CLM-P622

CLM-P622

UKAEA

Preprint

ELECTROSTATIC OSCILLATIONS AND PARTICLE TRANSPORT IN LOW-β PLASMAS II. DIFFUSION

A. THYAGARAJA
F. A. HAAS

CULHAM LABORATORY
Abingdon Oxfordshire

1981

CLM-P 622

ELECTROSTATIC OSCILLATIONS AND PARTICLE TRANSPORT IN LOW-β PLASMAS II. DIFFUSION

by

A. Thyagaraja and F. A. Haas

Culham Laboratory, Abingdon, Oxon, OX14 3DB, UK
(Euratom/UKAEA Fusion Association)

Abstract

The experimentally determined power spectra of density fluctuations in a particular low-β, low-current experiment are correlated with the observed particle transport using a two-fluid model. Whereas the electron diffusion is unaffected by fluctuations, the ion diffusion is a few times classical and is consistent with the experiment.

(Submitted for publication in Plasma Physics)

September 1980

1. INTRODUCTION

This work is a continuation of our attempt (Haas and Thyagaraja (1980)) to interpret a particular low-β, low-current experiment studied by Chung and Rose (1968). The choice of this experiment was determined by the relative completeness of its data, as well as the analytic simplicity resulting from the above conditions. We believe the present investigation to be a necessary precursor to studies of more complicated systems, such as tokamaks.

In our earlier work (Haas and Thyagaraja (1980)), we determined the natural modes of electrostatic oscillations in the plasma. In the present work we attempt to correlate the observed power-spectra of density fluctuations to the measured particle transport. Chung & Rose reported two regimes (I and II) of operation associated with distinct power spectra. They noted that although the fluctuation level influenced the particle diffusion in both regimes, the form of dependence differed. In our previous paper we were able to account for the existence of the two regimes. In the present paper we show that the diffusion does depend differently on the power spectra for the two cases. In particular even when their amplitudes are sizeable, frequencies small compared to the ion cyclotron frequency make a negligible contribution to the overall diffusion. Chung and Rose observed the diffusion to be a few times classical; our calculation leads to diffusion rates consistent with their results.

An unexpected and apparently new feature of our investigation is that it predicts the electron diffusion to be classical, that is, it is unaffected by electrostatic oscillations. This result is a direct consequence of the negligibility of electron inertia. The ions, however, are influenced by the fluctuations and diffuse at rates a few times classical. It follows that the overall diffusion must be non-ambipolar. For quasi-neutrality to be satisfied, we invoke the short-circuiting mechanism first proposed by Simon (1959) in a related but different context. Unfortunately, the possibility that the diffusion in the particular experiment might not be ambipolar, was not suspected by Chung and Rose. Therefore we have no direct experimental evidence for the existence or otherwise of Simon's mechanism.

For the conditions of the experiment, the two-fluid equations are expected to be valid. Our second assumption, the smallness of the fluctuations, is well supported by observation. The deduction of non-ambipolar diffusion is a direct consequence of these two assumptions.

2. ELECTRON DIFFUSION

Following the spirit of the preceding paper, we neglect electron inertial and dissipative effects; we also neglect fluctuations in the magnetic field. Adiabaticity is not required for the electrons and it is again assumed that the "mean" quantities T_e, p_e, as well as $\vec{B}$, are functions of x only. The y and z components of the fluctuated momentum equations are

$$0 = -T_e \frac{\partial}{\partial y}\left(\frac{\delta p_e}{p_e}\right) + \frac{eB}{c}\,\delta U_e - e\frac{\partial}{\partial y}\,\delta\phi \tag{1}$$

and

$$0 = T_e \frac{\delta p_e}{p_e} + e\,\delta\phi \tag{2}$$

respectively. Straightforward substitution of Eq. (2) into Eq. (1)

leads to

$$\delta U_e = 0 \ . \tag{3}$$

Thus to the level of our approximations, fluctuation effects make no contribution to electron diffusion. It follows that any electron diffusion which arises must be classical, being due, of course, to the presence of resistivity and viscosity in the basic equilibrium. Inclusion of these dissipative effects in the above flutter equations leads to a small correction to classical diffusion. For frequencies near the electron cyclotron frequency, ω_{ce}, inertial effects are significant and this could produce an important correction to the total electron diffusion. The frequency spectrum published by Chung & Rose, however, is in a range of frequencies much less than ω_{ce}, being of order ω_{ci}. Thus we conclude, that for their experiment, electron diffusion is essentially classical; the neglect of magnetic fluctuations and electron inertia appears to be very well justified. Note that the above argument is not directly applicable to tokamaks, since magnetic flutter plays an important role in these devices.

