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Abstract

The two-fluid equations are used to investigate a turbulence interpretation of transport in tokamaks. Radial energy balance equations are derived with mean energy fluxes and sources expressed in terms of correlations between the various fluctuating quantities. Ambipolarity and plasma diffusion are discussed, the latter in relation to tokamaks with negligible particle sources. It is argued that turbulence cannot significantly affect toroidal resistivity, and hence the present analysis is consistent with the experimentally 'observed' Spitzer value. Application to MACROTOR suggests that other gross features, including the energy confinement time, can also be qualitatively interpreted.

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1. INTRODUCTION

In this paper we continue our investigations of a two-fluid turbulence interpretation of transport in tokamaks (Thyagaraja et al. 1980; Cook et al. 1982). Appropriately averaging over the electron and ion energy equations, we have previously derived general forms for the mean radial energy transfer equations for both fluids (Haas et al, 1981); these lead to formal expressions for the heat fluxes in terms of correlations between the various fluctuating quantities. The analysis suggests that the radial anomalous electron thermal conduction can be interpreted in terms of temperature and magnetic fluctuations, both of which are known to occur in present tokamaks, and furthermore, that parallel electron thermal conduction is crucially involved. To proceed further, we considered a simple model in which density and velocity fluctuations were neglected (Haas et al 1981). With these assumptions and a suitably 'corrected' form for the parallel electron thermal conductivity, we demonstrated the effective radial electron thermal conductivity to be of the order of the experimental value for typical fluctuation levels. Our work showed that the enhanced thermal conductivity arises from the phase lag between magnetic and electron temperature fluctuations due to the parallel thermal conduction, (Haas et al. 1981). Furthermore, our results suggested that only frequencies of order 30-100 kHz could significantly affect the enhancement. Due to the large mass difference between electrons and ions, however, our work indicated the ion thermal conduction to be essentially unenhanced, thus supporting the belief that the ions behave neoclassically (Thyagaraja et al. 1980).

Our previous work suggests, therefore, that a two-fluid turbulence theory could plausibly interpret the transport phenomena observed in tokamaks. The neglect of density and velocity fluctuations, however, is a serious omission. The principal objective of the present paper is to remedy this defect, thus allowing our investigation to cover convection, turbulent heating, particle and momentum transport. Following a previous study of the dissipationless system (Thyagaraja and Haas, 1982), we linearise the full set of two-fluid equations: continuity, momentum, energy and Maxwell equations. The energy equations, however, now include both electron and ion parallel thermal conduction as a dissipative (and hence phase-shift producing) mechanism. Manipulation of the linearised relations leads to forms for the turbulent heating and particle and energy fluxes in terms of the power and wave-number spectrum of any one fluctuating quantity, which we shall assume to be known from experiment.

The results of the above calculational procedure will be used to provide at least a qualitative interpretation of some of the grosser experimental aspects of transport in tokamaks. These can be summarised as follows:

(1) The electron perpendicular thermal diffusivity is of order (0.5 to 5.0) x 10^4 cm^2 sec^{-1} for a wide variety of tokamaks.

(2) The ion perpendicular thermal diffusivity is a few times neoclassical, although with neutral injection heating the factor can be larger. In high density machines such as FT and ALCATOR it has been claimed that the ion thermal diffusivity is almost neoclassical.

(3) Particle transport is believed to be ambipolar in the central regions at least. Although difficult to measure, there appears to be some consensus (Coppi and Sharky, 1981; Strachan et al, 1982) that the particle flux can be written as the sum of an outward diffusive flux and an inward convective flux. This assumption, together with a diffusivity of order 10^4 cm^2 sec^{-1} and an inward velocity of order 10^2 to 10^3 cm sec^{-1}, allows a realistic simulation of density profile both with and without gas-puffing. Remarkably, it is even asserted that the above form of particle flux will interpret impurity transport (The ASDEX team, 1981) and the diffusion of added trace elements (Stodiek et al, 1980; Chrien et al, 1981).

(4) Present tokamak experiments suggest that the toroidal resistivity is essentially that given by the Spitzer value.

In section 2 of this paper we describe the underlying philosophy of our method. Section 3 contains the full set of linearised two-fluid equations. These are used to obtain relations which we subsequently use in deriving explicit expressions for the various energy and particle fluxes; the details of the eliminations are given in an appendix. In section 4 we discuss ambipolarity in relation to turbulence. This leads to an interpretation of the 'phenomenological' simulations of plasma diffusion (Coppi and Sharky, 1981). We follow this with a rederivation of the radial mean energy balance equations. Using results obtained in Section 3 we are able to derive a relation between the electron heat flux and the electron particle flux. Section 5 comprises a discussion of some features of our analysis, together with an application to MACROTOR (Zweben et al, 1979). Section 6 contains our conclusions.

2. THEORETICAL BASIS OF CALCULATIONS.

For our present purpose it is sufficient to consider a cylindrical model of tokamak with minor radius a and periodicity length $2\pi R$. When required we shall use cylindrical coordinates r, θ, z, based on the axis of the cylinder. Our analysis employs the two-fluid equations, namely, electron and ion continuity, momentum and energy, as well as Ampere's and Faraday's equations. These comprise fifteen independent equations for the variables $n_e(\underline{r},t)$, $T_e(\underline{r},t)$, T_i, $\underline{u}_e$, $\underline{u}_i$, $\underline{E}$ and $\underline{B}$, where the symbols have their usual meanings; note quasi-neutrality is assumed throughout. Furthermore, we assume all sources of mass, momentum and energy to be steady. The linear relations between the unknowns are given in Section 3. In this section, however, we give a very general discussion of our underlying procedure.

If we denote the physical variables $n_e(\underline{r}, t)$, $\underline{u}_e(\underline{r},t)$ etc. by the general symbol W, then their time evolution is described by

$$\frac{dW}{dt} = F(W,S), \qquad (1)$$

where F is a complicated operator which depends on W and its space derivatives. The quantity S represents steady external source parameters such as total current, auxiliary heating etc. Experiment suggests that $W(\underline{r},t)$ can be written as

$$W(\underline{r}, t) = \langle W\rangle + \delta W(\underline{r}, t) , \qquad (2)$$

where the average of any quantity $Q(r, t)$ is defined by

$$\langle Q\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d\theta \, \frac{1}{2\pi R}\int_0^{2\pi R} dz \lim_{t_M \to \infty} \frac{1}{t_M}\int_0^{t_M} Q \, dt \tag{3}$$

with t_M denoting the shot-time. Thus it follows that $< W >$ is a function of r only. Combining Eqs. (1) and (2) we derive

$$\frac{d}{dt}\,\delta W = F(\langle W\rangle + \delta W,\ S), \tag{4}$$

which on averaging, lead

$$0 = \langle F(\langle W\rangle + \delta W,\ S)\rangle \ . \tag{5}$$

These are the equations which determine $\langle W\rangle$ in terms of S and the various cross-correlations of δW. Returning to Eq.(4), we separate the right hand side into terms linear in δW and collect all higher order terms into a "source" Σ .

