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THE $J_{\parallel}$ KINK INSTABILITY IN A LOW PRESSURE PINCH DISCHARGE

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THE $J_{\parallel}$ KINK INSTABILITY IN A LOW PRESSURE PINCH DISCHARGE

by

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A B S T R A C T

A study has been made of the kink instability caused by $j_{\parallel}$ ($\equiv \underset{\sim}{j}.\underset{\sim}{B}/B$) in a pinch discharge whose pressure gradient is negligibly small. Since Newcomb's necessary and sufficient condition for stability can be applied only to specific field configurations, the energy principle is used to derive a number of general results on stability, valid for all configurations. Sufficient conditions for stability are obtained together with the approximate ranges of wave numbers which can be unstable and the radial extent of the instability. Localisation of the instability to an annular part of the discharge is possible but, unlike pressure driven modes, highly localised perturbations are stable.

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CONTENTS

		Page
1.	Introduction	1
2.	Minimisation of the Energy Integral and Compressibility	2
3.	Sufficient Condition for Stability	4
4.	The Unstable Wave Numbers	5
	The Wave Number m	5
	The Wave Number k_z	5
5.	The Radial Extent of the Instability	8
6.	Conclusions	11
7.	References	13

1. Introduction

In a moderately constricted linear pinch discharge with high plasma pressure ($\beta \equiv 2p/B^2 \sim 1$) the dominant destabilising mechanisms result from $j_\perp$ or ∇p, as considered elsewhere [1]. There is, however, a smaller destabilising effect which results from $j_\parallel$ and which is independent of both β and ∇p. (The subscripts $_\parallel$ and $_\perp$ are used to denote vector components in the direction of $\underline{B}$ and $\underline{B} \wedge \underline{r}$, respectively. As β tends to zero, the ∇p effects decrease in magnitude leaving the unchanged $j_\parallel$ effect to become the dominant one.

Following the Princeton group [2] the $j_\parallel$-instability is referred to here as the kink instability, but in some cases the expression $j_\parallel$-kink instability will be used to distinguish it from one of the ∇p modes which also causes bending (i.e. kinking of the plasma and the lines of force [1].

The kink instability is of considerable current interest. Firstly, in the recent Zeta experiments with high currents and low β the discharge has been observed to be considerably more stable for a period of time after peak current [3]. Secondly, in the stability theory of toroidal plasmas it has been shown [4] that for a toroidal pinch with current below approximately the Kruskal limit, the average field curvature gives stability against the ∇p modes, whereas the stability of the $j_\parallel$ mode, which depends on the torsion of the magnetic field, is approximately unchanged from the straight pinch case. The properties of the $j_\parallel$-kink instability are therefore of importance in the search for completely stable toroidal plasmas.

As is well known, the necessary and sufficient stability condition for the $j_\parallel$-kink mode is obtained by solving the Euler-Lagrange equation, i.e. the Newcomb criterion [5]. Since an analytical solution is possible only for special field configurations and otherwise numerical solution is involved, the stability conditions have been obtained for only a few special cases, e.g. for the Stellarator field by Johnson et al. [2] for the "paramagnetic" model by Kadomtsev [6] and Whiteman [7] and for the Bessel function model by Voslamber and Callebaut [8]. In this paper the kink instability is studied with the aid of the energy principle and a series of general properties are derived which are independent of the specific field configuration.

2. Minimisation of the Energy Integral and Compressibility

A straight cylindrical discharge is considered with perfectly conducting plasma extending out to a perfectly conducting wall of radius a. With cylindrical coordinates r,θ,z a plasma displacement $\underset{\sim}{\xi} \exp i\,(\omega t - m\theta - k_z z)$ is considered. The resulting energy integral is well known (see for example reference 5) and provided $\beta \neq 0$ the minimisation of the integral with respect to ξ_θ, ξ_z without restraint gives the usual condition $\nabla\cdot\underset{\sim}{\xi} = 0$ and

$$\left(\frac{m}{r}\,\xi_z - k_z\,\xi_\theta\right) = \frac{2ik_z B_\theta \xi_r}{k_{\parallel} rB} \qquad \ldots (1)$$

The resulting minimum integral can be put in various forms by integrating by parts; two of the standard forms are

$$\delta W = \int \frac{\pi r dr}{2}\left[k_{\parallel}^2\,B^2\,\xi_r^2 + \left(\frac{2\frac{m}{r}B_\theta\xi_r - K_{\parallel}B\,\frac{d(r\xi_r)}{dr}}{kr}\right)^2 - \frac{2j_z B_\theta \xi_r^2}{r}\right] \qquad \ldots (2A)$$

$$= \int \frac{\pi r dr}{2}\left[k_{\parallel}^2\,B^2\,\xi_r^2\left(1 + \frac{k_z^2 r^2 - m^2}{k^4 r^4}\right) + \frac{k_{\parallel}^2\,B^2\left(\frac{d\xi_r}{dr}\right)^2}{k^2} - \frac{4k_{\parallel}B\frac{m}{r}\,B_\theta k_z^2 r^2 \xi_r^2}{k^4 r^4} + \frac{2k_z^2\xi_r^2}{k^2 r}\,\frac{dp}{dr}\right] \qquad \ldots (2B)$$

where $\underset{\sim}{k} \equiv \frac{m}{r}\,\underset{\sim}{i}_\theta + k_z\,\underset{\sim}{i}_z$ and where rationalised e.m. units are being used.

