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COLLISIONLESS NON-LINEAR COUPLING OF ELECTROMAGNETIC WAVES NEAR THE ELECTRON GYRO-FREQUENCY BY PLASMA ELECTRONS

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by

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A B S T R A C T

A collisionless mechanism is suggested which can lead to the mixing of two electromagnetic waves in a magneto-plasma at frequencies near the electron gyrofrequency. Some preliminary experiments are described which support the predictions of the theory, namely that the difference frequency wave amplitude $\propto$ (difference frequency)$^{-2}$ when collisions are rare, but is independent of difference frequency when collisions occur frequently during a beat period.

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1. INTRODUCTION

The non-linear interaction of electromagnetic waves in a plasma was first discussed by Bailey and Martyn (1934) and has been more recently reviewed in detail by Ginzburg (1961) and, particularly for the case near gyro-resonance, by Bailey (1956). Numerous observations have been made in the ionosphere and in laboratory plasmas in which the effect of the non-linear (complex) conductivity of the plasma electrons has led to the generation of, for example, sum, difference and harmonic frequency waves. In general the observed non-linear properties of the plasma arise out of the variation of electron-atom collision frequency with electron mean energy, which itself depends on the instantaneous applied electric field strength. Although non-linear effects are predicted for sufficiently large wave amplitudes even in the absence of collisions (Ginzburg 1961, W.B. Thompson 1963), no detailed experimental verification is available.

In a magnetized plasma carrying waves of frequency ω_1, ω_2 near to the electron gyro-frequency resonance, i.e. $\omega_1 \approx \omega_2 \approx \ldots \omega_c$, the electron velocity $v_\perp$ transverse to the magnetic field $\underline{B}$ is a linear function of the combined wave amplitude $\underline{E} = \underline{E}_1 + \underline{E}_2 + \ldots$. The magnetic dipole moments of the orbiting electrons depend on $v_\perp^2/B$, and thus on E^2. The dipole moments therefore contain alternating components at the combination frequencies of ω_1, $\omega_2 \ldots$, and so waves will appear in the plasma, e.g. at $\omega_1 \pm \omega_2$. The effect may be regarded as if caused by an amplitude dependent permeability, instead of the more usual amplitude dependent permittivity, and does not rely on collisions. It will obviously be greatest under conditions in which the diamagnetic effect of the plasma electrons can become large, i.e. when the electrons have large transverse kinetic energy. A simple non-relativistic example is treated in the following section, in which two equal amplitude waves near gyro-frequency are shown to generate a time dependent at their difference frequency.

2. SIMPLE THEORY OF DIFFERENCE FREQUENCY GENERATION

A detailed discussion of electron motion in r.f. fields with $\underline{E}$ perpendicular to $\underline{B}$ has been given by Bailey and Martyn (1934), taking into account the effect of collisions on the instantaneous mean electron energy, by Townsend and Gill (1938) and by Linhart (1960).

Here we consider the motion of plasma electrons, assumed initially at rest, when two alternating fields $\underline{E}_1 \cos \omega_1 t$ and $\underline{E}_2 \cos \omega_2 t$ are applied, both $\underline{E}_1$ and $\underline{E}_2$ being perpendicular to a uniform applied magnetic induction $\underline{B}$, and $\omega_1 \approx \omega_2 \approx \omega_c = eB/m$, $\omega_1 - \omega_2 \ll \omega_c$.

For simplicity we take $E_1 = E_2 = E$, $\omega_1 - \omega_c = \omega_c - \omega_2 = \frac{\Omega}{2}$. Then the electrons are acted upon by an accelerating field.

$$\underline{E}\ (\cos \omega_1 t + \cos \omega_2 t) = \underline{E} \cos \tfrac{1}{2}\, \Omega t \,.\, \sin \omega_c t\ ,$$

i.e. by a resonant field which alternately accelerates and decelerates, so that the Larmor radii will oscillate between zero and some maximum value.

If we take coordinates Ox parallel to $\underline{E}$, Oz parallel to $\underline{B}$, and assume that collisions do not occur during a beat period $2\pi/\Omega$, i.e. the electron collision frequency $\nu \ll \Omega$, the equations of motion become:-

$$\frac{d^2x}{dt^2} = \frac{eE}{m}\ (\cos \omega_1 t + \cos \omega_2 t) + \omega_c \frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} = -\omega_c \frac{dx}{dt} \qquad \ldots (1)$$

$$\frac{d^2z}{dt^2} = 0$$

where e, and m are the charge and mass of an electron. Solving these for the velocity, using the condition that $\omega_1 \approx \omega_2 \approx \omega_c$:

$$v_x = \frac{dx}{dt} = \frac{eE}{m\Omega} \cdot \frac{1}{\omega_c}\ (\omega_1 \sin \omega_1 t - \omega_2 \sin \omega_2 t\ )$$

$$v_y = \frac{dy}{dt} = \frac{eE}{m\Omega}\ (\cos \omega_1 t - \cos \omega_2 t) \qquad \ldots (2)$$

$$v_z = \frac{dz}{dt} = 0$$

and the velocity perpendicular to $\underline{B}$, $v_\perp$, is given by:-

$$v_\perp^2 = v_x^2 + v_y^2 = 2\left(\frac{eE}{m\Omega}\right)^2 (1 - \cos \Omega t) \qquad \ldots (3)$$

