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T. C. HENDER
D. C. ROBINSON

CULHAM LABORATORY
Abingdon Oxfordshire

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T C Hender* and D C Robinson
Culham Laboratory, Abingdon, Oxon, OX14 3DB, UK
(Euratom/UKAEA Fusion Association)

ABSTRACT

The non-linear development of the m=0 resistive 'g' mode has been studied in the reverse field pinch. A saturation process is shown to exist in which the perturbations cause a quasi-linear flattening of the equilibrium pressure near the m=0 resonant surface. The mechanism for this flattening is discussed.

*Royal Holloway College, University of London

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ABSTRACT

The non-linear development of the m=0 resistive 'g' mode has been studied in the reverse field pinch. A saturation process is shown to exist in which the perturbations cause a quasi-linear flattening of the equilibrium pressure near the m=0 resonant surface. The mechanism for this flattening is discussed.

1. INTRODUCTION

The reverse field pinch[1] and Spheromak[2] have adverse curvature and as a consequence are unstable to the resistive interchange or 'g' mode. It is therefore particularly important to investigate the non-linear consequences of this mode. A numerical code has been developed to time advance a reduced set of MHD equations.

It is known from previous work that while 'g' modes with azimuthal mode number $m \geqslant 1$ may be stabilised by finite Larmor radius effects, the relatively long wavelength m=0 mode is unlikely to be stabilised[3]. Further we know from the work of Schnack[4] that the m=0 mode may continue to grow far into the non-linear regime. This provides our motivation for choosing to study the non-linear development of the m=0 'g' mode. This mode occurs about the point at which the longitudinal field reverses and is driven by the pressure gradient. To simplify the problem further we have assumed the plasma to be incompressible. This has the numerical advantage of precluding magneto-sonic waves which can lead to severe time-step restrictions.

2. EQUATIONS

We choose to study the 'g' mode in periodic cylindrical geometry. This means that the 'g' mode is driven 'correctly' by the interaction of pressure and curvature rather than by a fictitious 'g' term[5] which is needed in slab geometry. The incompressibility condition then allows us to define magnetic and velocity stream functions. Using these the resistive MHD equations can be reduced for the m=0 mode, to the two-dimensional form:

$$\frac{\partial\psi}{\partial t} + V_r \frac{\partial\psi}{\partial r} + V_z \frac{\partial\psi}{\partial z} = \eta\left(r\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial\psi}{\partial r}\right\} + \frac{\partial^2\psi}{\partial z^2}\right) \tag{1}$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial}{\partial z}\left(\frac{V_\theta}{r}\frac{\partial\psi}{\partial r} - V_z B_\theta\right) - \frac{\partial}{\partial r}\left(V_r B_\theta + \frac{V_\theta}{r}\frac{\partial\psi}{\partial z}\right) + \eta(\nabla^2\vec{B})_\theta \tag{2}$$

$$\rho\left(\frac{\partial V_\theta}{\partial t} + (\vec{V}.\nabla\vec{V})_\theta\right) = \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial B_\theta}{\partial z} - \frac{1}{r^2}\frac{\partial rB_\theta}{\partial r}\frac{\partial\psi}{\partial z} \tag{3}$$

where $V_r = -\frac{1}{r}\frac{\partial\phi}{\partial z}$, $V_z = \frac{1}{r}\frac{\partial\phi}{\partial r}$ and $B_r = -\frac{1}{r}\frac{\partial\psi}{\partial z}$, $B_z = \frac{1}{r}\frac{\partial\psi}{\partial r}$, η the resistivity, V the fluid velocity, B the magnetic field and J the current.

