CLM-P602

CLM-P602

UKAEA

Preprint

ENHANCED SPACE CHARGE COMPENSATION OF ION BEAMS USING MULTIPOLE MAGNETIC FIELDS

T. S. GREEN

CULHAM LABORATORY
Abingdon Oxfordshire

1980

CLM-P602

ENHANCED SPACE CHARGE COMPENSATION OF ION BEAMS USING MULTIPOLE MAGNETIC FIELDS

by T S Green

Culham Laboratory, Abingdon, Oxon OX14 3DB, UK.
(EURATOM/UKAEA Fusion Association)

ABSTRACT

Models of space charge compensation of medium current ion beams discussed in the literature show that the potential which develops across the beam is determined by the separate requirements of containment of electrons and of slow ions coupled to the need to maintain quasi space-charge neutrality. These analyses suggest that multipole magnetic fields which influence the containment may modify the potential. This possibility is analysed in the present paper and calculations of the modification of potential are presented for a particular model of space-charge compensation.

(Submitted for publication in Nuclear Instruments and Methods)

January 1980

1. INTRODUCTION

The problem of space charge compensation of medium to high current ion beams has recently been discussed extensively in the literature. The various treatments all show that the potential of the beam centre relative to the beam line walls relates to the need to contain the electrons required to neutralise the beam. The distribution of this potential has been shown to depend on the ratio of the Debye length to the beam dimensions. However, for many cases of interest this ratio is such that the potential is distributed continuously across the beam and vacuum and not only concentrated in the sheath; in such cases the resultant potential gradient acts to cause beam divergence.

In this note, we will examine the possibility that in such cases the potential and therefore the beam divergence can be reduced by the use of a multipole magnetic field configuration which confines the electrons and slow ions produced by collisions of the beam ions with the (background) neutral gas. The calculation to be performed is based on the assumption of charge neutrality on the beam axis which applies for a wide range of beam conditions as discussed by Gabovich[(1)] and Green[(2)] for ion beams and Dunn and Self for electron beams[(3)]. This condition allows one to equate the sum of the beam density and slow ion density to the electron density. The slow ion and the electron densities are calculated from the requirement that production rates balance loss rates. The treatment of the ion loss is straightforward, but that of the electron loss is less clear [Green[(2)]]. Three models have been discussed in the literature: each one will be considered below.

2. CALCULATION OF THE PLASMA POTENTIAL

2.1 The quasi-neutrality condition

This condition is written simply as

$$n_b + n_{io} = n_{eo}$$

where n_b is the density of ions in the beam
n_{io} is the density of slow ions on the beam axis
n_{eo} is the density of electrons on the beam axis

2.2 Calculation of slow ion density

The slow ion density is calculated by equating the production rate to the loss rate. The production rate per unit length of the beam is written simply as

$$n_b n_o \sigma_i v_b A_b \qquad (1a)$$

where n_o is the neutral gas density, v_b the velocity of the ions in the beam, σ_i the cross-section for production of slow ions by ionisation and charge exchange, A_b is the area of the beam (πa^2).

The loss rate is written as

$$n_{io} v_i A_w \tag{1b}$$

where v_i is an average velocity of the slow ions leaving the beam and A_{wi} is the area of the wall open to the ions, per unit length of beam. Normally v_i is taken to be $(\frac{2e\phi}{m_i})^{\frac{1}{2}}$ where ϕ is the plasma potential[1]. Clearly this is only an approximate value, the exact numerical factor depending on the distribution of potential: however, the experimental and theoretical study reported by Gabovich indicates that it is a reasonable approximation.

The wall area per unit length is usually given by $2\pi R$, R being the beam line radius. In this treatment we take the effective wall area to be reduced by a factor β_i by the effect of the multipole magnetic field (note that β_i will depend on v_i).

Equating the ion production and loss rates one derives the equation

$$\frac{n_{io}}{n_b} = \frac{a^2}{2R^2} \cdot \frac{\tau_{ti}}{\tau_{bi}} \cdot \frac{1}{\beta_i} \tag{2}$$

where τ_{ti} is the slow ion transit time $\left[R(\frac{m_i}{2e\phi})^{\frac{1}{2}}\right]$

and τ_{bi} is the time constant for ion production $\left[(n_o v_b \sigma_i)^{-1}\right]$.

2.3 Calculation of electron density

The electron density is also calculated by equating the rate of production to the rate of loss. The rate of production, S, is given simply by the relation

$$S = n_b n_o \sigma_e v_b A_b$$

where σ_e is the cross-section for electron production in collisions between beam ions and neutrals. Both surface effects and ionisation by the electrons themselves are neglected in this analysis.

