CLM-P607

UKAEA

Preprint

THE 'WALK-OFF' EFFECT IN FABRY-PEROT ETALONS LIMITATIONS TO 'SINGLE-SHOT' LINEWIDTH MEASUREMENTS AT LONGER WAVELENGTHS

J. M. GREEN

CULHAM LABORATORY
Abingdon Oxfordshire

1980

CLM-P 607

THE 'WALK-OFF' EFFECT IN FABRY-PEROT ETALONS LIMITATIONS TO 'SINGLE-SHOT' LINEWIDTH MEASUREMENTS AT LONGER WAVELENGTHS

J.M. Green

Culham Laboratory
Abingdon, Oxon OX14 3DB

Abstract

Except for light reflected at normal incidence, the number of multiple reflections that can be made between the plates of a Fabry-Perot etalon is limited, if only by the diameter of the plates themselves. A simple analysis of this 'walk-off' effect is presented and verified experimentally. It is shown to be an important consideration when etalons are used at longer wavelengths (say $\gtrsim$ 10μm) for such applications as 'single-shot' linewidth measurements or as intracavity elements in lasers.

(Submitted for publication in J. Phys. E., Sci. Instr.)

APRIL 1980

1. Introduction

One approach to high resolution linewidth measurements on a single shot basis is to use a detector array to record the dimensions of the ring pattern produced when a Fabry-Perot Etalon (FPE) is illuminated with a pulse of uncollimated light. However, under such conditions multiple reflections within the FPE produce a sideways expansion of the light, and so the number of reflections the light can make within the FPE is limited by the size of the FPE plates. This 'walk-off' effect can limit the finesse of the FPE as well as the intensity of the transmitted light. Because the angular diameters of FPE interference rings increase with increasing wavelength, this effect becomes more important at longer wavelengths, and can generally be neglected for measurements in the visible region. This possibly explains why a comprehensive treatment of the effect is not to be found in the standard reference works on FPE's. In this note a simple analysis of the 'walk-off' effect is presented and the results applied to linewidth measurements at 10μm wavelength.

2. The 'Walk-Off' Effect

The resolution of a Fabry-Perot Etalon (FPE) is generally derived from the relationship between the transmitted intensity I_T and the phase difference δ introduced between successive reflections between the FPE plates. Where an infinite number of such reflections is assumed to occur, I_T may be calculated by summing then squaring the infinite geometrical series of light amplitudes corresponding to successive reflections between the FPE plates, and can be expressed in the familiar form;

$$I_T = \frac{I_o}{1 + \frac{4r^2}{(1-r^2)^2} \sin^2 (\frac{\delta}{2})} \qquad (1)$$

where I_o is the initial intensity and r^2 is the FPE plate reflectivity. However, for non-normal illumination of the FPE equation (1) is no longer strictly valid since successive reflections within the FPE no longer completely overlap and only a finite number of multiple reflections can take place before the light reaches the edge of the FPE plates. If the effect of incomplete overlap is ignored then the influence of 'walk-off' can be quantified by taking the same geometrical series whose infinite sum gave rise to equation (1) but summing only a finite number of terms. The expression for I_T then becom

$$I_T = \frac{I_o\ (1-r^{2n})^2}{1 + \frac{4r^2}{(1-r^2)^2} \sin^2 (\frac{\delta}{2})} \{1 + \frac{4r^{2n}}{(1-r^{2n})^2} \sin^2 (\frac{n\delta}{2})\} \qquad (2)$$

where n is the number of round trips between the FPE plates. In equation (2) the first bracketed term in the numerator represents the effect of 'walk-off losses on the total transmitted intensity. The second bracketed term represents a broadening of the interference rings, since the fraction $(1-r^{2n})/(1-r^2)nr^{n-1}$ is always greater than unity.

3. Restrictions Imposed by the 'Walk-Off'

Figure (1) shows plots of I_T vs δ as given by equation (2) for various values of n and for r^2 values of 0.96 and 0.9. The figure illustrates the dramatic reduction and broadening of the intensity profile when n is low.