Defining the vorticity $\omega = (\nabla \times \vec{V})_\theta$

$$\rho\left(\frac{\partial\omega}{\partial t} + \frac{\partial}{\partial r}(V_r\omega) + \frac{\partial}{\partial z}(V_z\omega)\right) = \frac{\partial}{\partial r}(B_r J_\theta) + \frac{\partial}{\partial z}(B_z J_\theta) + \frac{1}{r}\left(\rho\frac{\partial V_\theta^2}{\partial z} - \frac{\partial B_\theta^2}{\partial z}\right) \tag{4}$$

and the system is closed by the relation

$$\omega = -\frac{1}{r}\frac{\partial^2\phi}{\partial z^2} - \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial r}\right) . \tag{5}$$

These equations are numerically advanced in time by a mixed ADI/explicit algorithm similar to that used by Waddell et al[6].

3. EQUILIBRIUM

The investigations have been conducted using the Tearing Mode Stable (TMS) equilibria[7]. This is an analytic reverse field pinch model for which the pitch, $P(r)$ and the Suydam value, C, are specified ($C = -\frac{dp}{dr} \Big/ rB_z^{\,2}\left(\frac{dP}{dr}/P\right)^2 \leqslant \frac{1}{8}$ is the Suydam stability criterion, where p is the plasma pressure). As a further refinement a vacuum region is included in which B_z = constant, and $B_\theta \propto 1/r$. To avoid discontinuities at the vacuum-plasma interface a matching zone is necessitated. In this region variables are matched smoothly to each other, in particular C is taken gradually to zero with zero gradients at both ends of the matching zone. In most of our runs we have taken the pitch in the core of the plasma as $P(r) = 2\left(1 - \frac{r^2}{8} - \frac{r^4}{400}\right)$, where the radius is defined such that the longitudinal field and current on axis are unity.

The tearing mode stability properties of the TMS model have been exhaustively studied by Robinson[7]. It is important for our purposes that the equilibrium be tearing mode stable, in order to isolate the non-linear 'g' mode behaviour. However the m=0 tearing mode is very weak for this configuration, even if wall stabilisation is not effective, and the stability should therefore be dominated by the 'g' mode behaviour. This property is fortunate, since in testing for tearing mode stability at finite beta we are left in the dilemma that the Δ' criterion[8] does not apply whenever the 'g' mode is dominant, in such a high β configuration. In an attempt to satisfy ourselves that our finite beta equilibria are tearing mode stable we have adopted two methods. Both involve reducing the pressure to zero while maintaining some property of the fields. In the first method we maintain the pitch constant, and in the seond we maintain the quantity $\sigma = \vec{J}.\vec{B}/|\vec{B}|^2$ constant. The second is probably preferable as it is the gradient of σ which drives the tearing mode. The reduced pressure configurations are tested for stability using a precise value of Δ' obtained from the code RCWALL, which is based upon the method developed by Robinson[7].

4. RESULTS AND DISCUSSION

The code is run by priming with linear eigenmodes from the code RIP4A[9]. It was found in all cases attempted, that if the non-linear code was primed with sufficiently small eigenmodes to keep it in the linear regime then it produces growth rates to within 2% of those of RIP4A. A typical magnetic island structure and velocity flow pattern, with which the non-linear run is started, is shown in Fig.1.

The growth rate defined as $\frac{1}{\psi}\frac{\partial\psi}{\partial t}$ at the singular surface is shown for 3 full non-linear runs in Fig.2. They are for S=1000 and 4000, with K_z=0.4, C=0.05 and for S=1000, C=0.05 and K_z=1.0 (where S is the magnetic Reynolds number). All three curves exhibit similar behaviour: a marked decrease in growth followed by an increase. The decrease in growth is caused by quasi-linear flattening of the zeroth-harmonic of pressure, in the region of the singular surface. This is shown for the C=0.05, S=1000, K_z=1.0 case in Fig.3. The increasing growth rate phase corresponds to resistive diffusion of the magnetic fields, which increase the average beta and drive the 'g' mode more unstable again.

The differences between the growth rate curves allows us to deduce some important properties of these modes. For the S=4000, K_z=0.4 case it is seen that the growth rate increases much more slowly after turning, than the corresponding S=1000 case. This is because the increase in pressure due to the resistive evolution of the equilibrium is slower for the S=4000 case. The different times to the turning points for the K_z=0.4 and K_z=1.0, S=1000, cases follow from a linear stability analysis of the equilibrium: it is found that growth rate of the K_z=1.0 mode increases much faster with pressure than does the K_z=0.4 case. Thus the decrease in pressure gradient, at the singular surface, is more quickly counteracted by the increase in pressure in the K_z=1.0 case.

