CLM-P621

CLM-P621

UKAEA

Preprint

ELECTROSTATIC OSCILLATIONS AND PARTICLE TRANSPORT IN LOW-β PLASMAS I. WAVES

F. A. HAAS
A. THYAGARAJA

CULHAM LABORATORY
Abingdon Oxfordshire

1981

CLM-P 621

ELECTROSTATIC OSCILLATIONS AND PARTICLE TRANSPORT IN LOW-β PLASMAS I. WAVES

by

F. A. Haas and A. Thyagaraja

Culham Laboratory, Abingdon, Oxon, OX14 3DB, UK
(Euratom/UKAEA Fusion Association)

Abstract

The two-fluid equations are used to investigate the waves observed in a particular low-β, low-current experiment for which a comprehensive set of data exists. Analysis leads to a self-consistent description of the published frequency spectra. The present paper (Part I) is a necessary precursor to establishing a theoretical interpretation of density fluctuations and particle transport in low-β plasmas. The latter theory is developed in Part II and then used to interpret the observed particle transport in the same experiment. Although we have considered a specific experiment, the basic ideas and results are believed to be applicable to purely electrostatic fluctuations in any plasma of sufficiently low-β and governed by the two-fluid equations.

(Submitted for publication in Plasma Physics)

September 1980

1. INTRODUCTION

In previous work (Thyagaraja et al (1980), Haas and Thyagaraja (1980)) we presented a thoretical interpretation of heat transfer in tokamaks and slow reversed field pinches in terms of temperature and magnetic fluctuations. The question arises whether particle transport could be similarly interpreted. Our ultimate aim is to correlate particle transport with observed density fluctuations. Since we believe particle transport to be a much more complicated problem than heat transfer, it behoves us to develop an interpretation for the simplest possible experiment, and for which the fullest data is available. Thus as a necessary preliminary to studying tokamak, in this set of papers (I and II) we shall investigate a low-β collisional experiment described by Chung and Rose (1968).

In spirit, the present work is exactly analogous to our treatment of temperature fluctuations and heat transfer. Thus we consider the effects of small amplitude fluctuations on the mean plasma properties. We note that the fluctuations can be coherent or random and may be due to saturated instabilities or external noise. In our earlier work we defined the conditions under which this separation of a mean and fluctuating part of any quantity is valid. In particular, we gave a precise meaning to the averaging procedure which defines the mean quantities (Thyagaraja et al (1980)). We assume that the same rather weak conditions apply in the present investigation. As we shall restrict ourselves to the collisional regime, we assume the plasma to be described by Braginskii's (1965) equations.

There are, however, significant differences of detail between our earlier work and the present one. In the mechanism of heat transfer, magnetic fluctuations play a vital role in enabling the large parallel thermal conductivity to enhance the perpendicular conductivity. In the present work the effect of magnetic fluctuations is neglected. This is justified for low-β, low current plasmas, and in which electrostatic oscillations are predominant. Although magnetic fluctuations do exist, they play an insignificant role in particle transport.

In their paper, Chung and Rose (1968) give detailed observations of the power spectra of density fluctuations, which they show to be correlated with the particle transport. The experiment was a hot cathode discharge using argon, the ions of which were greater than 99% singly ionised. The plasma was collisional with $\omega_{ci}\tau_i \sim 1$, that is, the ions would be unaffected by the applied uniform longitudinal magnetic field. The plasma was produced in a "source" region approximately 50 cm long. Their β was of order 10^{-3} while the axial current was 20A. For these parameters the effects due to the magnetic field fluctuations can be neglected in comparison with electrostatic effects. The power spectra and profiles were directly measured in an experimental region of up to two metres in length; these are reproduced in Figs.3 and 4. The results show the existence of two distinct regimes of operation (I and II). In regime I they show the power spectra to be independent of the conditions in the experimental region. In fact the dominant frequency occurs for $\omega \sim 1.2\omega_{ci}$, where ω_{ci} is the cyclotron frequency of the "source" region (Fig 1). In regime II their results indicate an m = 1 oscillation at frequencies smaller than ω_{ci}. Both regimes reveal frequencies very much smaller than ω_{ci}, and for which Chung and Rose offer no explanation. Temperature and density profiles are given for the experimental region only. Chung and Rose claim to observe correlations between fluctuations and particle flux for both regimes. In regime I the particle flux is more strongly affected by the density fluctuations than in II.

The purpose of the present work is to provide a self-consistent interpretation of the above observations. In paper I we attempt to derive the characteristic frequencies shown by the power spectra, and in doing this we neglect dissipation and non-linear effects. In paper II we attempt to interpret the observed particle transport. We show that dissipation is essential in our interpretation. To compare theory with experiment we have had to assume the profiles in the source to be the same as those measured in the experimental region. Fortunately, we have been able to show that the characteristic frequencies are relatively insensitive to details of the profiles. In principle, using numerical methods, the frequencies could be determined for arbitrary profiles. In deriving these spectra we have been led to a novel boundary condition at the plasma edge. While this is relatively unimportant in obtaining the frequencies, it is of crucial importance for evaluating the net outward diffusion, as will be shown in paper II.

