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ABSTRACT

Two melting problems are considered.
(i) The descent of an internally heated spherical particle of unit radius embedded in a low melting point solid is analyzed when the velocity of descent is very slow. When Q, a measure of the strength of internal heating, is less than unity, no melting occurs. When $Q = 1 + \varepsilon$, where $\varepsilon << 1$, the cavity surrounding the particle is spherical of radius Q, and the velocity of descent is equal to $3\varepsilon^3 U_o/4$ where U_o is the usual Stokes velocity.
(ii) The influence of the rear boundary condition on the melt-through of a plane slab ablated by a large and constant heat flux is analyzed by an approximate method which gives satisfactory agreement with numerical results over a wide range of parameter space. This method also allows the characterization of the logarithmic time singularity which occurs for sufficiently large heat transfer coefficient at the back face. Simple formulae for the onset of melting are also given.

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1. INTRODUCTION

A wide range of free boundary problems arise in reactor safety studies - in the movements of solid, liquid and gas phases through pools of coolant, in the quenching of hot solids and in the melting attack on solid structures. Of these, two are discussed in this paper viz. (i) the slow descent through a low melting point solid of a heavy particle which is internally heated by, for example, radioactive decay heat and (ii) the influence of the heat transfer coefficient at the rear of a plane slab on its ablation by a large and constant heat flux; this latter is relevant to the melting of a steel plate one face of which is cooled convectively and the other is strongly heated. Particular regimes are considered for idealized models which have interesting features.

2. THE DESCENT OF AN INTERNALLY HEATED PARTICLE THROUGH A LOW MELTING POINT SOLID

The essential characteristic of the descent through a low melting point solid of a hot particle is that it does so by melting a transient liquid cavity in which it falls under gravity. After an initial transient period of acceleration, a particle containing a constant heat source falls at a steady velocity with an invariant cavity shape. For a given weight of particle and strength of heat source, the fall velocity in a given bed material is determined. The weight of the particle is balanced by the drag exerted on it by the flow of liquid in the cavity. The drag depends on the shape of the particle and of the enveloping cavity, which is determined by the heat transfer from the particle; this is itself influenced by the rate of fall. This regelation problem is related to the classical experiment of a wire pulled through ice [1]; and to similar experiments with spheres [2]. However for a hot particle the melted cavity is produced by the sphere's internal heating and not by pressure melting. For ice skating, friction provides the heat source, which can produce the lubricating water film even when pressure melting is negligible [3].

2.1. Formulation

Here the particle is taken to be spherical, of radius a, and it is assumed that the thermophysical properties of the solid bed material (including the thermal diffusivity κ) do not change on melting. It is convenient to work with dimensionless variables, measuring length and time in multiples of a and a^2/κ respectively. The dimensionless temperature θ has been scaled so that $\theta = 1$ is the melting point of the solid and $\theta = 0$ is the ambient temperature far from the particle. The configuration is axisymmetric; spherical polar co-ordinates centred on the particle are taken with $\underline{r} \equiv (r,\phi)$ where r and ϕ are the radius and co-latitude respectively. When the particle is falling steadily, we have in the frame of reference of the particle

$$(\underline{u}.\nabla)\theta = \nabla^2\theta \qquad (r \geq 1) \qquad (2.1)$$

and in the liquid cavity, $\underline{u}$ satisfies the Navier-Stokes equation:

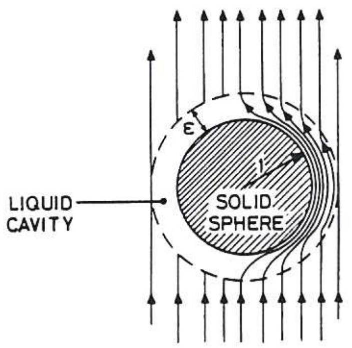


Fig.1 The motion of the melting substrate material in the frame of reference of the sphere

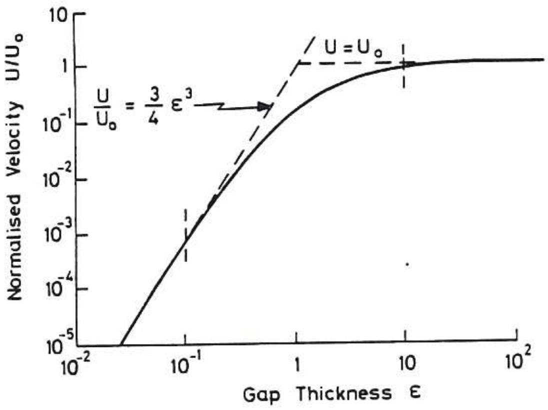


Fig.2 The fall velocity U as a function of the normalized gas thickness ϵ; U_o is the Stokes velocity.

