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Abstract

The equations of MHD equilibrium are solved for a large class of low-β stellarators with arbitrarily prescribed pressure and current density profiles by means of a large aspect-ratio ordering scheme. The fields and flux surfaces are found to sufficiently high order that the surface shaping and the axis-shift are recovered; the relation between the latter and the applied vertical field is given. Using these equilibria, we evaluate the resistive interchange stability criterion [11]. We apply these results to discussion of several special cases, and note the potential advantages of an $\ell = 2$ configuration.

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1. INTRODUCTION

A great deal of effort is currently being devoted to the study of stellarator-like configurations, with a view to finding systems with steady-state reactor potential [1,2]. The prospects for true steady-state operation have been strengthened by recent experimental success in the production of current-free plasmas [3,4], and it seems possible that the well-known problems of access and structural design associated with conventional stellarator windings can be overcome by use of modular coil systems [5,6]. The potentially serious loss rates associated with ion ripple trapping may be greatly reduced by an ambipolar radial potential, and thus could account for the unexpectedly high beam trapping efficiency observed in WVIIA [3].

Recent theoretical work has included the development of 3D codes to study MHD equilibrium and stability [7,8], the results of which complement those obtained by analytic means. The latter remain valuable in that a relatively simple and general description can be obtained. In an earlier paper [9], we employed an expansion based on an ordering introduced by Dobrott and Frieman [10], in order to obtain the equilibrium configuration for a general class of low-β, $\ell = 3$ stellarators. Making use of the same ordering scheme, we obtain here an analytical description of the equilibrium of low-β stellarators with general winding number ℓ, and investigate their stability to resistive interchanges [11,12] in several special cases of interest.

We discuss briefly the features of an $\ell = 2$ configuration which, when constructed using modular coils, appears to be capable not only of matching the tokamak as the basis for a reactor, but also of offering significant advantages.

In order to solve the equilibrium equations, we use the quasi-cylindrical co-ordinates (r, θ, ξ) of Fig. 1, and expand in powers of the inverse aspect-ratio, ε.

The main (toroidal) field, $\vec{B}_o$, is in the ξ-direction; the stellarator field is introduced in leading order $\varepsilon^{2/3}$.

The number of field periods around the torus is $p = \bar{p}\varepsilon^{-2/3}$, where $\bar{p} \sim O(1)$. These choices ensure that the vacuum rotational transform is of order unity, and that the toroidal and helical modulations of the field strength are of similar magnitude [10].

We make use of scaled variables $\rho = r/a$, (a being some convenient minor radius) and $\bar{s} = p\xi = \bar{p}\varepsilon^{-2/3}\xi$, and we define normalised magnetic field pressure and current density,

$$\vec{b} = \vec{B}/B_o \quad , \qquad \hat{P} = P/B_o^{\,2} \quad , \quad \vec{J} = \vec{j}\,ga/B_o \quad ,$$

where $g = 1 - \varepsilon\rho\cos\theta$. Then with $\hat{\nabla} = ag\nabla$, the M.H.D. equilibrium equations become

$$\hat{\nabla}\hat{P} = \vec{J} \times \vec{b} \quad (i); \qquad \vec{J} = \hat{\nabla} \times \vec{b} \quad (ii); \quad \hat{\nabla}\,.\,\vec{b} = 0 \quad (iii). \qquad (1)$$

The various quantities are now expanded in powers of $\lambda = \varepsilon^{1/3}$;

$$\hat{P} = \lambda^6 P + \ldots\ ; \qquad \vec{J} = \lambda^3\vec{J}^{(3)} + \ldots\ ; \quad \vec{b} = \hat{\xi} + \lambda^2\vec{b}^{(2)} + \ldots$$

where P now denotes the leading order normalised pressure. Thus $\beta \sim O(\varepsilon^2)$, and the plasma current gives rise to rotational transform of order unity, as do the vacuum fields.

It proves instructive to write the plasma current in the form

$$\vec{J} = g\,h\,\vec{b} + \frac{\vec{b} \times \hat{\nabla}\hat{P}}{\vec{b}^2} \qquad (2)$$

so that

$$g\,\vec{b}\cdot\hat{\nabla}h = -\,(\vec{b} \times \hat{\nabla}\hat{P})\cdot\hat{\nabla}\left(\frac{1}{\vec{b}^2}\right) \ . \qquad (3)$$

We divide the force-free current h into parts, representing the

Pfirsch-Schlüter current by h_p, and denoting by h_c the remaining part which contains any mean toroidal current. Then

$$g\vec{b}\cdot\hat{\nabla}h_p = -(\vec{b}\times\hat{\hat{\nabla}}P)\cdot\hat{\nabla}\left(\frac{1}{\vec{b}^2}\right) \tag{4}$$

and

$$g\,\vec{b}\,.\,\hat{\nabla}h_c = 0 \quad . \tag{5}$$

With our ordering, this results in

$$h = \lambda^3 h_c^{(3)} + \lambda^4 h_c^{(4)} + \lambda^5 h_c^{(5)} + \lambda^6 h_c^{(6)} + \lambda^6 h_p^{(6)} + \ldots .$$

2. EQUILIBRIUM CALCULATION

The exact solution of Laplace's equation in toroidal coordinates may be expanded in our quasi-cylindrical coordinates, using the ordering $p = \bar{p}\varepsilon^{-2/3}$; the result, for one harmonic, is

$$V_n^P = B_{p,n}(\varepsilon\rho)^n\left\{\cos\,(n\theta+\bar{s}) + \frac{\bar{p}^2\varepsilon^{2/3}\rho^2}{4(n+1)}\cos\,(n\theta+\bar{s})\right.$$

$$+ \frac{\varepsilon\rho}{4}\left\{(2n+1)\cos\,([n+1]\theta+\bar{s}) + \cos\,([n-1]\theta+\bar{s})\right\}$$

$$+ \frac{\bar{p}^4\varepsilon^{4/3}\rho^4}{32(n+1)(n+2)}\cos\,(n\theta+\bar{s}) + \frac{\bar{p}^2\varepsilon^{5/3}\rho^3}{16(n+1)}\left\{(2n+3)\cos\,([n+1]\theta+\bar{s})\right.$$

$$\left.\left. + 3\cos\,([n-1]\theta+\bar{s})\right\}\right\} + 0\,(\varepsilon^{n+2}) \quad . \tag{6}$$

In order to describe a stellarator with an $n = \ell$ winding which is slightly modulated, for instance in order to create a magnetic well or to 'centre' the separatrix with respect to the windings, we add in small amounts of $\ell = n \pm 1$,

$$V_{n+1}^P = B_{p,n}\left[\frac{\gamma}{4} - \frac{(2n+1)}{4}\right](\varepsilon\rho)^{n+1}\left\{\cos\,([n+1]\theta+\bar{s})\right.$$

$$\left. + \frac{\bar{p}^2\varepsilon^{2/3}\rho^2}{4(n+2)}\cos\,([n+1]\theta+\bar{s})\right\} \tag{7}$$

and

$$V^{p}_{n-1} = B_{p,n}\, \frac{\delta}{4}\, \varepsilon^2\, (\varepsilon\rho)^{n-1} \Big\{ \cos\,([n-1]\theta+\bar{s})$$

$$+ \frac{\bar{p}^2 \varepsilon^{2/3} \rho^2}{4n} \cos\,([n-1]\theta+\bar{s}) \Big\} \quad . \tag{8}$$

We set $B_{p,n} = \varepsilon^{(2/3\,-n)}\, \alpha B_o a$, where α is determined by the currents flowing in the helical windings, and leave γ, δ as free parameters.