If all the electrons can take part in this motion, and since by our assumption of initial rest they must all gyrate in phase, the diamagnetic magnetization $\underline{M}$ which opposes the original magnetizing force, H, of the plasma is:-

$$\underline{M} = - (\tfrac{1}{2}\, mnv_\perp^2/B)\, \hat{\underline{k}} \qquad \ldots (4)$$

where $\hat{\underline{k}}$ is a unit vector along Oz, and n is the electron density, and the induction in the plasma becomes, from equations (3) and (4)

$$\underline{B} = \mu_0 (\underline{H} - \underline{M}) = \mu_0 \left[\underline{H} - \frac{ne^2E^2}{mB\Omega^2} (1 - \cos \Omega t)\, \hat{\underline{k}} \right] \qquad \ldots (5)$$

where μ_0 is the permeability of free space. Thus the field in the plasma has a sinusoidal component of amplitude proportional to Ω^{-2} and the frequency $\Omega = \omega_1 - \omega_2$. Note that a result of the form of equation (5) implies that ω_c is a function of time and equations (1) are non-linear; this has been ignored in this simple treatment. [A first order approximation putting $B = (B_0 - \Delta B_0 + \Delta B_0 \cos \Omega t)$ indicates that the periodic field will contain harmonics of the beat frequency, Ω. As long as $\Delta B/B \ll \Omega/\omega_c$ the effect is negligible.]

If now we suppose collisions occur frequently during one beat period $2\pi/\Omega$ the free motion described above is lost. We can make a simple estimate of the effect of frequent collisions by assuming that, on average, $v_\perp \to 0$ after a collision, so that equations (2) apply between collisions (provided $\nu \ll \omega_c$). Then the change in $v_\perp$ between collisions at times t' and $t' + 1/\nu$ is approximately $\left(\frac{dv_\perp}{dt}\right) t' \cdot \frac{1}{\nu}$, where we assume $\frac{1}{\nu}$ is small enough for the acceleration to be regarded as a constant between collisions, and dependent only on the value of accelerating field at $t = t'$. In this way we can show that:-

$$v_\perp^2 = \frac{E^2 e^2}{2m^2\nu^2} (1 - \cos \Omega t) \qquad \ldots (6)$$

and the magnetic field in the plasma is:-

$$\underline{B} = \mu_o \left[\underline{H} - \frac{E^2 e^2 n}{4mB\nu^2} \; (1 - \cos \Omega t) \; \hat{\underline{k}} \right] \qquad \ldots (7)$$

i.e. there is still a periodic component of field in the plasma but its amplitude is reduced and is now independent of Ω.

Fig. 1 shows the expected magnitude of the effect in terms of maximum electron energy achieved in the beat field, and the corresponding variation of magnetic field ΔB and plasma parameter $\Delta\beta_\perp$

$$(\beta_\perp = \frac{\text{kinetic energy density of motion} \perp B}{\text{magnetic field energy density}} = \frac{2M}{\mu_o B}$$

for an assumed electron density of 10^{12} cm^{-3}, applied field $B_o = 3{,}200$ gauss. The solid line represents the idealized case in which electrons have free times larger than the period of the difference frequency which is here taken to be 1.6 Mc/s. The dashed line corresponds approximately to conditions in one experiment mentioned in the next section. The assumption of $n = 10^{12}$ cm^{-3} is based upon the experience that for microwave generated discharges the electron plasma frequency $\omega_p \sim \omega$.

Here we are not concerned with the effect of variation of ν with field amplitude, which has been discussed by Bailey (1937) for the case of waves near gyro-resonance and which leads to the more usual non-linear effects.

In the above equations relativistic effects have been ignored in the interests of simplicity. Their inclusion would make the particle motion much more complicated, since the Lorenz force equations (1) would become non-linear (ω_c) is no longer constant in time even for a single particle) and there will also be motion to consider in the z direction caused by the oscillating component of the magnetic field parallel to axis Oy (of the r.f. wave) interacting with the transverse velocity v_x. This force is of order $ev_x B_y = \frac{ev_x E_x}{c}$ for a transverse plane wave. For a single frequency Davidowskii (1963) has shown that these effects can cancel, since the parallel motion causes the applied field to be Doppler shifted in frequency at the same rate as the relativistic frequency shift, enabling a particle to stay in synchronism indefinitely. Such an analysis has not been attempted for the two frequency case, but a qualitative examination suggests that the electron will acquire a periodic motion back and forth along the field lines.

Although relativistic effects themselves may lead to the generator of a difference frequency field, the mechanism suggested here in no way depends on relativistic effects for its action. For practical purposes we may safely neglect the change in ω_c for $v_\perp/c < 0.1$. Since the maximum value of $v_\perp$ is $2eE/m$ (from equation 3), this applies for fields $\sim$ 10V/cm and $\Omega > 10^7$, which holds for most of our observations. (see Fig.1)

Another simplification lies in ignoring the initial velocity and phase of the particles. In the collision-free case at simple resonance the electrons gain energy so rapidly from the oscillating field that they pull into phase with the applied field (Linhart 1960) and attain energies only slightly different from the most favoured particles. The effect of initial phase on the energy gained from the field between electrons when a few collisions occur has been shown by Townsend and Gill (1938) to be insignificant. Inclusion of random phase and proper averaging would result in a numerical factor of order unity in the expressions for $\underline{M}$ and would not influence our simple argument.