3. ION DIFFUSION

Unlike the electrons, ion diffusion due to density fluctuations is dependent on inertia and dissipation - ion viscosity and resistivity. If these effects are neglected, then by the above arguments there will be no enhancement of the classical diffusion, which is contrary to experiment. Note that inclusion of ion inertia alone does not lead to enhanced diffusion, either. The physical reason for this is clear: in the absence of dissipation the momentum equations are reversible in time, and this implies that the net diffusion, $\langle \delta u\, \delta n \rangle$[1], vanishes. Dissipation plays an essential role in introducing a phase difference between δu and δn, and this leads to a positive value (outward diffusion) for $\langle \delta u\, \delta n \rangle$[2]. For purposes of illustration, we shall represent the dissipation schematically, that is, by an inverse momentum relaxation time, $1/\tau$. The validity of this approximation will be discussed later.

Using Fourier transform notation (Haas and Thyagaraja (1980)), the ion momentum equations can be written as

$$im_i\, \Omega\, \delta\hat{u}_i = -(1+\alpha)T_i \frac{d}{dx}\left(\frac{\delta\hat{n}}{n}\right) + \frac{\alpha}{L_p} T_i \frac{\delta\hat{n}}{n} + \frac{eB}{c}\delta\hat{v}_i - \frac{d}{dx}\left((1+\alpha)T_e \frac{\delta\hat{n}}{n}\right) \qquad (4)$$

and

$$im_i\, \Omega\, \delta\hat{v}_i = -(1+\alpha)T_i\, ik_y \frac{\delta\hat{n}}{n} - \frac{eB}{c}\delta\hat{u}_i - ik_y\,(1+\alpha)T_e \frac{\delta\hat{n}}{n} \qquad (5)$$

where α arises from the adiabaticity assumption (Haas and Thyagaraja (1980)), and the quantities Ω and L_p are defined by

$$\Omega = \omega - i/\tau \quad \text{and} \quad L_p^{-1} = -\frac{1}{nT_i}\frac{d}{dx}(nT_i) \,. \qquad (6)$$

Introducing

$$T = T_e + T_i \quad \text{and} \quad L_T^{-1} = -\frac{1}{T_e}\frac{dT_e}{dx} \,, \qquad (7)$$

the above equations can be expressed in the simpler forms

---

[1]The operation $\langle\ \rangle$ denotes an appropriate space - time average, which we shall define later.

[2]Strictly $\langle \delta u\, \delta n \rangle$ has the sign of $-\frac{1}{n}\frac{dn}{dx}$.

$$im_i \Omega \hat{\delta u_i} = -(1+\alpha)T \frac{d}{dx}\left(\frac{\hat{\delta n}}{n}\right) + (1+\alpha)\frac{T_e}{L_T}\frac{\hat{\delta n}}{n}$$

$$+ \frac{eB}{c}\hat{\delta v_i} + \alpha \frac{T_i}{L_p}\frac{\hat{\delta n}}{n} \tag{8}$$

and

$$im_i \Omega \hat{\delta v_i} = -(1+\alpha)T\, ik_y \frac{\hat{\delta n}}{n} - \frac{eB}{c}\hat{\delta u_i} \quad . \tag{9}$$

Eliminating $\hat{\delta v_i}$ between Eqs. (8) and (9), and defining

$$C_s^2 = \frac{T}{m_i} \qquad \omega_{ci} = \frac{eB}{m_i c} \tag{10}$$

we derive an expression for $\hat{\delta u_i}$, namely,

$$\hat{\delta u_i} = \frac{i\Omega}{\omega_{ci}^2 - \Omega^2}\left[(1+\alpha)C_s^2\left\{\frac{\hat{\delta n}}{n}\left(\frac{1}{L_T} - \frac{\omega_{ci}}{\Omega}k_y\right) - \frac{d}{dx}\left(\frac{\hat{\delta n}}{n}\right)\right\} + \frac{\alpha}{L_p}\frac{T_i}{m_i}\frac{\hat{\delta n}}{n}\right] . \tag{11}$$

It is now convenient to introduce the spectral representation for the density fluctuations, that is

$$\frac{\delta n}{n} = \sum_{k'_y}\sum_{k'_z}\int d\omega' \, \frac{\hat{\delta n}}{n}\left(k'_y, k'_z, \omega', x\right) e^{i\omega' t + ik'_y y + ik'_z z} \quad . \tag{12}$$