Suppose that the coils are wound on the surface $\rho = \rho_c$;

$$\left.\frac{d\phi}{d\theta}\right|_{coil} = \left.\frac{\varepsilon\rho_c j_\phi}{j_\theta}\right|_{\rho=\rho_c} = -\left.\frac{\varepsilon\rho_c b_\theta}{b_\phi}\right|_{\rho=\rho_c}$$

, where $b_{\theta,\phi}$ $(j_{\theta,\phi})$ correspond to the helical winding fields (currents) only.

We wish to describe a winding law $\frac{d\phi}{d\theta} \sim (1 + \varepsilon\Omega\cos\theta)$. From Eqs. (6), (7) and (8) with the above description of α,

$$b_\theta \sim \sin\,(n\theta+\bar{s}) + \frac{\varepsilon\rho_c}{4n}\left\{\gamma(n+1)\sin\,([n+1]\theta+\bar{s}) + \frac{\delta}{\rho_c^{\,2}}(n-1)\sin\,([n-1]\theta+\bar{s})\right\}$$

$$b_\phi \sim \sin\,(n\theta+\bar{s}) + \frac{\varepsilon\rho_c}{4}\left\{\gamma\,\sin\,([n+1]\theta+\bar{s}) + \frac{\delta}{\rho_c^{\,2}}\sin\,([n-1]\theta+\bar{s})\right\}.$$

Hence, in general

$$\frac{d\phi}{d\theta} \sim \left(1 + \frac{\varepsilon\rho_c}{4n\sin\,(n\theta+\bar{s})}\left\{\gamma\sin\,([n+1]\theta+\bar{s}) - \frac{\delta}{\rho_c^{\,2}}\sin\,([n-1]\theta+\bar{s})\right\} + 0(\varepsilon^2)\right),$$

but if $\gamma = -\frac{\delta}{\rho_c^{\,2}}$, then $\frac{d\phi}{d\theta} \sim \left(1 + \frac{\varepsilon\gamma\rho_c}{2n}\cos\theta\right)$;

Thus a winding law of the desired form is represented by choosing $\gamma = \frac{2n\Omega}{\rho_c}$ and $\delta = -\,2n\rho_c\Omega$. Of course, a field with the same properties could also be generated by means of appropriately shaped modular coils.

The equilibrium fields are found by solving (1,i - iii) order by order in λ, up to 10th order, and details of the method are given in [9] for $\ell = 3$. We present here the results of this calculation, and comment briefly on the structure of the fields.

The full set of fields is:

$$b_\rho = \lambda^2 \frac{n\alpha\rho^{n-1}}{\bar{p}} \cos(n\theta+\bar{s}) + \lambda^4\left\{\bar{p}\alpha\rho^{n+1}\frac{(n+2)}{4(n+1)} + U(\rho)\right\}\cos(n\theta+\bar{s})$$

$$+ \lambda^5\left\{\frac{\alpha}{4\bar{p}}\left[\left(\rho^n(n+1) + \delta\rho^{n-2}(n-1)\right)\cos([n-1]\theta+\bar{s}) + \gamma\rho^n(n+1)\cos([n+1]\theta+\bar{s})\right]\right.$$

$$\left. + V(\rho)\sin 2(n\theta+\bar{s})\right\}$$

$$+ \lambda^6\left\{A(\rho)\sin\theta + \left(f(\rho) + \frac{\bar{p}^3\alpha(n+4)\rho^{n+3}}{32(n+1)(n+2)}\right)\cos(n\theta+\bar{s}) + W(\rho)\cos 3(n\theta+\bar{s})\right\}$$

$$b_\theta = -\lambda^2\frac{n\alpha\rho^{n-1}}{\bar{p}}\sin(n\theta+\bar{s}) + \lambda^3 b(\rho) - \lambda^4\left\{\bar{p}\alpha\rho^{n+1}\frac{n}{4(n+1)} + \frac{1}{n}\frac{d}{d\rho}(\rho U(\rho))\right\}\sin(n\theta+\bar{s})$$

$$- \lambda^5\left\{\frac{\alpha}{4\bar{p}}\left[\left(\rho^n + \delta\rho^{n-2}\right)(n-1)\sin([n-1]\theta+\bar{s}) + \gamma\rho^n(n+1)\sin([n+1]\theta+\bar{s})\right]\right.$$

$$\left. - \frac{1}{2n}\frac{d}{d\rho}(\rho V(\rho))\cos 2(n\theta+\bar{s})\right\}$$

$$+ \lambda^6\left\{B(\rho)\cos\theta + \left(g(\rho) - \frac{\bar{p}^3\alpha n\rho^{n+3}}{32(n+1)(n+2)}\right)\sin(n\theta+\bar{s}) - \frac{1}{3n}\frac{d}{d\rho}(\rho W(\rho))\sin 3(n\theta+\bar{s})\right\}$$

$$b_\xi = 1 + \lambda^3(\rho\cos\theta - \alpha\rho^n\sin(n\theta+\bar{s}))$$

$$- \lambda^5\left\{\frac{\alpha\bar{p}^2\rho^{n+2}}{4(n+1)} + \frac{\bar{p}\rho}{n^2}\frac{d}{d\rho}(\rho U(\rho)) - \frac{\alpha\rho^n\sigma(\rho)}{\bar{p}}\right\}\sin(n\theta+\bar{s})$$

$$+ \lambda^6\left\{\rho^2\cos^2\theta + \left[\frac{\bar{p}}{n}\frac{d}{d\rho}(\rho V(\rho)) + \left(\frac{n\alpha\rho^{n-1}}{\bar{p}}\right)^2\frac{\sigma'(\rho)}{2\bar{p}}\right]\frac{\rho}{2n}\cos 2(n\theta+\bar{s}) + b_\beta(\rho)\right.$$

$$\left. - \frac{\alpha}{4}\left((2+\gamma)\rho^{n+1}\sin([n+1]\theta+\bar{s}) + (3\rho^{n+1} + 2\delta\rho^{n-1})\sin([n-1]\theta+\bar{s})\right)\right\}. \quad (9)$$

The poloidal field component $\lambda^3 b(\rho)$ is produced by the mean toroidal current density $\sigma(\rho)$, the remaining functions of radius in Eq. (9) being defined in the appendix.

