electrons do not escape along the field lines.

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Recently, several authors [1 - 3] have speculated that small magnetic fluctuations associated with drift waves may be responsible for anomalous electron energy losses in Tokamaks. These generally extend earlier work [4] on the effect of stochastic wandering of field lines on particle transport. In Refs. [1 - 3] the basic approach is to examine particle loss rates in an ambient magnetic field with a steady random [1 - 3] or oscillating [1] radial field perturbation. In order to get expressions for diffusion and thermal conduction, these authors make various assumptions about magnetic island width [1] and/or correlation lengths of the field fluctuations [1 - 3].

Three basic conclusions stem from this work. Firstly, very small magnetic perturbations can have a large effect on transport. Secondly, electrons are lost along the field lines much faster than the ions, due to their greater velocity. Thus the diffusion is not naturally ambipolar and large radial electric fields must be set up to confine the electrons. Finally, particles with larger parallel velocity are lost much faster than particles with smaller parallel velocity, simply because they move faster along the field lines. Although this point was not emphasised in Refs. [1 - 3], it is clearly pointed out in related work concerning diffusion by randomly wandering magnetic fields of energetic cosmic rays in astrophysical plasmas [5], and diffusion of energetic electrons in laser produced plasmas [6].

While the first result is certainly very interesting, the second is somewhat unsettling and the third is not borne out by experiment. Indeed, experiments generally show that the confinement time of runaway electrons in a Tokamak plasma is comparable to or longer than the overall energy confinement time [7].

The purpose of this note is to show that ordinary quasi-linear theory [8, 9] gives a quantitative description of particle diffusion and thermal conduction associated with magnetic flutter of drift waves. It is not entirely clear whether quasi-linear theory is valid; however it is not clear that it is invalid either. Refs. [1 - 3] all invoke some sort of coherent or steady random structure of the field. For instance Ref. [1] assumes a single drift wave on a rational surface and invokes an assumed island width. Refs. [2 and 3] assume steady magnetic field fluctuations. However, if, at each point, the turbulence has a wide spectrum of wave numbers and frequencies, as seems to be indicated by scattering experiments on ATC [10] and TFR [11], it may well be that ordinary quasi-linear theory is the more accurate description.

In any case, quasi-linear predictions of transport induced by magnetic flutter are simply derived for a known spectrum. This alone argues that the quasi-linear expressions should be recorded. A quasi-linear treatment also has the incidental advantage that it is not necessary to assume "ergodic" wandering of the magnetic field lines.

We show here that the quasi-linear expressions do indeed indicate that small magnetic flutter can enhance the electron energy flux. However they also show that the particle flux is ambipolar and that there is no enhanced transport of runaway electrons, which do not diffuse - simply because they cannot be resonant with any wave of the spectrum.

We also show how particle and energy fluxes are computed. Our calculation closely parallels that given in Ref. [8]. We begin by assuming a steady state and cylindrical geometry, but with arbitrary theta and z components of the average magnetic field. The averaged electron Vlasov equation is

$$v_r \frac{\partial\langle f_e\rangle}{\partial r} - \frac{e}{m}\frac{\partial}{\partial \underline{v}}\cdot\left[\left(\langle\underline{E}\rangle + \frac{1}{c}\,\underline{v}\times\langle\underline{B}\rangle\right)\langle f_e\rangle + \left\langle\left(\underline{\tilde{E}} + \frac{1}{c}\,\underline{v}\times\underline{\tilde{B}}\right)\tilde{f}_e\right\rangle\right] = 0\,, \tag{1}$$

where $\langle\ \rangle$ indicates an ensemble average, tildes indicate fluctuating parts, f_e is the electron distribution function and the other notation is standard. In Equation (1) collisional drag between electrons and ions is neglected as it does not contribute directly to wave induced particle flux. We multiply equation (1) by $\underline{v}$ and integrate over $\underline{v}$ to obtain

$$\frac{\partial}{\partial r}\int d^3v\,\underline{v}\,v_r\,\langle f_e\rangle + \frac{e}{m}\langle n_e\rangle\langle\underline{E}\rangle + \frac{e}{m}\langle\tilde{n}_e\underline{\tilde{E}}\rangle + \frac{e}{mc}\int d^3v\,\langle f_e\rangle\,\underline{v}\times\langle\underline{B}\rangle + \frac{e}{mc}\left\langle\int d^3v\,\tilde{f}_e\,\underline{v}\times\underline{\tilde{B}}\right\rangle = 0 \tag{2}$$

where n_e is the number density of electrons. We now take the θ-component of equation (2). As the plasma is in a steady state, $\langle E_\theta\rangle = 0$, as indicated by Maxwell's homogeneous curl equation. Also the contribution (to the θ-component of the equation) from the $\partial\langle f_e\rangle/\partial r$ term can easily be shown [8] to be smaller than the contribution from the $(\underline{v}\times\langle\underline{B}\rangle)\,\langle f_e\rangle$ term by the ratio of the electron Larmor radius to the macroscopic scale length. The remaining terms are

$$e\,\langle\tilde{E}_\theta\,\tilde{n}_e\rangle - \frac{1}{c}\,\langle\underline{\tilde{J}}_e\times\,\underline{\tilde{B}}\rangle_\theta = \frac{e}{c}\,B_o\int d^3v\;v_r\;\langle f_e\rangle\,, \tag{3}$$

where $\langle\underline{B}\rangle \equiv B_o\hat{\underline{z}}$ and $\underline{\tilde{J}}_e$, the fluctuating electron current density is defined by

$$\underline{\tilde{J}}_e \equiv -\,e\int d^3v\;\underline{v}\;\tilde{f}_e\,. \tag{4}$$