3. EXPERIMENTAL OBSERVATIONS

Some preliminary experiments have been performed to seek evidence of the mixing mechanism described above: in these, two equal amplitude microwaves of slightly differing frequencies are applied to a plasma and periodic changes sought in the plasma properties, e.g. the internal field B, or the electron kinetic energy, which occur at the appropriate difference frequency Ω. The variation of the amplitude of, for example, field changes as a function of Ω could then be compared with equations (5) and (7) under the appropriate condition of electron free time. The plasma used in these experiments was entirely produced and maintained by the microwave fields since experience has shown that the presence of r.f.

fields of the amplitude necessary to produce a significant effect (say, $E \sim 10$ volt cm^{-1}) will determine the plasma conditions regardless of any external means of production, provided $\omega_p \lesssim \omega$. For the same reasons it is not easy to check the variation of ΔB with E^2, since under these large-signal conditions the internal electric field adjusts itself to a value appropriate to the maintenance of a plasma with $\omega_p \sim \omega$, rather akin to conditions in the positive column of a d.c. discharge, but complicated by the limiting density condition which results from the screening of the wave by the plasma itself.

The microwave feed arrangement is shown schematically in Fig.2. The interacting microwaves were supplied from a single klystron amplifier tuned to amplify equally low-level signals from two reflex klystron oscillators within a frequency range equal to the pass-band of the amplifier (9375 ± 5 Mc/s). The frequency separation of the oscillators could be adjusted between 0.5 and 10 Mc/s and maintained within ~ 10 kc/s by means of a feed-back system based on digital counting techniques. At separations less than 0.5 Mc/s the electromagnetic coupling between the two oscillators caused them to synchronize.

(a) Plasma in a uniform magnetic field

In the first experiment (Fig.3) a 0.8 cm bore silica tube was inserted across a waveguide carrying the two waves, so that the electric vectors were perpendicular to the axis. The uniform magnetic field of an electromagnet was applied along the axis, so that $\underline{E}$ was perpendicular $\underline{B}$. When the magnetic field was adjusted for gyro-resonance ($\omega_c = \omega$) i.e. B = 3,200 gauss, breakdown occurred readily with the tube filled with helium or hydrogen, even at pressures as low as 10^{-5} torr (Townsend and Gill, 1938; Buchsbaum et al., 1961). About 10 watts of cw power could then be matched into the discharge, which was highly absorbing to the microwaves.

At pressures above about 10^{-3} torr the spectral line emission was sufficient to observe its time variation with a photomultiplier having a wide-band output. The spectral light was seen to be strongly modulated at the difference frequency, indicating that the mean energy of the exciting electrons was also varying at the difference frequency (Hamberger, 1963).

A further indication that the electron energy was fluctuating at frequency Ω was seen with an electric probe touching the outside wall of the tube, which indicated a periodic change in external wall potential ~ 1 volt, presumably caused by the fluctuating plasma potential.

In this arrangement the total high frequency flux changes in the plasma could be measured by a pick-up coil encircling one end of the tube outside the waveguide, whose side wall had a longitudinal slit to allow penetration of the changing flux. The voltage induced in this was approximately sinusoidal and at the difference frequency its amplitude was proportional to Ω in the available range $(0.5 < \frac{\Omega}{2\pi} < 10$ Mc/s), showing that the amplitude of the magnetic field fluctuation was independent of Ω in this range. From the coil dimensions the difference frequency component of B was found to be $\Delta B \approx 0.2$ gauss, corresponding to a periodic change in the value of $\beta_{\perp} \sim 10^{-4}$.

The independence of ΔB with Ω was found at all pressures at which the discharge could be maintained, even though at the lowest the estimated electron-atom collision frequency was much less than the lowest difference frequency used (e.g. at 10^{-5} torr, $\nu \sim 10^{4}$ sec^{-1}). This apparently serious disagreement with the theory is, however, explained by the absence of axial containment of the electrons; consequently, an electron having a thermal energy of, say, 2-3 eV, will drift to an end wall in an average time $\sim 3.10^{-8}$ sec., effectively destroying the beat motion in the r.f. field. Using the corresponding value of $\nu \sim 3.10^{7}$ sec^{-1} in equation (7) (curve II of Fig.1) we find a predicted value of $\Delta B \sim 0.2$ gauss, assuming reasonable values of $n \approx 10^{12}$ cm^{-3}, $E \approx 10$ V cm^{-1}. Despite the crude nature of these quantitative assumptions, the results seem consistent with the theory for the collision dominated case of gyro-resonance mixing.