We solve $\vec{b}.\hat{\nabla}\psi = 0$, as discussed in [9], for the fields (9), and find the expression for ψ :

$$\psi = \psi_0(\rho) - \lambda \frac{n\alpha\rho^{n-1}}{\bar{p}^2} \sin(n\theta+\bar{s})\psi' - \lambda^2\left(\frac{n\alpha\rho^n}{2\bar{p}^2}\right)^2 \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\psi_0'}{\rho}\right) \cos 2(n\theta+\bar{s})$$

$$+ \lambda^3\left\{ \rho\frac{\psi_0'}{b*} \cos\theta\left[A(\rho) + \alpha^2\rho^{2(n-1)} \frac{n}{4\bar{p}^3}\left(5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho^2}\right)\right]\right.$$

$$\left. + \pi_1(\rho)\sin(n\theta+\bar{s}) + \pi_2(\rho)\sin 3(n\theta+\bar{s})\right\} + 0(\lambda^4) \qquad (10)$$

where $\pi_1(\rho)$ and $\pi_2(\rho)$ are given in the appendix, and

$$b* = b(\rho) - \frac{n^2(n-1)\alpha^2}{\bar{p}^3}\rho^{2n-3}.$$

Substituting $\sigma(\rho)$ for $\psi_0(\rho)$ we obtain an expression for h_c ; from (4) we find $h_p^{(6)} = 2\rho\frac{dP}{d\rho}\cos\theta / b*$ and so the expressions for the fields, given in the appendix, may be understood by means of (1ii), (1iii) and (2).

3. EQUILIBRIUM QUANTITIES

The rotational transform is found, from the method of averaging [13], to be

$$i = \frac{b*}{\rho} = \frac{b(\rho)}{\rho} + i_{vac}$$

$$i_{vac} = -\frac{n^2(n-1)\alpha^2}{\bar{p}^3}\rho^{2n-4}. \qquad (11)$$

Having defined $f = \int_0^\rho b^*(\rho)\, d\rho$, we locate the magnetic axis by rewriting (10) as

$$\psi = \psi_o \left(f + \lambda^3 \rho \cos\theta \left\{ A(\rho) + \alpha^2 \rho^{2(n-1)} \frac{n}{4\bar{p}^3} \left(5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho^2} \right) \right\} \right) \tag{12}$$

where we have omitted helical terms, (valid for $n > 1$), and finding the point where $\nabla\psi = 0$.

To the required accuracy, its coordinates are (ρ_x, π) , where

$$b^*(\rho_x) = \lambda^3 \frac{d}{d\rho} (\rho A(\rho)) \Big|_{\rho=\rho_x} + \lambda^3 \frac{(2n-3)}{4n} \delta i_{vac} \quad . \tag{13}$$

The last term is only necessary for $\ell = 2$, where all the surfaces are shifted by this small amount; we neglect it henceforth.

If we also define $\Delta(\rho) = \frac{\rho A(\rho)}{b^*}$, then provided $\Delta(o) \sim 0(1)$, (13) becomes

$$b^*(\rho_x - \lambda^3 \Delta(\rho)) = 0 \quad . \tag{14}$$

The solution is then $\rho_x = \lambda^3 \Delta(o)$, to lowest order. In terms of $\Delta(\rho)$, Eq. (Ax) becomes

$$b^{*2}\Delta'' + b^*(2b^{*\prime} + \frac{b^*}{\rho})\, \Delta' + b^*\Delta \frac{d}{d\rho}\left(\frac{1}{\rho}\frac{d}{d\rho} (\rho b_o)\right)$$

$$= 2\rho P' - b^* b + \alpha^2 \rho^{2(n-1)} \frac{n}{4\bar{p}^3} \sigma' \left(5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho^2} \right) \tag{15}$$

with

$$b_o = - \frac{n^2 (n-1)\alpha^2}{\bar{p}^3} \rho^{2n-3} \quad ,$$

which is the toroidal shift equation [14], in the limit $\alpha = 0$, $\lambda^3\Delta(o)$ being the axis shift. A first integral of Eq. (15) can be obtained when $\ell = 2$, which reduces to the well-known result for tokamaks [14] when $\alpha = 0$.

Use of $\Delta(\rho)$ is not always appropriate; for instance, if $\sigma = 0$ and $A_{(o)} \neq 0$, $\rho_x = \varepsilon^{1/(2n-3)} \left| \dfrac{A_o \bar{p}^3}{n^2(n-1)\alpha^2} \right|^{1/(2n-3)}$

(Note, however, that for $\ell = 2$, $\Delta(\rho)$ is always appropriate).

For "tokamak-like" configurations, where $\Delta(\rho)$ can be used, (for $\sigma \neq 0$, or for $\ell = 2$), we invert (10), to obtain

$$\rho(\psi,\theta,\bar{s}) = \rho_o(\psi) + \lambda \frac{n\alpha\rho_o^{\,n-1}}{\bar{p}^{\,2}} \sin(n\theta+\bar{s}) + \lambda^2 \left(\frac{n\alpha\rho_o^{\,n-1}}{\bar{p}^2}\right)^2 \frac{(2n-1)}{2\rho_o} \sin^2(n\theta+\bar{s})$$

$$+ \lambda^3 \Bigg[- \left[\Delta(\rho_o) + \frac{\alpha^2 \rho_o^{\,2(n-1)} n}{b^*(\rho_o) 4\bar{p}^3} \left(5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho_o^{\,2}}\right)\right] \cos\theta$$

$$+ \left(\frac{U(\rho_o)}{\bar{p}} - \frac{n^2 \alpha \rho_o^{\,n-2}}{\bar{p}^3} b(\rho_o)\right) \sin(n\theta+\bar{s}) + - - \Bigg] \qquad (16)$$

where we have omitted those vacuum terms from order λ^3 which are of order i_{vac} or smaller.

In [9] it is shown how the 'virtual casing principle' is used when applying the boundary condition at infinity. This determines the external vertical field, which is given by

$$B_v = \frac{\varepsilon^2 B_o}{2} \Bigg\{ A(\rho) + \frac{d}{d\rho} (\rho A(\rho)) + b(\rho)\left(\frac{3}{2} + \ell n \left(\frac{\varepsilon\rho}{8}\right)\right)$$

$$- \frac{\rho\sigma(\rho)}{b^*} \left[A(\rho) + \frac{\alpha^2 \rho^{2(n-1)} n}{4\bar{p}^3} \left(5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho^2}\right)\right] \Bigg\}_{\rho=\rho_b} \qquad (17)$$

(ρ_b is the lowest-order radius of a (closed) surface, outside the plasma). When $\alpha = 0$, this also reduces to the tokamak result, [14].