Examination of the magnitudes of the perturbations in the non-linear runs shows that we are still in a quasi-equilibrium state. Thus

$$\nabla p_0 \simeq \vec{J}_0 \times \vec{B}_0 \tag{6}$$

where subscripts zero indicate zeroth harmonics. Now near the m=0 surface, B_{z0} must by definition be small. Hence the flattening of the pressure near the singular surface can almost totally be accounted for by quasi-linear modifications to the J_{z0}. This can easily be demonstrated to be correct: by simply altering the J_{z0} to that at a given timestep while maintaining $J_{\theta 0}$, $B_{\theta 0}$ and B_{z0} at their original values we obtain an almost exact fit to the pressure gradient in the region of singular surface at that timestep.

Using this knowledge we can obtain an order of magnitude estimate to the saturation width. We have at the singular surface, r_s

$$\eta J_{z0} = E_{z0} - (V_\theta B_r)_0 \tag{7}$$

since $V_r|_{r_s} = 0$. It is the $(V_\theta B_r)_0$ term which causes the flattening in pressure profile at r_s since if it were negligible the equilibrium would just diffuse resistively in the normal manner. Hence for a saturation process to occur we must have

$$\eta J_{z0}|_{orig} \sim (V_\theta B_r)_0 \tag{8}$$

where the subscript orig. implies the initial equilibrium value. Now at r_s from linear stability theory

$$\rho\omega V_{\theta 1} = J_{z0}|_{orig} B_{r1} \tag{9}$$

where ω is the linear growth rate and the subscript 1 indicates a linear perturbation term. Equations (8) and (9) then give

$$(V_\theta B_r)_0 = \frac{1}{2} V_{\theta 1} B_{r1} = \frac{1}{2} J_{z0}|_{orig} \frac{B_{r1}^2}{\rho\omega} \sim \eta J_{z0}|_{orig}$$

hence

$$\rho\eta\omega \sim \frac{B_{r1}^2}{2} \tag{10}$$

but from Furth, Killeen and Rosenbluth[5] the resistive layer thickness, ℓ, is given by

$$\ell = \left(\frac{\rho\eta\omega}{K_z^2 (B_{z0}')^2}\right)^{1/4} \quad . \tag{11}$$

Using (10) this then gives

$$\ell \sim \frac{1}{2^{\frac{1}{4}}}\left(\frac{B_{r1}}{K_z (B_{z0}')^2}\right)^{\frac{1}{2}} \sim \text{island width.} \tag{12}$$

Hence we can expect appreciable saturation by pressure flattening when the island width is comparable to the resistive layer thickness. This is indeed found to be the case in our non-linear calculations. A somewhat similar situation prevails when considering the non-linear behaviour of the tearing mode[10]. Our calculations show that the resistive layer thickness decreases as $S^{-0.22}$ for values of S up to 5 x 10^4 and the radial field perturbation decreases as $S^{-0.39}$. This is to be compared with the predictions from eq(11) for a pure resistive 'g' mode and eq(12) that the field perturbation decreases as $S^{-0.67}$.

We have also modified the equilibrium profiles in the linear code, RIP4A, to match the pressure profiles at various times of the non-linear evolution. The linear code then gives growth rates to within 5% of the corresponding non-linear calculation. Hence we conclude that we are dealing with a quasi-linear phenomenon in which the dominant effect is that of perturbations on their own equilibrium. The quasi-linear modification to J_{z0} is shown in Fig.4.

5. CONCLUSIONS

The decrease in growth rate has been shown to be due to the quasi-linear modification to the axial current, J_{z0}. The effect of this on the growth of the linear perturbations can be seen with reference to eq(9). Setting $J_{z0}|_{r_s}$ equal to zero (which is equivalent to flattening the pressure at r_s) essentially stops the growth of the azimuthal velocity perturbations, V_θ, at r_s. This is found to be the case on detailed examination of the results from the non-linear calculations. The coupling between the radial field perturbations and the azimuthal velocity has been broken and so the exponential linear growth will cease.

