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by

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A B S T R A C T

It is shown that anomalous damping of helicon waves propagat-
ing in a cylinder of hot plasma occurs when the sound speed is high
enough.   The damping mechanism is ohmic dissipation of currents
flowing at the plasma surface.

The theory is applied successfully to some recently published
experimental results which up to now had been explained only by
invoking a collisionless mechanism.

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In a recent paper (NAZAROV et al, 1963) experimental results have been given on the damping of high frequency oscillations of a plasma cylinder in an external magnetic field. The oscillations are those known as helicons (or low frequency whistlers). The measurements were made in hydrogen and helium plasmas bounded by a glass tube where

N = electron number density = 4 to $8 \times 10^{13} cm^{-3}$

T_e = electron temperature = 50 eV

$\dfrac{\omega}{2\pi}$ = wave frequency = 10 Mc/s

$\dfrac{2\pi}{k}$ = wavelength = 10 cm

B = external magnetic field = 900 gauss

a = radius of plasmas = 3cm.

For these conditions the damping length measured was 40 cm, whereas the damping length calculated for a helicon wave propagating in an infinite medium is 10^4 cm. Thus the wave attenuation is too large to be explained by collisions in the main body of the plasma. DOLGOPOLOV et al. (1963) have suggested a collision-less Cerenkov mechanism to explain the results and obtain a damping length between 60 and 100 cms. It is the purpose of this paper to show that the observed wave damping can be explained by the ohmic dissipation of surface currents flowing at the plasma boundary as described by KLOZENBERG et al. (1964) (abbreviated to KMT). In this theory wave propagation in a uniform cylindrical cold plasma in a uniform axial magnetic field in the z-direction is considered. The equation for the axial component of the perturbation magnetic field (assumed proportional to $e^{i(kz + m\theta - \omega t)}$ where t is the time, m the azimuthal mode number and θ the azimuthal co-ordinate and ion motion is neglected) takes the form:-

$$L_o \, b_z \equiv \left[-\frac{\eta'^2}{\omega^2 \mu_o^2} \nabla^4 + \left(\frac{H^2}{\omega^2} + \frac{2i\eta'}{\omega \mu_o} \right) \nabla^2 + 1 \right] b_z = 0 \qquad \ldots (1)$$

The condition for the neglect of ion motion is:-

$$\frac{\Omega_i}{\nu} \ll \frac{\omega}{\Omega_i} \qquad \ldots (2)$$

where

$$\eta' = \eta + \frac{i M_e \omega}{N e^2}$$

and

$$\eta = \text{plasma resistivity} = \frac{M_e \nu}{N e^2}$$

M_e = electron mass

e = charge on proton

ν = electron collision frequency

$H = \dfrac{kB}{Ne\mu_o}$ = helicon phase velocity in an infinite medium

Ω_i = ion gyro-frequency

Ω_e = electron gyro-frequency

It was shown that when

$$\nu \gg \omega \qquad \qquad \text{... (3)}$$

wave attenuation is anomalous (i.e. sensibly independent of collision frequency) under the following conditions:

$$\Omega_e \gg \nu \qquad \qquad \text{... (4)}$$

$$ak_r > \frac{3\nu}{\Omega_e} \qquad \qquad \text{... (5)}$$

where $k = k_r + ik_i$.

Condition (3) means that electron inertia may be neglected compared with resistivity, while (4) implies that wave damping in an infinite medium (where surface currents are negligible) is very small. Condition (5) gives a lower bound on the dimensionless wave number: there is also an upper bound on ak_r above which volume damping exceeds anomalous (or surface) damping. The theory has been confirmed by the experiments of LEHANE and THONEMANN (1964) for propagation in a plasma and HARDING and THONEMANN (1964) for a solid state plasma.

In the experiments of NAZAROV et al. (1963) conditions (2) and (3) are not satisfied. In this paper, however, an extension of the theory of KMT is given in which anomalous damping is present in such circumstances. It can be applied approximately under the experimental conditions reported by NAZAROV et al. (1963).