When $\sigma = 0$ Eq.(Ax) gives $-\dfrac{2\bar{p}^3 \rho^{2(3-n)}}{n^2(n-1)\alpha^2} \dfrac{dP}{d\rho} = \dfrac{d}{d\rho}\left(\rho^3 \dfrac{dA}{d\rho}\right)$; using $P \equiv P(f)$, we choose $P = P_o\left(1 - (\rho/\rho_a)^{2(n-1)}\right)$, and find

$A(\rho) = A_o - \frac{P_o}{i_a}\frac{(n-1)}{2}\left(\frac{\rho}{\rho_a}\right)^2$, where ρ_a is the plasma boundary radius, and i_a is the rotational transform at ρ_a .

Finally, (17) becomes

$$B_v = \frac{\varepsilon^2 B_o}{2}\left\{A(\rho) + \frac{d}{d\rho}(\rho A(\rho))\right\}_{\rho=\rho_b} ,$$

so

$$\frac{B_v}{\varepsilon^2 B_o} = A_o - \frac{P_o(n-1)}{i_a} .$$

Specific equilibrium surfaces were computed in [9] for the parameters of CLEO stellarator ($\ell = 3$) using Eq. (10). It was found that although equilibrium could be maintained by the helical fields alone, it is desirable to apply a suitably chosen vertical field in order to reduce the distortion and destruction of flux surfaces induced by the plasma pressure.

Since the rotational transform is constant across the minor cross-section in $\ell = 2$ stellarators, we would expect that the flux surfaces are less sensitive to the effects of finite pressure than is the case for $\ell = 3$ where the transform in the centre is small. For the case of $\ell = 2$, we have calculated equilibria with $\beta \sim 1 - 2\%$ for the parameters of WVII [3]. Typical results for current-free plasmas are depicted in Figs. 2 and 3, where we see that despite the absence of a vertical field the loss of confined plasma volume remains small with increase of pressure. It is interesting to note that in these calculations we observe a spiralling of the magnetic axis about the lowest-order position given by the planar shift $\lambda^3\Delta(0)$. By including the higher order corrections in determining ρ_x , we find $\rho_x \simeq \lambda^3\Delta(0)(1 - \frac{4\alpha\lambda}{\bar{p}^2}\sin\bar{s})$.

Since $\lambda \simeq \frac{1}{2}$, the spiral amplitude is about 50% of the lowest-order shift in agreement with the numerical results.

4. STABILITY

In order to determine whether a plasma equilibrium is stable or not, it is necessary to begin by studying the possible ideal M.H.D. instabilities; if the system proves stable to these, it is possible to proceed to investigate the influence of non-ideal effects, and particularly of resistivity, to see if they allow instability.

In a stellarator without a net longitudinal current, the expected ideal M.H.D. instabilities are localised pressure-driven modes. The Mercier criterion [15], written in terms of surface-averaged quantities, determines the stability of modes which are localised radially but not azimuthally, i.e. interchange-like. Ballooning modes [16], however, are able to localise azimuthally in regions of 'unfavourable curvature', when the plasma pressure is sufficiently high. Since the equilibria under discussion have low $\beta \sim \varepsilon^2$, the pressure-driven modes can exhibit ballooning character only in regions of locally enhanced pressure gradient [16]. In the absence of such steep gradients therefore, the ideal pressure-driven modes are interchanges and so their stability in the limit of large mode number is determined by the Mercier criterion.

Given stability to modes capable of growing on the ideal time-scale, it is then appropriate to test for stability to non-ideal modes.

Criteria for stability against resistive interchanges are given in [11], [12] and [17] although that in [17] results from the other two only when the poloidal "β" is small. In [12] it is shown that the resistive criterion derived is always more stringent than the Mercier criterion, and so it will be adequate for our purposes to evaluate the resistive criterion. Further, in the large aspect-ratio, large toroidal field case, the results given in [11] and [12] can be shown to be equivalent, and for our purposes the more convenient is that given in [11]:

$$W^{(o)} - A_2 > 0 \tag{18}$$

for stability against resistive interchanges.

$$W^{(o)} = \frac{1}{\phi'^2}\left(P'V'' + I'\chi'' - J'\phi''\right) - \frac{1}{\phi'^2}\left(\phi'\chi'' - \chi'\phi''\right)\langle\omega\rangle$$

$$- \left(\frac{2\pi P'}{\phi'}\right)^2 \left\langle \frac{\sqrt{g}}{\vec{B}^2} \right\rangle ,$$

$$A_2 = \left(\frac{2\pi}{\phi'}\right)^2 \left\langle \tilde{\omega}^2 \sqrt{g}\, \frac{\vec{B}^2}{g^{11}} \right\rangle .$$

$\phi(\chi)$ and $I(J)$ are the longitudinal (transverse) magnetic and current fluxes. $\omega = \frac{\vec{J}.\vec{B}}{\vec{B}^2}$. $\langle x\rangle$ is the average of x over a flux-surface, and $\tilde{x} = x - \langle x\rangle$. The g^{ik} are the metric coefficients of the flux-coordinate system employed, in which current and field lines are straight: $x^1 \equi a$, the radial coordinate, and $' \equiv \frac{d}{da}$.

In [17], (replacing averages over closed field-lines by surface averages), we have an expression for V^{**} :

$$V^{**} = V'' + \frac{I'\chi'' - J'\phi''}{P'} - \frac{(\phi'\chi'' - \chi'\phi'')}{P'} \frac{\langle\omega\vec{B}^2\rangle}{\langle\vec{B}^2\rangle}$$

$$- P' \left\langle (\vec{B}^2)^{-1} \right\rangle$$

where $'$ denotes the derivative taken with respect to some flux surface coordinate. For convenience we shall take this to be the toroidal flux ϕ.

When the main toroidal field is large, $\frac{\langle\omega\vec{B}^2\rangle}{\langle\vec{B}^2\rangle} \simeq \langle\omega\rangle$, and allowing for the fact that $\sqrt{g} \approx aR(1 + O(a/R))$ in [11], we see that

$W^{(o)} = \frac{dP}{da} \cdot V^{**} = P' 2\pi a B_\xi V^{**}$. Thus $W^{(o)} - A_2 > 0$ is equivalent to a modified form of the V^{**} criterion:

$$V^{**} - \frac{1}{P' 2\pi a B_\xi} \quad A_2 < 0 \, . \tag{19}$$

We may write this last expression in terms of our dimensionless variables by introducing the following dimensionless equivalents of the quantities above:

If $[x] = \int_0^{2\pi} d\theta \int_0^{2\pi} d\bar{s} \int_0^{\rho(\psi,\,\theta,\,\bar{s})} x\rho(1 - \varepsilon\rho \cos\theta) d\rho$, then $V = [1]$,

$U = [\vec{b}^2]$, $L = [(\vec{b}^2)^{-1}]$ and $\hat{\phi} = \frac{1}{2\pi}[b_\xi / (1 - \varepsilon\rho \cos\theta)]$, $' \equiv \frac{d}{d\hat{\phi}}$.