In the special case where $\beta = 0$, it follows from the equation of motion that $\xi_{\parallel} = 0$ and in this case ξ_θ and ξ_z are no longer independent. The single minimising condition is then

$$\nabla\cdot\underset{\sim}{\xi} = -\,\frac{2k_z k_{\perp} B_\theta \xi_r}{rk^2 B} + \frac{k_{\parallel}^2}{rk^2}\,\frac{d(r\xi_r)}{dr} \qquad \ldots (3)$$

Despite the different condition, however, the resulting minimum integral is identical with (2) so that the same δW applies, as also the Newcomb criterion which is based on the minimised δW.

The case of $\beta = 0$ may at first sight seem academic - the pressure is never zero in practice - but in fact, even when $\beta \neq 0$, only small departures from marginal stability will often change the value of $\nabla\cdot\underset{\sim}{\xi}$ from zero to that given in (4). When ω is non-zero but small $\nabla\cdot\underset{\sim}{\xi}$ is given by [1]

$$\nabla\cdot\underset{\sim}{\xi} = \frac{\omega^2\rho\left(2k_z k_\perp B_\theta B\xi_r + k_\parallel^2 B^2 \frac{d(r\xi_r)}{dr}\right)}{rk^2(\gamma p k_\parallel^2 B^2 - \omega^2\rho[B^2 + \gamma p])} \qquad \ldots (4)$$

which is comparable with (4) when

$$\omega^2\rho \;\underset{\sim}{>}\; \gamma p k_\parallel^2 \qquad \ldots (5)$$

In order to simplify the study of the kink mode the assumption is made that

$$\left|\frac{dp}{dr}\right| \ll \left|\frac{2m\, k_\parallel B\, B_\theta}{k^2 r^2}\right| \qquad \ldots (6)$$

throughout either most of the discharge or, if only part of the discharge is involved in the integral, as when there are singularities, throughout most of that part. The integral of the pressure gradient term is then negligible. (When there is appreciable magnetic shear $k_\parallel$ can be zero or very small for only part of the range, so that low β is all that is required for (6) to be satisfied. But for the constant pitch case the assumption involves the omission of those k's which are perpendicular to $\underset{\sim}{B}$. There is, in fact no $j_\parallel$-kink mode for such k's since all the terms other than the pressure term vanish.) With the assumption [7], the energy integral (2B) reduces to

$$\delta W = \int \frac{\pi r \xi_r^2}{2}\, dr \left[k_\parallel^2 B^2 \left\{ 1 + \frac{k_z^2 r^2 - m^2}{(m^2 + k_z^2 r^2)^2} + \frac{\left(\frac{r}{\xi_r}\frac{d\xi_r}{dr}\right)^2}{m^2 + k_z^2 r^2} \right\} - \frac{4k_\parallel B \frac{m}{r} B_\theta k_z^2 r^2}{(m^2 + k_z^2 r^2)^2} \right] \qquad \ldots (7)$$

where the less abbreviated form of k is now used so as to make the dependence of δW on m more explicit.

From the Newcomb criterion [5], if $k_{\parallel}$ is nowhere zero, the integral must be taken over the whole discharge. If $k_{\parallel}$ is zero at certain radii (the singular points), the integral must be applied separately to each of the sub-intervals between consecutive singular points and between the end singular points and the walls. Stability requires $\delta W > 0$ for each sub-interval.

3. Sufficient Condition for Stability

From the form of δW in (2A) one obtains immediately the well known sufficient condition for stability namely $j_z B_\theta \lessgtr 0$. Since here $j_\perp$ ($\equiv |\nabla p|/B$) is assumed small, the condition is

$$\frac{j_{\parallel} B_\theta B_z}{B} \lessgtr 0 \qquad \ldots (8)$$

This condition is unnecessarily restrictive, but it does show that both B_θ and B_z must be present for the $j_{\parallel}$-kink instability to occur and that for given B_θ, B_z only one sign of $j_{\parallel}$ is destabilising. (N.B. $j_{\parallel} \equiv \frac{B_\theta}{B} j_\theta + \frac{B_z}{B} j_z$.)

Thus the simple pinch discharge with only B_θ present is free of the $j_{\parallel}$-kink instability, although, of course, it is prone to both the interchange instability and the ∇p-kink mode. At the centre of a pinch discharge with B_z present, where with increasing radius $j_{\parallel}$ causes rotation of B from the z-direction to the θ-direction, the sign of $j_{\parallel}$ is destabilising. The normal direction for $j_{\parallel}$ in the inverse pinch is the stable direction.