(b) Plasma in a mirror field

In order to examine the main prediction of the argument, namely that $\Delta B \propto \Omega^{-2}$

in the absence of collisions, it was necessary to contain the electrons axially for times longer than the available beat periods (i.e. $\geqslant 10^{-6}$ sec). Since the use of a very long (i.e. several metres) container was not practical a weak magnetic mirror field was used. Although the theory presented applies strictly only to uniform fields, the work of Ard and his co-workers at Oak Ridge (1963) has shown that with a single frequency very pronounced gyro-resonant interaction occurs even though electrons have very different gyro-frequencies in different parts of the plasma. The arrangement shown in Fig.4 was therefore adopted, in which an axially varying magnetic field of mirror ratio 1.3:1 was used to contain electrons inside a plasma produced in a 4 cm diameter, 4 cm long fused silica vessel. The field variation (shown in Fig.4) was obtained by shaping the pole pieces of the electromagnet. The containing vessel was supported by quartz pegs inside a tunable brass cavity which was excited by the same waveguide system as before and adjusted to resonate in H_{013} mode, so that $\underline{E}$ was perpendicular to $\underline{B}$. With this arrangement it was not possible to measure total flux changes in the plasma, as the presence of conductors elsewhere than near the axis disturbed the r.f. properties of the cavity. It was, however, possible to introduce a very small air cooled axial probe coil (3 mm diameter) wound in a silica former, through a central hole in one end plate without seriously disturbing the r.f. fields, so that relative changes in plasma flux could be observed. The resonant frequency of the magnetic probe system was $\sim$ 30 Mc/s. Since any conductors entering the r.f. cavity would pick up both microwave signals, the probe leads were carefully filtered to render insignificant the generation of spurious beat frequency signals caused by mixing in the detection amplifier. With this arrangement up to about 100 watts of total continuous r.f. power could be matched into the cavity; at higher powers arcing occurred which destroyed the probe coil.

The general behaviour was similar to that reported by Ard et al.: a plasma could be maintained in helium at pressures greater than about 10^{-6} Torr when the magnetic field was adjusted to resonance in the mid-plane. It was impossible to maintain a steady pressure with hydrogen owing to the very rapid rate of clean-up on the walls of the vessel. At low pressures ($\lesssim 10^{-3}$ Torr) X-rays were copiously generated and detected through a hole in the cavity wall by a counter: the X-ray count was greatest when the cyclotron resonance was at the mid-plane (where the E field was also greatest). Subsidiary maxima occurred when $\omega = \omega_c$ at the outer field antinodes, which were just inside the ends of the vessel. From the attenuation produced by copper coils the mean X-ray energy was estimated to be about 25 keV.

Whenever a plasma was present (as indicated by the emission of X-rays at pressures below that at which the discharge was visible) a signal appeared on the magnetic probe at the difference frequency. To confirm that this signal was caused by a changing axial magnetic field, and not by some radial components, a second coil, whose axis could be rotated, was temporarily introduced at the mid-plane hole. A detectable signal was observed from this coil only when its axis was parallel to B.

In general the amplitude of the difference frequency signal was not steady. As in the case of the Oak Ridge experiments, stable operating conditions were reached only for a limited pressure range, in this case between 10^{-4} and 10^{-3} Torr. Below 10^{-4} Torr relatively large signals sometimes appeared, but alteration to the r.f. conditions (when attempting to vary Ω) would cause an abrupt change, so that variation of ΔB with Ω could not be measured. However, in the range 10^{-4} – 10^{-3} Torr the discharge would occasionally remain steady long enough ($\sim$ 30 min.) for the signal amplitude to be measured as the frequency difference was varied. The curve of Fig.5 is typical of the results obtained during such a "quiet" discharge. For differences greater than about 4 Mc/s, the field amplitude varied approximately as Ω^{-2}, as predicted; at lower frequencies the variation is less rapid and for $\frac{\Omega}{2\pi} \lesssim 1$ Mc/s, ΔB is approximately independent of Ω.

Such a result would be expected if the electrons have a free life-time of about $10^{-6} - 3.10^{-7}$ sec. ($\nu \sim 1 - 3$ Mc/s). At the pressure of helium used (2.5×10^{-4} Torr) this is about the electron-atom collision time. However, any mechanism which would disturb the ordered transverse electron motion, such as a plasma instability or asymmetry in the axial field (see below) would have the same effect.

A comment is needed about the use of a non-uniform field instead of the uniform field (for which the theory applies). The behaviour of electrons in a mirror-field subjected to resonant electric fields is very complex, and has recently been analyzed in some detail by Seidl (1963). One result of this theory shows that some electrons, i.e. those which perform small axial oscillations in the mirror field, behave as if they were in a uniform field. Those which have large amplitudes such that ω_c varies appreciably along their paths, can have only small fluctuations in $v_\perp$ periodic with their lateral motion in the mirror field, and so can not take part in the organized motion periodic at the (much lower) beat frequency as described by equation 3. The lifetime of even those favoured (low amplitude) electrons is also limited by axial asymmetry in the mirror field which cause the electrons to drift towards the weaker mirror and so spend only a short time in the central region. This may be the cause of the short containment time referred to above. It was observed that when the axial symmetry of the field was disturbed (by modifying one pole of the electromagnet) results such as those of Fig.5 could not be obtained.