Henceforth we work in terms of these quantities (and drop the ^ on $\hat{\phi}$).

If we put

$$V^{**} = V'' - \frac{V'}{U'} U'' - P'V' \left(\frac{V'}{U'} + \frac{L'}{V'} \right) ,$$

then taking the leading order forms of $\sqrt{g}$, g^{11} , in A_2 we find the criterion

$$V^{**} - 4\pi \hat{P}' q^2 < 0 \, , \tag{20}$$

with $q = \rho/b^*$.

With the present ordering, V', U' and L' are all $\sim 2\pi + O(\varepsilon^{4/3})$; so $V^{**} = V'' - U'' - 4\pi\hat{P}'$ to the required accuracy, which proves to be a convenient form, as $V - U = 2[1 - b] - [(b-1)^2]$, where $b = |\vec{b}|$.

The first part of this expression contains the destabilising magnetic hill term associated with the geodesic curvature of the stellarator field and the second contains stabilising terms arising from favourable average curvature. However the ordering is non-optimal with respect to the stability criterion, so that whereas the magnetic hill is $O(\varepsilon^{4/3})$, the curvature terms are $O(\varepsilon^2)$.

Higher order terms in general represent only corrections to those already described, although in the interesting special case of a current-free $\ell = 2$ stellarator we shall see that pressure-dependent effects cancel exactly in $O(\varepsilon^2)$ so that finite pressure terms are recovered only at $O(\varepsilon^{8/3})$. In this case we shall assume that $\beta_p = 8\pi P/(\varepsilon B_o i)^2$ is sufficiently small for the contribution of this term to be neglected.

For "tokamak-like" configurations, (16) can be rewritten in terms of coordinates (ρ',θ',ξ) centred on the magnetic axis by replacing $\Delta(\rho_o)$ by $\Delta'(\rho_o) = \Delta(\rho_o) - \Delta(o)$, and writing ρ' for ρ and θ' for θ. We shall work in terms of these coordinates throughout the rest of this section, although dropping the primes on ρ',θ' and Δ' .

Let us consider the terms in $V-U$; the second, $[(b-1)^2]$, is easily evaluated, and to lowest order is

$$\left[\varepsilon^2\left(\rho\cos\theta - \alpha\rho^n\sin(n\theta+\bar{s})\right)^2\right] + O\left(\varepsilon^{7/3}\right)$$

$$= \pi^2\varepsilon^2\left(\frac{\rho_o^{\,4}}{2} + \frac{\alpha^2\rho_o^{\,2(n+1)}}{(n+1)}\right) + O(\varepsilon^{7/3}) \qquad .$$

$[(1-b)]$ is less useful than $[(1-b)]'$, which is also simpler to evaluate, being given by

$$[(1-b)]' = \int_0^{2\pi} d\theta \int_0^{2\pi} d\bar{s}\ \frac{1}{2}\,\frac{d\rho^2}{d\phi}\,(1-\varepsilon\rho\cos\theta)\,(1-b) \ ,$$

where $\rho = \rho(\psi, \theta, \bar{s})$.

To evaluate this latter we need b and $\frac{d\rho^2}{d\phi}$;

$$b = \left(b_\xi^{\,2} + b_\rho^{\,2} + b_\theta^{\,2}\right)^{1/2} = b_\xi + \frac{1}{2}\left(b_\rho^{\,2} + b_\theta^{\,2}\right) + O\left(\varepsilon^{7/3}\right)$$

and so

$$- \left(1-\varepsilon \rho \cos \theta\right) (1-b) = \frac{1}{2} \left(b_\rho^{\,2} + b_\theta^{\,2}\right)$$

$$+ \varepsilon\left(\rho \cos \theta - \alpha \rho^n \sin (n\theta+\bar{s})\right) + \varepsilon^{5/3}\, b_\xi^{(5)} \sin (n\theta+\bar{s})$$

$$+ \varepsilon^2 \left\{b_\beta + - - - - \right\}$$

omitting periodic terms, which, on averaging, will vanish to this order.

$$b_\rho^{\,2} + b_\theta^{\,2} = \varepsilon^{4/3} \frac{n^2\alpha^2\rho_o^{\,2(n-1)}}{\bar{p}^2} \left[1 + \varepsilon^{1/3} \frac{2n\alpha\rho_o^{\,2(n-2)}}{\bar{p}^2} \sin (n\theta+\bar{s}) + 0(\varepsilon^{2/3}\alpha^2)\right] .$$

We find ϕ , from its definition, to be

$$\phi = \pi \left(\rho_o^{\,2} + \varepsilon^{2/3} \frac{n^3\alpha^2}{\bar{p}^4} \rho_o^{\,2(n-1)}\right) + 0\left(\varepsilon^{4/3}\right)$$

and, consequently,

$$\frac{d\rho^2}{d\phi} = \frac{1}{\pi} \Bigg(1 + \varepsilon^{1/3} n^2 \frac{\alpha}{\bar{p}^2} \rho_o^{\,n-2} \sin(n\theta + \bar{s}) - \varepsilon^{2/3} n^3 \frac{(n-1)}{\bar{p}^4} \alpha^2 \rho_o^{\,2(n-2)} \cos 2(n\theta + \bar{s})$$

$$- \frac{\varepsilon \cos \theta}{\rho_o} \frac{\partial}{\partial \rho_o} \left[\frac{\rho_o^{\,2}}{b^*(\rho_o)} \left(A(\rho_o) + \frac{n\alpha^2\rho_o^{\,2(n-1)}}{4\bar{p}^3} \left[5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho_o^{\,2}} \right]\right)\right]\Bigg)$$

$$+ 0 \left(\varepsilon\alpha^2\right) .$$

Finally, integrating the product in the expression for [1-b]′, we obtain our criterion in the form

$$\frac{d}{d\phi} D - \varepsilon^2 4\pi \frac{dP}{d\phi} q^2 < 0 \quad , \tag{21}$$

where

$$D = \varepsilon^{4/3} 2\pi \frac{n^2\alpha^2}{\bar{p}^2} \rho_o^{2(n-1)}$$

$$+ \varepsilon^2 2\pi \left(-\frac{\rho_o^{\,2}}{2} + 2\int_o^{\rho_o} b(\rho)\sigma(\rho)d\rho - b^2(\rho_o) \right.$$

$$\left. + \frac{1}{\rho_o}\frac{d}{d\rho_o}\left[\rho_o^{\,2}\Delta(\rho_o) + \frac{\alpha^2\rho_o^{\,2(n+1)}}{b^*(\rho_o)4\bar{p}^3}\left(5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho_o^2}\right)\right]\right)$$

$$+ \quad - - -$$

In obtaining this result we neglect the contribution in $O(\varepsilon^2)$ of terms proportional to i_{vac}, some of which represent corrections to the leading order magnetic hill, since they are expected to be small.