The minimum value of n that can be tolerated before broadening becomes significant can be simply deduced by solving equation (2) for the condition that I_T takes on half its maximum value, which occurs when;

$$(\frac{4r^2}{(1-r^2)^2}) \sin^2 (\frac{\delta}{2}) = 1 + (\frac{8r^{2n}}{(1-r^{2n})^2}) \sin^2 (\frac{n\delta}{2}) \qquad (3$$

The solution of equation (3) for δ as n approaches infinity yields the familiar expression for the reflectivity finesse, F_R, of the FPE i.e.

$$F_R = \pi r/(1-r^2) \qquad (4)$$

The increase in width of the I_T vs. δ profile associated with 'walk-off', is determined by the magnitude of the second term on the RHS of equation (3), and will periodically go to zero as n is decreased from infinity to a minimum value n_o given by

$$n_o = 2F_R \qquad (5)$$

However, for $n < n_o$ the broadening associated with 'walk-off' may still be acceptable. From the viewpoint of theoretical analysis, a suitable minimum value for n is

$$n_{min} = F_R \qquad (6)$$

It can readily be shown that for $n = n_{min}$ the term '$r^{2n}/(1-r^{2n})^2$' in equation (3) remains close to its asymptotic value (as r approaches unity) of exp $(-\pi)$ for all values of r of practical interest (say, $r > 0.8$). Correspondingly, for $n \geq n_{min}$ the broadening of the FPE interference pattern due to 'walk-off' will be less than 16%.

In order to relate the value of n to geometrical factors we note that the sideways displacement of a ray of light crossing between the FPE plates is $d \tan \theta$ where d is the plate separation and θ is the angle of incidence of the ray inside the FPE. In practice, the relationship between n and θ must be evaluated for each case in terms of the experimental geometry. However, to further quantify the effect of 'walk-off' we will take the extreme case where a sideways displacement equal to the diameter (D) of the FPE plates can be tolerated. Then from equation (6) it follows that for small values of θ,

$$\theta \leq D/2dF_R \qquad (7)$$

for less than a 16% reduction in reflectivity finesse.

4. Experimental

The finesse and transmission of a piezo electrically-tuned air-spaced FPE of 2.5 cm clear aperture (Technical Optics FPI-25) was measured as a function of the angle of incidence to the FPE plates of a 10.6 μm cw CO_2 laser beam with diffraction limited divergence. This was done by collecting the radiation transmitted by the FPE, focusing it onto a fast Mercury Cadmium Telluride detector, and displaying the detector output as the FPE plate separation was swept through a full free spectral range (FSR). As the FPE was tilted away from normal incidence, n was calculated by dividing the sideways displacement of light undergoing one round-trip in the FPE ($= 2d \tan\theta$) into the maximum allowed sideways displacement, in this case limited to 1 cm, the diameter of the CO_2 laser beam at the etalon. The (reduced) reflectivity finesse at some finite value of θ, F_R^θ was computed from the measured value of finesse F_m^θ by evaluating the expression

$$(F_R^{\theta})^{-1} = (F_m^{\theta})^{-1} - (F_D)^{-1} \qquad (8)$$

where F_D is the defect finesse which is associated with the parallelism and surface finish of the FPE plates and in this case also with the divergence of the incoming CO_2 laser beam. F_D was assumed to be independent θ and was evaluated at $\theta = 0$ using a computed value for F_R^{o} of 77 corresponding to the measured reflectivity of the FPE plates of 0.96 at 10.6 μm. In this case $F_D = 136$.

Fig (2) shows the measured and computed values for etalon transmission and finesse as a function of angle of incidence θ for a FPE plate separation $d = 2$ cm. The results demonstrate that for this case the predictions based on equation (2) are adequate, showing a dramatic reduction in the reflectivity finesse of the etalon even at small angles of incidence. It is interesting to note that in this example the value of θ corresponding to a FSR of the etalon is $1^{o}19'$ while the maximum value of θ given by equation (7) (using a value $D = 1$ cm) is only 11 minutes of arc.