We note that for $\frac{\delta n}{n}$ to be real,

$$\left[\frac{\hat{\delta n}}{n}\left(k_y, k_z, \omega, x\right)\right]_{c.c.} = \frac{\hat{\delta n}}{n}\left(-k_y, -k_z, -\omega, x\right) \quad . \tag{13}$$

Using Eq. (12), δu_i can be written as

$$\delta u_i = (1+\alpha)C_s^2 \sum_{k_y}\sum_{k_z}\int d\omega \frac{i\Omega}{\omega_{ci}^2 - \Omega^2}\left[\frac{1}{L_T}\frac{\hat{\delta n}}{n} - \frac{d}{dx}\left(\frac{\hat{\delta n}}{n}\right)\right.$$

$$\left. - \frac{\omega_{ci}k_y}{\Omega}\frac{\hat{\delta n}}{n} + \frac{\alpha}{(1+\alpha)C_s^2 L_p}\frac{T_i}{m_i}\frac{\hat{\delta n}}{n}\right] e^{i\omega t + ik_y y + ik_z z} \quad . \tag{14}$$

From the above results, we can now evaluate the net outward diffusion of ions due to density fluctuations. Introducing a space - time average we determine $\left\langle \delta u_i \frac{\delta n}{n} \right\rangle$, where

$$\left\langle \delta u_i \frac{\delta n}{n} \right\rangle = \frac{1}{2\pi a L t_M} \int_0^{t_M} dt \int_0^{2\pi a} dy \int_0^{L} dz \;\; \delta u_i \frac{\delta n}{n} \quad . \tag{15}$$

The time t_M is characteristic of the time-scale over which the experiment runs in steady-state, and is long compared with the "periods" of the observed fluctuations. Due to the integrations over y and z , $\left\langle \delta u_i \frac{\delta n}{n} \right\rangle$ is of course, a function of x only. As can be seen from its dimensions, $\left\langle \delta u_i \frac{\delta n}{n} \right\rangle$ is not the actual diffusion, but is closely related to it. To develop our analysis it is useful to introduce the spectral function $C^2(k_y,k_z,\omega,x)$, which is defined by

$$\left\langle \left(\frac{\delta n}{n}\right)^2 \right\rangle = \sum_{k_y} \sum_{k_z} \int_{-\infty}^{\infty} d\omega \, C^2 \left(k_y,k_z,\omega,x\right) \quad . \tag{16}$$

Thus substituting Eqs. (12) and (14) into Eq. (15) and performing the integrations, it follows that $\left\langle \delta u_i \frac{\delta n}{n} \right\rangle$ can be expressed as

$$\left\langle \delta u_i \frac{\delta n}{n} \right\rangle \equiv \Gamma^{th} + \Gamma^{diff} \tag{17}$$

with

$$\Gamma^{th} = \frac{(1+\alpha)C_s^2}{\omega_{ci}^{\;2}\tau} \left(\frac{\chi}{L_T} - \frac{1}{2}\frac{d\chi}{dx}\right) \tag{18}$$

$$\Gamma^{diff} = \frac{\alpha T_i}{m_i L_p} \, \frac{\chi}{\omega_{ci}^{\;2}\,\tau} \quad . \tag{19}$$

The quantity χ is defined to be

$$\chi = \sum_{k_y} \sum_{k_z} \int_{-\infty}^{\infty} d\omega \, \frac{C^2(k_y,k_z,\omega,x)\left(1 + \frac{\omega^2}{\omega_{ci}^2}\right)}{\left(1 - \frac{\omega^2}{\omega_{ci}^2}\right)^2 + 4\frac{\omega^2}{\omega_{ci}^2} \frac{1}{\omega_{ci}^2 \tau^2}} \, . \tag{20}$$

We note in passing, that due to its oddness in k_y , the term $\frac{\omega_{ci} k_y}{\Omega} \frac{\delta \hat{n}}{n}$ in Eq. (14) makes no contribution to the net diffusion.

4. COMPARISON WITH EXPERIMENT

The experimental results show the density fluctuation levels to be strongly dependent on the source magnetic field (see Figure 1). Chung and Rose found, however, that by optimizing the operating conditions, fluctuations on axis could be made $\lesssim 1\%$. Because of the low-level and the absence of coherent oscillations they described this state as 'quiescent', with the associated diffusion being apparently classical. As they varied the operating conditions away from 'quiescence' the fluctuation level increased and partially coherent oscillations were observed. Figure 1 shows two regimes (I and II) which are distinguished by different characteristics of the fluctuations. We described the waves and power-spectra associated with these branches in our first paper.