4. ACKNOWLEDGEMENTS

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5. REFERENCES

Ard, W.B., et al., (1963). Sixth International Conference on Ionization Phenomena in Gases, Paris, 1963. Proceedings, 4, 75.

Bailey, V.A. and Martyne, D.F., (1934) Phil. Mag. 18, 369.

Bailey, V.A. (1937) Nature, London 139, 68.

Bailey, V.A., (1956) Nuovo Cim. (Supplement) 4, 1430.

Buchsbaum, S.J., Gordon, E.I. and Brown, S.C., (1961) Plasma Physics (J. Nuc. Eng. Part C) 2, 164.

Davydovskii, V. Ya., (1963) Soviet Physics - JETP, 16, 629.

Ginzburg, V.L., "Propagation of Electromagnetic Waves in Plasmas", Oxford, Pergamon Press, 1964.

Hamberger, S.M., (1963) Plasma Physics (J. Nuc. En. Part C), 5, 73.

Linhart, J.G., "Plasma Physics" Amsterdam, N. Holland Publishing Co., 1960.

Seidl, M., "High frequency heating of electrons in a mirror machine", CLM- P 29. (1963) (to be published).

Thompson, W.B., (1963) Culham Laboratory. Private Communication.

Townsend, J.S. and Gill, E.W.B., (1938) Phil. Mag. 26, 290.

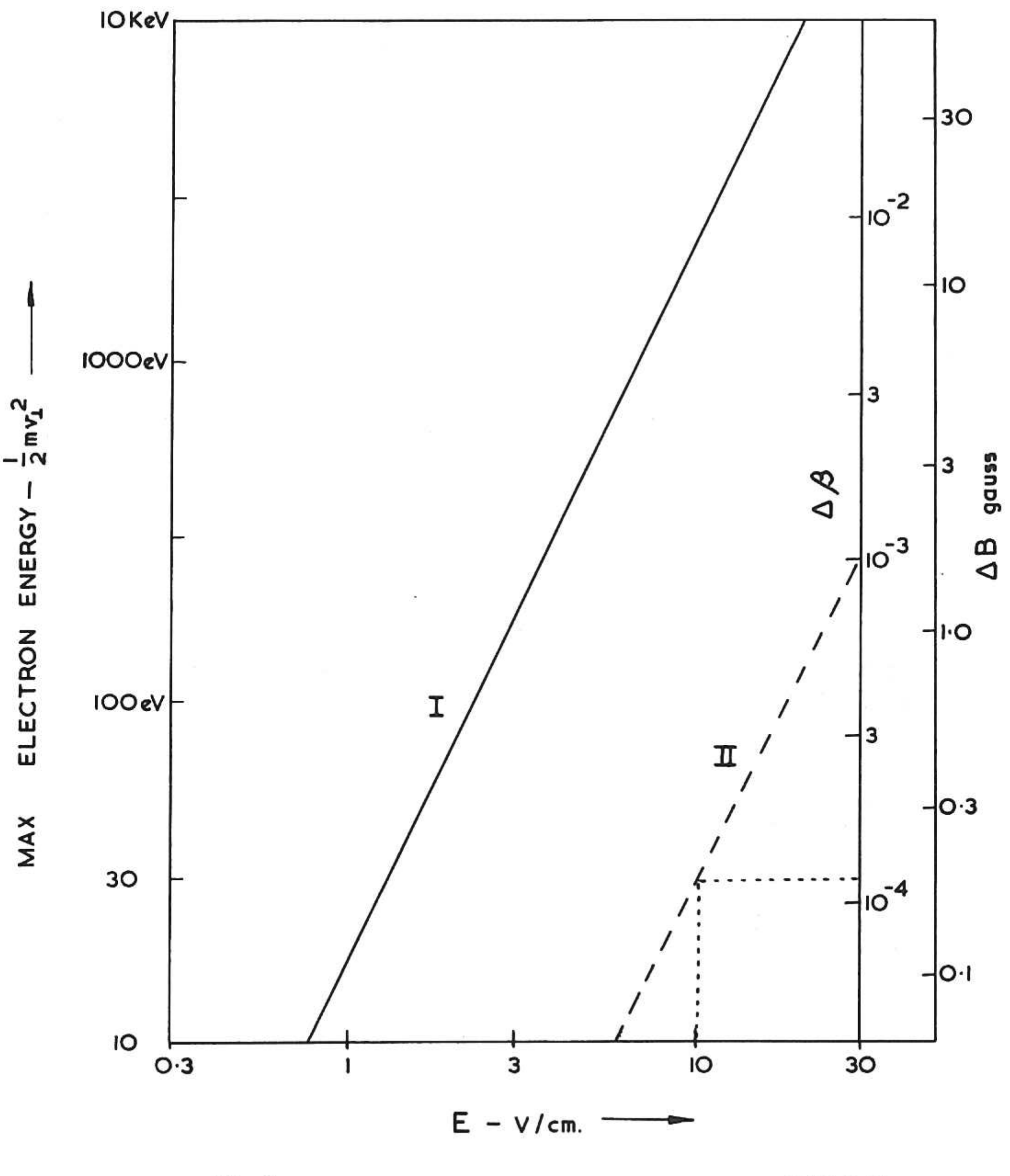

Fig. 1 (CLM-P 63)