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Firstly if $\nu < \omega$ (4) and (5) must be replaced by the more general conditions

$$\Omega_e \gg |\nu + i\omega| \qquad \qquad \dots (4')$$

$$\left| ak_r - \frac{\omega}{\nu} ak_i \right| \geqslant 3 \, \frac{|\nu + i\omega|^2}{\Omega_e \nu} \qquad \qquad \dots (5')$$

Since KMT have not given solutions of their dispersion relation for $\nu < \omega$, (5') must be considered an apostetori condition for the occurrence of anomalous damping. It is satisfied by the results of NAZAROV et al. (1963) as is (4'). Secondly, it will now be shown that for a sufficiently incompressible plasma, ion motion has only a weak effect on surface currents, so that anomalous damping may occur even when (2) is not valid. The theory depends on including plasma pressure in the basic equations which in linearised form (with zero current and electric field in equilibrium) are as follows:-

$$\rho_o \frac{\partial \vec{v}}{\partial t} = \vec{j} \times \vec{B} - \nabla p$$

$$\vec{E} + \vec{v} \times \vec{B} = \eta \vec{j} + \frac{1}{Ne} \vec{j} \times \vec{B} + \frac{M_e}{Ne^2} \frac{\partial \vec{j}}{\partial t} - \frac{1}{Ne} \nabla P_e$$

$$P = c_s^2 \rho$$

$$\frac{\partial \rho}{\partial t} + \rho_o \, \text{div} \, \vec{v} = 0 \qquad \qquad \dots (6)$$

$$\text{curl} \, \vec{E} = -\frac{\partial \vec{b}}{\partial t}$$

$$\text{curl} \, \vec{b} = \mu_o \vec{j}$$

The equation of state in equilibrium has been taken as $p_o \propto \rho^\gamma$ where $\gamma = \frac{5}{3}$. The perturbation mass density, velocity, current density, pressure, electric field and magnetic field are denoted by ρ, $\vec{v}$, $\vec{j}$, p, $\vec{E}$, $\vec{b}$ respectively, ρ_o is the equilibrium mass density, p_e the electron pressure perturbation and $c_s = \sqrt{\frac{\gamma NkT}{\rho_o}}$ is the sound speed.

The equation for the axial component of the perturbation magnetic field assumed proportional to $e^{i(kz + m\theta - \omega t)}$ is found from equation (6) to be given by:

$$L \, b_z = 0 \qquad \qquad \dots (7)$$

where
$$L = L_1 + \frac{C_S^2}{\omega^2} \nabla^2 L_2$$

is the wave operator, and

$$L_1 = \frac{i\eta'}{\omega\mu_0}\left(\frac{A^2}{\omega^2} + \frac{i\eta'}{\omega\mu_0}\right)\nabla^4 + \left[\left(\frac{A^2}{\omega^2} + \frac{i\eta'}{\omega\mu_0}\right)\left(1 - \frac{k^2A^2}{\omega^2}\right) + \frac{H^2}{\omega^2} + \frac{i\eta'}{\omega\mu_0}\right]\nabla^2 + 1 - \frac{k^2A^2}{\omega^2}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \dots (8)$$

$$L_2 = \frac{-\eta'^2}{\omega^2\mu_0^2}\nabla^4 + \left[\frac{H^2}{\omega^2} + \frac{2i\eta'}{\omega\mu_0}\left(1 - \frac{k^2A^2}{\omega^2}\right)\right]\nabla^2 + \left(1 - \frac{k^2A^2}{\omega^2}\right)^2$$

where $A = \dfrac{B}{\sqrt{\mu_0\rho_0}} = $ Alfvén speed.

The results of KMT are recovered by putting $C_S = 0$ and $\nu \gg \omega$ in equation (8). Since for helicons the wave velocity is large compared to the Alfvén speed, (8) then becomes

$$L \approx L_0$$

provided the condition $A^2 \ll \dfrac{\omega\eta}{\mu_0}$ (which is the same as (2)) is satisfied. If the latter condition is not satisfied but the sound speed is sufficiently large so that

$$A^2 \ll C_S^2 + H^2 \qquad\qquad \dots (9)$$

then the wave operator may be written in the approximate form

$$L \approx \left(\frac{C_S^2}{\omega^2}\nabla^2 + 1\right)L_0 + H^2 C_S^2 f \nabla^4 \qquad\qquad \dots (10)$$

L_0 is given by equation (1) and f is the coupling factor which determines the proportion of sound wave present when a helicon wave is propagated (and vice versa). The coupling factor is given by

$$f = \frac{A^2}{C_S^2 H^2}\frac{\omega\eta'}{\mu_0} \qquad\qquad \dots (11)$$

For sound speeds high enough to satisfy

$$f \ll 1 \qquad\qquad \dots (12)$$

the last term in (10) may be neglected compared to the term in ∇^4 in the product. In the limit of vanishingly small f, the sound wave and helicon wave may propagate independently. In the case of helicon propagation, the wave operator is then

the same as that given by KMT and their theory is applicable. In Fig.1 below the damping factor ak_i is given as a function of ω/ω_0 for the $m = 1$ mode, in the limiting case $\dfrac{\Omega_e}{\nu} = \infty$, where $\omega_0 = \dfrac{B}{Ne\mu_0 a^2}$ and $\dfrac{1}{k_i}$ is the e-folding length.