In order to correlate the fluctuations shown in Figure 1 with particle transport it is necessary to have a plot of the observed particle flux against the source magnetic field. Unfortunately such a plot is not available in the Chung-Rose paper. These authors publish instead a plot of the density e-folding length as a function of the source magnetic field in the two regimes (see Figure 2). Following Simon (1959), the e-folding length is taken to be proportional to the square root of the effective diffusion coefficient. Thus

Figure 2 can be taken to be a plot of the square root of the effective diffusion coefficient as a function of the source magnetic field. The curves in the two figures enable us to relate the diffusion rates directly with the fluctuation levels.

In our previous paper we gave a plausibility argument for

$$\frac{1}{L_T}\,\frac{\delta\hat{n}}{n} - \frac{d}{dx}\left(\frac{\delta\hat{n}}{n}\right) = 0 \tag{21}$$

to be the boundary condition at $x = a$. Since this implies that

$$\frac{\chi}{L_T} - \frac{1}{2}\frac{d\chi}{dx} = 0 \tag{22}$$

at $x = a$, it follows that $\Gamma^{th} = 0$ at the plasma boundary. Thus the total rate of loss of ions through the boundary is $2\pi aLn(a)\,\Gamma^{diff}(a)$, where

$$\Gamma^{diff}(a) = \frac{\alpha T_i}{m_i L_p}\,\frac{1}{\omega_{ci}^{\,2}\tau}\,\chi(a) \quad . \tag{23}$$

Defining $\sigma = \frac{\omega}{\omega_{ci}}$ and $\varepsilon = \frac{1}{\omega_{ci}\tau}$, then

$$\chi(a) = \omega_{ci}\sum_{k_y}\sum_{k_z}\int_{-\infty}^{\infty} d\sigma\, \frac{C^2\left(k_y,k_z,\omega,a\right)(1+\sigma^2)}{(1-\sigma^2)^2 + 4\varepsilon^2\sigma^2} \tag{24}$$

and from Eq. (16)

$$\left\langle\left(\frac{\delta n}{n}\right)^2\right\rangle = \omega_{ci}\sum_{k_y}\sum_{k_z}\int_{-\infty}^{\infty} d\sigma\; C^2\left(k_y,k_z,\omega,a\right) \quad . \tag{25}$$

Now experiment indicates that $\varepsilon \ll 1$, and hence the main contribution to the integral in Eq. (24) comes from $\sigma \simeq 1$. Thus to a good approximation

$$\Gamma^{\mathrm{diff}}(a) \simeq \frac{\pi}{2} \frac{\alpha T_i}{L_p m_i} \sum_{k_y} \sum_{k_z} C^2(k_y, k_z, \omega_{ci}, a) \tag{26}$$

or alternatively,

$$\Gamma^{\mathrm{diff}}(a) \simeq \frac{\pi\alpha T_i}{2L_p m_i \omega_{ci}} \left\langle \left(\frac{\delta n}{n}\right)^2 \right\rangle \frac{\sum_{k_y} \sum_{k_z} C^2(k_y, k_z, \omega_{ci}, a)}{\sum_{k_y} \sum_{k_z} \int_{-\infty}^{\infty} C^2(k_y, k_z, \omega, a)\, d\sigma} . \tag{27}$$

This is the formula which we shall compare with experiment.

We consider regime II first. The power-spectrum for this regime of operation is shown in Figure 4 of our earlier paper. Crudely speaking, the spectrum consists of a single sharp peak superimposed on a roughly uniform "background of noise" of frequency range of order the ion cyclotron frequency ($\omega_{ci} \simeq 5 \times 10^5$ rads/sec). This suggests the simple model shown in Figure 3 of the present paper. Thus denoting the height (intensity) of the peak by h , then the observations indicate the height of the background to be of order $h/10$. The width of the peak is $O(2/\tau)$, where τ is the characteristic damping time due to resistivity $\left(\tau_{res} = \frac{m_i \tau_e}{3m_e}\right)$ or viscosity $\left(\tau_{vis} = \frac{m_i a^2}{T_i \tau_i}\right)$. Substitution of parameters typical of the experiment leads to $\tau_{res} \simeq 1.2 \times 10^{-3}$ sec and $\tau_{vis} \simeq 0.6 \times 10^{-3}$ sec. Since these times are very comparable we can take either as characterising the width of the peak. For definiteness, however, we shall suppose viscosity to be the responsible process since it has the shorter relaxation time. It follows that

$$\frac{\sum\limits_{k_y}\sum\limits_{k_z} C^2(k_y,k_z,\omega_{ci},a)}{\sum\limits_{k_y}\sum\limits_{k_z}\int_{-\infty}^{\infty} C^2(k_y,k_z,\omega,a)\,d\sigma} \simeq \frac{\frac{h}{10}\,\omega_{ci}}{\frac{h}{\tau_{res}} + \frac{h}{10}\omega_{ci}} \tag{28}$$

which for the values of ω_{ci} and τ_{vis} quoted above is of order one, and hence

$$\Gamma^{diff}(a) = \frac{\pi}{2}\frac{\alpha T_i}{\omega_{ci} m_i L_p} \left\langle\left(\frac{\delta n}{n}\right)^2\right\rangle . \tag{29}$$

