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ABSTRACT

Sawtooth oscillations are observed in the small tokamak TOSCA with a period of 100 - 200 μs. The q = 1 singular surface occurs at a radius of 1 - 1.5 cm. The sawtooth oscillations are not suppressed by shaping the plasma into a triangle. By comparing our results with those of larger tokamaks we obtain an empirical scaling law for the repetition time. It is demonstrated that this scaling law is compatible with ohmic heating and a resistive instability whose growth rate is dependent upon the decrease in q below unity.

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Internal disruptions or sawtooth oscillations have been observed and investigated in many large Tokamak experiments[1]. Theoretical descriptions of the process have been given[2,3] and numerical simulations support the theory[4]. These investigations have now been extended to the small Tokamak TOSCA [5], where the effects of shaping the plasma cross-section on the internal m = 1, n = 1 mode, and the sawtooth period are studied. Data from a large number of Tokamaks which exhibit sawtooth oscillations are used to derive a scaling law for the sawtooth repetition time.

The internal disruptions are observed with a surface barrier detector (Ortec CR-019-50-100) that is sensitive to soft x-rays in the photon energy range between 0.5 and 10 keV, and a Si(Li) detector (Ortec 7000 series) with a sensitivity to x-rays between 0.8 and 30 keV. The soft x-ray signal obtained with the surface barrier detector when there is a peaked current distribution is shown in Figure 1 for a triangular plasma. The sawtooth oscillation represents approximately a 3.5% variation in temperature, its period is typically 100 → 200 μs. The sawtooth oscillation has a modulation superimposed on it with a frequency of 40 → 60 kHz. This modulation is identified, with the two x-ray detectors which are at different positions in the toroidal and poloidal directions, as an m = 1, n = 1 mode.

Spatial resolution of the plasma column is obtained by using an angled aperture which can be rotated to scan over a radius of 4 cm. This system is used to measure the radius at which the sawtooth oscillation inverts, r_s.

A comparison of sawtooth oscillations in circular and triangular plasmas has been carried out. According to theory [6] triangularity at an appropriate level[7] should have a stabilising effect on the internal ideal MHD m = 1, n = 1 mode, though it should be noted that the mode may be stable even for a circular plasma depending on the central value of the poloidal beta. From our experiments, we find that sawteeth are still present with a triangular shaped plasma. The general features of the sawtooth oscillations are the same except that the radius for inversion is larger (1 cm → 1.5 cm) and the period is slightly longer (on average 120 μs → 140 μs).

It should be noted that sawtooth oscillations are not always present on this device. Stable operation at low values of q (< 3) and Z_{eff} is obtained without current peaking and no sawtooth behaviour is observed.

From a comparison of the sawtooth period on other devices, it is apparent that the maximum period of the sawtooth oscillations on TOSCA is much shorter. This fact and the results from other ohmically heated Tokamaks is used to obtain a scaling law for the period of the sawtooth. This scaling law permits us to understand the nature of the processes causing the relaxation.

A systematic analysis has been carried out in which we assume the sawtooth repetition time can be expressed in the form

$$t_{repetition} = C\, t_1^a\, t_2^b\, t_3^c \,\ldots \tag{1}$$

where C is a constant and t_1, t_2, t_3 .... are various characteristic times of the different devices. The data set used for this is shown in Table 1. The results for two particular choices of parameters obtained on the computer are shown in Table 2. $\tau_R = \frac{r_s^2}{\eta}$ is the resistive 'skin' time, $\tau_A = \frac{R\rho^{\frac{1}{2}}}{B_\phi}$ the Alfven transit time, $\tau_h = \frac{3nT_e}{2\eta j^2}$ the heating time, and $\omega_* = \frac{dp}{dr} / eB_\phi r_s n$ the diamagnetic drift frequency. ρ is the mass density, R the major radius, B_ϕ the toroidal field, η the resistivity j the central current density and T_e the electron temperature. It is clear that the process is not strongly dependent on any one of the characteristic times.

Several theories [2,3,8] have been put forward to explain the sawtooth relaxation. Resistive heating, accompanied by a reduction in q, is followed by the growth of an m = 1 instability which could be an ideal MHD mode, internal resistive kink or a tearing mode. Ideal MHD predicts a growth rate (if the central $\beta_p > 0.3$) for the m = 1 mode that varies by only a factor of three for all the devices. However the observations show that the growth rates differ by about two orders of magnitude (TOSCA ∿ 30 μs, PLT ∿ 2 ms). Therefore, the mode must be resistive in origin.