Thus $$\frac{d}{dt}\,\delta W = \frac{\partial F(\langle W\rangle,\ S)}{\partial\langle W\rangle}\,\delta W + \Sigma\,(\langle W\rangle, \delta W, S). \tag{6}$$

Note that $\frac{\partial F}{\partial\langle W\rangle}$ is a symbolic expression for the linear operator acting on δW; the linear part is, in general, dependent on both δW and its spatial derivatives. Since $\langle\delta W\rangle = 0$ it follows that $\langle\Sigma\rangle$ also vanishes.

In general, the correlations of δW in Eq.(5) can only be calculated when the complete solution to the non-linear equation, Eq.(6), is fully known. Since this involves a knowledge of $< W >$, our problem, is at the very least, extremely difficult. Given that a self-consistent solution to the above problem exists, in order to make further progress we make two important assumptions. First, taking $\langle W\rangle$ as known from experiment, we assume that the correlations of δW can be derived using only the linear terms in Eqs. (6). A necessary condition for this is that $|\delta W| \ll |\langle W\rangle|$. Second, we assume that the spatial variation and power spectrum of a particular component of δW - the radial magnetic fluctuation say - is completely known from experiment. It follows that if overdeterminacy is to be avoided then one equation of the linear system (d/dt δW = linear terms) must be discarded. In the remaining linear equations this known fluctuation appears as a driving source and all other fluctuations can be determined in terms of it. Thus the correlations and fluxes which occur in Eq.(5) can then be expressed entirely in terms of the power spectrum of the assumed fluctuation.

We now discuss the implications of the above procedure. We remark that even if $\langle W \rangle$ is completely known the linearised equations alone cannot possibly provide an explanation of experiment. The reasons are obvious: a linear dissipative system has in general, only amplifying and decaying solutions, and furthermore, the amplitudes are themselves indeterminate. Thus it is the non-linear source Σ which effectively saturates any instabilities and leads to a final steady state of stationary turbulence. Although non-linear terms are essential in maintaining stationary turbulence, our investigations indicate that these terms enter mainly through the momentum equations, and are due to the Lorentz force. This suggests that the momentum balance be used to determine the unknown non-linear momentum sources, Σ. Thus it is the total momentum equation which we choose to discard. With a complete specification of the magnetic fluctuations (say) the linear equations can be used to express all other fluctuating quantities in terms of $\frac{\delta B_r}{B_o}$. With these relations, all the correlations required by our theory are now calculable. It should be remarked, however, that phase-shifts between the various fluctuations are essential for the occurrence of non-zero particle and energy fluxes. The various dissipative terms (electron and ion thermal conduction, viscosity, resistivity etc.) which are present in the full two-fluid equations can provide mechanisms for such phase-shifts. Investigation suggests, however, that for frequencies ($\lesssim$ 0.5 MHz) and modes (m $\lesssim$ 100) of interest, the electron and ion parallel thermal conductivities are the most important phase-shift producing effects. Thus for simplicity we neglect all other dissipative terms.

Although the above procedure is crude, it should be good enough to yield results which are at least qualitatively correct. A completely satisfactory investigation of our two-fluid turbulence interpretation of transport requires the solution of the full set of non-linear equations. This problem, which is currently under consideration, should give the spatial dependences and frequency spectra of all fluctuating quantities. Evaluation of the corresponding steady particle and energy fluxes would then indicate the validity or otherwise of the fluid turbulence interpretation.

3. THE LINEAR EQUATIONS

In this section, we consider the linearised equations of our quasi-neutral two-fluid system. Following convention, we denote ions and electrons by the suffices i and e, and equilibrium quantities by the suffix zero. In equilibrium, the ions are assumed to be at rest* and all equilibrium quantities are taken to be functions of r alone. Thus the current density, $\underline{j}_o(r)$, is given by

$$\underline{j}_o(r) = - e n_o(r) \; \underline{u}_{eo}(r), \tag{7}$$

and the pressure balance is

$$\frac{dp_o}{dr} = \frac{1}{c} \left(j_{o\theta} B_{oz} - j_{oz} \; B_{o\theta} \right) \tag{8}$$

where p_o is the total plasma pressure, $p_o = p_{eo} + p_{io}$. We write every fluctuating quantity, δf, in the form

$$\delta f = \delta \hat{f}(r) \exp \left[i(\omega t + m\theta + nz/_R) \right] , \tag{9}$$

where for convenience we suppress the frequency and mode (m,n) dependence of the amplitudes.

For the ions, the components of the linearised momentum equation

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* This assumption is not essential and is only made for mathematical simplicity.

are

$$m_i n_o i\omega\ \delta\hat{u}_{ir} = en_o\left(\delta\hat{E}_r + \frac{\delta\hat{u}_{i\theta}B_{oz}}{c} - \frac{\delta\hat{u}_{iz}B_{o\theta}}{c}\right) - \frac{d}{dr}\delta\hat{p}_i + eE_{or}\delta\hat{n} \tag{10}$$

$$m_i n_o i\omega\ \delta\hat{u}_{i\theta} = en_o\left(\delta\hat{E}_\theta - \frac{\delta\hat{u}_{ir}B_{oz}}{c}\right) - \frac{im}{r}\delta\hat{p}_i \tag{11}$$

$$m_i n_o i\omega\ \delta\hat{u}_{iz} = en_o\left(\delta\hat{E}_z + \frac{\delta\hat{u}_{ir}B_{o\theta}}{o}\right) - \frac{in}{R}\delta\hat{p}_i \tag{12}$$

and the continuity equation is

$$i\omega\delta\hat{n} + n_o\nabla\cdot\underline{\delta\hat{u}_i} + \delta\hat{u}_{ir}\frac{dn_o}{dr} = 0, \tag{13}$$

where

$$\nabla\cdot\underline{\delta\hat{u}_i} \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\delta\hat{u}_{ir}\right) + \frac{im}{r}\delta\hat{u}_{i\theta} + \frac{in}{R}\delta\hat{u}_{iz} \tag{14}$$

Following the discussion of Section 2, the ion energy balance equation is taken to be

$$\frac{3}{2}n_o\left(i\omega\delta\hat{T}_i + \delta\hat{u}_{ir}\frac{dT_{io}}{dr}\right) + n_oT_{io}\ \nabla.\underline{\delta\hat{u}_i}$$

$$= ik_\parallel K_{\parallel i}\left(ik_\parallel\delta\hat{T}_i + \frac{\delta\hat{B}_r}{B_o}\frac{dT_{io}}{dr}\right), \tag{15}$$

where

$$k_\parallel = \frac{mB_{o\theta}}{rB_o} + \frac{nB_{oz}}{RB_o},$$

$$B_o^2 = B_{o\theta}^2(r) + B_{oz}^2(r)$$

Neglecting electron inertia, the components of the electron momentum balance are

$$0 = -en_o\left(\delta\hat{E}_r + \frac{\delta\hat{u}_{e\theta}}{c}B_{oz} + \frac{u_{eo\theta}}{c}\delta\hat{B}_z - \frac{\delta\hat{u}_{ez}}{c}B_{o\theta} - \frac{u_{eoz}}{c}\delta\hat{B}_\theta\right) - \frac{d}{dr}\delta\hat{p}_e - e\delta\hat{n}\ E_{or} \tag{16}$$