In making the comparison with experiment it is instructive to consider the ratio of $\Gamma^{diff}(a)$ to the equivalent classical quantity $\Gamma^{cl}(a)$. Thus we write

$$\frac{\Gamma^{diff}(a)}{\Gamma^{cl}(a)} = \frac{\frac{\pi}{2}\frac{\alpha T_i}{\omega_{ci} m_i L_p}\left\langle\left(\frac{\delta n}{n}\right)^2\right\rangle}{\frac{1}{a\,\omega_{ce}^2\,\tau_e}\,\frac{T_e}{m_e}} . \tag{30}$$

Taking parameters from the experiment $(B \sim 2kG)$ gives

$$\frac{\Gamma^{diff}}{\Gamma^{cl}} \simeq 3.0 \times 10^2 \left\langle\left(\frac{\delta n}{n}\right)^2\right\rangle . \tag{31}$$

From Figure 1 the fluctuation level corresponding to $B \sim 2kG$ is approximately 4%. Thus we obtain

$$\frac{\Gamma^{diff}}{\Gamma^{cl}} = \frac{1}{2} . \tag{32}$$

Hence the total diffusion $(\Gamma^{cl} + \Gamma^{diff})$ is of order $1.5\,\Gamma^{cl}$, whereas from Figure 2 the total diffusion is approximately $2.0\,\Gamma^{cl}$. This is reasonable agreement considering that the comparison can be little more than qualitative. The principal source of uncertainty is the lack of precise knowledge of $\left\langle\left(\frac{\delta n}{n}\right)^2\right\rangle$ at the boundary of the plasma. Thus Figure 1 applies to $r = 1.0$ cm as does Figure 2, whereas Eq. (31) refers to the net plasma diffusion at the boundary. Also our calculations are restricted to slab geometry and therefore do not account for cylindrical effects. Nevertheless the theory does predict the correct qualitative behaviour of the diffusion coefficient as a function of density fluctuation level.

In regime I there is a substantial amount of power, namely, much larger than the background at around ν_{ci} (60kHz). This implies qualitatively that for the same value of $\left\langle\left(\frac{\delta n}{n}\right)^2\right\rangle$ the diffusion coefficient due to regime I fluctuations is larger than regime II fluctuations. This qualitative deduction from our model is indeed supported by the Chung-Rose observations. Ultimately the difference between the two regimes can be traced to a velocity resonance at ω_{ci}, shown clearly in Eq. (11). Since density fluctuations in regime I (mainly associated with frequencies $\omega \sim 1.25\omega_{ci}$) have a sizeable power at this resonance in contrast to regime II, the diffusion is correspondingly stronger.

Thus we are able to deduce the observed diffusion due to the two branches of electrostatic oscillations reported by Chung and Rose. Summarising, branch I has resonant behaviour in the vicinity of ω_{ci}. Since in our model the anomalous diffusion is determined by the power-spectral density at or near ω_{ci}, these fluctuations have a very significant effect on the particle transport as observed by Chung and Rose. The low-frequency branch II on the other hand is non-resonant near ω_{ci} and it is the background fluctuations which are responsible for the transport in this case. Since the background power spectrum can be expected to rise proportionately to the total power, namely, non-resonantly, the diffusion depends less sensitively on the fluctuation level, and this is observed experimentally. Thus we can categorise branch I type diffusion as resonant, while branch II is non-resonant. A clear prediction of our model is the fact that very low frequency oscillations (associated with strong peaks in the power-spectra) are not directly responsible for particle transport, as seen from Eq. (26).