In the form of δW given in (7), the only two terms in the square brackets which can be negative are

$$- \frac{m^2 k_{\parallel}{}^2 B^2}{(m^2 + k_z^2 r^2)^2} \qquad \ldots (9a)$$

and

$$- \frac{4k_{\parallel} B \frac{m}{r} B_{\theta} k_z^2 r^2}{(m^2 + k_z^2 r^2)^2} \qquad \ldots (9b)$$

Since the magnitude of (9a) can never exceed $k_{\parallel}^2 B^2$, which is one of the stabilising terms, it follows that a sufficient condition for stability is that (9b) should be zero or positive throughout either the whole discharge or the appropriate sub-interval, so that any of the following conditions are sufficient for stability

$$m = 0, \quad B_{\theta} = 0, \quad k_z = 0 \qquad \ldots (10a)$$

$$- k_{\parallel} B \frac{m}{r} B_{\theta} \geqslant 0 \qquad \ldots (10b)$$

the $B_{\theta} = 0$ case having been found already.

4. The Unstable Wave Numbers

4a. The Wave Number m

As already seen the case $m = 0$ is stable. Of the remaining m numbers, it follows from the same argument as that given by Newcomb [5] that $m = 1$ is the most unstable. In the case of a toroidal pinch, it is possible that all the values of the wavelength $2\pi/k_z$ for which the $m = 1$ mode is unstable are less than the torus circumference. (This occurs when the current is below the Kruskal limit.) In this case the most unstable m number will usually be the lowest one for which L/n is an unstable wavelength, where L is the torus circumference and n an integer. It will not always be the lowest value of m satisfying this condition, since for a particular mode, L/n may lie near one end of the unstable wavelength range where the growth rate will be small; a higher m could then have larger growth rate.

4b. The Wave Number k_z

Substituting for $k_{\parallel} \left(\equiv \left[\frac{m}{r} B_{\theta} + k B_z \right] / B \right)$ in (10b) gives

$$- (\frac{m}{r} B_{\theta})^2 - \frac{m}{r} k_z B_{\theta} B_z \geqslant 0 \qquad \ldots (11)$$

Dividing by $(m\, B_\theta/r)^2$ and writing $2\pi r\, B_z/B_\theta = \lambda_B$, the pitch of the magnetic field helix and $-\ 2\pi m/k_z = \lambda_i$, the pitch of the instability helix (the minus sign ensures that λ_i has the same sign as λ_B when the instability and field helices have the same sense,) a sufficient stability condition is

$$\frac{\lambda_B}{\lambda_i} - 1 \geqslant 0 \qquad \ldots (12)$$

Thus modes whose helices have the same sense as the magnetic field and shorter pitch are stable. The quantity $(\lambda_B/\lambda_i) - 1$ will have the same sign over each sub-interval since the equality $\lambda_B = \lambda_i$ defines a singular point and ends the sub-interval. Hence the condition (12) will either be satisfied throughout the whole of a sub-interval or violated throughout.

An upper limit to the range of λ_i for which instability may occur can be found by omitting only the term contain $d\xi_r/dr$ in δW. Since this term is positive definite, a sufficient stability condition is

$$(k_{\parallel}B)^2\left[1 + \frac{k_z^2r^2 - m^2}{(m^2 + k_z^2r^2)^2}\right] - \frac{4k_{\parallel}B\,\frac{m}{r}\,B_\theta k_z^2r^2}{(m^2 + k_z^2r^2)^2} \geqslant 0 \qquad \ldots (13)$$

Making the same substitution in terms of λ_B and λ_i as before and assuming that condition (12) is violated, i.e. $(\lambda_B/\lambda_i) < 1$, the stability condition reduces to

$$\frac{\lambda_B}{\lambda_i} \leqslant 1 - \frac{\frac{4}{m^2}\left(\frac{2\pi r}{\lambda_i}\right)^2}{1 - \frac{1}{m^2} + \left(\frac{2\pi r}{\lambda_i}\right)^4 + \left(2 + \frac{1}{m^2}\right)\left(\frac{2\pi r}{\lambda_i}\right)^2} \qquad \ldots (14)$$

This condition will be studied for the cases $m = 1$ and $m > 1$ separately.

<u>Case (a) $m = 1$</u>

In this case the condition (14) reduces to

$$\frac{\lambda_B}{\lambda_i} \leqslant \frac{\left(\frac{2\pi r}{\lambda_i}\right)^2 - 1}{\left(\frac{2\pi r}{\lambda_i}\right)^2 + 3} \qquad \ldots (15)$$

The equality in (15) gives a cubic equation for λ_i. If 19.4 $\lambda_B^2 > (2\pi r)^2$ there is only one real root and this lies in the range from -3 to $-\infty$ times λ_B. The ranges of possibly unstable wavelengths from (10), (12) and (15) is thus

$$1 > \frac{\lambda_B}{\lambda_i} > 0 \text{ and } 0 > \frac{\lambda_B}{\lambda_i} > -\frac{1}{3} \qquad \ldots (16)$$

In those cases where, of necessity, $d\xi_r/dr$ is non-zero in regions where $k_{\parallel}B$ is appreciable (see section 5 below), the long wavelengths $|\lambda_i| \gg 2\pi r$ will be stabilised by the $d\xi_r/dr$ term since all the other terms contain the factor $2\pi r/\lambda_i$ and will become small in comparison.