$$0 = - en_o \left(\delta\hat{E}_\theta - \frac{\delta\hat{u}_{er}}{c} B_{oz} + \frac{u_{eoz}}{c} \delta\hat{B}_r \right) - \frac{im}{r} \delta\hat{p}_e \tag{17}$$

$$0 = - en_o \left(\delta\hat{E}_z - \frac{u_{eo\theta}}{c} \delta\hat{B}_r + \frac{\delta\hat{u}_{er}}{c} B_{o\theta} \right) - \frac{in}{r} \delta\hat{p}_e \tag{18}$$

and the continuity equation is

$$i\omega\delta\hat{n} + \underline{u}_{eo} \cdot \nabla\delta\hat{n} + n_o \nabla \cdot \delta\hat{\underline{u}}_e + \delta\hat{u}_{er} \frac{dn_o}{dr} = 0 . \tag{19}$$

The appropriate energy balance equation is

$$\frac{3}{2} no \left(i\Omega\delta\hat{T}_e + \delta\hat{u}_{er} \frac{dT_{eo}}{dr} \right) + n_o T_{eo} \nabla \cdot \delta\hat{\underline{u}}_e$$

$$= ik_{\parallel} K_{\parallel e} \left(ik_{\parallel} \delta\hat{T}_e + \frac{\delta\hat{B}_r}{B_o} \frac{dT_{eo}}{dr} \right) \tag{20}$$

where $\Omega = \omega + \frac{m}{r} u_{eo\theta} + \frac{n}{R} u_{eoz}$.

These equations must, of course, be supplemented by those of Maxwell. Thus the radial and θ- components of Ampere's law are

$$\frac{im}{r} \delta\hat{B}_z - \frac{in}{R} \delta\hat{B}_\theta = \frac{4\pi en_o}{c} (\delta\hat{\mu}_{ir} - \delta\hat{u}_{er}) \tag{21}$$

$$\frac{in}{R} \delta\hat{B}_r - \frac{d}{dr} \delta\hat{B}_z = \frac{4\pi en_o}{c} (\delta\hat{u}_{i\theta} - \delta\hat{u}_{e\theta}), \tag{22}$$

and the three components of Faraday's law are

$$\frac{im}{r} \delta\hat{E}_z - \frac{in}{R} \delta\hat{E}_\theta = - \frac{i\omega}{c} \delta\hat{B}_r \tag{23}$$

$$\frac{in}{R} \delta\hat{E}_r - \frac{d}{dr} \delta\hat{E}_z = - \frac{i\omega}{c} \delta\hat{B}_\theta \tag{24}$$

$$\frac{1}{r}\frac{d}{dr}(r\delta\hat{E}_\theta) - \frac{im}{r}\delta\hat{E}_r = -\frac{i\omega}{c}\delta\hat{B}_z . \qquad (25)$$

Noting that $p_e = nT_e$ and $p_i = nT_i$, the above independent equations suffice to determine the fifteen unknowns $\delta\hat{n}$, $\delta\hat{T}_e$, $\delta\hat{T}_i$, $\delta\hat{\underline{u}}_e$ $\delta\hat{\underline{u}}_i$, $\delta\hat{\underline{E}}$ and $\delta\hat{\underline{B}}$. Before proceeding, however, it is useful to define the following quantities

$$\left.\begin{aligned}
\hat{\xi} &= \frac{1}{k_\parallel} \cdot \frac{\delta B_r}{B_o} \\
\delta\hat{p}(r) &\equiv \delta\hat{p}_i(r) + \delta\hat{p}_e(r) \\
\Delta_i &\equiv i\,\frac{\delta\hat{T}_i}{T_{oi}} + \frac{1}{T_{oi}}\frac{dT_{oi}}{dr}\hat{\xi} \\
\Delta_e &\equiv i\,\frac{\delta\hat{T}_e}{T_{oe}} + \frac{1}{T_{oe}}\frac{dT_{oe}}{dr}\hat{\xi} \\
\Delta_n &\equiv i\,\frac{\delta\hat{n}}{n_o} + \frac{1}{n_o}\frac{dn_o}{dr}\hat{\xi} \\
\Delta_p &\equiv i\,\frac{\delta\hat{p}}{p_o} + \frac{1}{p_o}\frac{dp_o}{dr}\hat{\xi}
\end{aligned}\right\} \qquad (26)$$

where $p_o = n_oT_o$ with $T_o(r) = T_{oe}(r) + T_{oi}(r)$. Carrying through the eliminations (see Appendix) we find the equations to reduce to a coupled first and second order differential equation in the variables $\hat{\zeta}$ and Δ_p. These equations correspond to forms of the ion continuity and total momentum balance, respectively. As we described in Section 2, however, we assume a single fluctuating quantity to be known as a function of r, ω, m, n, and discard the total momentum equation. We now summarise the linear relations which we shall require to evaluate the various correlations.

Thus we have

$$\delta\hat{u}_{er} = \Omega\hat{\xi} \tag{27}$$

and

$$\delta\hat{u}_{ir} = \omega\hat{\xi} + \frac{p_o}{p_o'} \omega^*\Delta_p + 0\left(\frac{\omega}{\omega_{ci}}\right), \tag{28}$$

where the diamagnetic frequency ω^* is defined to be

$$\omega^* = -\frac{cT_o}{eB_o} \cdot \frac{p_o'}{p_o}\left(\frac{mB_{oz}}{rB_o} - \frac{nB_{o\theta}}{RB_o}\right), \tag{29}$$

the prime denoting differentiation with respect to r. With the definitions of Eq (26), the electron and ion energy balance equations can be expressed very simply as

$$\Delta_e = \frac{\frac{2}{3}\Delta_n}{1 - \frac{2}{3} i \frac{K_{\|e} k_\|^2}{n_o\Omega}} \tag{30}$$

and

$$\Delta_i = \frac{\frac{2}{3}}{1 - \frac{2}{3} i \frac{K_{\|i}}{n_o}\frac{k_\|^2}{\Omega}} \cdot \left\{\Delta_n - \frac{\omega^*}{\omega}\frac{p_o}{p_o'}\Delta_p\left(\frac{3}{2}\frac{T_{oi}'}{T_{oi}} - \frac{n_o'}{n_o}\right)\right\} \tag{31}$$

We also note that the definitions of Eq (26) imply the identity

$$\Delta_p = \Delta_n + \frac{T_{oi}}{T_o}\Delta_i + \frac{T_{oe}}{T_o}\Delta_e \tag{32}$$