Thus,

$$\frac{dD}{d\phi} - 4\pi\varepsilon^2 q^2 \frac{dP}{d\phi} = \varepsilon^{4/3} 2n^2 (n-1) \frac{\alpha^2}{\bar{p}^2} \rho_o^{2(n-2)}$$

$$+ \varepsilon^2 \left(-1 + 2\left(\frac{b(\rho_o)}{\rho_o}\right) + \left[\Delta'' + \frac{3\Delta'}{\rho_o} - \frac{2\rho_o P'}{b^{*2}}\right] + \frac{1}{\rho_o}\frac{d}{d\rho_o}\frac{1}{\rho_o}\frac{d}{d\rho_o}\Omega(\rho_o)\right) \quad (22)$$

where

$$\Omega(\rho_o) = \frac{n\alpha^2\rho_o^{\,2n}}{4\bar{p}^{\,2}}\, q\left[5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho_o^{\,2}}\right] ,$$

and

$$q = \frac{\rho_o}{b^*(\rho_o)} .$$

$\left(\Delta'' + \frac{3\Delta'}{\rho_o} - \frac{2\rho_o P'}{b^{*2}}\right)$ can be found from Eq. (15); using $\frac{q'}{q} = \left(\frac{1}{\rho} - \frac{b^{*\prime}}{b^*}\right)$ we have

$$\Delta'' + \frac{3\Delta'}{\rho_o} - \frac{2\rho_o P'}{b^{*2}} = -\frac{b}{b^*} + \frac{\sigma'}{\rho_o^2 b^*}\Omega(\rho_o) + 2\Delta'\frac{q'}{q} - \frac{\Delta}{b^*}\frac{d}{d\rho_o}\frac{1}{\rho_o}\frac{d}{d\rho_o}(\rho_o b_o)$$

and for $\ell = 2$ we are able to integrate this to give

$$\Delta' = \frac{1}{\rho_o b^{*2}} \int_0^{\rho_o} \left[2\rho P' - b^* b + \frac{\sigma'}{\rho^2} b^* \Omega(\rho) \right] \rho d\rho \quad ,$$

as the term in Δ vanishes.

From Eq. (22) we see that for the general $\ell = 2$ stellarator, negative shear $(q'/q > 0)$ is stabilising, as has been shown for tokamaks [11]; however in the absence of an axial current, the shear vanishes for $\ell = 2$, and as noted previously, the pressure dependent terms cancel in $O(\varepsilon^2)$ of the stability criterion (21), which then yields

$$\frac{8\alpha^2}{\bar{p}^2}\varepsilon^{4/3} - (11 + 3\gamma)\ \varepsilon^2 = \varepsilon^2\left(2p\,|i_{vac}| - (11 + 3\gamma)\right) < 0 . \qquad (23)$$

The first term arises from the destabilising "magnetic hill" generated by the stellarator field, while the remaining stabilising term results from the effects of toroidal curvature. Since the flux-surface ellipticity $\eta = \frac{4\alpha\lambda}{\bar{p}^2}$, this result coincides in the limit of low β_p with the ideal M.H.D. criterion (67) given in ref. [18] which was obtained by means of an expansion in radius. The pressure dependent term in $O(\varepsilon^{8/3})$, omitted from (23) is found in ref. [18] to be destabilising and is given by $\varepsilon^2 P'(\rho)\,\frac{i_{vac}\beta_p}{2\pi p}$. Thus, provided $\beta_p \le 1$ this term is indeed negligible in (23). It is interesting to note that the already substantial margin of stability obtained with a pure $\ell = 2$ winding $(\gamma = 0)$ can be enlarged significantly by an appropriately phased poloidal modulation $(\gamma > 0)$ of the ℓ-winding pitch, as discussed in section 2.

In the case of current-free stellarators with $\ell \ge 3$ the axis shift can be large and this prevents us from using the above analysis which assumes $\rho_x \sim O(\varepsilon)$. However, in the special case where $A_o = 0$, corresponding to a particular choice of vertical field, this assumption remains valid and then for $\ell = 3$ stellarators (without coil modulation) we find the criterion

$$2pi_{vac}(\rho) - 6 - \frac{2}{\rho}\frac{P'(\rho)}{i^2_{vac}(\rho)} < 0 \qquad (24)$$

showing that finite pressure is destabilising, especially near the axis. In the limit of vanishing pressure the criterion reduces to $V^{**} < 0$, which has been discussed previously in [10], although we find the stabilising term to be somewhat larger. A comparison of (23) and (24) thus suggests that $\ell = 2$ stellarators without plasma current are more stable to resistive interchanges than their $\ell = 3$ counterparts, but this neglects the well-deepening to be expected with a larger axis shift [19] and to examine this in general it would be necessary to evaluate criterion (20) for equilibria with $A_o \neq 0$. A numerical approach to this question should be possible, but lies outside the scope of the present investigation.

5. DISCUSSION

Although numerous variants of the stellarator concept have been proposed, the engineering problems of designing, constructing and maintaining a device of reactor size impose strict constraints which virtually exclude the more complex systems. As is the case with tokamak reactor designs, cost and output power considerations lead to low aspect ratio, and the many problems associated with conventional ℓ-windings have led to several proposals for the use of coils with a modular construction [5,6]. The solution proposed by Wobig and

Rehker [5] to this problem is particularly attractive in that a single set of coils, which could be individually demountable for maintenance, would provide both the toroidal and helical equilibrium fields, and leave adequate access for neutral injection or R.F. heating, and so on.

The choice of ℓ-number in the stellarator field is effectively limited to $\ell = 2$ or $\ell = 3$ by the difficulty of achieving satisfactory rotational transform in a low aspect ratio device by means of coils which necessarily must be separated from the vacuum vessel by a substantial thickness of blanket material. Since high rotational transform in the centre of the plasma as well as at the edge is desirable to ensure good confinement of energetic ions, $\ell = 2$ would appear to be preferable, and as the equilibrium calculations in section 3 have shown, the $\ell = 2$ flux surfaces are not sensitive to finite plasma pressure, obviating the need for a vertical field system, which would be desirable for an $\ell = 3$ stellarator [9], and essential in a tokamak. Although the detrimental effect of the stellarator "magnetic hill" on interchange stability increases with rotational transform, and tends therefore to be stronger in the centre of an $\ell = 2$ stellarator than in the centre of an $\ell = 3$ device, we have seen that there is nonetheless a substantial margin of stability due to toroidal effects. From the criterion (23) for $\ell = 2$ current-free stellarators we see that if $i_{vac} \sim 0.5$ which is likely to be the situation encountered in practice, then provided the number of winding periods p is not too large, say $p \lesssim 8$, then a sufficient well can be formed and stability against resistive interchanges is possible over a useful range of poloidal beta values. In addition, it has been shown that a modular $\ell = 2$ system exhibits substantial negative shear [5], even in the absence of a plasma current, which may be beneficial for interchange stability.