5. DISCUSSION

As we have previously noted, our novel boundary condition (Eq. (21)) causes Γ^{th} to vanish at the plasma edge. Although both Γ^{th} and Γ^{diff} vanish at the centre, their ratio $\Gamma_{th}/\Gamma_{diff}$ is finite there, being of order T_e/T_i. The sign of $\Gamma^{th}(x)$ depends on the relative magnitudes of the temperature scale length and the scale-length associated with the statistical function χ. Interestingly, the simple boundary condition $\frac{\delta n}{n} = 0$ at $x = a$ causes the net diffusion to vanish. The boundary condition which we have used was suggested by consideration of the recombination layer at the edge of the plasma (Haas and Thyagaraja (1980)); we note that it has the form of the most general type of boundary condition, namely

$$\frac{\delta n}{n} \propto \frac{d}{dx}\left(\frac{\delta n}{n}\right) . \qquad (33)$$

Physically, it signifies that the fluctuating electric field at the edge of the plasma is zero. Although strictly only a heuristic assumption, this boundary condition does lead to results consistent with the published data. It would be desirable to have an independent experimental check of this boundary condition, in view of its central importance.

In the absence of dissipation it is impossible for any net diffusion to occur. This is due to the fact that in a dissipationless system the phase relation between $\delta u\ \delta n$ is such that the correlation integral, $\langle \delta u\ \delta n \rangle$, vanishes. This can also be directly inferred from our formulae by taking the limit $\tau \to \infty$. This limit should be taken with the proviso that the power spectral density at ω_{ci} is zero. If this were not the case the theory would become invalid since the flutter velocity would be infinite. It should be noted that Eq. (26) does not show the τ dependence of $\Gamma^{diff}(a)$ explicitly. This is because Eq. (26) is the result of taking $\omega_{ci}\tau$ large but <u>finite</u>, and <u>not</u> due to setting $\frac{1}{\tau} = 0$. The C^2 function implicitly contains any τ dependence. However, this function is taken as <u>given</u> in the present work. A determination of this function is a matter for experiment, or for a more general non-linear analysis.

In the present application we have shown that electrostatic oscillations make an insignificant contribution to electron diffusion, which therefore, remains essentially classical. This result, however will be generally valid for all low-β, low axial current systems in which the "mean" temperature and density are constant on "mean" pressure surfaces. The same result applies to ions provided that the frequencies of oscillation are small compared with the ion cyclotron frequency. Now under the appropriate conditions, Chung and Rose find the particle diffusion to be a few times larger than classical. This indicates that the ion diffusion is predominantly determined by the power spectrum of density fluctuations at or near ω_{ci} .

To simplify matters we have assumed the fluctuations to be adiabatic. While this may be correct for frequencies $\omega \gtrsim \omega_{ci}$, we anticipate that our assumption will fail for lower frequencies. The latter, however, are not expected to contribute significantly to the diffusion. Furthermore, the frequencies themselves are not very sensitive to the value of α. Deviations from adiabaticity are due entirely to electron and ion thermal conduction, and a proper investigation of the density-temperature correlation would require the fluttered energy equations. This procedure is not open to us, since Chung and Rose do not report any temperature fluctuation measurements. In view of this we are impelled to make the adiabatic approximation. We note, however, that our ultimate results are qualitatively consistent with those observed.

For the conditions of the experiment we have shown that electrostatic fluctuations do not significantly affect the electron diffusion, but leave it classical. The ions, on the other hand, are affected, their diffusion being several times classical. This difference in behaviour is entirely due to the large mass difference. It follows that the total diffusion cannot be ambipolar. For consistency with quasi-neutrality it is necessary that a means exist for removing charge separation. Such an effect has been suggested by Simon (1959); he pointed out that short-circuiting electron currents can arise in a plasma, leading to non-ambipolar diffusion in a classical context, that is, in the absence of fluctuations. This effect could be particularly important in experiments with ends, or where toroidal symmetry is broken, for example, by divertors or limiters. The present analysis indicates the necessity of Simon currents in the Chung and Rose experiment, but unfortunately the latter workers give no direct evidence which supports our prediction.

The importance of low-β and low axial currents consists in the fact that under these conditions the magnetic fluctuations are insignificant in their dynamical effects. This important simplification distinguishes the present model from tokamaks. Although the latter are low-β machines their large currents lead to large fluctuating Lorentz forces which could play a significant role in particle diffusion.

Formally Eq. (29) for the diffusion flux at the edge resembles the Bohm diffusion flux in the sense that it is inversely proportional to the magnetic field. However, the analogy stops there. Bohm's formula makes no reference to fluctuation intensity, furthermore it conventionally contains the electron temperature. Our general expressions for diffusion, Eqs. (18) and (19), refer to the ions, and as far as we are aware the thermal part, Γ^{th}, has never been previously proposed. Although it vanishes at the edge (by virtue of our boundary condition) and does not contribute a net particle loss, it is nevertheless an important component of diffusion within the plasma.