From Eqs. (30), (31) and (32) we deduce that

$$\Delta_p = \frac{\omega}{\omega^*} \cdot \frac{p_o'}{po} G(r, \omega)\, \Delta_n \tag{33}$$

where

$$G(r,\omega) = \frac{\dfrac{p_o}{p_o'}\dfrac{\omega^*}{\omega}\left[1 + \dfrac{\dfrac{2}{3}\dfrac{T_{oi}}{T_o}}{1 - \dfrac{2}{3}\, i\, \dfrac{K_{\parallel i}}{n_o\omega}\, k_\parallel^{\,2}} + \dfrac{\dfrac{2}{3}\dfrac{T_{oe}}{T_o}}{1 - \dfrac{2}{3}\, i\, \dfrac{K_{\parallel e}}{n_o\Omega}\, k_\parallel^2}\right]}{1 + \dfrac{\dfrac{2}{3}\dfrac{T_{oi}}{T_o}}{1 - \dfrac{2}{3} i \dfrac{K_{\parallel i}}{n_o\omega} k_\parallel^2}\cdot\dfrac{\omega^*}{\omega}\dfrac{p_o}{p_o'}\left(\dfrac{3}{2}\dfrac{T_{oi}'}{T_{oi}} - \dfrac{n_o'}{n_o}\right)} \tag{34}$$

The equation of continuity for the ions then gives

$$\frac{1}{r}\frac{d}{dr}(r\hat{\xi}) = -\frac{1}{r}\frac{d}{dr}\left(rG(r,\omega)\Delta_n\right) + H(r,\omega)\Delta_n \tag{35}$$

where

$$H = \frac{p_o'}{p_o}\frac{\omega}{\omega^*}\left[\frac{k_\parallel^2 v_{th}^2}{\omega^2} - \frac{p_o n_o'}{p_o' n_o}\cdot\frac{\omega^*}{\omega}\right]G(r,\omega) - 1 , \tag{36}$$

and $v_{th}^2 = T_o/m_i$. Given the profiles of the mean quantities (T_{oe}, T_{oi}, n_o etc), ω, m, n and $\hat{\xi}$ (say) as a function of r , then Eq. (35) can be used to determine $\delta n/n_o$ and hence all other fluctuating quantities. It follows that we can then evaluate the correlations arising in the particle and energy fluxes defined later.

4. ANALYTIC CALCULATION OF TRANSPORT

(a) Ambipolarity

We recall that in equilibrium, the ions are at rest and currents flow in the θ and z-directions only, that is $j_{or}(r) = 0$. By Ampere's equation it follows that

$$\langle j_r\rangle = \langle\delta j_r\rangle = 0 . \tag{37}$$

This implies, that in the mean, magnetic fluctuations in a closed system like tokamak can only lead to ambipolar transport. This result, which is independent of the size of the fluctuations, implies interesting consequences in two-fluid theory. We note that in general

$$j_r = en(u_{ir} - u_{er}) , \tag{38}$$

and hence averaging we derive

$$<j_r> = e <\delta n\, \delta u_{ir}> - e <\delta n\, \delta u_{er}> . \tag{39}$$

It is a consequence of the mass difference of electrons and ions, however, that $<\delta n\, \delta u_{ir}>$ is not generally equal to $<\delta n\, \delta u_{er}>$. Thus there appears to be a contradiction between Eqs. (37 and (39). This apparent paradox is simply resolved by observing that although u_{iro} and u_{ero} can be taken to be zero, we must allow the possibility that $<u_{ir}>$ and $<u_{er}>$ are non-vanishing. Actually, following standard practice in turbulence, we require $<u_{ir}> = u_{2ir}$ and $<u_{er}> = u_{2er}$, where the twos denote second-order in the amplitude levels. Thus the content of Eqs. (37) and (39) should be expressed as

$$<j_r> = e\left\{ <\delta n\, \delta u_{ir}> + n_o u_{2ir} - <\delta n\, \delta u_{er}> - n_o u_{2er} \right\} = 0 . \tag{40}$$

These considerations indicate that the electron and ion particle fluxes are given by

$$\left.\begin{aligned} F_{er} &= <\delta n\, \delta u_{er}> + n_o u_{2er} \\ &\text{and} \\ F_{ir} &= <\delta n\, \delta u_{ir}> + n_o u_{2ir} . \end{aligned}\right\} \tag{41}$$

For reasons which will become clear later, we shall refer to the quantities $\Phi_{er} \equiv <\delta n\, \delta u_{er}>$ and $\Phi_{ir} \equiv <\delta n\, \delta u_{ir}>$ as the nominal particle fluxes. If the plasma is in a stationary state then the mean continuity equation for the ions is

$$\frac{1}{r}\frac{d}{dr}(r F_{ir}) + S_{op}(r) = 0 , \tag{42}$$

where $S_{op}(r)$ is the particle source. If the latter is known then Eq. (42) determines F_{ir}. Since - as we show later - the nominal fluxes can be evaluated, it follows that u_{2ir} can be determined. The quasi-neutrality condition $(F_{ir} = F_{er})$ then yields u_{2er}.

It should be noted that the above discussion of ambipolarity rests on the concept of periodicity. This should be appropriate within the body of the plasma. In the proximity of limiters or divertors, however, periodicity will be broken and the fluxes need not be ambipolar.

We should emphasise that our view of ambipolarity within the body of the plasma rests firmly on two considerations. Firstly, the mean state of the plasma is not time dependent. Secondly, Ampère's law being a linear relation between fields and currents, it must result in zero mean current perpendicular to the mean magnetic surfaces in any closed periodic system. Indeed, if these currents were non-zero (that is, particle transport is not ambipolar) the system could not be in a steady-state since $\langle\nabla\cdot j\rangle \neq 0$. The usual arguments for ambipolarity invoke radial electric fields. In our view radial electric fields do not drive radial velocities but lead to flows in the poloidal and toroidal directions only.

(b) Particle Diffusion

We consider tokamaks which are (i) sufficiently large or dense so that the neutral atom mean free path is short compared with the plasma minor radius, and (ii) characterised by low levels of impurities, that is $Z_{eff} \simeq 1$. Under these conditions the particle source in Eq. (42) can be neglected in the central regions of the discharge, and this implies that both the radial electron and ion fluxes must vanish $(F_{er} = F_{ir} = 0)$. Furthermore, during steady operation the plasma density in these regions is typically parabolic. Coppi and Sharky (1981) have pointed out that such experiments can be readily interpreted in terms of a simple phenomenological procedure. Thus they assume - without specifying whether ions or electrons are being considered - that the particle flux can be written as

$$F_r = - D \frac{dn_o}{dr} + n_o \dot{u}_r . \qquad (43)$$

The first term represents an anomalous outward diffusion $(D \sim 10^4 \text{ cm}^2 \text{sec}^{-1})$ and the second an anomalous inward particle flow $(u_r \sim -10^3 \text{ cm sec}^{-1})$.

With this approach it is possible to correctly simulate PLT, T-10, Alcator-A and ASDEX (The Asdex Team, 1981). Furthermore, the same prescription leads to successful simulations of gas-puffing experiments (Coppi and Sharky, 1981; Strachan et al., 1982).