6. CONCLUSION

We have studied a general class of low-beta toroidal stellarator equilibria in which plasma pressure and current density profiles may be prescribed arbitrarily as functions of the mean flux surface radius. Between the general stellarator equilibrium and the low-beta tokamak which forms a special case, there is a strong similarity made evident by our calculation. By evaluating the resistive interchange criterion it is seen that notwithstanding the destabilising effect of the magnetic hill a substantial margin of stability is available in $\ell = 2$ stellarators, which if constructed using a modular coil design would appear to be an attractive alternative to the tokamak, especially in view of the possibility of steady-state operation.

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APPENDIX

In stating the expressions for the fields, (7), we have employed the following definitions:

$h_c^{(3)} \equiv h_c^{(3)}(\rho) = \sigma(\rho)$ is the toroidal current, and the associated poloidal field is $b(\rho)$, given by

$$\frac{1}{\rho}\frac{d}{d\rho}(\rho b) = \sigma(\rho) \qquad \text{(Ai)}$$

$$-\frac{1}{n\rho}\frac{d}{d\rho}\left(\rho\frac{d}{d\rho}(\rho U)\right) + \frac{nU}{\rho} = -\frac{n\alpha\rho^{n-1}}{\bar{p}^2}\sigma' \qquad \text{(Aii)}$$

$$\frac{1}{2n}\frac{d}{d\rho}\left(\rho\frac{d}{d\rho}(\rho V)\right) - 2nV = -\left(\frac{n\alpha\rho^{n}}{2\bar{p}^2}\right)^2\frac{d}{d\rho}\left(\frac{\sigma'}{\rho}\right) \qquad \text{(Aiii)}$$

$$\frac{db_\beta}{d\rho} = -\left(\frac{dP}{d\rho} + b(\rho)\sigma(\rho) + \left\{\left(\frac{n\alpha\rho^{n-1}}{\bar{p}^2}\right)^2\frac{\bar{p}\sigma'}{2}\right\}\right) . \qquad \text{(Aiv)}$$

If we write

$$\sigma_1(\rho) = \left\{-\frac{\alpha\rho^{n+1}}{4}\frac{(n+2)}{(n+1)} + \frac{n\alpha\rho^{n-2}}{\bar{p}^3}b(\rho) - \frac{U(\rho)}{\bar{p}}\right\}\sigma' - \alpha\rho^n\sigma$$

$$+ \left(\frac{n\alpha\rho^{n-1}}{2\bar{p}^2}\right)^3\left[4n\frac{d}{d\rho}\left(\frac{\sigma'}{\rho}\right) + \rho^2\frac{d}{d\rho}\left(\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\sigma'}{\rho}\right)\right)\right] \qquad \text{(Av)}$$

and

$$\sigma_2(\rho) = \frac{1}{3}\left(\frac{n\alpha\rho^{n}}{2\bar{p}^2}\right)^3\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\sigma'}{\rho}\right)\right) \qquad \text{(Avi)}$$

then

$$\frac{1}{\rho}\frac{d}{d\rho}(\rho g) + \frac{nf}{\rho} = \sigma_1(\rho) \qquad \text{(Avii)}$$

and

$$\frac{1}{\rho}\frac{d}{d\rho}(\rho f) + \frac{ng}{\rho} = -\left\{\alpha\rho^n\sigma(\rho) - \frac{\bar{p}^2}{n^2}\rho\frac{d}{d\rho}(\rho U)\right\} \qquad \text{(Aviii)}$$

$$- \frac{1}{3n\rho} \frac{d}{d\rho} \left(\rho \frac{d}{d\rho} (\rho W) \right) + \frac{3n}{\rho} W = \sigma_2 (\rho) \quad . \qquad \text{(Aix)}$$

Finally,

$$- b^*(\rho A'' + 3A') + \rho A\sigma' + 2\rho P' - b^*b$$

$$+ \frac{n\alpha^2 \rho^{2n-1}}{4\bar{p}^3} \sigma' \left[5n + (n+1)\gamma + \frac{(n-1)\delta}{\rho^2} \right] = 0 \qquad \text{(Ax)}$$

and

$$\frac{1}{\rho} \frac{d}{d\rho} (\rho A) - \frac{B(\rho)}{\rho} = - b(\rho) \quad . \qquad \text{(Axi)}$$

The surface function ψ involves two functions as yet unspecified:

$$\pi_1 (\rho) = \left\{ - \frac{\alpha\rho^{n+1}}{4} \frac{(n+2)}{(n+1)} + \frac{n\alpha\rho^{n-2}}{\bar{p}^3} b(\rho) - \frac{u(\rho)}{\bar{p}} \right\} \psi_o'$$

$$+ \left(\frac{n\alpha\rho^{n-1}}{2\bar{p}^2} \right)^3 \left[4n \frac{d}{d\rho} \left(\frac{\psi_o'}{\rho} \right) + \rho^2 \frac{d}{d\rho} \left(\frac{1}{\rho} \frac{d}{d\rho} \left(\frac{\psi_o'}{\rho} \right) \right) \right] \Bigg\} \qquad \text{(Axii)}$$

and

$$\pi_2 (\rho) = \frac{1}{3} \left(\frac{n\alpha\rho^n}{2\bar{p}^2} \right)^3 \frac{1}{\rho} \frac{d}{d\rho} \left(\frac{1}{\rho} \frac{d}{d\rho} \left(\frac{\psi_o'}{\rho} \right) \right) .$$

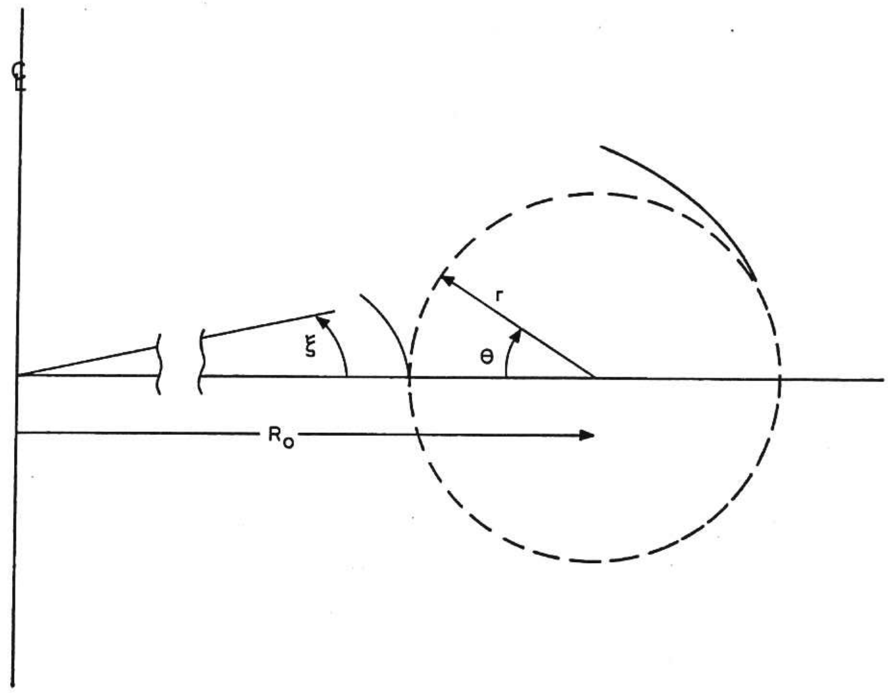

Fig.1 Coordinate geometry.