Amplitudes of electron energy, magnetic field and β variations

I Ideal curve for $\nu = 0$, $\Omega = 10^7$ sec^{-1} } $B_0 = 3{,}200$ G

II Collisional case, $\nu = 3 \times 10^7$ sec^{-1}, $\Omega \ll \nu$ } n = 10^{12} cm^3

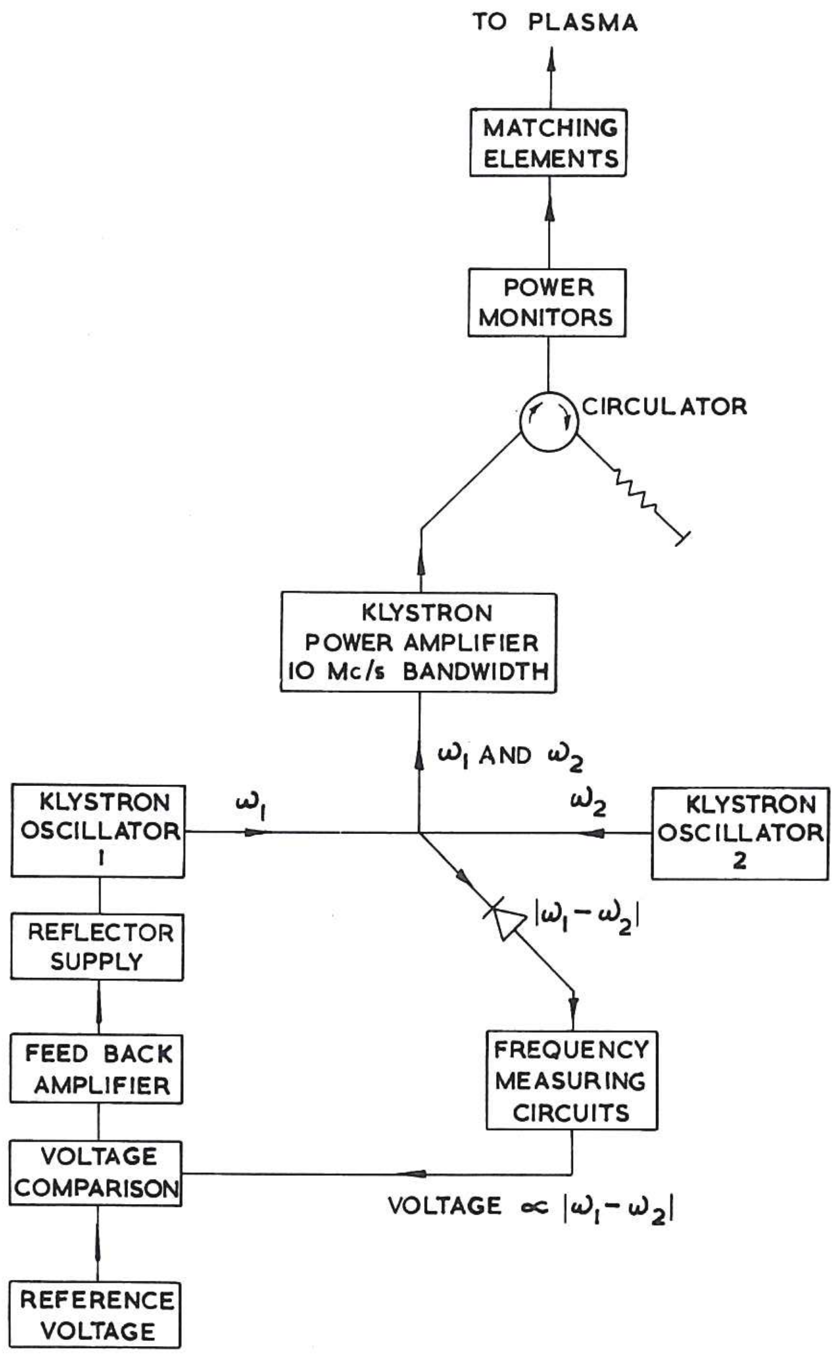

Fig. 2 Two-frequency feed arrangement (CLM-P 63)