The theoretical basis of the above method can be understood from our present work. Thus if we can assume that the nominal particle fluxes can be written in the forms

$$\Phi_{er} = - D \frac{dn_o}{dr} + n_o \tilde{u}_{er} \quad (44)$$

$$\Phi_{ir} = - D \frac{dn_o}{dr} + n_o \tilde{u}_{ir} , \quad (45)$$

then we can identify u_r with $\tilde{u}_{er} + u_{2er}$, or equivalently, $\tilde{u}_{ir} + u_{2ir}$. Thus the inward phenomenlogical velocity is capable of a turbulence interpretation.

Another possibility, not envisaged by Coppi and Sharky, is to replace Eq. (43) by separate fluxes for the electrons and ions, that is

$$\left. \begin{aligned} F_{er} &= - D_e \frac{dn_o}{dr} + n_o u_{er} \\ F_{ir} &= - D_i \frac{dn_o}{dr} + n_o u_{ir} \end{aligned} \right\} \quad (46)$$

Setting $F_{er} = F_{ir} = 0$ then $D_i/D_e = u_{ir}/u_{er}$. Again, the inward ion and electron radial velocities can be given a turbulence interpretation

(c) Energy Fluxes

In an earlier paper (Haas, Thyagaraja, Cook, 1981) we derived electron and ion energy balance equations for the mean temperatures, T_{oe} and T_{oi}. Taking account of the second-order velocities u_{2er} and u_{2ir}, we rederive these equations and present the resulting expressions in a more physical form. The electron energy balance is

$$\frac{3}{2} n \left(\frac{\partial T_e}{\partial t} + \underline{u}_e \cdot \nabla T_e\right) + n T_e \nabla \cdot \underline{u}_e$$

$$= \nabla \cdot (K_{\parallel e} \nabla_{\parallel} T_e + \overline{K}_{\perp e} \nabla_{\perp} T_e) + \frac{3}{2} \frac{m_e}{m_i} \frac{n}{\tau_{ei}} (T_i - T_e) + S_e , \qquad (47)$$

where S_e signifies all sources and sinks not explicity shown. Note that $K_{\parallel e}$ and $\overline{K}_{\perp e}$ denote the instantaneous values for the parallel and perpendicular thermal conductivity coefficients (assumed known); the bar over the latter is to distinguish it from the effective perpendicular thermal conductivity, $K_{\perp e}$ (see Eq. (55)). An analogous equation pertains for the ions. Following our earlier paper, we fluctuate all quantities, retain terms to second-order and perform the average defined in Eq. (3). The mean energy equations then take the forms

$$- \frac{1}{r} \frac{d}{dr} (r Q_e(r)) + S^*_{oe} (r) = 0 \qquad (48)$$

and

$$- \frac{1}{r} \frac{d}{dr} (r Q_i(r)) + S^*_{oi} (r) = 0 , \qquad (49)$$

where Q_e, Q_i denote the effective heat fluxes and S^*_{oe}, S^*_{oi} are the effective sources. The latter can be expressed as

$$S^*_{oe} = S_{oe}(r) + 3 \frac{m_e}{m_i} \frac{n_o}{\tau_e} (T_{oi} - T_{oe}) - <\delta p_e \nabla \cdot \underline{\delta u}_e >$$

$$- p_{oe} \frac{1}{r} \frac{d}{dr} (r u_{2er}) - \frac{3}{2} F_{er} \frac{d T_{oe}}{dr} - \frac{3}{2} <\delta T_e \delta S_p > \qquad (50)$$

and

$$\hat{S}^*_{oi} = S_{oi}(r) + 3\,\frac{m_e}{m_i}\frac{n_o}{\tau_e}\,(T_{oe} - T_{oi}) - \langle \delta p_i\, \nabla\cdot \delta \underline{u}_i \rangle$$

$$- \, p_{oi}\,\frac{1}{r}\frac{d}{dr}\,(r\ u_{2ir}) - \frac{3}{2}\,F_{ir}\,\frac{d\,T_{oi}}{dr} - \frac{3}{2}\,\langle \delta T_i\, \delta\, S_p \rangle\ , \qquad (51)$$

where $S_{oe}(r)$ and $S_{oi}(r)$ are the mean energy sources and include ohmic heating, radiation losses, etc. The symbol $\delta\, S_p$ denotes a fluctuating particle source and arises from use of the fluctuated continuity equation. As before, the particle fluxes F_{er} and F_{ie} are defined to be

and

$$\left.\begin{aligned} F_{er} &= \langle\, \delta n\ \delta u_{er} \rangle + n_o\ u_{2er} \\ F_{ir} &= \langle\, \delta n\ \delta u_{ir} \rangle + n_o\ u_{2ir} \end{aligned}\right\} \qquad (52)$$

The effective electron energy flux, $Q_e(r)$, is

$$Q_e(r) = Q_e^{\ cond}(r) + Q_e^{\ conv}\,(r), \qquad (53)$$

where the conductive part is given by

$$Q_e^{\ cond} = -\,K_{\perp e}\,\frac{d\,T_{oe}}{dr}\ , \qquad (54)$$

with

$$K_{\perp e} = \langle K_{\perp e} \rangle + \langle K_{\parallel e} \rangle\ \Gamma_e \qquad (55)$$

and

$$\Gamma_e = \left\langle \left(\frac{\delta B_r}{B_o}\right)^2 \right\rangle + \left\langle \frac{\delta B_r}{B_o}\ \Delta_\parallel(\delta T_e) \right\rangle \left(\frac{d\,T_{oe}}{dr}\right)^{-1} . \qquad (56)$$

Note that $\langle \bar{K}_{\perp e} \rangle$ and $\langle K_{\parallel e} \rangle$ denote $\bar{K}_{\perp e}$ and $K_{\parallel e}$ evaluated in terms of the mean quantities n_o , T_{oe} , B_o etc. The convective part, $Q_e^{conv}(r)$, can be written in the form

$$Q_e^{conv}(r) = \frac{3}{2} n_o < \delta T_e \, \delta u_{er} > . \tag{57}$$

Analogous expressions can be derived for $Q_i(r)$. Throughout the rest of this paper we shall assume the fluctuating particle sources δS_p to be negligible.

We now derive a relation between $Q_e(r)$ and the nominal electron particle flux, Φ_{er}. As in Section 3 we assume a mode of given m, n and ω. We have already defined the radial displacement amplitude $\hat{\xi}$ in Eq. (26); we similarly define

$$\hat{\nu} \equiv \frac{\delta\hat{n}}{n_o} \quad \text{and} \quad \hat{\phi} \equiv \frac{\delta\hat{T}_e}{T_{oe}} . \tag{58}$$