2. THEORETICAL MODEL

We consider a slab geometry in which the z - coordinate is defined to be in the direction of a constant uniform magnetic field, B. All equilibrium quantities are taken to be functions of x only; thus x represents the "radial" coordinate. The fully ionised plasma is assumed to lie between the planes x = 0 and x = a, the former denoting the 'centre' and the latter representing the plasma edge. We assume the 'poloidal' - coordinate y to have the period $2\pi a$. For the experiment being considered, $\lambda_D \ll a$, and hence strict charge neutrality ($n_e = n_i \equiv n$) is appropriate. For simplicity, we choose the density and electron temperature profiles to be $n = n_o \exp(-x/L_n)$ and $T_e = T_o \exp(-x/L_T)$ where, in general, L_n and L_T are not equal to a. We further assume the ion and electron temperatures to have the same x - dependence, such that $T_i(x) \sim 10^{-1} T_e(x)$ in the region $x < a$. Thus defining the plasma edge to be that place where recombination becomes significant, we take $T_i \simeq T_e \lesssim 1$ eV throughout $x \geq a$. We shall take account of ion and electron streaming both in the x - direction (U_i, U_e), and the y-direction (V_i, V_e), but assume streaming along the Z-axis to be absent[1]. In fact, we could have formulated the present problem in cylindrical geometry. The additional mathematical complexity, however, would only obscure the ideas to which we wish to draw attention. We now write down the equilibrium equations in the absence of fluctuations. The x-component of the ion-momentum equation is

$$0 = -\frac{d}{dx}(nT_i) + \frac{enB}{c} V_i + enE_x + \frac{m_i n}{\tau_{ei}} (U_e - U_i) \tag{1}$$

where the convective term has been neglected. The last term represents resistivity, τ_{ei} being the mean time between successive collisions of an electron with ions. Making allowance for viscosity, the y-component of the ion-momentum equation can be written as

$$0 = -\frac{enB}{c} U_i + \frac{m_i n}{\tau_{ei}} (V_e - V_i) + \frac{d}{dx}\left(\mu_i \frac{dV_i}{dx}\right) \tag{2}$$

where $\mu_i = n\, T_i\, \tau_i$ is the viscous coefficient in the absence of a

---

[1] We note that in the experiment the axial electron velocity is not zero, but this is unimportant for our purposes.

magnetic field. This is the relevant form since $\omega_{ci}\tau_i \sim 1$, and hence the ions are unaware of B. For the electrons, viscosity is definitely negligible and the corresponding equations are

$$0 = -\frac{d}{dx}(nT_e) - \frac{enB}{c}V_e - en\,E_x - \frac{m_i n}{\tau_{ei}}(U_e - U_i) \qquad (3)$$

and

$$0 = \frac{enB}{c}U_e + \frac{m_i n}{\tau_{ei}}(V_i - V_e)\,. \qquad (4)$$

If we assume $\frac{n_i - n_e}{n} \ll 1$, then using the two continuity equations it is straightforward to express Poisson's equation as

$$\frac{dE_x}{dx} = -8\pi en\frac{(U_i - U_e)}{U_i + U_e}\,. \qquad (5)$$

Given n, B, T_e and T_i we now have five equations for the unknowns U_i, U_e, V_i, V_e and E_x. To a good accuracy these equations are satisfied by

$$\left.\begin{aligned} V_i &= 0 \\ V_e &= \frac{c}{L_p eB}(T_e + T_i) \\ U_i = U_e &= \frac{c(T_e + T_i)}{L_p e\,B\,\omega_{ce}\tau_{ei}} \\ \text{and}\qquad E_x &= -\frac{T_i}{eL_p} \end{aligned}\right\} \qquad (6)$$

where the scale-length L_p is defined by

$$\frac{1}{L_p} = -\frac{1}{nT_e}\frac{d}{dx}(nT_e) = -\frac{1}{nT_i}\frac{d}{dx}(nT_i) \qquad (7)$$

and note further, that $\frac{1}{L_p} = \frac{1}{L_n} + \frac{1}{L_T}$. (8)

We conclude that the equilibrium exhibits outward classical (ambipolar) diffusion. Equations (1) to (5) are completed by the continuity equations

$$\frac{d}{dx}(nU_e) = \frac{d}{dx}(nU_i) = S(x) \quad . \tag{9}$$

Thus given the form for n(x) the source function S(x) can be determined.

The experiment suggests that the above solution is not appropriate. This is due to the fact that, in practice, a true steady-state is not established. More realistically, experiments are in a state of fluctuation about a mean. Thus we stipulate that any physical quantity is to be expressed as the sum of a mean part which is a function of x only, and a fluctuating part. For example, the density is written as

$$n = n_o(x) + \delta n \tag{10}$$

where typically $\frac{\delta n}{n_o}$ is of order 4 to 20%. We emphasise that this separation implies only that the time variations of the "fluctuating" quantities are rapid compared to the times over which the "mean" values are sensibly constant. In practice, it is not necessary to specify whether the fluctuations are random or coherent. By definition, the space-time averages of any fluctuating quantity over the "mean" or macroscopic scales must vanish.

In what follows we identify the natural modes of oscillation of the system.