The work of this paper has required only that the fluctuations be small compared to a suitably defined mean value. We have not considered the origins of the fluctuations. In principle they could be the result of non-linearly saturated instabilities, or they could be due to unsteadiness in the sources. Furthermore, our analysis does not _have_ to invoke instability mechanisms. We conclude, that taking the density fluctuation level as given, the measured and the deduced diffusion rates are in qualitative agreement.

REFERENCES

CHUNG, K. and ROSE, D. J. (1968) Proceedings of the Symposium on Turbulence of Fluids and Plasma, Microwave Research Institute Symposium Series, XVIII, Polytechnic Press of the Polytechnic Institute of Brooklyn, New York, 311.

HAAS, F. A. and THYAGARAJA, A. (1980) Paper I, Submitted to Plasma Physics. (CLM-P 621).

SIMON, A. (1959) An Introduction to Thermonuclear Research, International Series of Monographs on Nuclear Energy, Pergamon Press, London, p. 150.

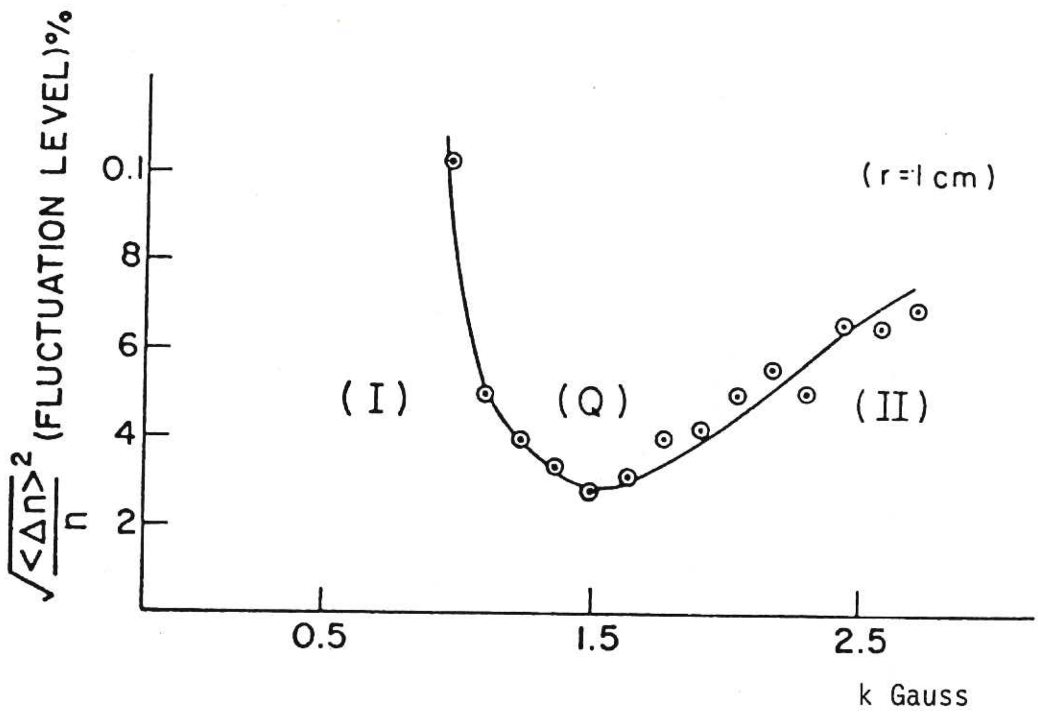

Fig.1 Density fluctuations at r = 1 cm versus the source magnetic field.