If 19.4 $\lambda_B{}^2 < (2\pi r)^2$ there are two further real roots for λ_i, both of which lie in the range between λ_B and $2\pi r\lambda_B/|\lambda_B|$. In this case the range given by the first part of (16) is divided into two ranges by an internal range of stable λ_i, the limits of the stable range being given by these two extra roots.

Case (b) $m \geqslant 2$

For these higher values of m the right hand side of (14) is positive for all values of λ_i and hence the lower limit for λ_B/λ_i is always positive; all negative values are stable.

These higher m values are of interest only for toroidal discharges with currents below the Kruskal limit (otherwise the more unstable $m = 1$ mode is possible) and since in that case the inequalities $|\lambda_B| > L > 2\pi r$ must hold the approximation $|\lambda_i| \sim |\lambda_B| \gg 2\pi r$ will generally be valid. In which case a good approximation for (14) is

$$\frac{\lambda_B}{\lambda_i} \leqslant 1 - \frac{4\left(\frac{2\pi r}{\lambda_B}\right)^2}{m^2 - 1} \qquad \ldots (17)$$

so that the range of possibly unstable wavelengths is given by

$$1 > \frac{\lambda_B}{\lambda_i} > 1 - \frac{4\left(\frac{2\pi r}{\lambda_B}\right)^2}{m^2 - 1} \qquad \ldots (18)$$

which becomes increasingly small as m increases.

5. The Radial Extent of the Instability

The radial range over which ξ_r is appreciably different from zero depends, of course, on the radial variation of the δW integrand. This in turn depends to a large extent on the radial variation of λ_B and in particular, the presence of a singular point has a dominant effect. Here we consider only the simple cases where $d\lambda_B/dr$ has the same sign throughout the discharge and where, at most, only one zero in λ_B occurs. In addition only the $m = 1$ case is treated. The extension of the following arguments to more complex variations of λ_B with radius and to higher m is straight forward.

Case (a) λ_B everywhere positive and $\frac{d\lambda_B}{dr} < 0$.

This is the condition that occurs in many pinch discharges. It is assumed that λ_B decreases from a maximum value λ_{max} at $r = 0$ to a minimum value λ_{min} at $r = a$ as illustrated in figure 1. If λ_i lies outside the range λ_{min} to λ_{max} there will be no singularity and any instability can extend over the whole tube (see $\xi_r^{(1)}$). If, however, λ_i lies within this range, such that $\lambda_i = \lambda_B$ at the radius r_s, then for $r < r_s$ the integrand in δW will be positive from condition (12) and the contribution to δW for these radii for non-zero ξ_r will be positive. Since ξ_r can go to zero at the singularity, the most unstable mode will have $\xi_r = 0$ throughout the inner sub-interval. The instability will be restricted to the outer annular region as illustrated by $\xi_r^{(2)}$ in figure 1.

The radial variation of the integrand in δW for $m = 1$, and where $\lambda_B/\lambda_i < 1$, can be seen by rewriting δW in the following form

$$\delta W = \int \frac{\left(1 - \frac{\lambda_B}{\lambda_i}\right) k_z^2 r^3 \left(\frac{B_\theta}{r}\right)^2 dr}{(1 + k_z^2 r^2)^2} \left\{ \left[k_z^2 r^2 \left(1 - \frac{\lambda_B}{\lambda_i}\right) - \left(1 + 3\frac{\lambda_B}{\lambda_i}\right) \right] \xi_r^2 + \frac{1 + k_z^2 r^2}{k_z^2} \left(\frac{d\xi_r}{dr}\right)^2 \right\} \quad \ldots (19)$$

Examining the terms in the curly bracket shows that the stabilising terms (the positive terms) contain either the factor r^2 or the factor $(\frac{d\xi_r}{dr})^2$ which also tends to zero as $r \to 0$. The destabilising term $(1 + \frac{3\lambda_B}{\lambda_i})$ is non-zero at $r = 0$ and is a decreasing function with radius. Hence the integrand will be negative for small radii but positive for large radii. An upper limit for the radius r_c where the change over occurs in given by equating the curly bracket to zero and omitting the $d\xi_r/dr$ term. This is condition (15) again and putting it in a different form, the approximate value of r_c is given by

$$\left(\frac{2\pi r_c}{\lambda_i}\right)^2 \simeq \frac{1 + 3\,\frac{\lambda_B}{\lambda_i}}{1 - \frac{\lambda_B}{\lambda_i}} \qquad \ldots (20)$$

This has only one solution for r_c^2 and there will always be a solution since the right hand side of (20) is a decreasing function with radius in this case. Since δW is positive for $r > r_c$, the most unstable modes will have ξ_r small for such radii (see figure 1).