It is more difficult to distinguish between the internal resistive kink and the tearing mode. Theory indicates that for most of the devices the internal resistive kink should dominate but depending on the ideal MHD potential energy and $\frac{\tau_R}{\tau_A}$, the weaker tearing mode may grow. The repetition time will depend on the time scale for the reduction of q below unity and the dependence of the growth rate of the resistive mode on shear, with an allowance for diamagnetic drifts. From the field diffusion equation for $\frac{1}{q} = \mu$

$$\frac{\partial \mu}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\eta}{r} \frac{\partial}{\partial r} (r^2 \mu) \right) \tag{2}$$

and the heating equation

$$n \frac{\partial T_e}{\partial t} = \eta j^2 - Q_{ie} - \frac{n\,T_e}{\tau_{E_e}} \qquad (3)$$

Q_{ie} is the electron-ion heat transfer rate, τ_{E_e} the central electron energy confinement time. We can solve for the small decrease in q below unity, with $\mu = 1 + \mu_1\ (r,t)$ using $\eta_1 = -\frac{3}{2}\,\eta\,\frac{T_{1e}}{T_{oe}}$ for the perturbed resistivity, T_{1e} is the perturbed electron temperature. Equations (2) and (3) become

$$\frac{\partial \mu_1}{\partial t} = \frac{\eta}{r}\left(3\,\frac{\partial \mu_1}{\partial r} + r\,\frac{\partial^2 \mu_1}{\partial r^2}\right) + \frac{2}{r}\,\frac{\partial \eta_1}{\partial r}$$

$$(4)$$

$$\frac{\partial \eta_1}{\partial t} = \frac{3}{2}\ \frac{\eta}{\tau_h}\left(-\frac{\eta_1}{\eta_o} - 1 + Q_{ie} - \frac{1}{r}\,\frac{\partial}{\partial r}\,(r^2\mu_1)\right)$$

if τ_{E_e} is much greater than the repetition time. These equations have solution

$$\mu_1 \propto e^{\lambda t}\,\frac{J_1(\sqrt{C}\,r/r_s)}{r}\ , \quad \text{with}\quad C = \tau_R \cdot \frac{-\frac{3}{2}\frac{\lambda}{\tau_h} - \lambda^2}{(\lambda - \frac{3}{2\tau_h})}$$

which with the boundary condition $\mu_1\ (r_s) = 0$ determines the value of λ. As $\frac{\tau_R}{\tau_h}$ varies from 0.5 to 10 for the devices so the root λ varies from $\frac{1.4}{\tau_h}$ to $\frac{0.6}{\tau_h}$. Hence we obtain $\mu_1 \propto \frac{t}{\tau_h}$ to a good approximation. The sawtooth repetition time is then determined by using this variation in the dependence of the growth rate on shear and assuming that the heating is terminated when the m = 1 island width, produced by the resistive mode, grows to r_s. From the different growth rate scalings [6,9] we can present the dependence of the repetition time on the characteristic times. Table 3 shows the predictions for the different modes. (We have taken ω_* to be independent of the sawtooth period). Figure 2 indicates that the tearing mode scaling without the inclusion of diamagnetic drift effects appears to give a good fit to the data. This does not differ very significantly from the empirical fit given in Table 2 , this scaling is similar to that obtained by Waddell et al.[3]

Our results show that triangularity does not suppress the internal m = 1 instability. However there are theoretical results [6] which indicate that, at sufficiently high temperatures and at an appropriate rotation frequency , stabilisation of the internal resistive mode will occur. The growth of the m = 1 instability and the sawtooth repetition time varies by two orders of magnitude from the smallest to the largest devices. The empirical scaling and the theoretical scalings for the repetition time indicate that the internal m = 1 instability is resistive in origin.

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TABLE 1

Machine	r_s cm	R cm	B_ϕ kG	V Volts	$T_e(0)$ keV	$n_e(0)$ $x10^{13}$	Zeff	T ms	gas	Ref
PLT	9	130	35	2	0.85	14	2	9	He	[1]
T-10	12	150	35	2	0.8	8	1.5	13	D_2	[2]
TFR	5.5	98	25	2.5	1.2	4.5	6	2.5	H_2	[3]
T-4	5.5	100	26	5.5	0.7	6	5	1	H_2	[4]
Pul	2.5	70	27	2.5	0.5	16	1.2	1.7	H_2	[5]
Ormak	2.5	80	13.5	3.5	0.75	2.5	7	0.5	H_2	[6]
ST	1.5	109	37	2	0.75	6	3.8	1.7	D_2	[7]
DITE	5	117	13.4	3	0.6	1.2	5	1	H_2	[8]
Al	2.0	54	60	2	0.8	70	1	1.5	D_2	[9]
TOSCA	1.5	30	4	2	0.2	1.5	1.5	0.12	H_2	

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TABLE 2

Computer Fit

	Parameters used	Indices found	Correlation Coefficient
C1	τ_R τ_A τ_h	$\tau_R^{0\cdot42}$ $\tau_A^{0\cdot14}$ $\tau_h^{0\cdot44}$	0.985
C2	τ_R τ_A τ_h ω_*	$\tau_R^{0\cdot35}$ $\tau_A^{0\cdot04}$ $\tau_h^{0\cdot44}$ $\bar{\omega}_*^{-0\cdot16}$	0.986

TABLE 3

$$\left(\Delta q \equiv \mu_1,\ S = \frac{\tau_R}{\tau_A},\ \alpha = \left.\frac{dq}{dr}\right|_{r=r_s}\right)$$