The increase in growth rates caused by resistive diffusion of the equilibrium reflects an unfortunate facet of our reduced MHD formalism. By using an incompressible formalism we lose independent control of the plasma energy content, which could have been used to control the diffusive rise in pressure. The inclusion of more physical effects such as parallel thermal conduction, to control this pressure rise, would allow the saturation process to continue. This view is confirmed by the difference between the S=1000 and 4000 mode evolution.

In redistributing the pressure to achieve the flattening at the singular surface, the m=0 mode increases the pressure gradient markedly in the core of the plasma and to some extent in the outer regions of the plasma, Fig.3. This effect is further enhanced by resistive diffusion. We have examined the linear effect of this on the m=1 modes in the core and find that their growth rate increases considerably. This could be an important effect, but we have not yet examined the non-linear effects of the m=1 and m=0 mode interaction, which would require a three-dimensional calculation.

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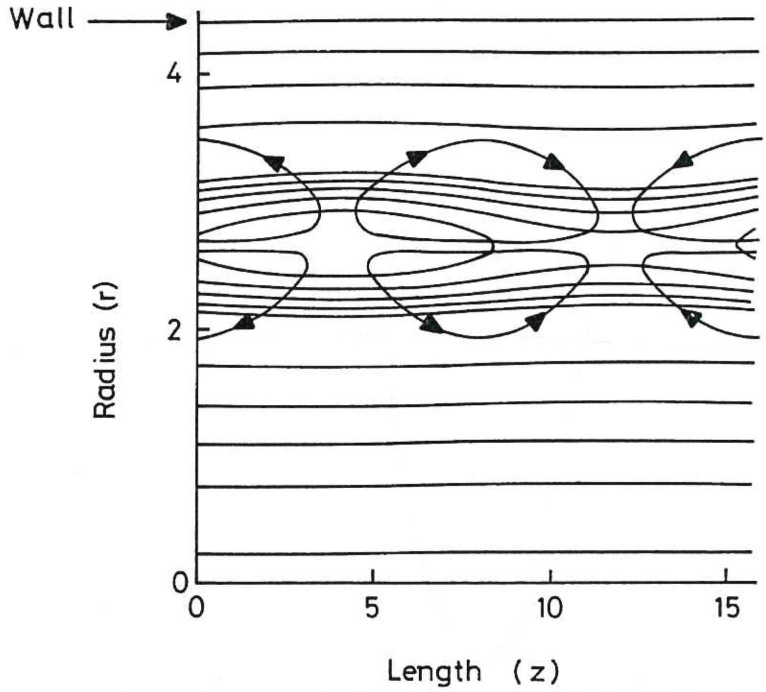

Fig.1 Magnetic fields and velocity flows (arrowed lines) with which non-linear run is primed.