Thus the instability will be localised either to an inner region $r < r_c$ or, if there is a singularity, to the annular region $r_s < r < r_c$. (If, of course, r_c is greater than the wall radius, the r_c limit is of no importance.) This localisation is much less marked than in the case of pressure driven instabilities in a sheared field, such as the Suydam mode or the resistive interchange. Such perturbations, which are highly localised near the singularity, will always be stable for the $j_\parallel$-kink mode since the destabilising terms, as well as most of the stabilising terms, vanish as $k_\parallel B$ and the radial extent of the perturbation tend to zero. The one exception is the $d\xi_r/dr$ stabilising term which remains large and becomes dominant.

Case (b) λ_B everywhere positive $\frac{d\lambda_B}{dr} > 0$

Here λ_B is assumed to increase from λ_{min} at $r = 0$ to λ_{max} at $r = a$, as shown in figure 2. If λ_i lies between λ_{min} and λ_{max} the effect of the singularity, this time, is to confine the instability to the inner region $r < r_s$, which is now the region violating condition (12).

For this case, no conclusions can be drawn regarding possible solutions to equation (20) without knowing the exact form of λ_B as a function of r_s. (Both sides of (20) are now increasing functions with radius.)

However, in the case where there is a singularity and the well in λ_B is sufficiently shallow for there to be no real solution for r_c^*, (i.e. the δW integrand is negative everywhere for the ξ_r^* terms) an important effect results from the fact that the instability region is bounded on both sides by singular points ($\pm\, r_s$). In the case (a) above, it is necessary for $d\xi_r/dr$ to be non-zero in regions where $(k_\parallel B)$ is appreciably different from zero. (In the singularity case the region where $d\xi_r/dr > 0$ can be localised near the singularity, but if ξ_r is to be appreciable away from the singularity and give the instability, then of necessity $d\xi_r/dr$ must have appreciably negative values away from the singularity in order to make ξ_r zero at the wall.) In the present case with $m = 1$, only negative values of $d\xi_r/dr$ occur and these can be localised near the singularity so that ξ_r will have an approximately constant value for all other r. (see figure 2.)

If the radial range for which $d\xi_r/dr$ is appreciable is δr, the integral of the $d\xi_r/dr$ term in δW is of the order

$$\frac{\xi_{ro}^* r_s \delta r}{k^*} \left[\frac{d(k_\parallel B)}{dr} \right]^*_{r = r_s}$$

where ξ_{ro} is the constant value of ξ_r for smaller radii. Since δr is small compared with r, this term is negligible compared with the others in δW. Stability depends in this case only on the ξ_r^* terms.

<u>Case (c) λ_B has one reversal of sign</u>

λ_B is assumed to change monotonically from λ_o at $r = 0$ to λ_a at $r = a$, where λ_o and λ_a have opposite signs (see figure 3). (It does not matter which of λ_o, λ_a has a particular sign.)

If λ_i lies between λ_o and zero and equals λ_B at $r = r_s^{(1)}$, the singularity will cause any instability to be localised in the outer region of the discharge $(r > r_s^{(1)})$ as in Case (a), whereas if λ_i lies between 0 and λ_a and equals λ_B at $r = r_s^{(2)}$, the singularity will cause any instability to occur only in the inner region $(r < r_s^{(2)})$.

Owing to the large magnetic shear in this case, most values of λ_i will be stable. However, if λ_i lies between 0 and λ_a, the $d\xi_r/dr$ term will not contribute to δW. Secondly, if $|\lambda_a|$ is greater than $|\lambda_o|$ so that λ_i can satisfy also the condition $\lambda_B/\lambda_i < -\frac{1}{3}$ for most of the range $r < r_s^{(2)}$ then from condition (16) δW can be negative for most of this range and an instability is possible.

In their study of the stability of the Bessel function model Voslamber and Gallebaut [8] found such a mode to be unstable. With the wall placed in a position equivalent to that illustrated in figure 3, their work shows that the only unstable values of λ_i lie between $-1.8\,|\lambda_o|$ and λ_a. The instability is a weak one and has a growth rate small compared with normal MHD growth rates.

If λ_B does not go very far negative i.e. $|\lambda_a|$ is small compared with $|\lambda_o|$, then $\lambda_B/\lambda_i > -\frac{1}{3}$ will hold for most of the range $r < r_s^{(2)}$, making δW positive, and an instability with λ_i between 0 and λ_a will not occur.

6. Conclusions

Although in many cases the answer to whether a particular discharge is stable or not requires the solution of the Euler-Langrange equation, it has been shown that with the aid of the energy principle a number of conclusions can be drawn which have general validity. The main results are summarized in the following table.