For the lowest value of the density range in the example cited above

$$\omega \sim \frac{1}{200}\,\Omega_e$$

$$\nu \sim \frac{1}{20}\,\omega$$

$$ak_r \sim 1.8$$

$$f \sim 0.1$$

$$A^2/(C_s^2 + H^2) \sim 0.2$$

Condition (9) is therefore approximately satisfied so that equation (10) may be used to an accuracy of roughly 20%. Since $f \sim 0.1$ a further inaccuracy of 10% occurs because a helicon wave necessarily involves a sound wave of amplitude at least 10% that of the helicon wave. Finally, conditions (4′) and (5′) are satisfied, where in the latter the value of ak_i is obtained from Fig.1, and it follows that the theory of KMT may be used. Since $\dfrac{\Omega_e}{\nu} \sim 4 \times 10^3 \gg 1$, Fig.1 is applicable and gives $ak_i \approx 0.1$, i.e. the damping length is 30 cms. With the 30% error the actual theoretical value is found to lie between 20 and 40 cms. This is more than adequate to explain the experimental results.

REFERENCES

DOLGOPOLOV, V.V., ERMAKOV, A.I., NAZAROV, N.I., STEPANOV, K.N., TOLOK, V.T. (1963) Nuclear Fusion, 3, 251.

HARDING, G.N., THONEMANN, P.C. (1964) Proc. Phys. Soc. (In the press). (Culham Laboratory preprint CLM- P 54).

KLOZENBERG, J.P., McNAMARA, B., THONEMANN, P.C. (1964) J. Fluid Mech. (In the press). (Culham Laboratory preprint CLM - P 55).

LEHANE, J.A., THONEMANN, P.C. (1964). (Culham Laboratory preprint CLM - P 64).

NAZAROV, N.I., ERMAKOV, A.I., DOLGOPOLOV, V.V., STEPANOV, K.N., TOLOK, V.I. (1963) Nuclear Fusion, 3, 255.
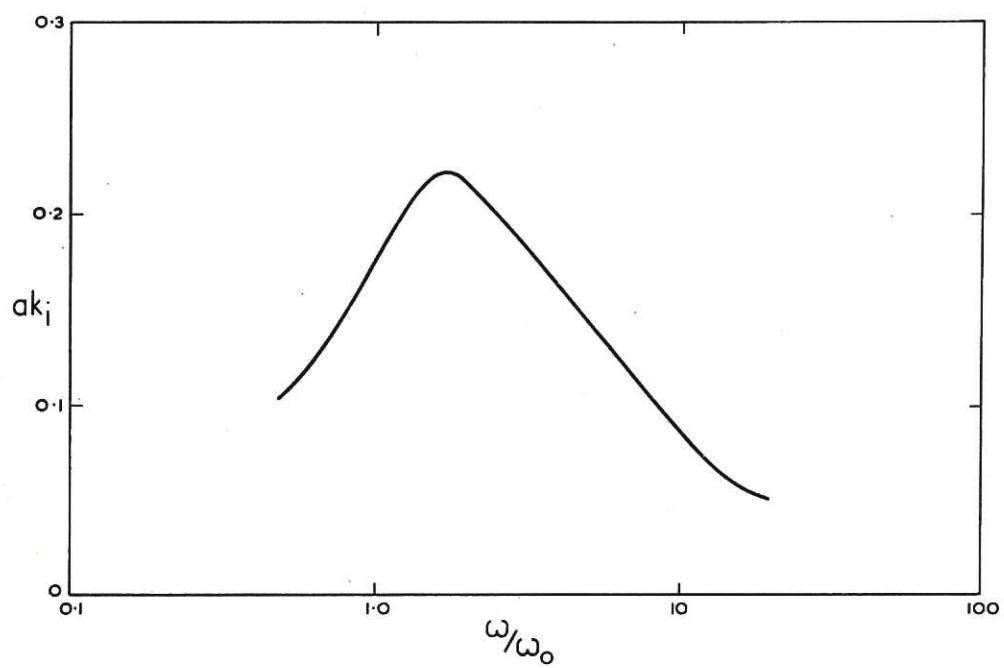
CLM - P 65   Fig. 1

Attenuation due to skin damping for $m=1$ mode and $\Omega_e\tau = \infty$