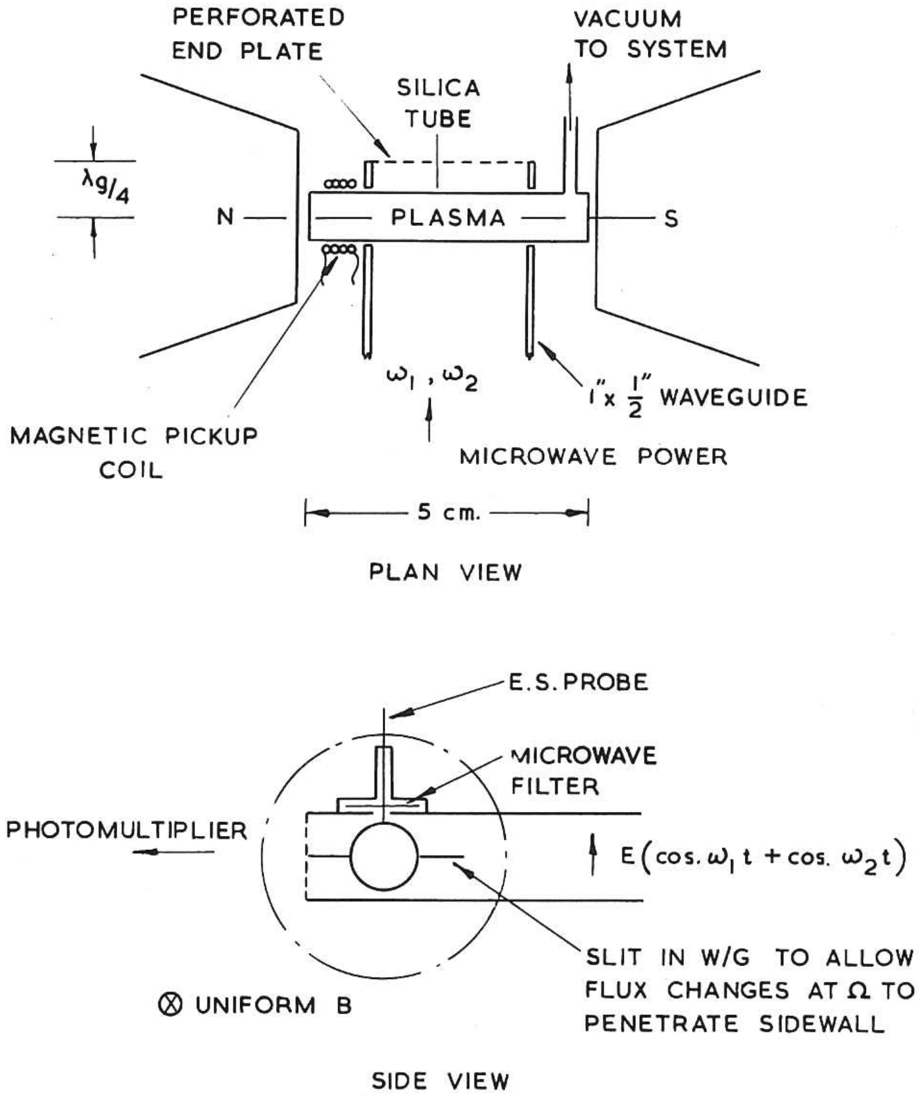

Fig. 3 Schematic of uniform field experiment (CLM-P63)