Scaling	Assumptions	Characteristic time
1	$\Delta q \sim \frac{t}{\tau_h}$ $\gamma = \gamma_T\ ,\ \gamma_T = \frac{\alpha^{\frac{2}{5}}}{S^{\frac{3}{5}}\ \tau_A}$	$\tau_R^{\frac{3}{7}}\ \tau_A^{\frac{2}{7}}\ \tau_h^{\frac{2}{7}}$
2	$\Delta q \sim \frac{t}{\tau_h}$ $\gamma = \frac{\gamma_T^{\frac{5}{3}}}{\omega_{*i}^{\frac{1}{3}}\ \omega_{*e}^{\frac{1}{3}}},\ \gamma_T = \frac{\alpha^{\frac{2}{5}}}{S^{\frac{3}{5}}\ \tau_A}$	$\tau_R^{\frac{3}{5}}\ \tau_A^{\frac{2}{5}}\ \tau_h^{\frac{2}{5}}\ \bar{\omega}_*^{\frac{2}{5}}$

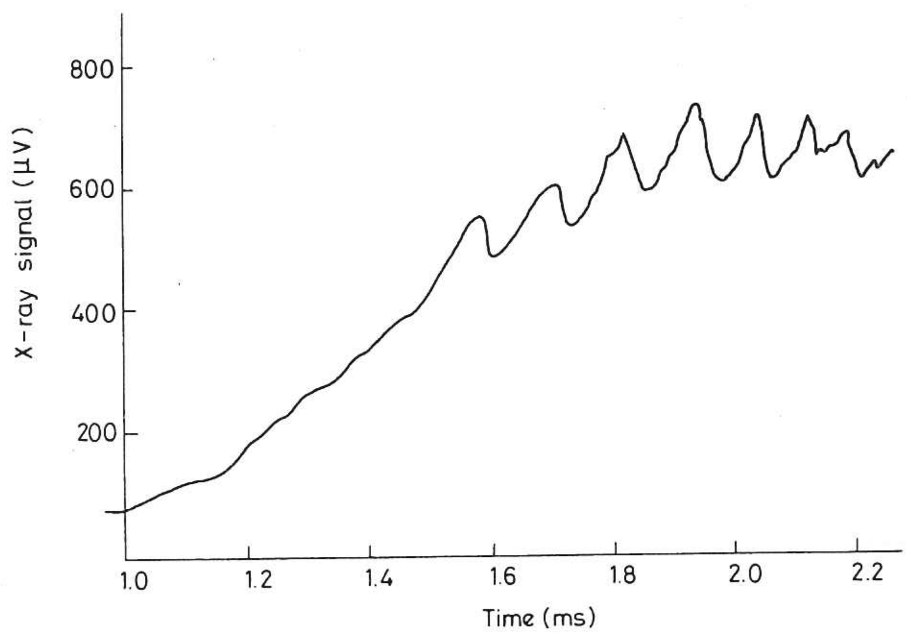

Fig. 1. Soft x-ray emmission as a function of time for a triangular plasma.

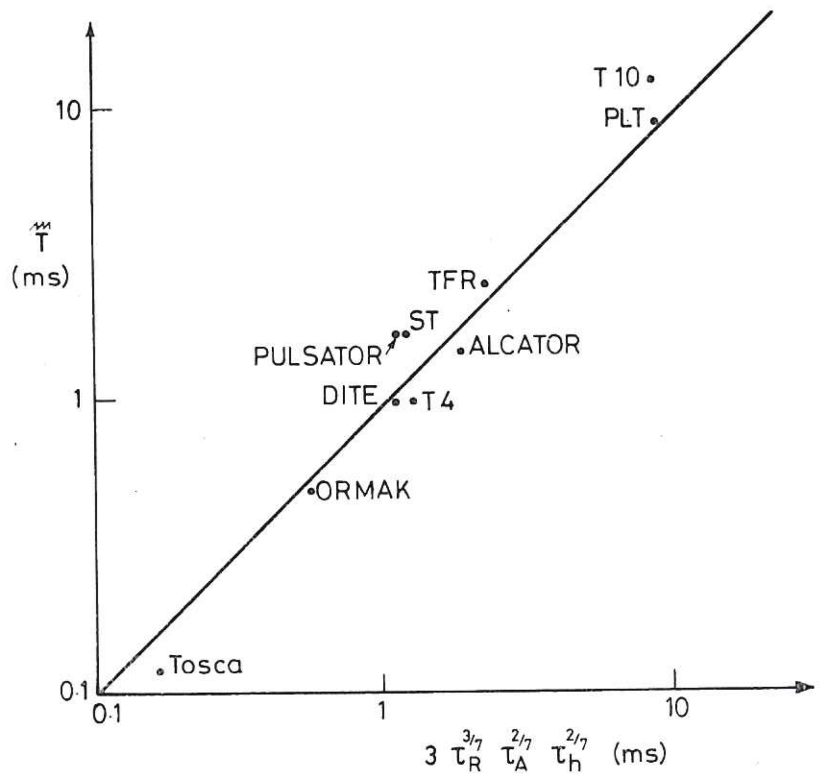

Fig. 2. Sawtooth period as a function of theoretical time for heating and termination by a tearing mode.