5. Application to the FPE Detector Array Combination

5(a) Limitations Imposed by the FPE

In the usual arrangement of a FPE/detector array combination, a lens is used to image a portion of the FPE interference pattern corresponding to a full FSR onto the array. If the effect of 'walk-off' is to be minimised the values of θ corresponding to the imaged portion of the interference pattern must therefore lie in the range

$$0 \le \theta \le (\eta d \tilde{\nu})^{-\frac{1}{2}} \qquad (9)$$

where η is the refraction index of the medium between the FPE plates and $\tilde{\nu}$ is the mean frequency (in wave numbers) of the light being analysed. Combining equations (7) and (9) yields the condition that for a negligible 'walk-off' effect over a full FSR, the FPE plate separation must be less than the value d_{max} where

$$d_{max} = \tilde{\nu}\, D^{2}\eta/4F_{R}^{2} \qquad (10)$$

Equation (10) shows directly that the broadening associated with 'walk-off' becomes increasingly important at longer wavelengths. Figure (3) shows d_{max} plotted against r^{2} for 2.5 cm diameter air spaced FPE plates when $\tilde{\nu} = 1000$cm
In this case the restrictions on d are seen to be severe for all r^{2} values above 0.9. It follows that with the restriction $d \le d_{max}$ the resolution, $\delta\tilde{\nu}$, achievable by the FPE alone (its FSR ($=1/2\eta d$) divided by its finesse $F^{\dagger}$) may also be limited.

For a given etalon, higher resolution (smaller $\delta\tilde{\nu}$) can only be achieved by increasing d and restricting measurements to a frequency interval a fraction, f, of a full FSR (starting from the position $\theta = 0$) where

$$f = d_{max}/d \qquad (11)$$

---

† $F^{-1} = F_{R}^{-1} + F_{D}^{-1}$ where F_{D} is the defect finesse see equation (8).

It follows that if a resolution of $\delta\tilde{\nu}$, is required over a width $\Delta\tilde{\nu}$, then (using equations (10) and (11))

$$\frac{\tilde{\nu}\,\eta^2 D^2}{2} \quad \left(\frac{F}{F_R}\right)^2 \geq \frac{\Delta\tilde{\nu}}{(\delta\tilde{\nu})^2} \qquad (12)$$

From equation (12) it can be seen that the choice of D (and not the finesse) is critical in specifying a FPE for some applications. Consider, for example, the use of an etalon to resolve the axial mode structure of a pulsed CO_2 TEA laser operating at $\tilde{\nu}$ = 1000 cm^{-1}. The total spectral width of the laser output might be ~ 0.03 cm^{-1} while for a 1.5 m long laser cavity, the axial mode spacing would be one tenth of this width. A resolution of 0.001 cm^{-1} might therefore be desired. For an air-spaced FPE and with $F \simeq F_R$, equation (12) is only satisfied if D exceeds 7.75 cm.

5(b) Limitations Imposed by the Detector Array

Suppose a portion of a FPE interference pattern corresponding to values of θ (internal angles) between $\theta = 0$ and $\theta = \theta_a$ is imaged onto a detector array comprising N equally spaced elements. Then if θ_a is small, and the curvature of the FPE interference pattern can be ignored, the p th element of the array will intercept light corresponding to a wavenumber interval $\delta\tilde{\nu}_p$ given by

$$\delta\tilde{\nu}_p \simeq \tilde{\nu}\,(p - 0.5)\,\left(\frac{\theta_a}{N}\right)^2 \qquad (13)$$

where the element p = 1 intercepts light corresponding to $\theta = 0$. The wavenumber interval resolved by the array increases with increasing p value, and thus the ratio R of the total bandwidth displayed across the array (from one order of the FPE interference pattern) to the bandwidth intercepted by a single element, will lie in the range

$$N^2 \geq R \geq \frac{N}{2} \qquad (14)$$

Following on from the analysis in section (5a), it is clear that if the array is not to limit the overall (FPE + array) resolution over the whole range $\Delta\tilde{\nu}$, then

$$N > 2\Delta\tilde{\nu}/\delta\tilde{\nu} \qquad (15)$$

In the example given above, an array comprising sixty or more elements would be required for a single-shot analysis of the spectral content of a pulsed CO_2 TEA laser.