From these definitions, the equations of Section 3, and the immediate preceding expressions, it is straightforward to show that

$$Q_e^{cond} = \frac{-\frac{2}{3} T_{oe} K_{\parallel e} k_\parallel^2}{1 + \frac{4}{9}\left(\frac{K_{\parallel e} k_\parallel^2}{n_o \Omega}\right)^2} \left[\frac{2n_o'}{n_o} \hat{\xi}\hat{\xi}^* + i(\hat{\xi}^*\hat{\nu} - \hat{\xi}\hat{\nu}^*) - \frac{2}{3}(\hat{\xi}\hat{\nu}^* + \hat{\xi}^*\hat{\nu}) \frac{K_{\parallel e}}{n_o} \frac{k_\parallel^2}{\Omega} \right] \tag{59}$$

and

$$Q_e^{conv} = \frac{2}{3} \frac{p_{oe} \Omega}{1 + \frac{4}{9}\left(\frac{K_{\parallel e} k_\parallel^2}{n_o \Omega}\right)^2} \left\{ \frac{K_{\parallel e} k_\parallel^2}{n_o \Omega} \left[\frac{2n_o'}{n_o} \hat{\xi}\hat{\xi}^* + i(\hat{\xi}^*\hat{\nu} - \hat{\xi}\hat{\nu}^*) - \frac{2}{3} \cdot \frac{K_{\parallel e}}{n_o} \frac{k_\parallel^2}{\Omega} (\hat{\xi}\hat{\nu}^* + \hat{\xi}^*\hat{\nu}) \right] + \frac{15}{4}\left(1 + \frac{4}{9}\left(\frac{K_{\parallel e} k_\parallel^2}{n_o \Omega}\right)^2\right)(\hat{\xi}\hat{\nu}^* + \hat{\xi}^*\hat{\nu}) \right\} - \frac{3}{2} T_{oe} \Phi_{er} . \tag{60}$$

Combining these equations we obtain

$$Q_e(r) = \frac{5}{2} p_{oe} \,\Omega\, (\hat{\xi}\,\hat{\nu}^* + \hat{\xi}^*\hat{\nu}) - \frac{3}{2} T_{oe}\, \Phi_{er}. \tag{61}$$

However, since

$$\Phi_{er} = n_o \,\Omega\, (\hat{\xi}\,\hat{\nu}^* + \hat{\xi}^*\hat{\nu}), \tag{62}$$

it follows that

$$Q_e = T_{oe}\, \Phi_{er} \equiv T_{oe} \langle \delta n \delta u_{er} \rangle \tag{63}$$

That is, the total electron heat flux as defined in Eq. (53), is equal to the energy carried by the nominal electron particle flux. We note that this result has been established for a single mode (m,n) and frequency (ω). Since, however, the ratio $Q_e/\langle \delta n\, \delta u_{er} \rangle$ is independent of m,n,ω, it immediately follows that the same relation can be derived between the total heat flux (over all m,n,ω) and the total nominal particle flux. This is a striking result and has an important bearing on the interpretation of experiment. Indeed, it can be used to interpret the ideas of Coppi and Sharky. If we assume that $K_{\perp e}^{experiment}$ is related to $Q_e(r)$ by

$$Q_e(r) = - K_{\perp e}^{exp} \frac{dT_{oe}}{dr}, \tag{64}$$

then using Eq. (63) this implies that

$$\Phi_{er} = - K_{\perp e}^{exp} \frac{1}{T_{oe}} \frac{dT_{oe}}{dr} \equiv - \frac{1}{T_{oe}} \cdot \frac{T'_{oe}}{n_o} K_{\perp e}^{exp} \frac{dn_o}{dr}$$

This enables us to write F_{er} in the form

$$F_{er} = - \frac{1}{T_{oe}} \frac{T'_{oe}}{n'_o} K_{\perp e}^{exp} \frac{dn_o}{dr} + n_o u_{2er}. \tag{65}$$

This immediately suggests that the electron flux takes the Coppi and Sharky form with

$$D_e = \frac{1}{T_{oe}} \cdot \frac{T'_{oe}}{n'_o} K_{\perp e}^{exp} \text{ and } u_e = u_{2er}. \tag{66}$$

Thus we conclude that the measured perpendicular electron thermal conduction and the electron particle diffusion are simply related and have the same order of magnitude (as assumed by Coppi and Sharky). To the extent that Coppi and Sharky's phenomenological values for D and u agree with the experimentally determined $K_{\perp e}^{exp}$, the above deductions check the theory independently of any assumption about profiles, mode numbers, amplitudes and frequencies.

The simplicity of the relation, Eq. (63), is ultimately due to the neglect of electron inertia and other higher-order effects in Ohm's law. For the ions the inertial terms are not negligible (Thyagaraja and Haas, 1983) and hence we cannot derive a result as simple as that for the electrons. It turns out, however, that $\frac{Q_{ir}}{T_{oi}\Phi_{ir}}$ is O(1). This further suggests that D_i and D_e must be different, and similarly for u_i and u_e. Coppi and Sharky's approach, however, implicitly assumes these quantities to be the same.

(d) Total Momentum Transport

As with energy transport we can fluctuate the total momentum balance equation and derive mean momentum transport equations for the r, θ and z directions. In view of the lack of detailed knowledge of the power spectrum, we touch on this only briefly.

Let us suppose that the radial magnetic field fluctuation $\delta B_r/B_o$ is expressed as follows

$$\frac{\delta B_r}{B_o} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{ \Psi_{mn}^{(1)}(r,t)\cos m\theta \cos\frac{nZ}{R} + \Psi_{mn}^{(2)}(r,t)\cos m\theta \sin\frac{nZ}{R} \right.$$

$$\left. + \Psi_{m,n}^{(3)}(r,t)\sin m\theta \cos\frac{nZ}{R} + \Psi_{m,n}^{(4)}(r,t)\sin m\theta \sin\frac{nZ}{R} \right\}, \qquad (67)$$

with δB_θ and δB_z taking similar forms. The Fourier coefficients in these expansions will be related by $\nabla\cdot\underline{B} = 0$, and possibly by other relations involving the dynamics of the turbulence. It is now straightforward to express the current fluctuations, $\delta\underline{j}$, in terms of the $\delta\underline{B}$ expansions using Ampère's law. With these expressions the mean Lorentz force $< \frac{\delta \underline{j} \times \delta \underline{B}}{c} >$, can be calculated in terms of time averages like $\frac{1}{t_M} \int_0^{t_M} \left(\Psi_{mn}^{(1)} (r,t) \right)^2 dt$. In general, a mode of given m,n can contribute to this force only through an interaction with itself. Thus two modes of different m,n cannot give a non-zero contribution to the average force. However, it is important to recognise that in the presence of dissipation, the amplitudes $\Psi^{(1)}$, $\Psi^{(2)}$, $\Psi^{(3)}$, $\Psi^{(4)}$ and their counterparts in δB_θ and δB_z will be correlated. In general, these correlations give rise to forces in the r, θ and z directions. The force in the r - direction is rather small compared with the equilibrium Lorentz force $\frac{\underline{j}_o \times \underline{B}_o}{c}$, although it is possible - particularly with large amplitude modes - that the $< \frac{\delta \underline{j} \times \delta \underline{B}}{c} >_r$ force is of the same order as $\frac{dp_o}{dr}$. On the other hand, in the poloidal and toroidal directions there is no force due to the mean pressure gradient or $\frac{\underline{j}_o \times \underline{B}_o}{c}$. Thus it follows, that in the poloidal and toroidal directions the mean Lorentz force due to the turbulence must drive mean ion velocities. For typical values of turbulence, the mean Lorentz force can be balanced by ion viscosity, giving rise to poloidal and toroidal flows small compared with the ion sound speed. It appears, however, that under disruptive conditions the size of these flows could become comparable with the ion sound speed. In this situation, ion inertia becomes important and could lead to a breakdown in equilibrium.