The loss rate of the electrons is more difficult to calculate since it depends in detail on the behaviour of the electrons with regard to the development of their energy distribution. Three cases previously considered in the literature in discussion of space charge problems are as follows.

Case (a) The electrons are assumed to be in thermal equilibrium with the high energy tail always filled. Depletion of the tail by loss of electrons out of the potential well is compensated for in less than an electron transit time. It follows that the electron loss rate, per unit length of beam line, is

$$\frac{n_{eo}v_e}{4} \cdot A_{we} \cdot \exp\left(\frac{-e\phi}{kT}\right)$$

where A_{we} is the area of the wall open to the electrons and is written, as above, as $2\pi R\beta_e$. T is the electron temperature, and v_e is the electron thermal velocity.

Equating this loss rate to the production rate, one derives the result

$$\frac{n_{eo}}{n_b} = \frac{a^2}{R^2} \frac{2\,\tau_{te}}{\tau_{be}} \cdot \frac{1}{\beta_e} \exp\left(\frac{e\phi}{kT}\right) \qquad (3a)$$

where τ_{te} is the electron transit time $(\frac{R}{v_e})$ and τ_{be} is the electron production time, $(n_o\sigma_e v_b)^{-1}$.

<u>Case (b)</u> The electrons are again assumed to be in thermal equilibrium with a Maxwellian distribution, <u>but</u> the distribution is truncated at an energy corresponding to the plasma potential. Electrons are scattered into the tail at a rate determined by the electron-electron collision time τ_{ee} which is temperature dependent. The temperature is maintained against the associated heat loss by energy input from the beam ions via ion-electron collisions[4,5].

The loss rate, per unit length of beam line, is

$$\frac{n_{eo}\pi R^2}{\tau_{ee}} \cdot \exp\left(\frac{-e\phi}{kT}\right)$$

Again equating loss rate to production rate, one derives the result

$$\frac{n_{eo}}{n_b} = \frac{a^2}{R^2} \cdot \frac{\tau_{ee}}{\tau_{be}} \cdot \exp\left(\frac{e\phi}{kT}\right) \qquad (3b)$$

<u>Case (c)</u> The electrons are assumed to have the energy spectrum with which they are created in the ionisation events, truncated at an energy corresponding to the plasma potential. Electrons gain energy from the ion beam to raise them above the plasma potential and thus escape from the potential well. From a calculation of both particle and energy balance, Gabovich et al.[1] derive an equation for n_{eo} which can be written in the form

$$\frac{n_{eo}}{n_b} = \frac{\phi_i}{U} \frac{\tau_{ei}}{\tau_{be}} \left[\left(\frac{\phi}{2\phi_i} + 1\right) - \left(1 + \frac{\phi_i}{\phi}\right) \ln\left(1 + \frac{\phi}{\phi_i}\right) \right] \qquad (3c)$$

where ϕ_i is the ionisation potential

U is the potential through which the beam ions are initially accelerated

and τ_{ei} is the time constant for the electrons to gain energy from the beam ions, such that eU/τ_{ei} is the rate of gain of energy of each electron.

Gabovich et al. give for τ_{ei} the following expression:

$$\frac{\tau_{ei}}{eU} = \left(\frac{2eU}{m_b}\right)^{\frac{1}{2}} \frac{m_e}{n_b e^4 \; 4\pi \; \ln \Lambda} \quad .$$

2.4 Evaluation of the potential

We may now derive a set of equations for the plasma potential by substituting the calculated values of electron and ion densities (equations 2 and 3) into the charge neutrality equation (equation 1). The general form of the equation becomes

$$\frac{1}{n_o \sigma_e} + \left(\frac{U}{\phi}\right)^{\frac{1}{2}} \frac{a^2}{2R} \cdot \frac{\sigma_i}{\sigma_e} \cdot \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \cdot \frac{1}{\beta_i} = f(\phi) \qquad (4)$$

For Case (a) $\quad f(\phi) = \left(\frac{m_e}{m_b}\right)^{\frac{1}{2}} \left(\frac{eU}{kT}\right)^{\frac{1}{2}} \frac{a^2}{2R} \cdot \frac{1}{\beta_e} \exp\left(\frac{e\phi}{kT}\right)$

Case (b) $\quad f(\phi) = \frac{a^2}{R^2} \cdot \tau_{ee} \cdot v_b \exp\left(\frac{e\phi}{kT}\right)$