$$p^{-1}(\underline{u}.\nabla)\underline{u} = \nabla^2\underline{u} - \nabla\Pi, \qquad \text{div } \underline{u} = 0 \qquad (2.2)$$

where Π is the normalized deviation of the pressure from its ambient value and p is the Prandtl number. The boundary conditions are (i) $\theta \to 0$ as $r \to \infty$. (ii) $\underline{u} = \underline{U}$, $\theta = 1$ and $-[\partial\theta/\partial n] = S\ \underline{U}.\hat{\underline{n}}$ on the cavity boundary $\underline{r} \equiv \underline{s}$. Here S is the Stefan number, [.] means jump across discontinuity, and $\underline{U}$ is the steady vertical velocity with which the bed approaches the particle (in the latter's frame of reference). (iii) The total (normalized) rate of heat production rate in the particle is Q. Below we use $\partial\theta/\partial r = -Q/4\pi$ on $r = 1$, but in general the particle's temperature distribution depends on α, the ratio of particle thermal conductivity to that in the bed material. (iv) $\underline{u} = 0$ on $r = 1$. In steady fall, the balance between the reduced weight of the particle and the drag exerted on it can be written in dimensionless form as $12pU_o = c_D U^2$ where U_o is the Stokes terminal velocity of the particle in an infinite medium, and is a measure of the particle's weight. c_D is the drag coefficient.

If $\{\psi\}$ represents the set of parameters $\{Q,S,p,U_o,\alpha\}$ then it is clear that $\underline{u} = \underline{u}(\underline{r};\{\psi\})$, $\theta = \theta(\underline{r};\{\psi\})$, $\underline{s} = \underline{s}(\phi,\{\psi\})$ and $U = U(\{\psi\})$. Depending on the values of the parameters in $\{\psi\}$ a number of different regimes may be discerned.

2.2. Solutions

A. No melting. One solution regime involves conduction through the bed material without melting. If the heat source distribution is spherically symmetric, then outside the particle, $\theta = Q/r$; thus $\theta < 1$ on $r = 1$, provided $Q < 1$. For this regime the remainder of the parameters in $\{\psi\}$ are not relevant.

B. Slow Descent. A second solution regime is that for which U is sufficiently small that (i) equation (2.1) reduces to Laplace's equation; (ii) the Reynolds number (U/p) is small so that Stokes theory is valid in the cavity; and (iii) the discontinuity in the temperature gradient at the cavity interface can be neglected so that the cavity remains spherical. If $Q = 1 + \epsilon$, then $\theta = (1 + \epsilon)/r$ is the solution of (2.1); thus the cavity radius is $1 + \epsilon$, and the cavity gap thickness is ϵ. To relate the heat source strength to the fall velocity it is necessary to calculate the velocity field in the cavity and thus the drag on the particle. The configuration is shown in Fig.1.

An axisymmetric Stokes stream function ψ may be introduced such that

$$r\sin\phi(u_r,u_\phi) = (r^{-1}\partial/\partial\phi,\partial/\partial r)\psi \qquad (2.3)$$

where ϕ is the co-latitude. The velocity $\underline{u}$ satisfies (2.2) with the l.h.s.neglected and the boundary conditions $\underline{u} = 0$ on $r = 1$ and $\underline{u} = \underline{U}$ on $r = Q$. The solution is $\psi = \psi_1 + \psi_2 + \psi_3$ where

$$\left.\begin{aligned}\psi_1 &= \tfrac{1}{2}\lambda_1 U\sigma^2[1 + \tfrac{1}{2}r^{-3} - 3r/2]\\ \psi_2 &= \tfrac{1}{2}\lambda_2 U\sigma^2[1 - r^{-3}]\\ \psi_3 &= \tfrac{3}{4}\lambda_2 U\sigma^2[1 - r^2]\end{aligned}\right\} \qquad (2.4)$$

in which $\sigma = r\sin\phi$, $\lambda_1 = 4Q(Q^5 - 1)/D$, $\lambda_2 = 2Q(Q^2 - 1)/D$ and $D=(Q-1)^4(4Q^2+7Q+4)$. The first term ψ_1 corresponds to the viscous flow around a rigid stationary sphere of radius unity with uniform free stream velocity $\lambda_1 U$[4,p231]. The second term ψ_2 represents an irrotational flow around the unit sphere with free stream velocity $\lambda_2 U$[4,p452]. The third term ψ_3 corresponds to a distribution of vorticity $\omega = 15\lambda_2 U\sigma/2$ (cf Hill's spherical vortex [4,p526].) The values of λ_1 and λ_2 are determined by the outer boundary condition only,since λ_1 and $(\lambda_2 + \lambda_3)$ satisfy the boundary conditions on $r = 1$ independently. The vorticity has only an azimuthal component of magnitude $\tfrac{1}{2}U\sin\phi(15\lambda_2 r - 3\lambda_1 r^{-2})$ and the corresponding pressure distribution is $-\tfrac{1}{2}U\cos\phi(30\lambda_2 r + 3\lambda_1 r^{-2})$.