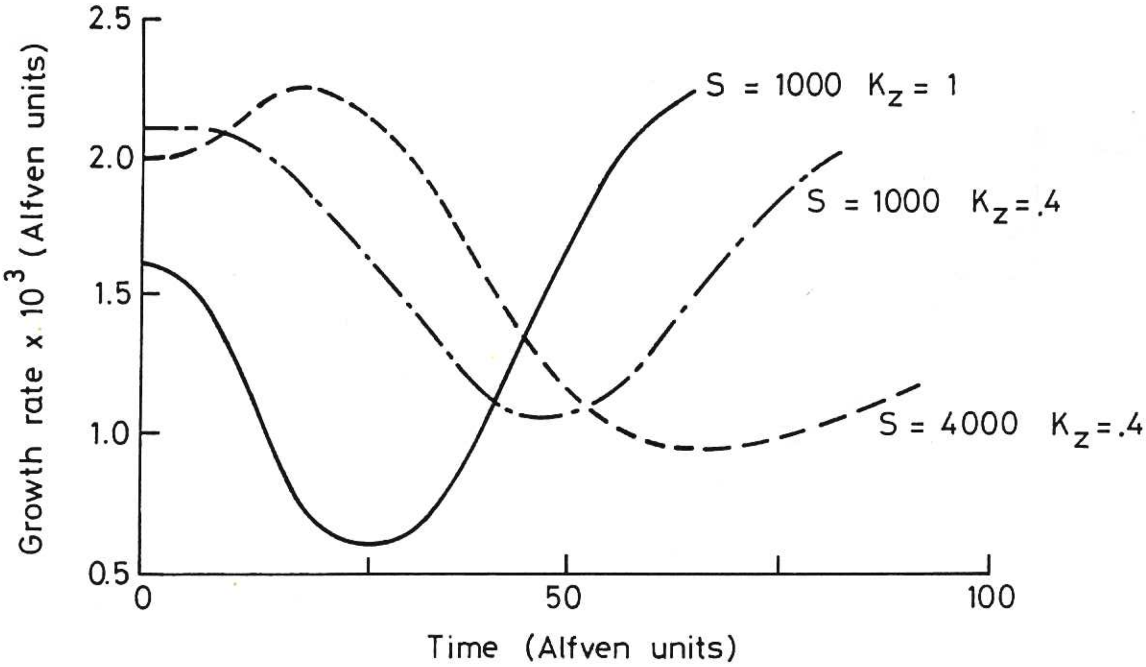

Fig.2 Growth rate curves for reconnected flux at the singular surface. All 3 curves are for TMS equilibrium with a Suydam value, C = 0·05.