3. WAVE EQUATION FOR DENSITY FLUCTUATIONS

In determining the natural modes of oscillation we neglect dissipation. All fluctuating quantities q(x,y,z,t) are expressed in the form $\hat{q}(x)\exp(i\omega t + ik_y y + ik_z z)$, the periodic boundary conditions in y and z being satisfied for allowed values of k_y and k_z only; ultimately, we shall replace k_y, k_z by $\frac{m}{a}$, $\frac{2\pi n}{L_s}$ where L_s is the length of the source region and m,n are the mode numbers in cylindrical geometry. The boundary conditions appropriate to x are discussed in Section 4. The ion and electron densities can be written as

$$n_i = n + \delta n + \tfrac{1}{2}\,\Delta n$$

$$\text{and } n_e = n + \delta n - \tfrac{1}{2}\,\Delta n,$$

where δn and Δn denote the quasi-neutral and charge imbalance parts of the fluctuation, respectively. Since the Debye length λ_D is very small compared to all relevant scale-lengths, $\Delta n \ll \delta n \ll n$, that is, quasi-neutrality is an excellent approximation. We assume that the temperature and density fluctuations are adiabatically related, that is

$$\frac{\delta T_i}{T_i} = \frac{\delta T_e}{T_e} = \alpha\,\frac{\delta n}{n}, \tag{11}$$

where $\alpha = \gamma - 1 = ^2\!/_3$. We further assume that all electromagnetic terms are unimportant, so that we do not have to consider fluctuations in B. These approximations are readily justified in the present context, and we shall review them in Section 7.

Using the equilibrium relations as required, the x,y,z components of the linearised ion momentum equation are

$$i m_i \omega\, \delta\hat{u}_i = -\,(1+\alpha)T_i\,\frac{d}{dx}\left(\frac{\delta\hat{n}}{n}\right) + \frac{\alpha}{L_p}\,T_i\,\frac{\delta\hat{n}}{n} + \frac{eB}{c}\,\delta\hat{v}_i + e\,\frac{d}{dx}\,\delta\hat{\phi} \tag{12}$$

$$i m_i \omega\, \delta\hat{v}_i = -\,(1+\alpha)T_i\, i k_y\,\frac{\delta\hat{n}}{n} - \frac{eB}{c}\,\delta\hat{u}_i + e i k_y\,\delta\hat{\phi} \tag{13}$$

$$i m_i \omega\, \delta\hat{w}_i = -\,(1+\alpha)T_i\, i k_z\,\frac{\delta\hat{n}}{n} + e i k_z\,\delta\hat{\phi} \tag{14}$$

where $\delta\hat{\phi}$ is the fluctuation electric potential. For the electrons inertia is always negligible. In deriving the dispersion equation, only the z-component of the electron equation of motion is required, that is

$$0 = - ik_z \left[(1 + \alpha) \, T_e \frac{\delta\hat{n}}{n} + e \, \delta\hat{\phi} \right] \quad .$$

If $k_z \neq 0$ this implies

$$(1 + \alpha) \frac{\delta\hat{n}}{n} = - e \frac{\delta\hat{\phi}}{T_e} \tag{15}$$

thus enabling us to eliminate $\delta\hat{\phi}$ from the ion momentum equations. The ion continuity equation is given by

$$i\omega \frac{\delta\hat{n}}{n} - \frac{1}{L_p} \delta\hat{u}_i + \left[\frac{d}{dx} \left(\frac{\delta\hat{u}_i}{T} \right) + ik_y \frac{\delta\hat{v}_i}{T} + ik_z \frac{\delta\hat{w}_i}{T} \right] T = 0 \tag{16}$$

where $T = T_i + T_e$.

Using the above equations it is straightforward to derive the eigenfunction equation, namely

$$\frac{d^2}{dx^2}\left(\frac{\delta\hat{n}}{n}\right) - \left(\frac{1}{L_p} + \frac{1}{L_T^*}\right) \frac{d}{dx} \left(\frac{\delta\hat{n}}{n}\right)$$

$$+ \frac{\delta\hat{n}}{n} \left[- \left(\frac{1}{L_p} - \frac{1}{L_T^*} \right) k_y \frac{\omega_{ci}}{\omega} - k_y^2 + \frac{1}{L_p \, L_T^*} + \frac{k_z^2}{\omega^2} (\omega_{ci}^2 - \omega^2) - \frac{(\omega_{ci}^2 - \omega^2)}{(1 + \alpha)} \frac{m_i}{T(x)} \right] = 0 \tag{17}$$

where L_T^* is given by

$$\frac{1}{L_T^*} = \frac{1}{L_T} \frac{Te}{T} + \frac{\alpha}{(1 + \alpha)} \frac{T_i}{T} \frac{1}{L_p} \quad .$$

4. BOUNDARY CONDITIONS

We now consider the boundary conditions appropriate to the eigenfunction equation ; by Eq (15) $\frac{\delta\hat{n}}{n} \propto \frac{\delta\hat{\phi}}{T_e}$, and hence these may be discussed in terms of the electric potential. In the cylindrical problem, the poloidal electric field E_θ must be finite at r = 0.