SUFFICIENT CONDITIONS FOR STABILITY	$\frac{j_{\parallel} B_\theta B_z}{rB} \leqslant 0$ at all radii in sub-interval, $m = 0,\ k_z = 0,$ $\frac{\lambda_B}{\lambda_i} \geqslant 1$ ($\lambda_B \equiv 2\pi r\, B_z/B_\theta$, $\lambda_i \equiv 2\pi m/k_z$) Any one of these four conditions is sufficient.	
POSSIBLY UNSTABLE WAVE NUMBERS (All other wavenumbers are stable. The most unfavourable value of λ_B or r present in discharge must be taken.)	<u>Wave Number m</u> $m \geqslant 1$ <u>Wave Number $k_z \equiv - 2\pi m/\lambda_i$</u> <u>For m = 1</u> $1 > \frac{\lambda_B}{\lambda_i} > 0$ and $0 > \frac{\lambda_B}{\lambda_i} > -\frac{1}{3}$. For $19.4\ \lambda_B^2 < (2\pi r)^2$ an inner part of first range is stable.	<u>For $m \geqslant 2$</u> $1 > \frac{\lambda_B}{\lambda_i} > 1 - \frac{4\left(\frac{2\pi r}{\lambda_i}\right)^2}{\left[m^2 - 1 + m^2\left(\frac{2\pi r}{\lambda_i}\right)^4 + (2m^2+1)\left(\frac{2\pi r}{\lambda_i}\right)\right]^2}$ For $\lvert\lambda_i\rvert \sim \lvert\lambda_B\rvert \gg 2\pi r$ the smaller limit is approximately $1 - \frac{4\left(\frac{2\pi r}{\lambda_i}\right)^2}{m^2 - 1}$
RADIAL EXTENT OF INSTABILITY FOR m = 1 AND SIMPLE CASES OF MAGNETIC SHEAR.	(a) <u>$\frac{1}{\lambda_B}\ \frac{d\lambda_B}{dr} < 0$ for all $r > 0$</u> with no singularity the radial range is $0 < r < x$ where x is the smaller of a and $r_c = \frac{\lvert\lambda_i\rvert}{2\pi}\left(\frac{1 + 3\frac{\lambda_B}{\lambda_i}}{1 - \frac{\lambda_B}{\lambda_i}}\right)^{1/2}$ with singularity $r_s < r < x$ (b) <u>$\frac{1}{\lambda_B}\ \frac{d\lambda_B}{dr} > 0$ for all $r > 0$</u> with no singularity $0 < r < x$ with singularity $0 < r < r_s$	

It has been found that the instability can often be localised to a limited range of radii. This is because, firstly, the instability can occur on only one side of a singular point and, secondly, because the stabilising terms dominate at large radii. The localisation cannot be too marked, either near a singularity or elsewhere, since otherwise the $d\xi_r/dr$ stabilising term will dominate and cause stability. In addition, since the instability is caused by line bending ($k_{\parallel}B \neq 0$), ξ_r must have appreciable amplitude away from a singularity for a significant instability to occur.

The particular position of a localised instability has been found to depend on the variation of the magnetic pitch λ_B ($\equiv 2\pi r\, B_z/B_\theta$) with radius and, in particular, on the sign of $\frac{1}{\lambda_B}\frac{d\lambda_B}{dr}$. In the special case where a region having $(\lambda_B/\lambda_i) < 1$ is bounded on both sides by singularities ($\lambda_B = \lambda_i$) the contribution to the energy integral of the $d\xi_r/dr$ term may be negligible.

The localisation of the kink instability to an annular region has been observed experimentally[9], and the comparisom between theory and experiment will be the subject of a separate paper.