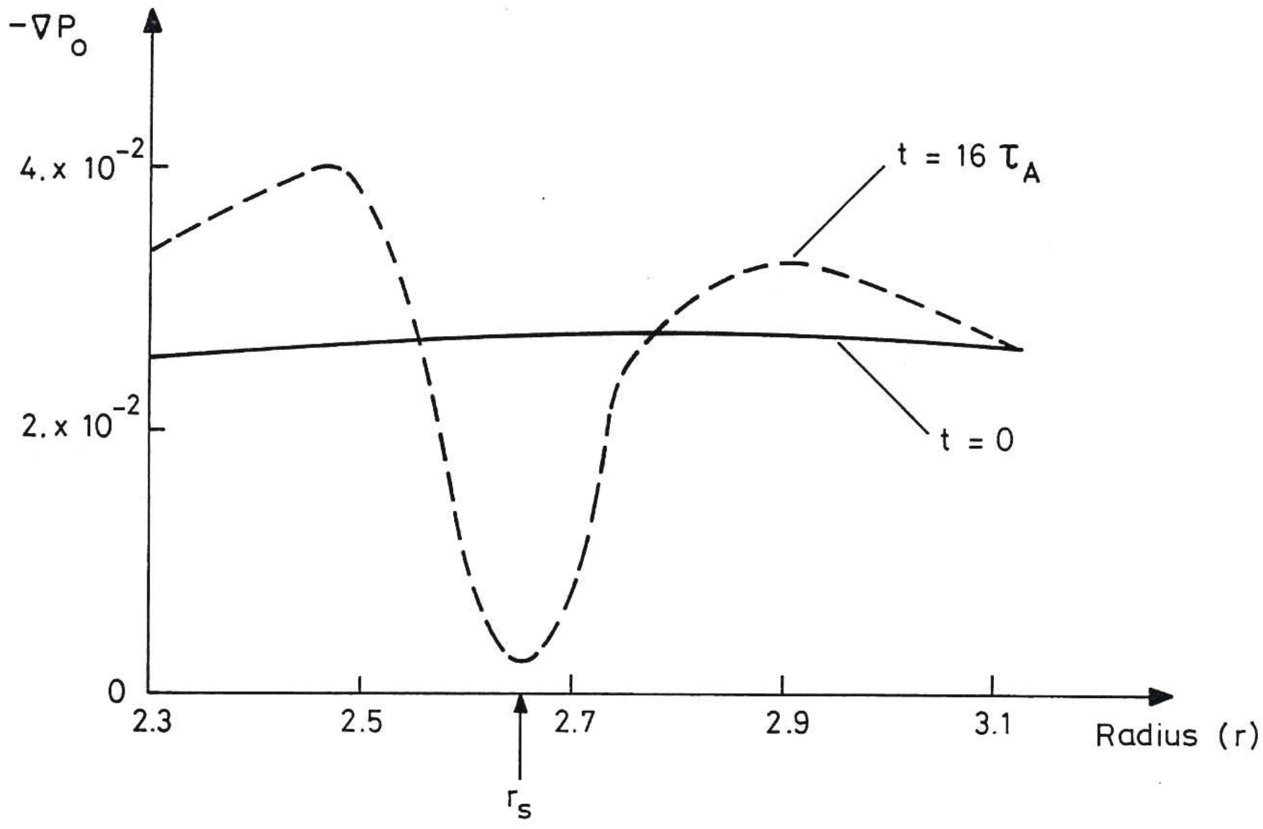

Fig.3 Non-linear development of zeroth harmonic of pressure gradient for S = 1000, C = 0·05 and K_Z = 1·0.

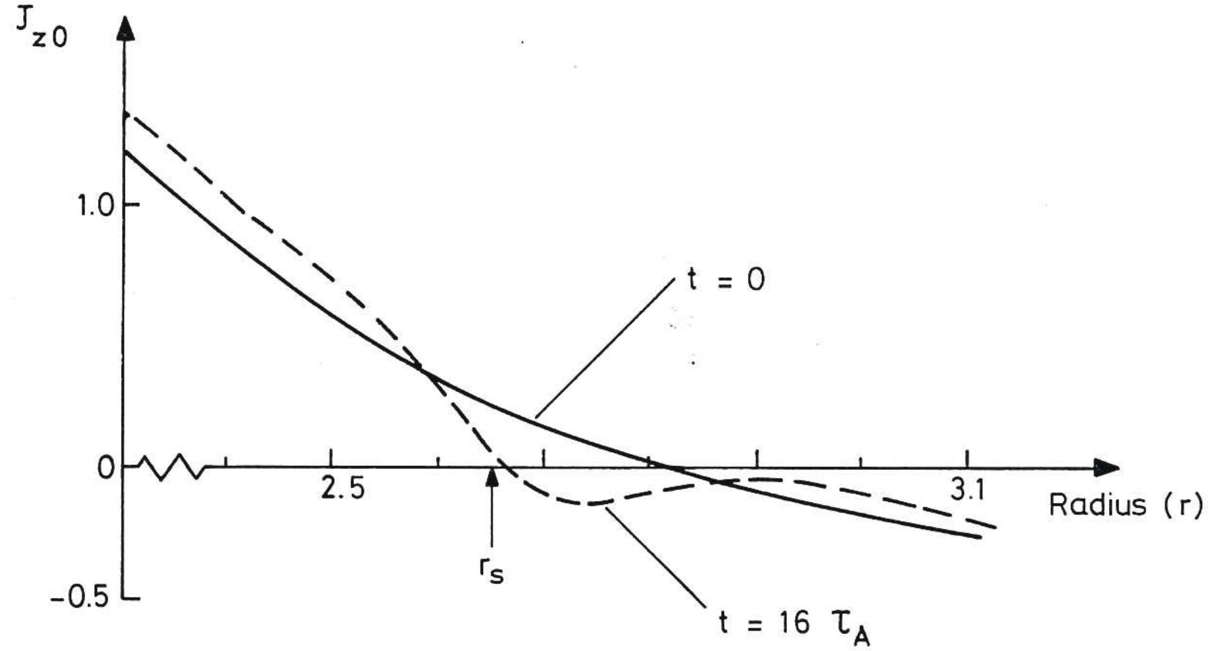

Fig.4 Non-linear development of zeroth harmonic of axial current for S = 1000, C = 0·05 and K_Z = 0·4.