Since $E_\theta = im \frac{\delta\hat{\phi}}{r}$, it follows that for $m \neq 0$ the electric potential $\delta\hat{\phi}$ must vanish at $r = 0$. By analogy, for the slab model we require $\delta\hat{\phi} = 0$ at $x = 0$. For $m = 0$, or equivalently $k_y = 0$, we must return to the basic equations. Thus consideration of the ion continuity equation, Eq (16), shows that $\delta\hat{u}_i = 0$ at $x = 0$. It follows from the y-component of motion for the ions that $\delta\hat{v}_i$ also vanishes at the origin. Using these values, Eqs (12) and (15) lead to the required boundary condition, namely,

$$\frac{d}{dx}\left(\frac{\delta\hat{n}}{n}\right) - \frac{1}{L_T^*} \cdot \frac{\delta\hat{n}}{n} = 0 \quad . \tag{18}$$

We now consider the boundary condition at the plasma edge, $x = a$. Fig 2 gives a schematic diagram of the temperature profiles through the plasma, neutral gas and transition regions, the latter being of width Δx. We suppose the wall to be an equipotential, that is $\delta\hat{\phi}$ = constant at $x = A$. In the outer region $T_e \simeq 0$, and since $e \frac{\delta\hat{\phi}}{T_e} \lesssim 1$, it follows that $\delta\hat{\phi} \simeq 0$ at $x = a$. For this region $(x > a)$, $\delta\hat{\phi}$ must satisfy $\nabla^2\delta\hat{\phi} = 0$. This implies that $E_x = \frac{d}{dx}\,\delta\hat{\phi} = 0$ as x approaches a from the outer region. In the transition layer $\delta\hat{\phi}$ is determined by Poisson's equation. Since T_e and T_i both fall to zero in the layer at essentially the same rate, the net space charge must be small compared to n. Thus any surface charge which appears must be insignificant. It follows from Gauss' theorem that E_x at $x = a - \frac{1}{2}\Delta x$ must vanish. In terms of the density fluctuations this leads to the boundary condition

$$\frac{d}{dx}\left(\frac{\delta\hat{n}}{n}\right) - \frac{1}{L_T}\,\frac{\delta\hat{n}}{n} = 0 \tag{19}$$

at $x = a$. The above argument establishes the plausibility of Eq (19); actual justification depends on the adequacy of our theory in describing the experimental results.

5. SOLUTIONS OF EIGENVALUE PROBLEM

In the previous sections we have derived an eigenfunction equation along with its attendant boundary conditions. To proceed further we introduce the transformation

$$\frac{\delta\hat{n}}{n} = \Phi \exp\left[\frac{1}{2}\left(\frac{1}{L_p} + \frac{1}{L_T^*}\right) x\right], \qquad (20)$$

and the eigenfunction equation now becomes

$$\Phi'' + \Phi\left[-\frac{1}{4 L_n^{*2}} - \frac{1}{L_n^*} k_y \frac{\omega_{ci}}{\omega} - k_y^{\,2} + \frac{k_z^{\,2}}{\omega^2}\left(\omega^2_{ci} - \omega^2\right) - \frac{m_i}{T(x)} \frac{(\omega^2_{ci} - \omega^2)}{(1 + \alpha)}\right] = 0 \qquad (21)$$

where

$$\frac{1}{L_n^*} = \frac{1}{L_p} - \frac{1}{L_T^*} .$$

To facilitate the analysis we replace $T(x)$ by an average value $\overline{T}$. This approximation will be discussed in the next section. A solution which satisfies the boundary conditions at $x = 0$ for $k_y \neq 0$ (ie $m \neq 0$) is

$$\Phi = \sin \frac{\mu x}{a} \qquad (22)$$

where μ is determined from the boundary condition at $x = a$, that is

$$\mu = -\frac{a}{2} \tan \mu \left(\frac{1}{L_T^*} - \frac{1}{L_T} + \frac{1}{L_n}\right) . \qquad (23)$$

For $k_y = 0$ ($m = 0$) the appropriate solution is now

$$\Phi = \sin\left(\mu\frac{x}{a}\right) + k \cos\left(\frac{\mu x}{a}\right) , \qquad (24)$$

where k is an arbitrary constant and μ has to be re-determined. Use of the relevant boundary conditions leads to

$$k = -2L_n^* \frac{\mu}{a} , \qquad (25)$$

with μ determined from

$$\frac{\frac{\mu}{a}\left(1 - L_n^*\left[\frac{1}{L_T^*} + \frac{1}{L_p} - \frac{2}{L_T}\right]\right)}{2L_n^*\left(\frac{\mu}{a}\right)^2 + \frac{1}{2}\left(\frac{1}{L_p} + \frac{1}{L_T^*}\right) - \frac{1}{L_T}} = -\tan\mu \ . \tag{26}$$

We observe that for $T_e \gg T_i$ the numerator is of order T_i/T_e and hence in this limit $\mu \simeq \pi$ is the first acceptable root.