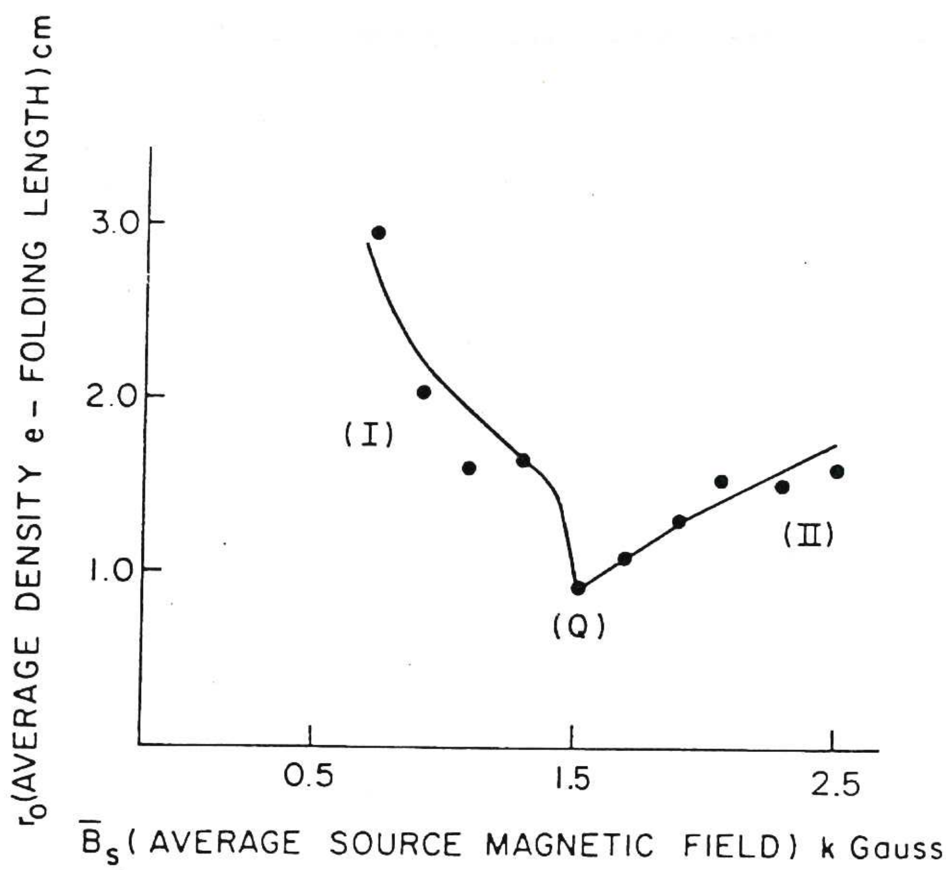

Fig.2 Average density e-folding length in the different operating regimes.

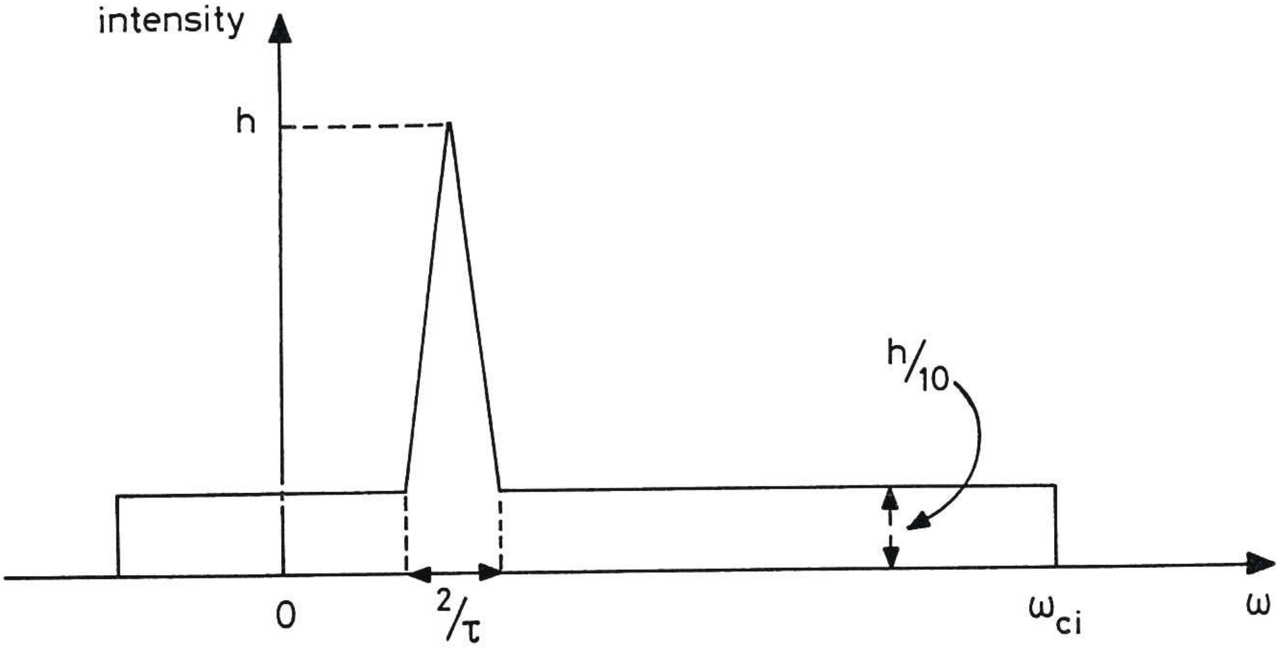

Fig.3 Simple model of power-spectrum for regime II.