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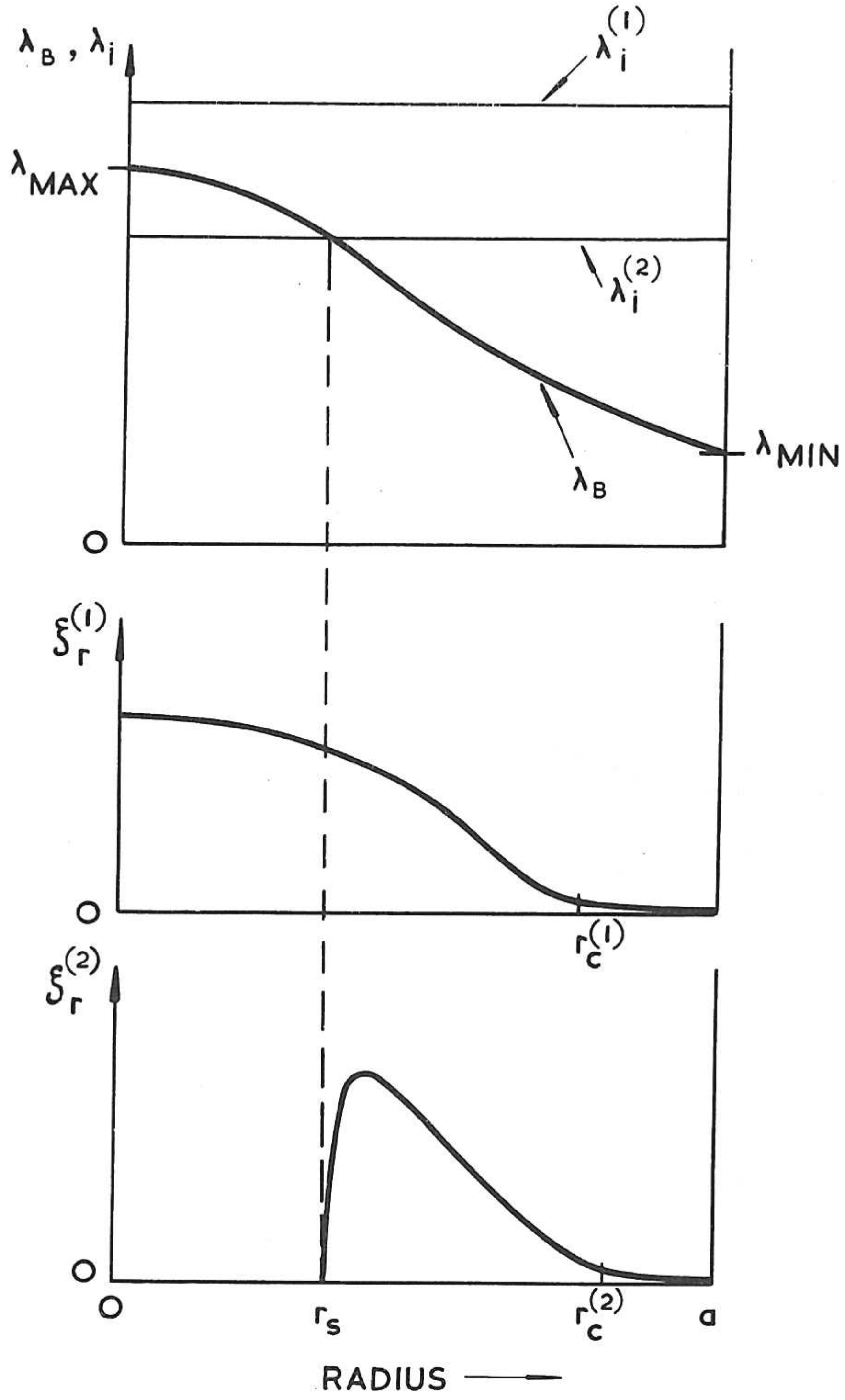
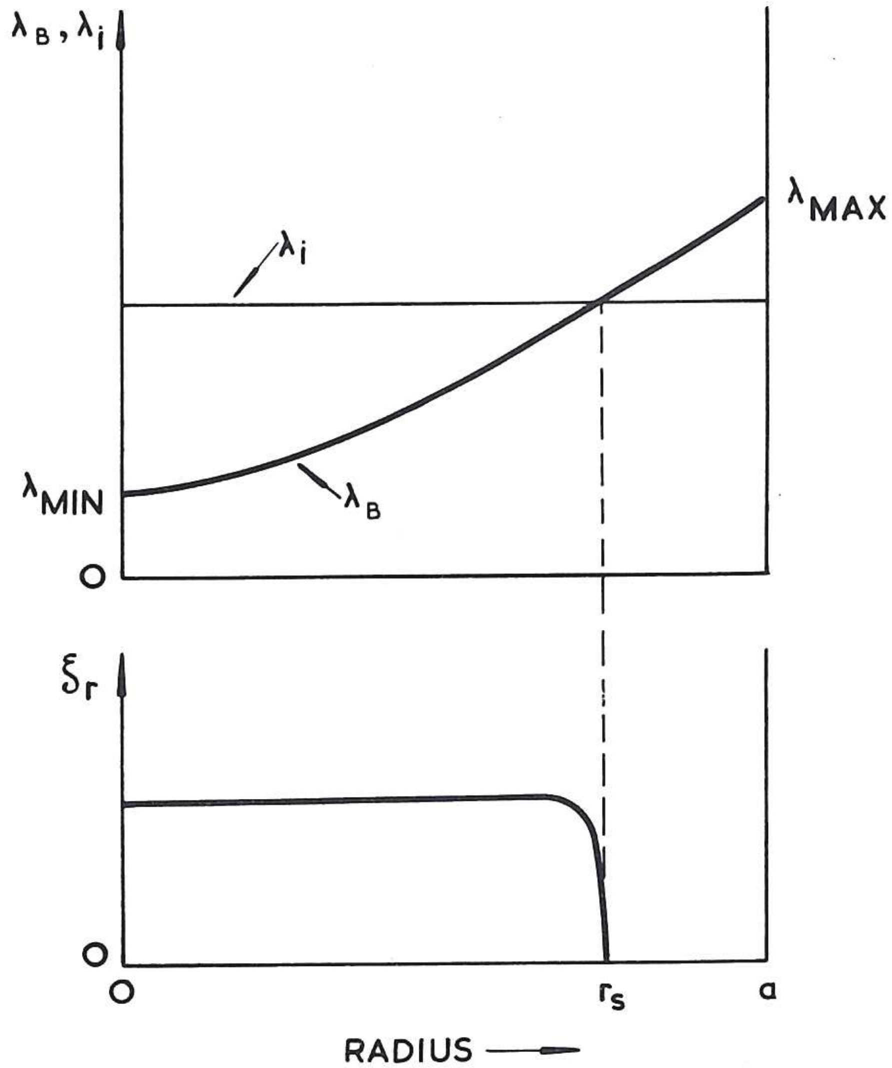