Recalling $k_y = m/_a$ and defining $f = \omega/\omega_{ci}$, we derive the dispersion equation for all m, namely

$$\frac{m_i\ \omega_{ci}^2}{(1+\alpha)\ T} f^4 - \left(\frac{\mu^2}{a^2} + \frac{m^2}{a^2} + k_z^2 + \frac{m_i\ \omega_{ci}^2}{(1+\alpha)\ T} + \frac{1}{4L_n^{*2}}\right) f^2 - \frac{m}{aL_n^*} f + k_z^2 = 0. \tag{27}$$

This equation has only real roots so that the modes are purely oscillatory. Note that changing the sign of both m and f leaves the equation unaltered. For m = 0, Eq (27) reduces to a biquadratic, which for small k_z leads to the low-frequency result

$$f = \pm \frac{1}{(1 + \mu^2/a^2k_z^2)^{\frac{1}{2}}} \ . \tag{28}$$

For $m \geq 1$ the dispersion equation reduces to a cubic for small k_z, with a low frequency solution given by

$$f = \frac{ak_z^2}{m} L_n^* \ . \tag{29}$$

As described in our brief review of the experiment, the results indicate two regimes of operation for the device. Using the parameters relevant to these, we investigate our dispersion equation to determine the appropriate frequencies. Taking $k_z = \frac{n\pi}{L_s}$ where L_s is the 'length' of the source region ($L_s \simeq 50$ cm), we consider only low n, that is, small k_z. Note that L_n, L_n^* and L_T, L_T^*, are of comparable order. For each of the two cases $m = 0$ and $m \geq 1$, we only consider the first 'radial' eigenfunction. Thus $\mu_1 \simeq \pi$ for $m = 0$, while $\mu_1 \simeq 2{\cdot}0$ (Regime I) and $\mu_1 \simeq 2{\cdot}2$ (Regime II) for $m \geq 1$. We have only evaluated frequencies $\nu\left(= \frac{\omega}{\pi}\right)$ for low m, each corresponding to a band of low n; in this matter we have been guided by experiment. Table I gives a summary of our results which show three distinct frequency ranges: low frequency $\nu \ll \nu_{ci}$, medium frequency $\nu < \nu_{ci}$, high frequency $\nu \gtrsim \nu_{ci}$.

6. EFFECT OF SPATIAL TEMPERATURE VARIATION ON FREQUENCIES

In principle, the eigenvalue equation, Eq (21), can be solved exactly and the frequencies computed. Thus for $\omega > \omega_{ci}$ the eigenfunctions are expressible in terms of Bessel functions. For $\omega < \omega_{ci}$ the solution can be expressed in terms of Whittaker functions. However, this procedure is complicated, and in view of the number of assumptions made in deriving Eq (21), not warranted. Using a much simpler method, we shall find that allowance for spatial temperature dependence only has a small effect on the frequencies. As an illustration we only consider the case $m \neq 0$.

We observe that for very low frequencies (first column of Table I) the temperature term in Eq (21) is unimportant; this is highlighted by Eqs (28) and (29). Any effects due to the temperature profile are small and can be calculated by a perturbation method. In any case, such effects cannot be distinguished experimentally.

For medium and high frequencies we write Φ as

$$\Phi = \sum_{k=1}^{\infty} A_k \sin(\mu_k x/a) \tag{30}$$

where the μ_k satisfy

$$\mu_k = -\, a/_2 \tan \mu_k \left(\frac{1}{L_T^*} - \frac{1}{L_T} + \frac{1}{L_n} \right). \tag{31}$$

Substituting in Eq (21) we obtain

$$A_k \left\{ - \frac{\mu_k^2}{a^2} - \frac{1}{4L_n^{*2}} - \frac{k_y}{L_n^*} \frac{\omega_{ci}}{\omega} - k_y^2 + \frac{k_z^2}{\omega^2} (\omega_{ci}^2 - \omega^2) - \frac{m_i(\omega_{ci}^2 - \omega^2)}{\overline{T}\,(1+\alpha)} \right\}$$

$$+ \frac{m_i(\omega_{ci}^2 - \omega^2)}{1+\alpha} \sum_{k'=1}^{\infty} A_{k'} \frac{\int_0^a \sin(\mu_k\, x/a) \sin(\mu_{k'}\, x/a) \left(\frac{1}{\overline{T}} - \frac{1}{T(x)}\right) dx}{\int_0^a \sin^2(\mu_k\, x/a)\, dx} = 0 \quad (32)$$

for k = 1, 2.... . Introducing $T(x) = T_c \exp(-K\, x/a)$, then $\overline{T}$ is defined through

$$\frac{1}{\overline{T}} = \frac{1}{T_c} \frac{\int_0^a \exp(K\, x/a) \sin^2\left(\mu_1 x/a\right) dx}{\int_0^a \sin^2\left(\mu_1 x/a\right) dx} \quad . \quad (33)$$

To proceed it is convenient to write Eq (32) in the form

$$A_k D_k + \sum_{k'=1}^{\infty} A_{k'} V_{kk'} = 0 \quad . \quad (34)$$

In our original determination of the frequencies we took account of the first radial eigenfunction, $\sin\left(\mu_1 x/a\right)$, only. To obtain the zeroth approximation to the frequency from Eq (34), we set $A_{k'} = 0$ for $k' \neq 1$.