(e) Resistivity

We now consider the experimental fact that the toroidal resistivity in a tokamak is of order the Spitzer value. This is despite the fact that electron particle and energy loss rates are known to be much greater than neoclassical. Thus for our approach to be consistent with experiment, it is necessary that toroidal resistivity be unaffected by turbulence. It is this point which we now investigate.

We have shown elsewhere (Thyagaraja and Haas, 1983) that the resistive terms in the electron momentum balance equation are smaller than the reactive terms, $\frac{\underline{v}_e \times \underline{B}}{c}$ and $\frac{1}{en} \nabla p_e$, by a factor of 10^{-3} or less under typical tokamak conditions. In the absence of resistivity we show below that the turbulent correlations must exactly cancel and therefore cannot drive any mean currents. When perturbations due to resistive terms are taken into account, owing to the smallness of the resistivity, the phase-shifts of the turbulent correlations are correspondingly small and hence any turbulent-driven current is of higher order. It follows, therefore, that the mean current is principally driven by the externally applied electric field. We now give the argument.

The electric field in the plasma can be decomposed into three components. Thus

$$\underline{E} = \underline{E}_{ext} + \underline{E}_o + \tilde{\underline{E}}_{turb} , \qquad (68)$$

where $\underline{E}_{ext}$ is the applied electric field resulting from the transformer. The component $\underline{E}_o$ is the time-independent electrostatic field which is determined by the equilibrium or mean state of the plasma; due to axisymmetry it has no toroidal component. $\tilde{\underline{E}}_{turb}$ denotes the turbulent or fluctuating part of the electric field; it has the property that $\langle\tilde{E}_{turb}\rangle = 0$. We note that the resistive time-scale, τ_R, of the plasma is much longer than the typical turbulence time-scale, which according to our interpretation, is of order the electron collision time, τ_e. Typically, τ_R and τ_e are related through $\tau_R \sim \omega_{pe}^2 \frac{a^2}{c^2} \tau_e$. We assume that the time-scale of $\underline{E}_{ext}$ is also of the order τ_R, that is, $E_{ext} \sim \eta_{spitzer} \langle j_o\rangle$. The following considerations apply only in this circumstance. In order to simplify the discussion we neglect electron inertia and viscosity and take η to be scaler. Thus the electron momentum equation (Ohm's law) takes the form

$$\underline{E}_{ext} + \underline{E}_o + \tilde{\underline{E}}_{turb} = \left\{- \frac{\underline{v}_e \times \underline{B}}{c} - \frac{1}{en} \nabla p_e\right\} + \eta j , \qquad (69)$$

In the limit $\tau_R/\tau_e \gg 1$,

$$\frac{E_{ext}}{\langle\eta\rangle} \rightarrow j_{o\ tor} \qquad (70)$$

and the equation becomes

$$\underline{E}_o + \tilde{\underline{E}}_{turb} = - \frac{\underline{v}_e \times \underline{B}}{c} - \frac{1}{en} \nabla p_e . \qquad (71)$$

Taking the toroidal component, noting the axisymmetry, and averaging, we find the left-hand side of Eq. (71) to vanish. It follows that the average of the toroidal component of the right-hand side also vanishes. It is important to note that the individual terms on the right-hand side, for example $\left\langle \frac{1}{en} \hat{\underline{e}}_{tor} \cdot \nabla p_e \right\rangle$, do not vanish. This is because the phase-shifts between δn and δp_e, say, are created by other dissipative processes, such as parallel electron and ion thermal conduction. These proceed on the turbulence time-scale and in our interpretation lead to the enhanced perpendicular electron thermal conduction.

Returning to Eq. (69), we now have

$$\underline{E}_o + \tilde{\underline{E}}_{turb} = -\frac{\underline{v}_e \times \underline{B}}{c} - \frac{1}{en} \nabla p_e + \eta j - \langle \eta \rangle \, j_{o\ tor} \, . \tag{72}$$

Taking the toroidal component and averaging, as before, we have

$$\left\langle \hat{\underline{e}}_{tor} \cdot \left(\frac{\underline{v}_e \times \underline{B}}{c} + \frac{1}{en} \nabla p_e \right) \right\rangle = \langle \eta \rangle \langle j_{tor} \rangle_2 + \langle \delta\eta \; \delta j_{tor} \rangle \, , \tag{73}$$

where $\langle j_{tor} \rangle_2$ denotes the second-order part of the mean current. Thus our analysis is consistent with the view that the toroidal resistivity in a tokamak is given by the Spitzer value, and that turbulence only leads to an unimportant correction.

Finally, we note the analogy between the above argument and our treatment of the electron energy balance equation. If we take this equation to be $\nabla \cdot \underline{q}_{e\parallel} = 0$, that is, we neglect inertia, work done etc, then we cannot derive an anomalous $\chi_{\perp e}$ since Γ_e (in Eq. (56)) vanishes. Thus the average of $\underline{q}_{e\parallel}$ is determined not only by the size of the fluctuations but also by phase-shift creating mechanisms such as electron inertia, for example. To the extent that the latter are small, the effective normal heat-flux is also small. Returning to the generalised Ohm's law in the limit $\tau_R/\tau_e \gg 1$, the effective phase-shift creating mechanism is the resistivity, and to the extent that this is small, turbulence does not significantly modify the toroidal (Spitzer) resistivity.

5. DISCUSSION AND APPLICATION TO MACROTOR

We clarify a number of issues relating to the work of this paper and consider some applications. Firstly, for our interpretation we require complete knowledge of one fluctuating quantity, $\frac{\delta B_r}{B_o}$, say. The mean quantities are governed by the mean transport equations which we have derived. However, they cannot be completely evaluated without a knowledge of the fluctuations. While in reality both the mean and fluctuating quantities are self-consistently determined by the full non-linear equations, our interpretation is based on approximate relationships between different fluctuating quantites; these take advantage of the experimental fact that the fluctuations in tokamaks are generally of small amplitude. Paying due regard to the non-linearity, a careful choice of the equations enables us to express all fluctuating quantities in terms of $\frac{\delta B_r}{B_o}$. Although these calculations are formally linear, since they involve the undetermined mean profiles, they are actually non-linear. Thus the turbulent correlations appearing in the mean equations are very complicated functions of the mean profile and the assumed power-spectrum of $\frac{\delta B_r}{B_o}$. In this sense our theory is non-linear. It must be remembered, however, that in reality the fluctuations are related non-linearly, and to the extent that we use linearised relations, our theory is only approximate and needs to be checked against a full non-linear calculation; the latter is necessarily numerical and remains for the future. Even without such a calculation our approximate equations provide an interesting qualitative description of certain aspects of tokamak transport.