Where the array limits the overall resolution of the system, special attention must be given to the width of the detector array strip, which must be apertured to less than twice the length of a single detector element if the maximum resolution of the array is to be achieved.

We also note that for small angles of incidence (θ) the dispersion of a FPE is inversely proportional to θ. The output from the detector array must therefore be suitably processed before a linewidth can be deduced. This processing must also take into account the angular distribution of intensity of th light entering the FPE coupled with any angular dependent 'walk-off' losses produced by the FPE.

6. Conclusions

A simple analysis has demonstrated that the reduced finesse and transmission of an etalon due to 'walk-off' effects can be important in single-shot linewidth measurements at longer wavelengths. The restrictions imposed by 'walk-off' can be minimised by using large diameter FPE plates or else a solid etalon of high refractive index.

It is clear that the above analysis can also be applied to the case where an etalon is located inside a laser cavity to tune the laser output (by tilting the etalon). For infrared lasers the above analysis shows that except for very thin etalons the effective finesse of a high reflectivity etalon would be greatly reduced before a tilt corresponding to a full FSR of the etalon could be achieved, especially since in this case the maximum sideways displacement of multiple reflections that can be tolerated will typically be a few millimetres (the diameter of the TEM_{00} mode). Large transmission losses are also predicted.

Acknowledgements

The author would like to acknowledge the many useful discussions he has had with members of the Laser Applications Group at Culham Laboratory. Special thanks are due to Dr. D.W. Stacey of the Clarendon Laboratory, Oxford, for reading the manuscript and suggesting several important improvements.