The integral on the right hand side of equation (3) is just the radial electron particle flux, Γ_e, say. Rearranging equation (3) we find

$$\Gamma_e = c\left\{\left\langle \tilde{n}_e \frac{\tilde{E}_\theta}{B_o}\right\rangle - \frac{1}{e B_o} \langle \underline{\tilde{J}}_e \times \underline{\tilde{B}}\rangle_\theta\right\}. \qquad (5)$$

An analogous expression for the ion flux may be derived by the same method (provided the ratio of the ion Larmor radius to the macroscopic scale length is small). It is

$$\Gamma_i = c\left\{\left\langle \tilde{n}_i \frac{\tilde{E}_\theta}{B_o}\right\rangle + \frac{1}{e B_o} \langle \underline{\tilde{J}}_i \times \underline{\tilde{B}}\rangle_\theta\right\}. \qquad (6)$$

Note that equations (5) and (6) are the standard expressions for the particle fluxes [9], augmented by a contribution from the magnetic flutter [1]. Since the plasma is quasi-neutral ($\langle n_i\rangle = \langle n_e\rangle$ and $\tilde{n}_i = \tilde{n}_e$) it is obvious that the particle flux arising from the electric field is ambipolar. It is not obvious that the particle flux arising from the fluctuating magnetic field is ambipolar, since $\underline{\tilde{J}}_e \neq -\underline{\tilde{J}}_i$ in general. However, if we take the cross-product of Maxwell's inhomogeneous curl equation with $\underset{\sim}{\tilde{B}}$ and average, we obtain

$$\langle \underline{\tilde{B}} \times (\underline{\nabla} \times \underline{\tilde{B}})\rangle = \frac{4\pi}{c}\langle \underline{\tilde{B}} \times (\underline{\tilde{J}}_e + \underline{\tilde{J}}_i)\rangle. \qquad (7)$$

The left hand side can be reduced to the divergence of the electromagnetic momentum stress tensor. Since there is no variation of any averaged quantity in the θ or z directions, the θ-component of equation (7) reduces to

$$-\frac{1}{r^2}\frac{d}{dr} r^2 \langle \tilde{B}_r \tilde{B}_\theta\rangle = \frac{4\pi}{c}\langle \underline{\tilde{B}} \times (\underline{\tilde{J}}_e + \underline{\tilde{J}}_i)\rangle_\theta .$$

Whatever the radial eigenfunction, it does not seem reasonable to us that the average force the plasma exerts on itself can be balanced by $\frac{1}{r^2}\frac{d}{dr} r^2 \langle \tilde{B}_r \tilde{B}_\theta\rangle$. Consequently $\langle \underline{\tilde{B}} \times \underline{\tilde{J}}\rangle = 0$ and the particle diffusion is ambipolar.

Thus the wave frequency and polarisation are picked out so that $\tilde{n}_e = \tilde{n}_i$ and $\langle \underline{\tilde{B}} \times \underline{\tilde{J}}_e \rangle = - \langle \underline{\tilde{B}} \times \underline{\tilde{J}}_i \rangle$. This is analogous to the case of purely electrostatic waves, where the polarisation is known but where the frequency is picked out so that $\tilde{n}_e = \tilde{n}_i$.

We now turn to the problem of the anomalous electron energy flux driven by the magnetic flutter associated with drift waves. The calculation again parallels that given in Ref. [8]. Taking the $\frac{1}{2} mv^2 v_\theta$ moment of the Vlasov equation, averaging over wave phases, and neglecting the $[\frac{\partial}{\partial t} + \underline{v}.\underline{\nabla}] \langle f_e \rangle$ terms on the same grounds as in our treatment of the particle fluxes, we find

$$0 = \frac{e}{m}\int d^3v \, \frac{mv^2 v_\theta}{2} \left\{ \left\langle \underline{\tilde{E}} . \frac{\partial \tilde{f}_e}{\partial \underline{v}} \right\rangle + \left\langle \frac{1}{c} (\underline{v} \times \underline{\tilde{B}}) . \frac{\partial \tilde{f}_e}{\partial \underline{v}} \right\rangle + \frac{1}{c} (\underline{v} \times \underline{B}_o) . \frac{\partial \langle f_e \rangle}{\partial \underline{v}} \right\} \quad (8)$$