On the particle surface, the normalized force F per unit area exerted on the sphere is $-\Pi\underline{\hat{n}} - (\underline{\hat{n}}\wedge\omega)$ [4,p178]; the total drag exerted by the fluid can be obtained by integrating this expression over the surface of the sphere. It is however more illuminating to consider separately the three contributions ψ_1, $\psi_2$and ψ_3 to the stream function. From d'Alembert's Paradox, the irrotational flow ψ_2 cannot contribute to the net drag. For ψ_3,the vertical component of the force acting at a point on the particle surface is $15\lambda_2 U.P_2(\cos\phi)$ where P_2 is a Legendre polynomial; when this is integrated over the surface, the resultant vanishes. The sole contribution to the drag consequently arises from ψ_1, which is the stream function for the regular Stokes problem. Thus $U_o = \lambda_1 U$, or

$$U/U_o = (Q-1)^4(4Q^2+7Q+4)/4Q(Q^5-1) \qquad (2.5)$$

which is the required relationship between U and Q. When $\varepsilon \to \infty$, U tends to the Stokes velocity U_o. When $\varepsilon \ll 1$, $(U/U_o) = \tfrac{3}{4}\varepsilon^3$. A cubic dependence on ε is also obtained in [5] for the thin spherical cavity, and in [6] for lubrication flow beneath a plane circular disc. Fig.2 shows (U/U_o) as a function of ε; the limiting forms for $\varepsilon \gg 1$ and $\varepsilon \ll 1$ are good approximations except when $10^{-1} < \varepsilon < 10$.

In the solid, the Peclet number is simply U. In the spherical shell of liquid, the flow is most rapid past the equator of the particle, and the average flow velocity U_m there satisfies $\pi U_m(Q^2-1) = \pi UQ^2$. If $\varepsilon<1$,$U_m\varepsilon$ is the appropriate liquid Peclet number. When ε is small, $U_m\varepsilon = \tfrac{1}{2}U$; when $\varepsilon = 1$, $U_m\varepsilon = 4U/3$. Thus the condition $U \ll 1$ is sufficient to ensure that the l.h.s. of (2.1) is negligible in both solid and melt. Similarly the condition $U \ll p$ ensures that lubrication theory is valid in the liquid shell. From boundary condition (ii), the jump in the temperature gradient at the melt-front is negligible provided $SU << 1$. Finally eqn.(2.5) implies $U < U_o$. Hence the condition for the slow descent solution to be valid is

$$U << U^* \equiv \min [1, p, S^{-1}, U_o] \tag{2.6}$$

holds and (2.5) is true.

2.3 Comment

Note that for a sufficiently light particle in a viscous fluid with a low Stefan number (i.e. $U_o << 1, S < 1, p > 1$), the two regimes - (A) no melting and (B) slow descent - constitute the complete solution for all values of Q. For other values of the parameter set $\{\psi\}$, the non-linearities in this free boundary problem cannot be circumvented; these will be considered elsewhere. Some experimental results for the descent of internally heated spheres are presented in [7].

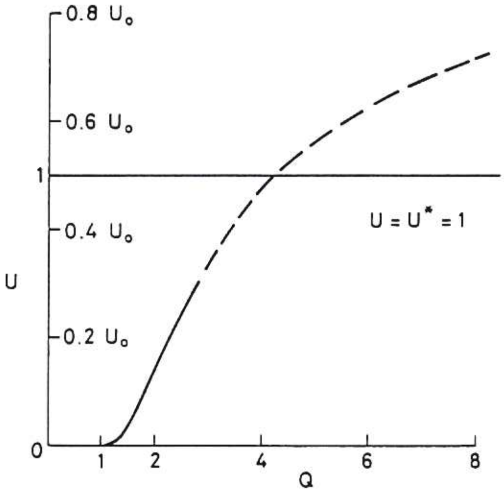


Fig.3 The fall velocity U as a function of heating rate Q when the Stokes velocity $U_o = 2$. For $p > 1$ and $S < 1$, $U^* = 1$. The solution is valid for $U \ll U^*$.

Note also that whereas the Stokes analysis for an isolated sphere in an infinite medium is only valid for $U_o << p$, the analysis in §2.2B is valid for $\varepsilon^3 U_o << p$ i.e. it is the gap size which is restricted, not the particle size.