Case (c) $\quad f(\phi) = \left[\left(\frac{\phi}{2\phi_i} + 1\right) - \left(1 + \frac{\phi_i}{\phi}\right) \ln \left(1 + \frac{\phi}{\phi_i}\right)\right] \times \frac{\phi_i}{U} \cdot v_b \tau_{ei} \; .$

These equations show the influence of the multipole magnetic field on the potential via the two terms β_i and β_e. The following points may be made.

i. The influence on the slow ions via the term β_i is only important if the slow ion density is comparable with, or greater than, the fast ion density. The ratio of these two terms is the ratio of the terms on the LHS of equation 4.

ii. The influence on the electrons via the term β_e is only effective in case (a).

iii. As a consequence of (ii) it follows that in cases (b) and (c) the first order effect of magnetic confinement of the ions and electrons is actually to increase the slow ion density and consequently the potential required to maintain the electron density equal to the ion density.

3. NUMERICAL RESULTS FOR CASE (a)

Equation (4) may be rewritten in the form

$$\exp(\eta) - \frac{\beta_e}{\beta_i} \cdot \frac{\sigma_i}{\sigma_e} \cdot \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} = \beta_e \cdot \frac{2R}{a^2} \cdot y$$

where η is the reduced parameter $(\frac{e\phi}{kT})$ and y is the beam line parameter $\left(\frac{m_b kT_e}{m_e eU}\right)^{\frac{1}{2}} \cdot \frac{1}{n_o \sigma_e}$. We can say that $\left(\frac{m_b kT_e}{m_e eU}\right)^{\frac{1}{2}}$ is approximately equal to unity being the ratio of the electron and ion velocities $(\frac{v_e}{v_b})$. Thus y is approximately equal to the mean free path for ionisation of the background gas.

Figure 1 shows solutions of this equation for protons in molecular hydrogen gas. The beam energy is not specified but is assumed to be sufficiently high that σ_e equals σ_i (ie charge exchange is negligible). R is taken to be equal to 5 cm and a to 2.5 cm as typical values for medium current beams.

Three cases are solved for different assumptions concerning β_e, β_i.

i. $\beta_e = \beta_i = 1$ - no multipole field is applied

ii. $\beta_e = \beta_i = 10^{-2}$ - multipole fields are applied to reduce the open wall area to 1% and it is assumed that this is the same for both ions and electrons

iii. $\beta_e = 10^{-2}$, $\beta_i = \beta_e \left[\frac{m_i}{m_e} \cdot \eta\right]^{\frac{1}{2}}$ - multipole fields are applied to reduce the open wall area for electrons to 1% and it is assumed that open area for the ions scales relatively with the ion momentum

The calculations show that at high pressures (> 3 millitorr) the potential is essentially determined by the slow ion containment being the solution of

$$\exp \eta = \frac{\beta_e}{\beta_i} \frac{\sigma_i}{\sigma_e} \cdot \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \eta^{-\frac{1}{2}}$$

and is therefore sensitive to the ratio of β_e to β_i.

At lower pressures the results indicate that electron containment dominates

$$\exp(\eta) \simeq \beta_e \frac{2R}{a^2} y .$$

In both pressure extremes the change in potential is due to the improved containment of the electrons. At the higher pressure this is enhanced if the electron loss area is less than that of the slow ions.

4. DISCUSSION

These calculations show that multipole magnetic fields can produce a decrease in the reduced plasma potential under conditions that

a. the slow ion loss rate through cusps scales as their momentum;

b. the electrons are thermalised.

One might expect condition (a) to be satisfied if the Debye length in the beam plasma is large compared with the cusp width in order that local electric fields due to space charge do not modify particle motion in the cusp.

Condition (b) is more suspect since the classical electron-electron collision time is longer than the electron containment time in general. Indeed it is this factor which has led to the development of such differing models of space charge neutralisation.

The analysis presented above suggests that an important test of this assumption of electron thermalisation would be to measure the variation of reduced potential with and without containing multipole magnetic fields.

There are two important limitations to the present anlysis which will be considered in future reports:

It is assumed that the potential is distributed through the beam. This corresponds to the condition that the Debye length is $\sim$ beam radius[2,6] which is true for beams $\lesssim$ several hundred milliamps. It does not apply to multi-ampere beams[6].

The analysis only involves the calculation of the reduced potential. To complete the discussion it is necessary to consider the thermal balance and how the electron temperature is influenced.