The relation $D_1 = 0$ now corresponds to the dispersion equation originally obtained for $m \neq 0$. To estimate the corrections due to the hitherto neglected temperature profile, we take the normalisation $A_1 = 1$, and obtain

$$A_1' = - \frac{V_{k'1}}{D_{k'}} \quad (35)$$

for $k' \neq 1$. Thus it follows that

$$D_1 - \sum_{k'}^{\infty} \frac{V_{k'1} V_{1k'}}{D_{k'}} = 0 , \tag{36}$$

or to first approximation

$$D_1 = \frac{V_{12}^2}{D_2} . \tag{37}$$

Writing out Eq (37) we find

$$\frac{m_i \omega_{ci}^2 a^2}{\bar{T}(1+\alpha)} \left(1 + \Delta\left(\frac{\omega}{\omega_{ci}}\right)\right) \left(\frac{\omega}{\omega_{ci}}\right)^3 - \left(\mu_1^2 + \frac{a^2}{4L_n^{*2}} + m^2 + \frac{m_i \omega_{ci}^2 a^2}{\bar{T}(1+\alpha)} \left(1+\Delta\left(\frac{\omega}{\omega_{ci}}\right)\right)\right) \frac{\omega}{\omega_{ci}}$$

$$- \frac{ma}{L_n^*} = 0 \tag{38}$$

where

$$\Delta\left(\frac{\omega}{\omega_{ci}}\right) = \frac{a^2 m_i \omega_{ci}^2}{\bar{T}(1+\alpha)} \frac{\left(1 - \frac{\omega^2}{\omega_{ci}^2}\right)}{\mu_1^2 - \mu_2^2} \frac{\left(\int_0^a \sin\left(\mu_1 \frac{x}{a}\right) \sin\left(\mu_2 \frac{x}{a}\right) \frac{\bar{T}}{T(x)} dx\right)^2}{\int_0^a \sin^2\left(\mu_1 \frac{x}{a}\right) dx \int_0^a \sin^2\left(\mu_2 \frac{x}{a}\right) dx} . \tag{39}$$

We note that for $\omega > \omega_{ci}$ the correction Δ is positive, while for $\omega < \omega_{ci}$ it is negative. In estimating the size of Δ we substitute the values of ω/ω_{ci} obtained from $D_1 = 0$. For regime I, the μ values corresponding to the first and second radial eigenfunctions are $\mu_1 = 2.0$ and $\mu_2 = 4.9$. By way of example, we consider two temperature profiles, namely, K = 1.2 and K = 3.0; these lead to values for Δ of 0.02 and 0.1, respectively. Restricting ourselves to the larger Δ, the original frequencies of 94.0 and − 11.0 kHz for m = + 1, become modified to 91.0 and − 10.5 kHz, respectively. Thus we conclude that spatial temperature dependence does not have a significant effect on the determination of frequency.

7. DISCUSSION

For both regimes of operation, I and II, the power spectra given by Chung and Rose clearly show frequencies $\nu \simeq 0$ and for which they offer no comment or explanation. Our theory, however, straightforwardly predicts oscillations of this frequency. In regime I the data shows dominant fluctuations at $\omega \simeq 1.2\ \omega_{ci}$, the cyclotron frequency being the value appropropriate to the source region (see Fig 3). Our theory is again successful, predicting oscillations at 72 kHz which correspond to $\omega = 1.25\ \omega_{ci}$; these are associated with m = 0 and low but arbitrary n. Neighbouring this frequency, theory predicts m = 1, 2 oscillations which also lie in the region of observation. Although they make no comment, the results of Chung and Rose show signals at $\nu \simeq -20$ kHz, which could be interpreted in terms of medium-range frequencies for m = 1, 2, 3, (see Table I). Similarly, a signal at 125 kHz can be interpreted as an m = 3 mode. For regime II (see Fig 4) only m = ± 1 are clearly observed, and our prediction of $\nu = \pm 16$ kHz is consistent with peaks which occur in the power spectrum.

The theory being linear and dissipationless, cannot of course, give the relative intensities of the various modes. Only a sophisticated non-linear theory could explain why all the modes predicted by linear theory are not actually seen. We have demonstrated that the predicted frequencies are insensitive to details of temperature and density profiles. The choice of boundary conditions is not expected to be important, influencing the calculated frequencies only through the numerical value of μ. We now briefly review the caveats underlying the present formulation.

In the absence of dissipation there are two extreme values for the exponent α, namely, $\alpha = 2/3$ and zero, corresponding to adiabatic and isothermal fluctuations, respectively. We have assumed adiabaticity throughout although, strictly, this is invalid for low frequencies. Fortunately, however, we have found these frequencies to be independent of α to a leading approximation. In principle the relationship between $\frac{\delta n}{n}$, $\frac{\delta T_i}{T_i}$, $\frac{\delta T_e}{T_e}$ is determined by the two energy equations, but for calculating frequencies in the absence of dissipation, we assume the present procedure to be adequate.

For the low - β , low axial current values of the experiment, it can be checked that electromagnetic effects are small compared with electrostatic effects. The validity of the procedure can be demonstrated by comparing the magnetic fields produced by the observed density fluctuations with the main magnetic field. In a similar manner, any induced electric fields are small in comparison with the electrostatic fields. More precisely, electromagnetic effects are unimportant provided that $1 \gg ak_z \gg \beta$; this rules out consideration of $k_z = 0$.

We note that the theory predicts the intensity of density fluctuations, $\left(\frac{\delta n}{n}\right)^2$, to be an increasing function of x for the first eigenfunction. Interestingly, such an effect has been observed in the outer region of tokamaks (Paul (1979)), suggesting that the physics of this region may be similar to that discussed here. However, it is important to note that tokamaks, in contrast to the present experiment, are low - β, high current devices. It can be shown that it is never permissible to neglect magnetic fluctuations in comparison with electrostatic oscillations for such machines. Therefore, an uncritical application of the above ideas to tokamaks would lead to incorrect results.