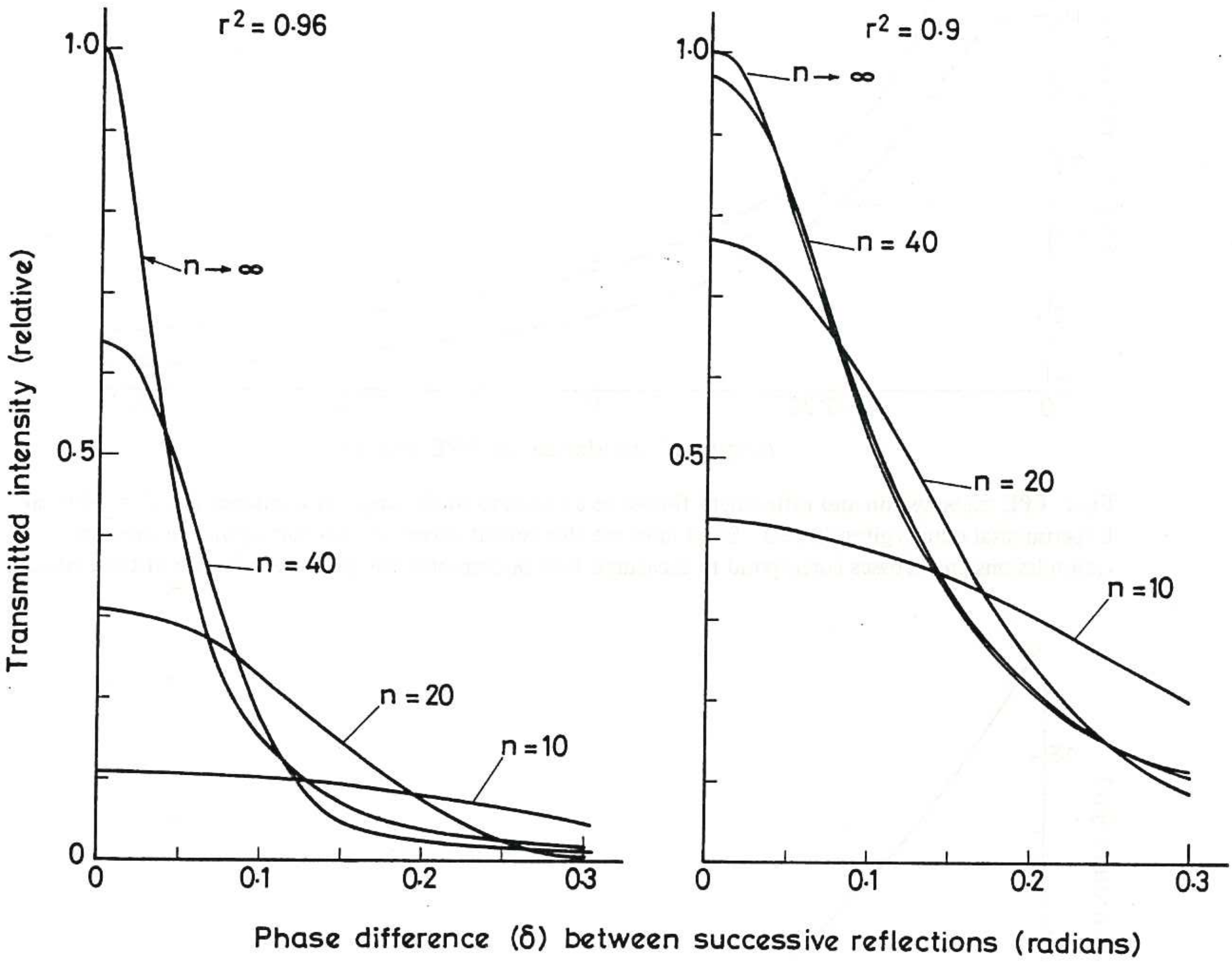

Fig.1 Intensity profile of a FPE interference pattern plotted for various values of the number (n) of round trips between plates: two sets of curves are shown for FPE plate reflectivities of 0·96 and 0·9.