References

1. Gabovich M D, Katsubo L P and Soloshenko I A, Sov.J.Plasma Phys. 1, 162 (1975).

2. Green T S, Proc.Symp. on Prod. and Neutralisation of Negative Hydrogen Ions and Beams, Brookhaven BNL50727, p.228 (1977).

3. Dunn D A and Self S A, J.Appl.Phys. 35, 113 (1964).

4. Hamilton G W, Symp.Ion Sources and Beam Formation, Brookhaven BNL50310, 171 (1971).

5. Holmes A, Culham Laboratory Preprint CLM P539 (1978).

6. Anderson O A and Hooper E B Jr, Proc.Symp. on Production and Neutralisation of Negative Hydrogen Ions and Beams, Brookhaven BNL50727, 205 (1977).

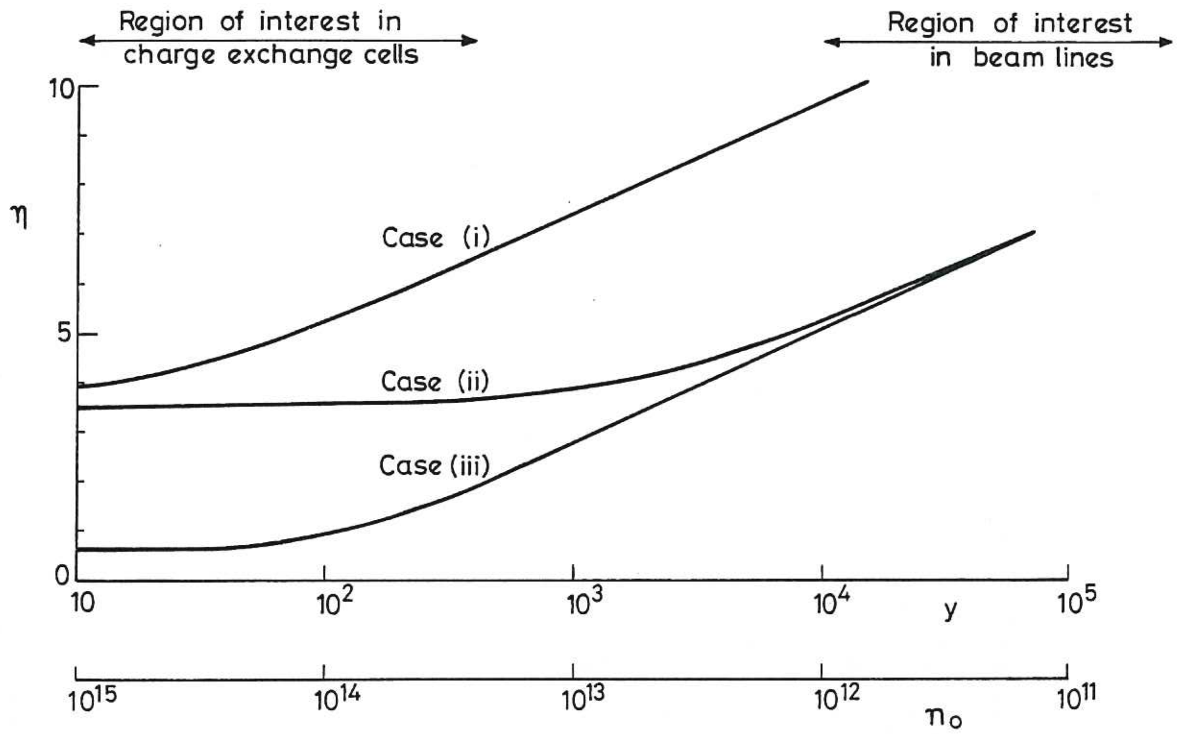

Fig.1 Scaling of the reduced plasma potential ~~η~~ with the beam line parameter y

$$y \text{ equals} \left(\frac{m_b}{m_e} \cdot \frac{kT_e}{eU}\right)^{1/2} \frac{1}{n_o \sigma_e} \simeq \lambda_{eo}$$

(also shown are approximate values of gas density n_o calculated for $\sigma_e = 10^{-16}$ cm² and for

$$\left(\frac{m_b}{m_e} \cdot \frac{kT_e}{eU}\right)^{1/2} \text{ equal to unity)}$$

Case (i) No electron or ion containment.

(ii) Electron and ion containment equal. $\beta = 10^{-2}$.

(iii) Electron containment improved to $\beta_e = 10^{-2}$. Ion containment scales relative to electron as the ion momentum.