3. THE ABLATION OF A PLANE SLAB

The one-dimensional melting of a slab of finite thickness, when the melt is instantaneously removed, is a classical problem for which accurate numerical solutions are available and for which uniqueness has been established [8]. A number of different regimes may be discerned, depending on the governing parameters, and approximate methods may be used in those which enable the functional dependence of the solution on the characteristic dimensionless parameters to be clarified. One such method "*the pseudo-separation of variables*" is discussed and used in this context below in §3.3.

3.1. Formulation and Melting Regimes

If a normalization is chosen so that the slab is initially of unit thickness and zero temperature (equal to that of the coolant), that the melting point of the slab material is unity, and that time is measured in units of the thermal diffusion time for the slab, then the equations for this one phase Stefan problem are

Fourier Equation : $\theta_\tau = \theta_{xx} \quad X(\tau) < x < 1$ (3.1)

Front face : $\Psi = Su - \theta_x$ (3.2a), $\theta = \theta_o(\tau)$ (3.2b), at $x = X(\tau)$, $\tau \geq 0$

Back face: Newton's law of cooling : $\theta_x + H\theta = 0$ (3.3)

Initial Conditions : $\theta = 0 \quad 0 < x < 1, \tau = 0$ (3.4)

$X(\tau)$ is the location of the front face of the solid, and $u \equiv dX/d\tau$. During the initial heat-up phase ($0 \leq \tau \leq \tau_p$), θ_o is below the melting point, $X = 0$, and $u = 0$. For $\tau > \tau_p$, $\theta_o \equiv 1$ and melting occurs so that $X > 0$ and $u > 0$. The configuration is shown in fig.4.

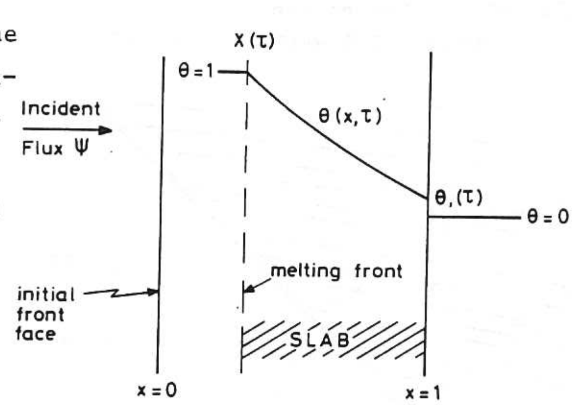


Fig.4 The configuration for the melting of a plane slab.

From these governing equations it is apparent that θ and X have the functional forms $\theta \equiv \theta(x,\tau;\Psi,H,S), X \equiv X(\tau;\Psi,H,S)$ where Ψ is the dimensionless heat flux at the front face, H is the Biot number i.e. the dimensionless heat transfer coefficient at the rear face, and S is the Stefan number, the dimensionless latent heat of melting. The time for melt-through τ_m is given functionally by $\tau_m \equiv \tau_m(\Psi,H,S)$.

When the slab has completely melted, the sensible and latent heat absorbed by the slab is $1 + S$. The rate at which heat is conducted into the coolant is $H\theta_1$ where $\theta_1(\tau)$ is the temperature at the rear face. The energy balance at melt-through thus gives

$$\Psi . \tau_m = 1 + S + H \int_o^{\tau_m} \theta_1(\tau) d\tau \tag{3.5}$$

If $Q \equiv \Psi/(1+S)$, then $\tau_m = Q^{-1}$ for $H = 0$. However there are some ranges of Ψ and H for which τ_m does not exist. It is straightforward to show [9] (i) melting can only occur if and only if $\Psi > H/(H+1)$ (ii) complete melting can only occur if and only if $\Psi > H$ (iii) when equilibrium is reached with only partial melting, the fraction melted $\gamma = 1 + H^{-1} - \Psi^{-1}$.

Numerical integration of (3.1) - (3.4) has been carried out using the MELTIN code [10] which uses Crank-Nicolson with front tracking. Sufficiently small space and time steps have been used to assure the accuracy of the method. Fig.5 shows temperature profiles when $S = 1$ and $H/\Psi = 0.25$. These illustrate clearly the marked difference in