From our investigation we conclude that the theory presented is consistent with the published data. In the succeeding paper, we show how the density fluctuations together with the above spectrum can be correlated with the observed particle transport. The value of the above ideas lies not so much in explaining the measurements in a particular experiment but in serving as a necessary preliminary to interpreting particle diffusion in tokamaks.

REFERENCES

BRAGINSKII, S.I. (1965) Reviews of Plasma Physics, Consultants Bureau, New York, I.

CHUNG, K. and ROSE, D. J. (1968) Proceedings of the Symposium on Turbulence of Fluids and Plasma, Microwave Research Institute Symposium Series, XVIII, Polytechnic Press of the Polytechnic Institute of Brooklyn, New York., 311.

HAAS, F.A. and THYAGARAJA, A. (1980) UKAEA Report CLM-P 606.

PAUL, J. W. M., (1979) 9th European Conference on Controlled Fusion and Plasma Physics, Oxford, 371.

THYAGARAJA, A., HAAS, F. A. and COOK, I. (1980) Nuclear Fusion 20, 611.

TABLE I

FREQUENCY SPECTRUM PREDICTED BY THE DISPERSION EQUATION

Regime Characteristics	Low Frequency $\nu \ll \nu_{ci}$		Medium Frequency $\nu < \nu_{ci}$		High Frequency $\nu \gtrsim \nu_{ci}$	
	m	ν (kHz)	m	ν (kHz)	m	ν (kHz)
Regime I	0	±2.5	0	None	0	±72
$B = 1.5$ kG			+1	-11	+1	94, -82
$a = 3.0$ cm		$(k_z = \frac{\pi}{L_s})$	-1	+11	-1	-94, 82
$L_n = 1.5$ cm			+2	-17	+2	108, -91
$\bar{T} = 6$ eV			-2	+17	-2	-108, 91
$\nu_{ci} = 57$ kHz			+3	-19	+3	127, -108
$L_s = 50$ cm			-3	+19	-3	-127, +108
$\mu = 2.0$ $(m \geq 1)$						
$\mu = \pi$ $(m = 0)$						
$k_z = n\pi/L_s$						
Regime II	0	±3.0	0	None	0	±111
$B = 2.0$ kG			+1	-16	+1	125, -106
$a = 3.0$ cm	+1	-1.0	-1	+16	-1	-125, +106
$L_n = 1.0$ cm	-1	+1.0	+2	-27	+2	140, -114
$\bar{T} = 9$ eV			-2	+27	-2	-140, +114
$\nu_{ci} = 76$ kHz			+3	-31	+3	160, -129
$L_s = 50$ cm			-3	+31	-3	-160, +129
$\mu = 2.2$ $(m \geq 1)$						
$\mu = \pi$ $(m = 0)$						
$k_z = n\pi/L_s$						

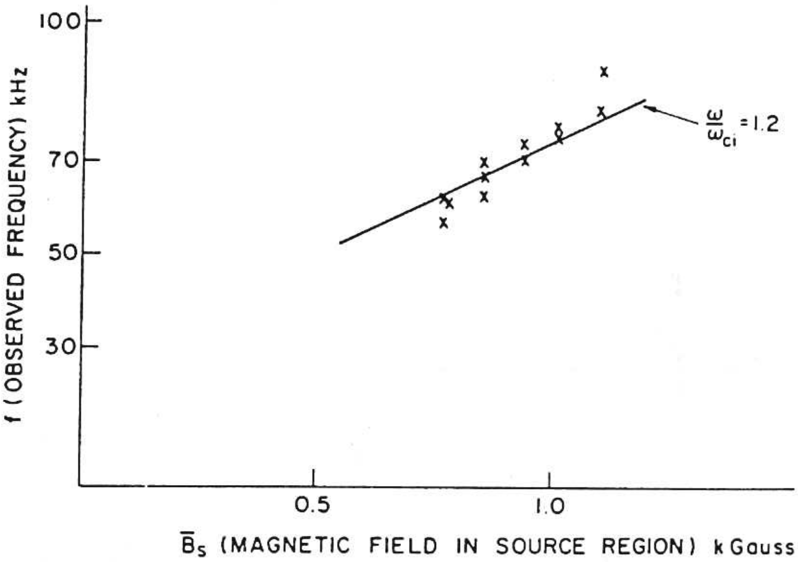

Fig.1 Frequency dependence of Mode (I) on the magnetic fields (Chung and Rose (1968)).

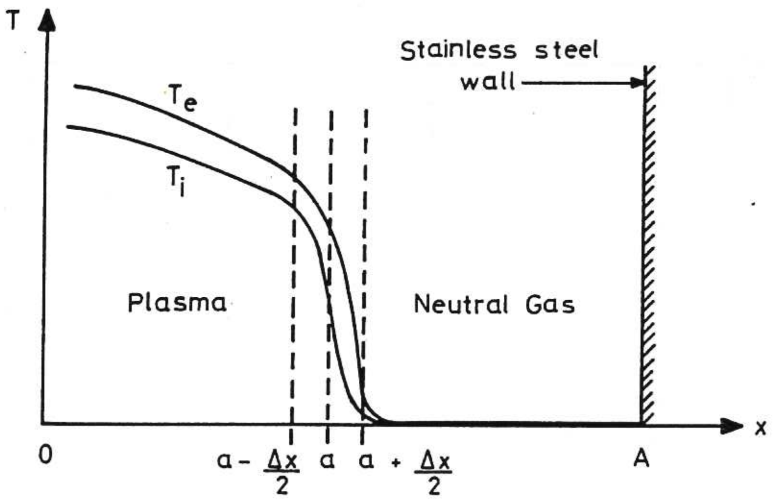

Fig.2 Temperature profiles through the plasma, neutral gas and transition regions.