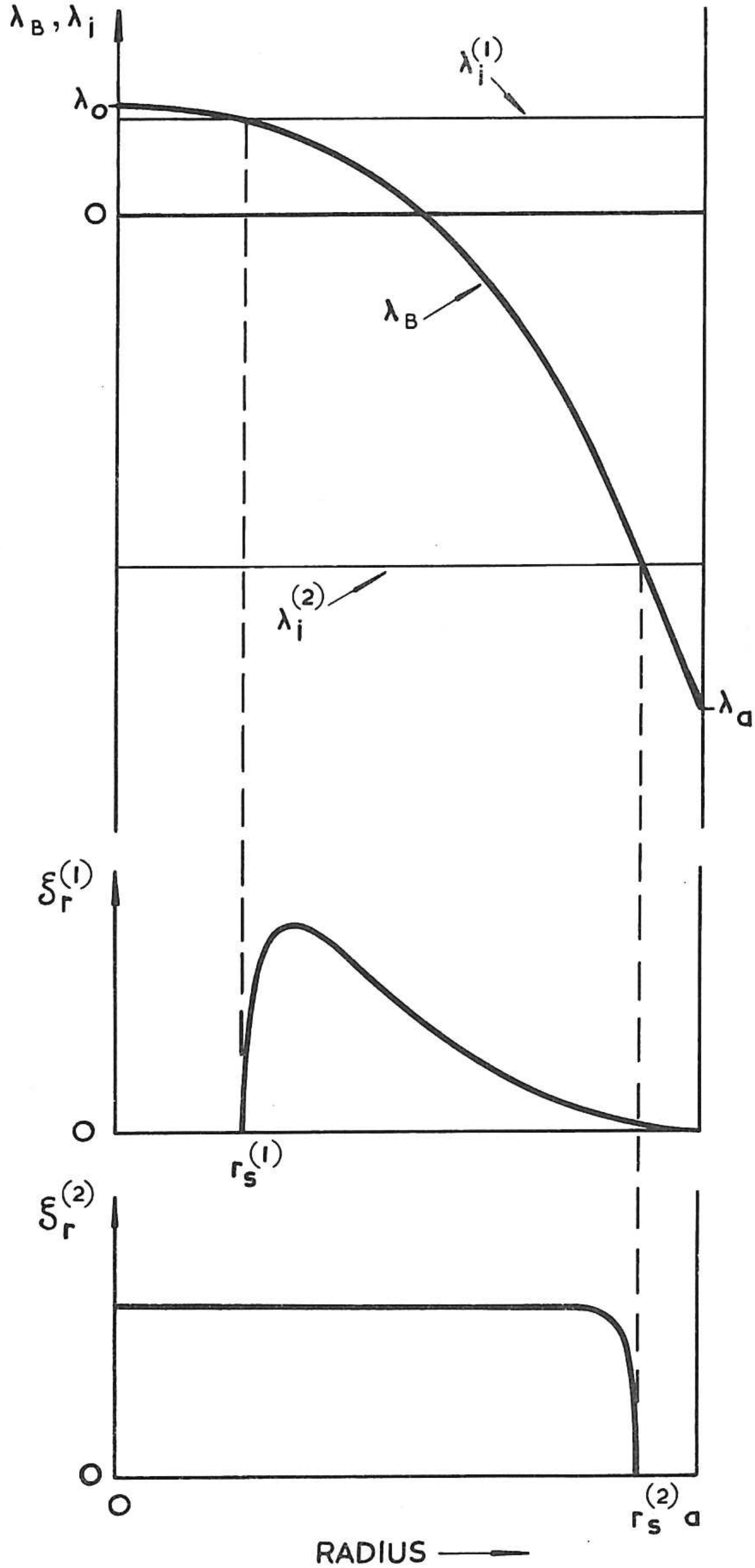
CLM-P61 Fig. 1

The lower figures illustrate the expected form of ξ_r when $\frac{1}{\lambda_B} \frac{d\lambda_B}{dr} < 0$, as shown in the top figure. Case (1): no singularity. Case (2): with singularity at $r = r_s$



CLM - P 61 Fig. 2

Expected form of ξ_r when $\frac{1}{\lambda_B}\frac{d\lambda_B}{dr} > 0$ and a singularity occurs at $r = r_s$



CLM-P61 Fig. 3

Expected forms of ξ_r when λ_B changes sign.

Case (1): when λ_i lies between 0 and λ_o.

Case (2): when λ_i lies between 0 and λ_a.