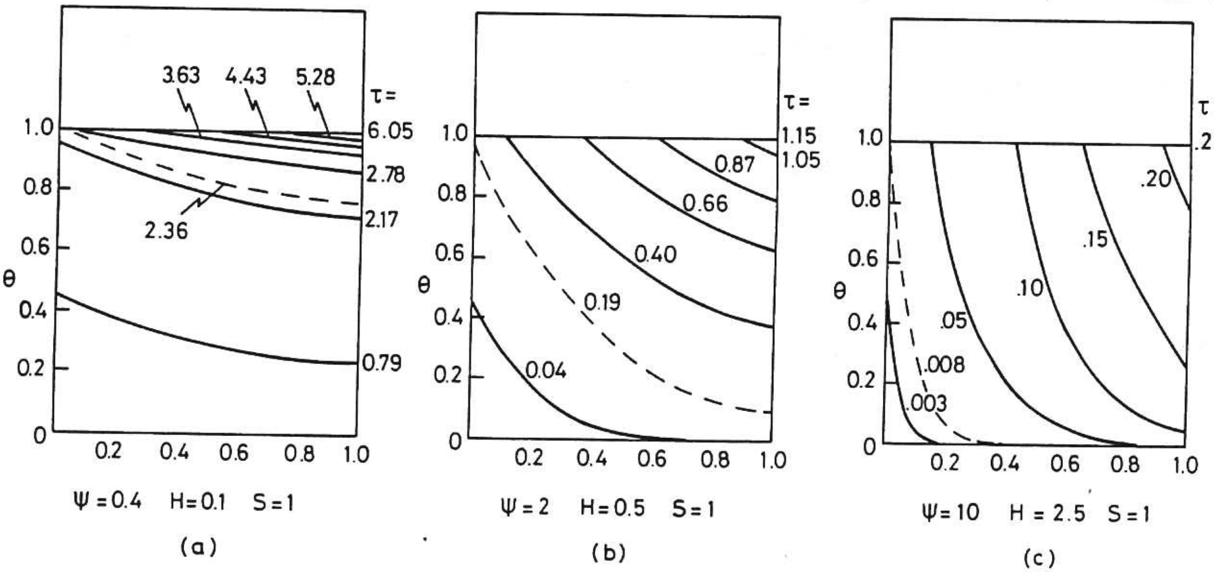


Fig.5 Evolution of temperature profiles for 3 cases when $H/\psi = 0.25$.

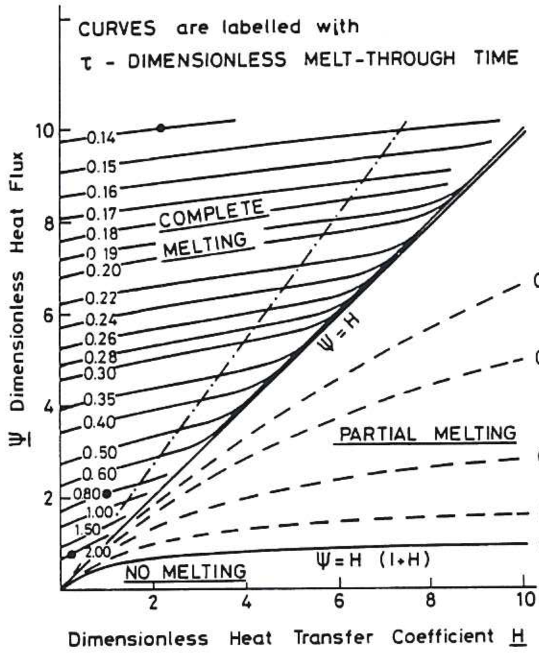


Fig.6 Dependence of melt through time τ_m on H and ψ. The dashed curves (---) indicate the fractions melted when melting is incomplete. The line (- - -) is $\psi = H(1+S)$.

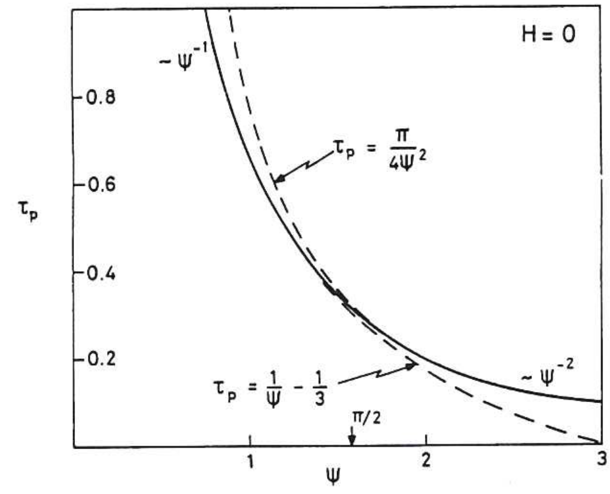


Fig.7 Time to first melting when H = 0.

the evolution of the temperature profiles when $\Psi << 1$ and $\Psi >> 1$. The dashed profile shows the temperature distribution when melting first begins. A parametric study was carried out [9] using MELTIN for S = 0.37, a value appropriate for steel. The lines of constant melt-through time ('isochrones') obtained therein are reproduced in Fig.6.