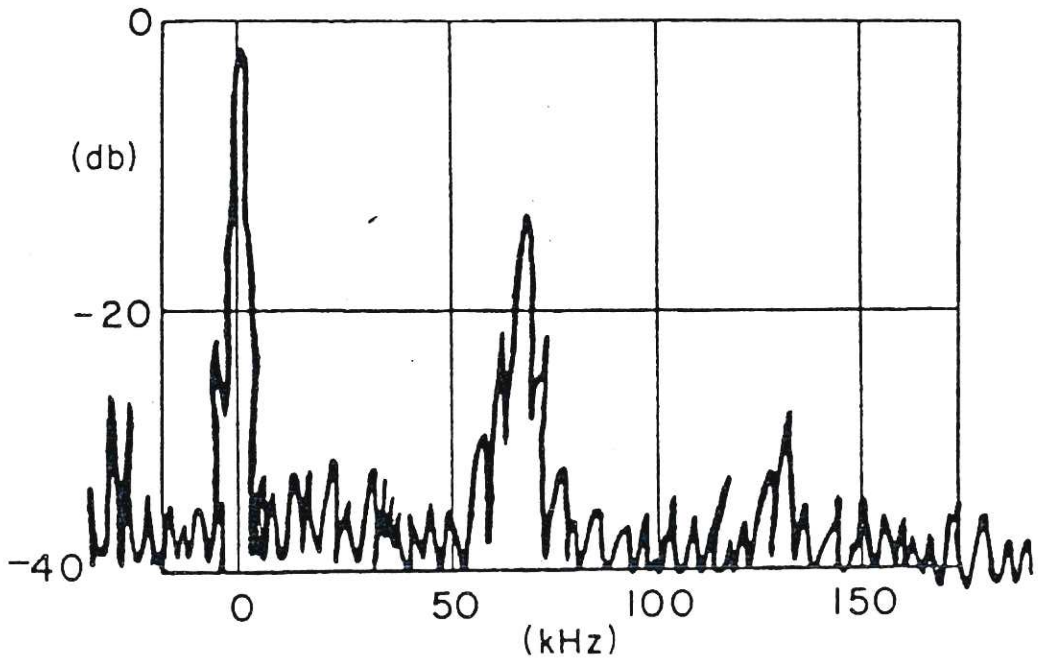

Fig.3 Power spectrum in regime (I) directly obtained by spectrum analyser (Chung and Rose (1968)).

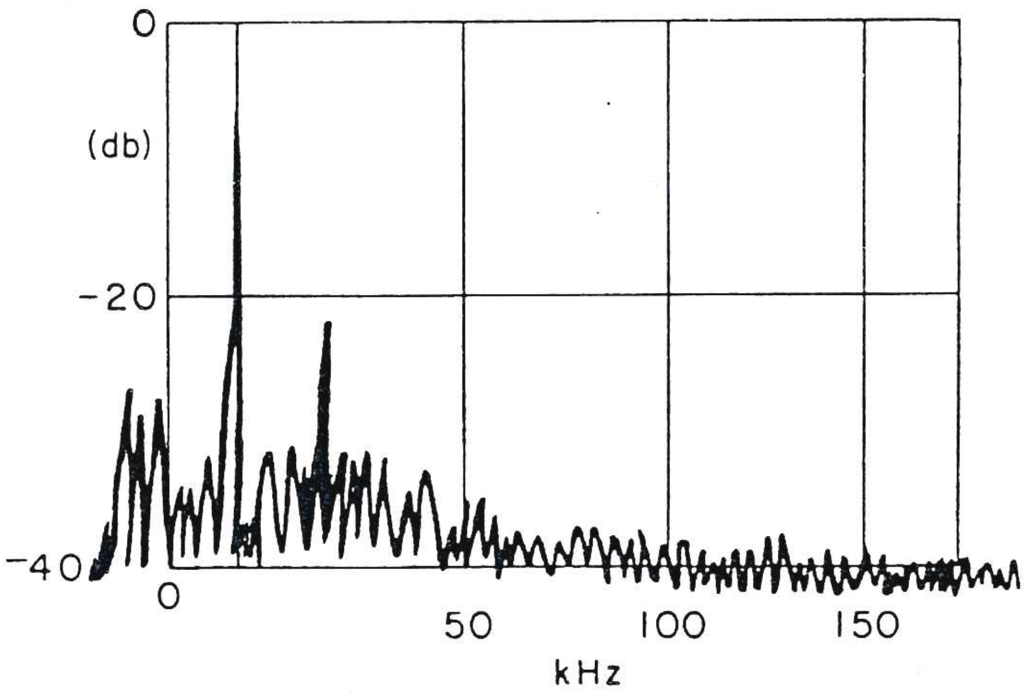

Fig.4 Power spectrum in regime (II) directly obtained by spectrum analyser (Chung and Rose (1968)).