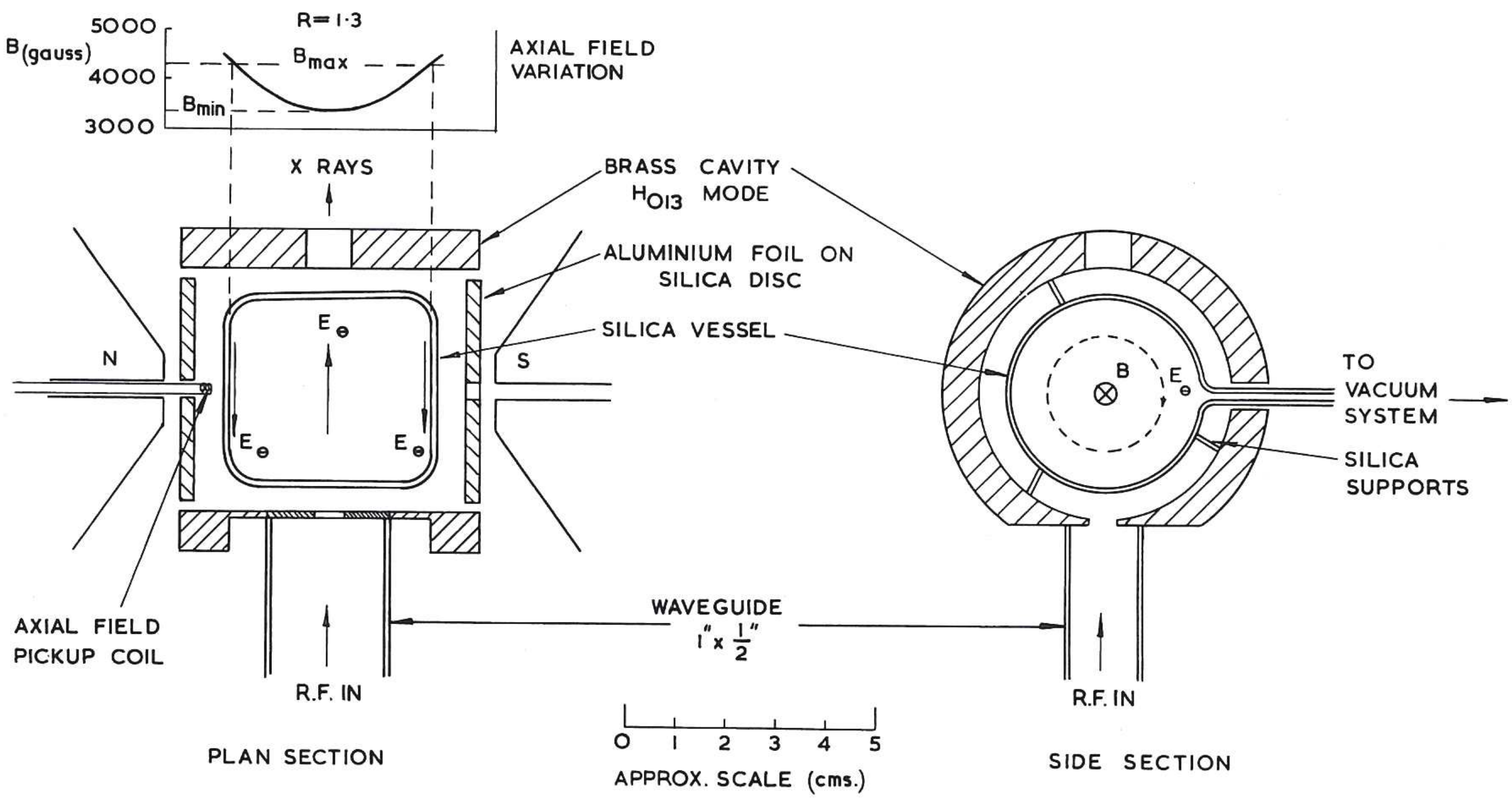

Fig. 4 Mirror field experiment (CLM-P 63)

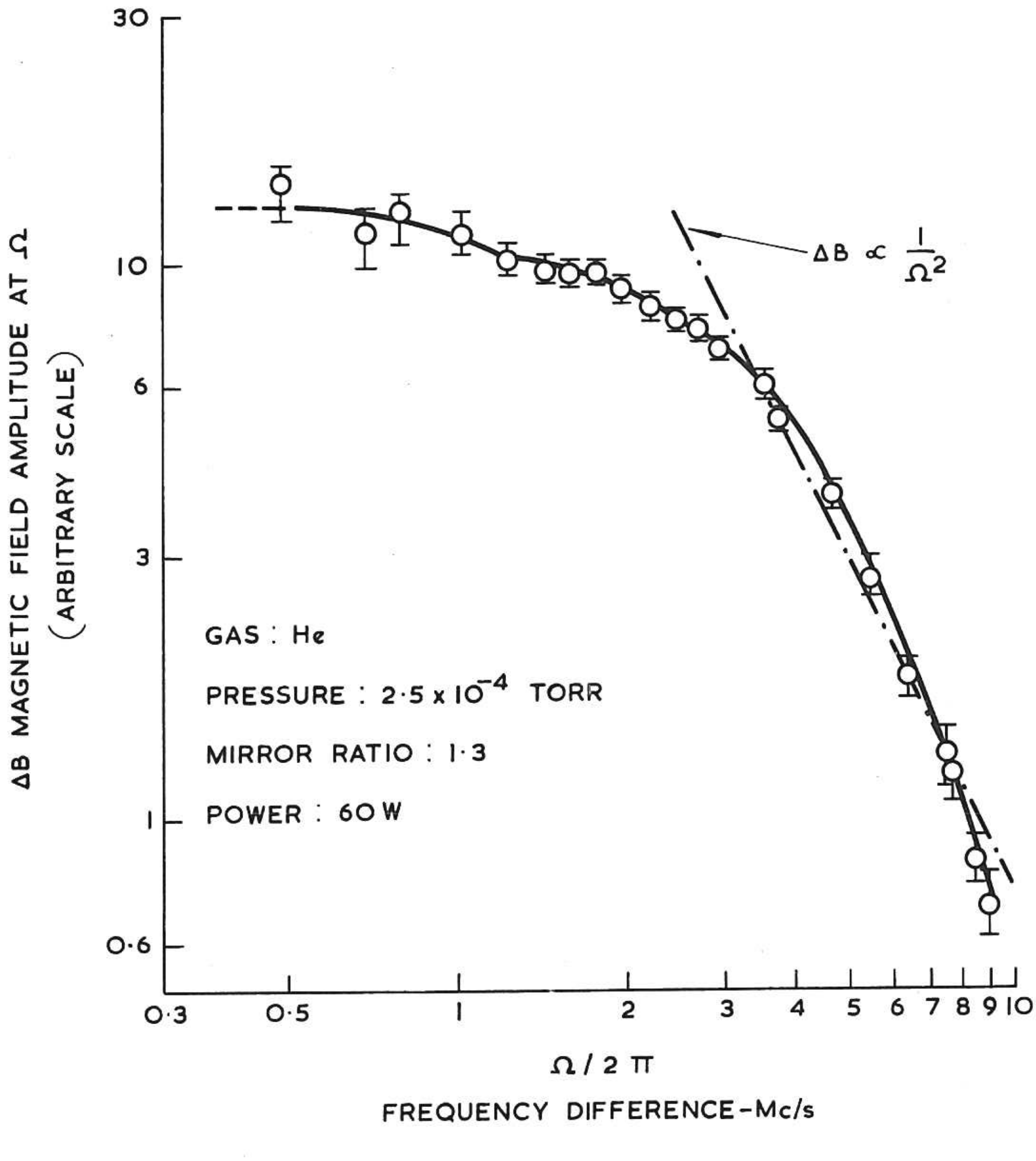

Fig. 5 (CLM-P 63)
Amplitude of axial field fluctuation as a function of different figures