3.2. Premelting

Prior to melting, the temperature profile can be calculated as a Fourier sum [11,p125.] The time τ_p before melting is implicitly given by

$$\gamma = \sum_{n=o}^{\infty} \beta_n \exp(-\alpha_n^2 \tau_p) \qquad (3.6)$$

where $\gamma = 1 + H^{-1} - \Psi^{-1}$ as above, $\beta_n = 2(\alpha_n^2 + H^2)/\{\alpha_n^2(H + H^2 + \alpha_n^2)\}$ and the α_n are the roots of $\alpha \tan \alpha = H$ [11,p491].

When $H \to 0$ and $\Psi \lesssim \pi/2$, then τ_p is sufficiently large for all except the first term in the Fourier sum to be negligible and the profile at the onset of melting is essentially parabolic (see also [12]).

$$\theta = \Psi\{\frac{1}{3} - x) + (\frac{x^2}{2} + \tau_p)\} = 1 + \Psi[\frac{x^2}{2} - x] \qquad (3.7)$$

where $$\tau_p = \frac{1}{\Psi} - \frac{1}{3} : \qquad \Psi \lesssim \pi/2 , \ H = 0 \qquad (3.8)$$

When $\Psi >> 1$, the rapid temperature rise prior to melting occurs only in a thin boundary layer and is thus independent of H. The temperature profile for an infinite half space [11,p75] is appropriate in this case. It follows that

$$\tau_p = \pi/(4\Psi^2) \qquad \Psi \gtrsim \pi/2, \quad H = 0 \qquad (3.9)$$

The heat-balance integral method [12] underestimates (3.9) by $\sim$ 15%, giving $2/(3\Psi^2)$. Fig.7 shows that (3.8) and (3.9) together cover the whole range of Ψ and match well with a difference of $\sim$ 5% at $\Psi = \pi/2$.

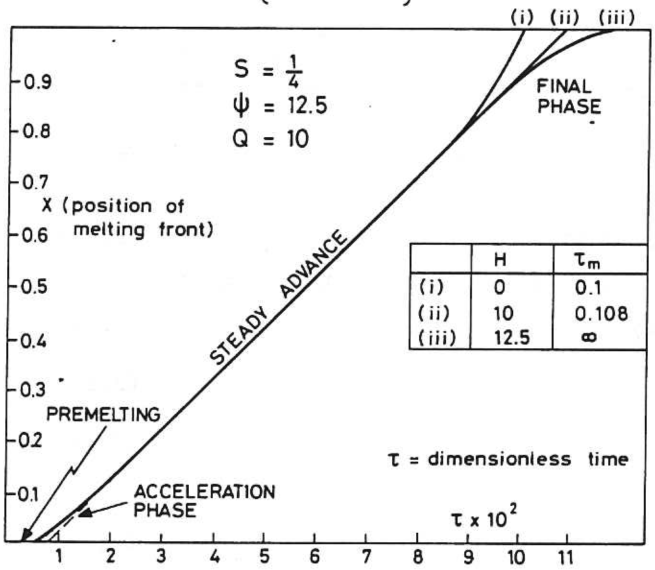
3.3. High Heat Flux: $Q >> 1$.

It is convenient to renormalize into variables $z \equiv Q(1-x)$, $\sigma = Q^2\tau$, $U \equiv dZ/d\sigma$ where Z is the position of the melting front. Then (3.1) to (3.4) become

$$\left.\begin{aligned} &\theta_\sigma = \theta_{zz} \quad 0 < z < Z(\sigma); \; \theta_z = \mu\theta \text{ on } z = 0; \\ &\theta = 1 \text{ and } 1 + S = -SU + \theta_z \text{ on } z = Z; \end{aligned}\right\} \quad (3.10)$$

where $\mu \equiv H/Q$, the slab thickness is now Q, and the 'applied flux' is 1+ S. In this normalization the time σ_m to melt completely a slab with insulated rear face (μ=0) is Q.

When Q is large, the melting of the slab can be divided into four stages. (i) premelting, lasting $\left\{\pi/4(1+S)^2\right\} < 1$ - see above; (ii) an acceleration stage during which the melting front speed increases to its steady value. This has been analysed when $S >> 1$, by Andrews and Atthey [13] and their solution shows that $|U|$ approaches 1 when $\sigma \sim 1$. The solution by Landau [14,15] confirms this for the full range of S. (iii) a period $\sim$ Q during which the front advances steadily towards the rear face (iv) a final stage in which the rear boundary condition exerts a significant influence. This stage can be rather short if $H \to 0$, or infinite as $H \to \Psi$. These phases are clearly discernible in Figure 8 for Q = 10. We are concerned here with the analysis of the final stage.