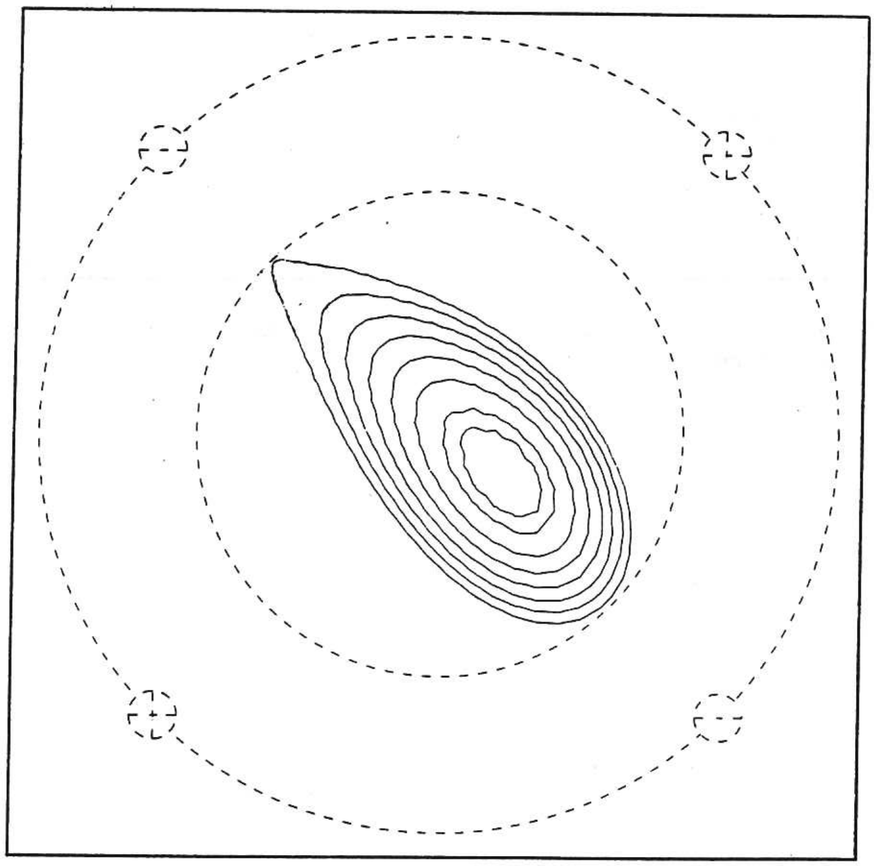

Fig.2 WVII flux surfaces for $i = 0{\cdot}5$, $\beta \sim 1\%$, $B_v = 0$. The torus axis is to the left of the figure, the location of the stellarator windings in this cross-section and their polarity (+ into the paper) being indicated on the outer circle (dashed). The limiter radius is given by the inner dashed circle.

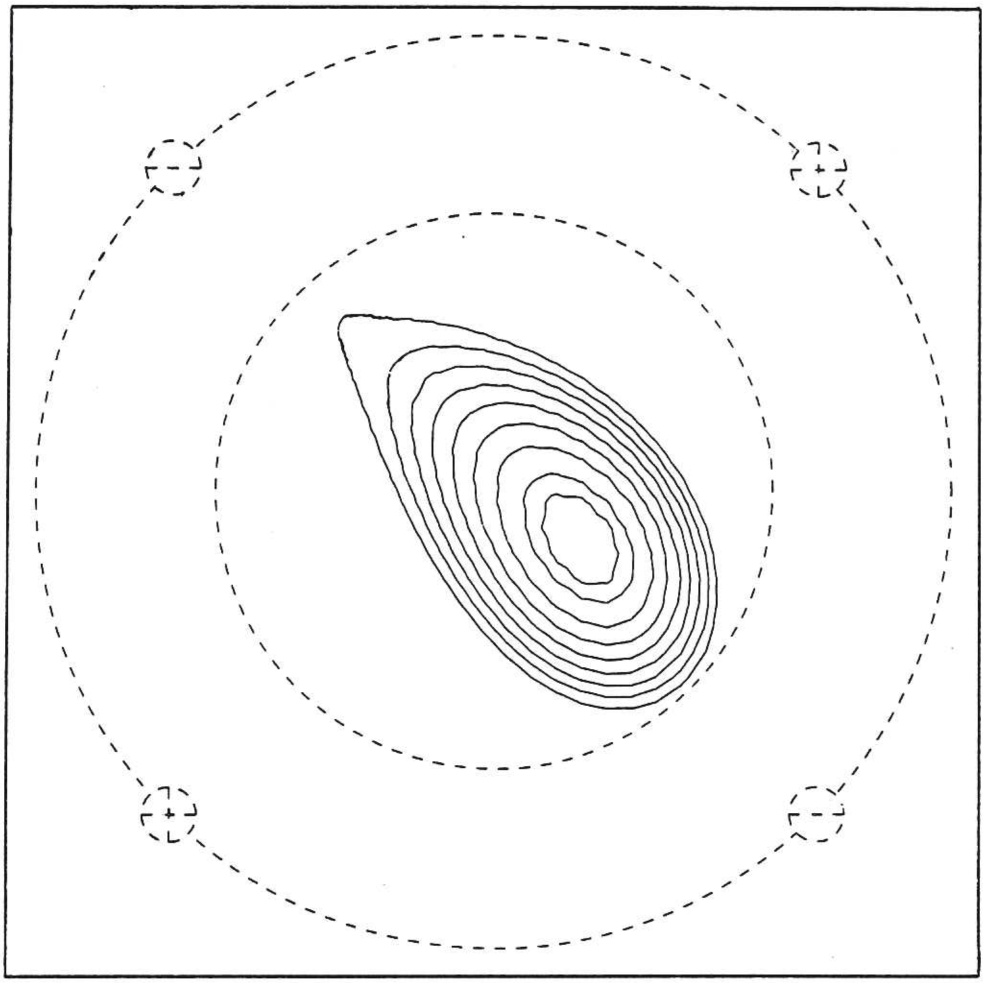

Fig.3 WVII flux surfaces for i = 0·5, $\beta \sim 2\%$, $B_V = 0$. Although β is twice as large as in the case of Fig.2, the flux surface displacements are only slightly greater.