By various partial integrations, the last term in equation (8) can be manipulated into eB/mc times the radial electron energy flux, W_e. The other terms in equation (8) can also be simplified by partial integrations, so that equation (8) reduces to

$$W_e = \frac{c}{B}\int d^3v \left\{ \frac{mv^2}{2} \langle \tilde{E}_\theta \, \tilde{f}_e \rangle + \langle m \, v_\theta \underline{v} . \underline{\tilde{E}} \, \tilde{f}_e \rangle + \frac{mv^2}{2} \frac{v_z}{c} \langle \tilde{B}_r \, \tilde{f}_e \rangle \right\} , \quad (9)$$

where, in deriving equation (9), we have neglected terms like $\frac{v_r}{c} \tilde{B}_z$ because we are only interested in electron energy flux driven by radial magnetic flutter. Thus equation (9) is an expression for the energy flux in terms of a given wave spectrum.

Let us assume that linear theory gives the result

$$\tilde{f}_e(\underline{k}, \underline{v}, \omega) = \underline{\tilde{E}}(\underset{\sim}{k}) . \underline{F}(\underline{k}, \underline{v}, \omega) \quad (10)$$

in the absence of magnetic flutter. In the presence of radial magnetic flutter, $\underline{\tilde{E}}$ in the Vlasov equation is replaced by $\underline{\tilde{E}} + \frac{1}{c} \underline{v} \times \underline{\tilde{B}}$, so that in the presence of magnetic flutter :

$$\tilde{f}_e(\underline{k}, \underline{v}, \omega) = \left(\tilde{\underline{E}}(\underline{k}) + \frac{1}{c}\,\underline{v} \times \tilde{\underline{B}}(\underline{k})\right) . \underline{F}(\underline{k}, \underline{v}, \omega) \tag{11}$$

Assuming that the θ-component of the force is dominant with or without magnetic flutter and also that $\tilde{B}_z = 0$, the equation for energy flux reduces to

$$W_e = \Sigma \frac{c}{B} \int d^3v \left\{ \frac{mv^2}{2} \left\langle \left| \tilde{E}_\theta(\underline{k}) + \frac{v_z}{c}\,\tilde{B}_r(\underline{k}) \right|^2 \right\rangle + m\,v_\theta^2 \left\langle \tilde{E}^*_\theta(\underline{k}) \left(\tilde{E}_\theta(\underline{k}) + \frac{v_z}{c}\,\tilde{B}_r(\underline{k}) \right) \right\rangle \right\} F(\underline{k}, \underline{v}, \omega) + \text{c.c.} \tag{12}$$

Obvious generalisations of equation (12), which relax our simplifying assumptions, can be made.

Equation (12) shows the energy flux, with or without the radial magnetic flutter. Clearly, a measure of the importance of magnetic flutter is simply the ratio $v_z\,\tilde{B}_r(\underline{k})/c\,\tilde{E}_\theta(\underline{k})$. Adopting the result of Ref. [1] :

$$\frac{\tilde{B}_r(\underline{k})}{B} \sim \frac{i\omega}{k_{\parallel}} \cdot k_\perp \rho_i \cdot \sqrt{\frac{T_e}{M} \cdot \frac{4\pi n M}{B^2}} \cdot \left(1 + \frac{T_i}{T_e}\frac{\omega_{*e}}{\omega}\right) \cdot \frac{e\varphi(\underline{k})}{T_e} \,, \tag{13}$$

one can show that for Tokamak parameters $v_z\,\tilde{B}/c\,\tilde{E}$ can indeed be of order unity or larger.

As is apparent from equation (12), only the real part of $F(\underline{k}, \underline{v}, \omega)$ contributes to the energy flux. Depending on whether a particle is magnetically trapped or not, the real part of $F(\underline{k}, \underline{v}, \omega)$ goes as the imaginary part of $(\omega + i\,\nu_{eff} - \omega_D)^{-1}$ or $(\omega - k_{\parallel} v)^{-1}$, where ν_{eff} is the effective collision frequency, $\omega_D \approx \omega_* \frac{L_n}{R} \cdot \frac{v^2}{V_e^2}$, L_n is the density gradient scale length, R is the major radius of the torus, ω_* is the drift frequency and V_e is the electron thermal velocity [8, 9, 12].

For the untrapped particles, the real part of F is non-zero only for those particles with $v_{\parallel} = \omega/k_{\parallel}$. However, for drift waves, $\omega/k_{\parallel} \ll V_e$, so that untrapped runaway electrons are not affected by the magnetic flutter. For the trapped electrons, the real part of F can result either from $\omega = \omega_D$ or else from the $i\nu_{eff}$ term in the resonant denominator. Since the wave frequency is of order ω_* or less, trapped electrons with energy larger than 3 or 4 times the thermal energy cannot be in resonance. Similarly, as electron energy increases, ν_{eff} decreases so that the real part of F from either resonance or collisions is very small. Thus runaway electrons are not resonant with any wave in the spectrum and are not preferentially diffused by the magnetic flutter associated with drift waves.

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