	H	τ_m
(i)	0	0.1
(ii)	10	0.108
(iii)	12.5	∞

Fig.8 Position of melting front as a function of time.

An important special case is $\mu = 1$ (i.e. $H = \Psi/(1+S)$). After a time period $\sigma \sim O(1)$, the front will have accelerated and the temperature profile relaxed to

$$\theta = \exp(z - Z) \quad (3.11)$$

giving U = -1. This profile satisfies the rear boundary condition at all times when $\mu = 1$, and so the speed of the front is unchanged as it approaches the rear face. The normalized enthalpy in the thermal front is 1 (unity), and this is equal to that conducted through the rear face prior to complete melting. Thus

$$(1 + S)\sigma_m = Q(1 + S) + 1 \qquad (\mu = 1) \qquad (3.12)$$

For general μ, the method of *pseudo-separation of variables* may be used. In this we write $\theta = \theta(\sigma_1 z) = G(\sigma) F(nz)$ where $n = n(\sigma)$ is a slowly varying function of time, to be determined. Substitution into the Fourier equation (3.10) gives $G_\sigma/G + F_\sigma/F = F_{zz}/F$ or

$$G_\sigma/(n^2G) + (n_\sigma z/n^2)F_y/F = F_{yy}/F \quad (3.13)$$

where $y \equiv nz$. Provided the second term on the left hand side is negligible then a kind of separable solution can be obtained. Introduction of a separation constant C^2, gives $F \sim \exp(\pm Cnz)$. Since n is to be determined, C can be absorbed in n and chosen to be unity. The second term is then negligible provided $n_o z/n^2 << 1$.

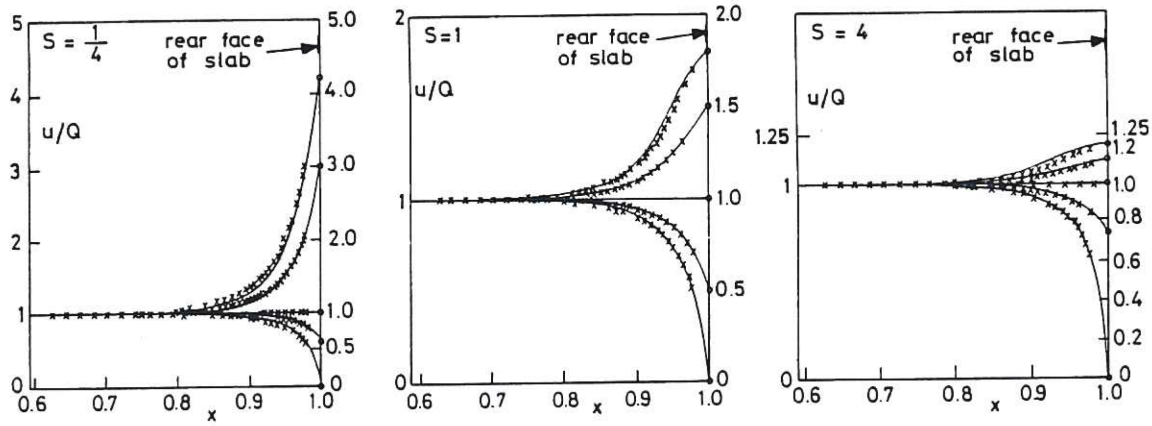


Fig.9 Plots of the melt front velocity against position. The initial position of the front face is x = 0. Smooth curves are from (3.20); for comparison the accurate results from MELTIN are denoted by xxx.

In order to satisfy the rear boundary condition at all times, the following Ansatz is made for θ:

$$\theta = \theta_1(\sigma)\left\{\cosh(nz) + (\mu/n)\sinh(nz)\right\} \tag{3.14}$$

where θ_1 can be identified as the temperature at the rear face. At the melt-front

$$1 = \theta_1\left\{\cosh(nZ) + (\mu/n)\sinh(nZ)\right\} \tag{3.15}$$

Let
$$\Gamma = \frac{\sinh(nZ) + (\mu/n)\cosh(nZ)}{\cosh(nZ) + (\mu/n)\sinh(nZ)} \tag{3.16}$$

then
$$\theta_z = n\Gamma \text{ at } z = Z \tag{3.17}$$

and the Stefan condition becomes

$$1 + S + SU = n\Gamma \tag{3.18}$$

Since $\theta = 1$ at the front at all times then in the frame of reference of the front, $U\theta_\zeta + \theta_{\zeta\zeta} \to 0$ as $\zeta \to 0$ where $\zeta = z - Z$. Insertion of the ansatz (3.14) gives