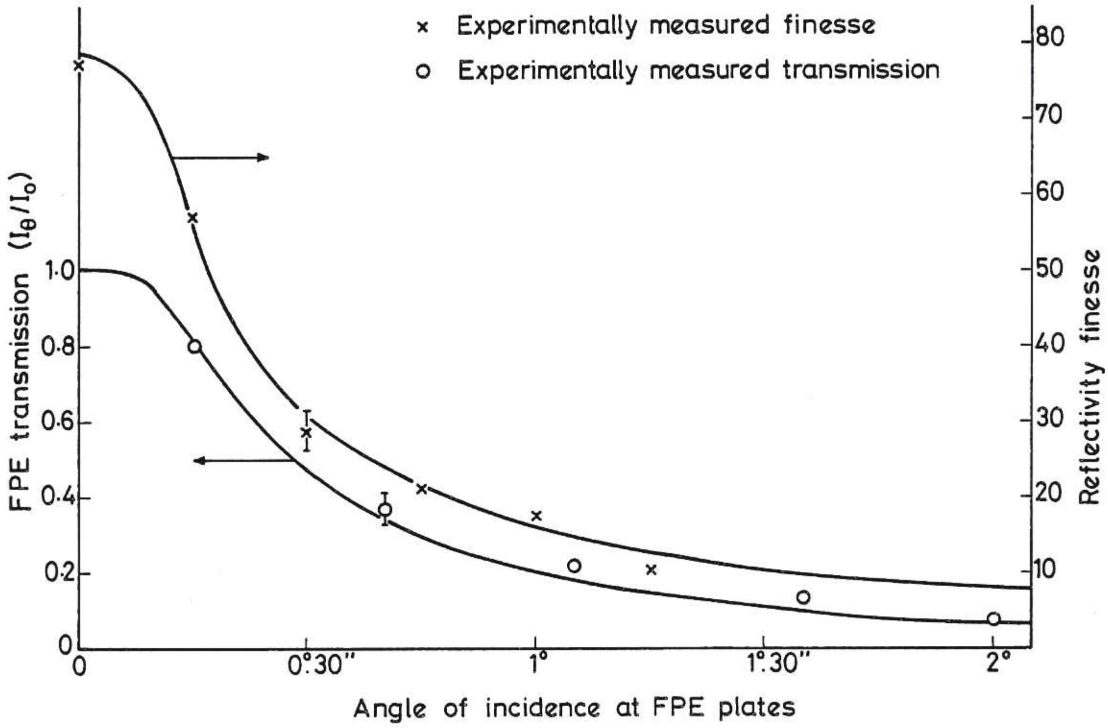

Fig.2 FPE transmission and reflectivity finesse as a function of the angle of incidence for $\tilde{\nu}$ = 1000 cm^{-1}. Experimental details given in text. Solid lines are theoretical curves, circles correspond to measured transmissions and crosses correspond to measured finesse corrected for the defect finesse of the etalon.

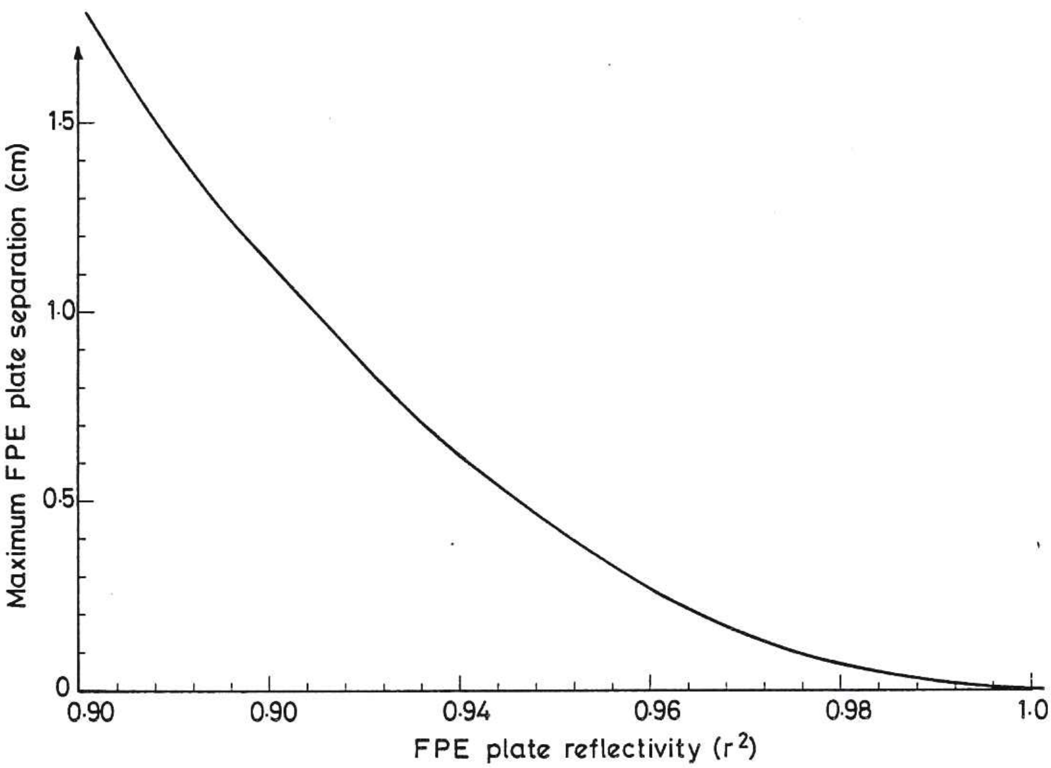

Fig.3 For a 1″ diameter air-spaced FPE operating at $\tilde{\nu}$ = 1000 cm^{-1}, the maximum FPE plate separation d_{max} is plotted as a function of FPE plate reflectivity, (see equation 10).