$$Un\Gamma + n^2 = 0 \tag{3.19}$$

The 3 equations (3.16), (3.18), (3.19) can be manipulated into the form

$$\left.\begin{aligned} U &= -(1 + S)/(\Gamma^2 + S) \\ n &= -U\Gamma \\ Z &= (1/2n)\log\left(\left\{\frac{\Gamma+1}{\Gamma-1}\right\}\left\{\frac{\mu-n}{\mu+n}\right\}\right) \end{aligned}\right\} \tag{3.20}$$

which for fixed S and μ give U and Z parametrically in terms of Γ. Fig.9 shows U as a function of Z for a range of values of μ. The smooth curves have been calculated using (3.20), the 'crosses' are the exact solution calculated using MELTIN [10]. The agreement in general is good. It is simple to confirm that those solutions which differ from the true ones to some significant extent are precisely those where the coupling terms in (3.13) is not negligible. The steadily advancing solution corresponds to $\mu = \Gamma = n = -U = 1$ giving Z indeterminate. When $\mu \neq 1$, the solution far from the rear face is also a steady advance : $\Gamma = n = -U = 1$.

When U is small, expansions in the small parameter Γ^{-1} are appropriate. For n and U, only the first term is needed; for Z it is necessary to keep the first two terms. Elimination of n and Γ gives the approximate formula :

$$Z + \sigma_a U = Q(1 - \gamma) \equiv Z_o \quad (3.21)$$

where $\sigma_a \equiv \{S\Delta + (1 - \Delta^3)/3\}/(1 + S)^2$ and $\Delta \equiv \Psi/H$. The solution of (3.21) is

$$Z = Z_o + B \exp(-\sigma/\sigma_a) \quad (3.22)$$

where B is a positive constant determined by matching to the appropriate solution at earlier times. When $Z_o < 0$, the rear face is reached in finite time with velocity $U = Z_o/\sigma_a$. For $Z_o > 0$, the slab reaches equilibrium when $Z = Z_o$ after a logarithmically infinite time with only the fraction γ melted. When $Z_o = 0$, both U and Z decay exponentially, and the slab takes an infinite time to melt completely. When Z_o is small and negative i.e. Ψ is close to H, the dominant contribution to the time for melt-through is

$$\sigma_m \sim - \left\{S/(1 + S)^2\right\}\log(\Psi - H) \quad (3.23)$$

Thus the Ansatz (3.14) leading to equations (3.20) gives a good representation of the solution as the melting front approaches the rear face, except when μ is small.

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REFERENCES

[1] Drake L D and Shreve R L (1973) Pressure melting and regelation of ice by round wires, Proc. Roy. Soc. A 332, 51-83.

[2] Townsend D W and Vickery R P (1967), An experiment in regelation, Phil.Mag.16, 1275-1280.

[3] Evans D C B, Nye J F and Cheeseman K J (1976) The kinetic friction of ice, Proc. Roy.Soc.A 347, 493-512.

[4] Batchelor G K (1967) An Introduction to Fluid Dynamics. Cambridge U.P.

[5] Nye J F (1967) Theory of regelation, Phil.Mag.16, 1249-1266.

[6] Landau L D and Lifshitz E M (1959) Fluid Mechanics, Pergamon Press, p70.

[7] Peckover R S, Dullforce T A and Turland B D (1979) Models of core melt behaviour after a postulated reactor vessel melt-through, in Proc.Conf. on "Fast Reactor Safety Technology", American Nuclear Soc. pp 2150-2171.

[8] Boley B A (1978) in Moving Boundary Problems (ed. by Wilson D G, Solomon A D and Boggs P T) Academic Press, pp 205-231.

[9] Palentine J E and Turland B D (1980), "The melting of horizontal stainless steel plates subjected to constant flux thermal loads" Proc. of the ICHMT Conf. on Nuclear Reactor Safety Heat Transfer. Hemisphere Press.

[10] Turland B D and Peckover R S(1981) MELTIN: A 1D Melting code for plane slabs (UKAEA Report).

[11] Carslaw H S and Jaeger J C (1959) Conduction of heat in solids, Clarendon Press, Oxford, 2nd Edition 510pp.

[12] Goodman T R and Shea J J (1960), The melting of finite slabs, J.Appl.Mechanics 27, 16-24.

[13] Andrews J G and Atthey D R (1975) Analytical and numerical techniques for ablation problems, in Moving Boundary Problems in Heat Flow and Diffusion (ed.Ockenden J R and Hodgkins W R) Clarendon Press, Oxford, pp.38-53.

[14] Landau H G (1950) Quart.Appl.Maths 8, 81-94.

[15] Bankoff S G (1964) Heat conduction or diffusion with change of phase, Advances in Chemical Engineering 5, 75-150.