We now discuss the nature of the mean energy equations, Eqs. (48) and (49). In fact we shall only consider the electron energy equation, as the discussion for the corresponding ion equation is similar. We remark that turbulence contributes two distinct effects. Thus the heat flux $Q_e(r)$ involves turbulence correlations like those contained in Γ_e and Q_e^{conv} (see Eqs. (56) and (57)). It also contributes terms such as $- < \delta p_e \nabla \cdot \underline{\delta u}_e >$ and $- p_{oe} \frac{1}{r} \frac{d}{dr} (r\, u_{2er})$ to the effective source (Eq. 50).

Both these effects can be of importance. The question arises as to whether all turbulence contributions should be incorporated into a single effective heat flux, Q_{eff}, with a source comprising only those terms which depend solely on the mean quantities, T_{oe}, n_o etc. Thus we can write

$$-\frac{1}{r}\frac{d}{dr}\left(rQ_{eff}(r)\right) + S_{oe}(r) + 3\,\frac{m_e}{m_i}\frac{n_o}{\tau_e}\,(T_{oi} - T_{oe}) = 0, \tag{74}$$

where $\quad Q_{eff} = Q_e + Q_s, \qquad (75)$

with Q_s to be derived from

$$\frac{1}{r}\,\frac{d}{dr}\,(rQ_s) = \langle \delta p_e \;\nabla.\delta \underline{u}_e \rangle + p_{oe}\,\frac{1}{r}\frac{d}{dr}\,(ru_{2er}) + \frac{3}{2}\,F_{er}\,\frac{d}{dr}\,T_{oe} + \frac{3}{2}\,\langle \delta T_e \delta S_p \rangle \tag{76}$$

In this form, however, we observe that the effective heat flux cannot be expressed entirely in terms of local correlations. By a local correlation, for example $\langle \delta n\ \delta u_{er} \rangle$, we mean one which involves fluctuations at a particular radius, r. We note that $Q_s(r)$ involves an r- integration over local correlations, and in this sense, is a non-local quantity. In our view, the mean energy equations are most naturally written in terms of local correlations, which reveal the physical origins of the various terms. There are, in fact, two physically acceptable ways of doing this. The first, which we have already given in Eqs. (48) and (50), refers to the transport of the internal energy of the electron fluid ($C_v T_e \equiv \frac{3}{2}\, n\ T_e$). The second, which we give below, refers to the transport of enthalpy ($C_p T_e \equiv \frac{5}{2}\, n\ T_e$). In this form, Q_e^{cond} is unchanged but Q_e^{conv} becomes

$$Q_e^{conv}(r) = \frac{5}{2}\, n_o\ \langle \delta T_e\ \ \delta u_{er} \rangle. \tag{77}$$

It follows that $Q_e = \langle \delta p_e \delta u_{er} \rangle$. The appropriate effective electron energy source is now written as

$$S^{*}_{oe} = S_{oe}(r) + 3\,\frac{m_e}{m_i}\frac{n_o}{\tau_e}\,(T_{oi} - T_{oe}) + \langle \delta \underline{u}_e . \nabla \delta p_e \rangle + u_{2er}\,\frac{d}{dr}\,p_{oe} - \frac{5}{2}\,F_{er}\,\frac{d}{dr}\,T_{oe} + S_{op} T_{oe} - \frac{3}{2}\,\langle \delta T_e \delta S_p \rangle. \tag{78}$$

However, our original form for the mean electron energy equation, Eq. (48), seems closer to intuition, since in this case Q_e is directly related to the nominal electron particle flux, Φ_{er}.

We now draw attention to the significance of u_{2er}. Returning to our earlier discussion of the mean energy transfer, Eq.(63) can be written as

$$Q_e = T_{oe} F_{er} - p_{oe} u_{2er}, \tag{79}$$

which is a consequence of the linearised phase relations. For those tokamaks where particle sources are negligible in the central regions, $F_{er} = 0$. Thus if Q_e is to be identified as an outward energy flux, then u_{2er} must be inward. Indeed this shows the crucial role played by u_{2er} in our interpretation of these experiments. More generally, it suggests the importance of turbulence in explaining experiments.

Turbulence affects the mean energy balance in two distinct ways. Firstly, it leads to an energy flux Q_e expressible in terms of local correlations, and secondly, it provides turbulent sources. Experimentalists, however, only include mean sources in their energy balance studies. Thus an experimentalist really measures the heat flux Q_{eff} as defined in Eq.(74). From the Q_{eff} so calculated the experimental thermal conduction coefficient, $K_{\perp e}^{exp}$, is obtained from the definition

$$Q_{eff}(r) \equiv - K_{\perp e}^{exp} \frac{dT_{oe}}{dr} . \tag{80}$$

While estimates suggest that Q_e (as defined in Eq.(53)) and Q_{eff} tend to be of the same order, there is no reason to believe that they are exactly equal. Thus Eq.(64) is only correct in an order-of-magnitude sense, as is Eq.(66). To establish a more precise relation between Q_e and Q_{eff} requires a complete knowledge of the power spectrum.

It is of interest to apply some of the ideas developed in this paper to an actual experiment. The MACROTOR observations of Zweben et al (1979) provide a useful test. MACROTOR has a minor radius $a = 45$ cm and major radius $R = 90$ cm. The toroidal field is of order 3 kG and the machine runs with a density of order 5×10^{12} cm^{-3} and $T_e \sim 100$ eV, at a current of 50 kA and a power of 100 kW. The bulk electron energy confinement time is estimated to be 1 ms. The machine shows both coherent and incoherent density and magnetic fluctuations. Zweben et al do not give much information concerning the coherent fluctuations except to say that they are of low frequency (7kHz), of large amplitude and consistent with an m = 2 tearing mode. The high frequency components (30 - 100 kHz) form the incoherent part of the spectrum, although well above noise level,

$$\sum_{\nu} \frac{|\tilde{B}_r|}{B_T} \simeq 10^{-4} \text{ to } 10^{-5} \tag{81}$$

for frequencies, ν, in the above range. The corresponding fluctuation spectrum was also measured and is reproduced in Fig. 1.

It is of interest to examine whether the incoherent component alone is sufficient to explain the observed electron energy confinement time. We first consider the electron energy transport implied by the theories of Callen (1977) and Rechester and Rosenbluth (1978). Thus according to them we find

$$\chi_{\perp e} = \pi R v_{the} \sum_{\nu} \frac{|\tilde{B}_r|^2}{B_T^2} \sim 10^3 \text{ cm}^2 \text{ sec}^{-1} \tag{82}$$

Hence $\tau_{eE} \sim \frac{a^2}{\chi_{\perp e}} \sim 2.5$ sec., which is inconsistent with 1 ms. Conditions in MACROTOR are such that the Knudsen number ($Kn = \frac{v_{the}\tau_e}{2\pi R}$) is of order unity, and hence this machine is operating in the collisionless/collisional regime. Thus replacing the collisionless form of $\chi_{\parallel e}$ ($= \pi R v_{the}$) by the collisional form ($v_{the}^2 \tau_e$) should lead to roughly the same value for τ_{eE}; in fact we find $\tau_{eE} \sim 1$ sec. Thus the $\chi_{\perp e}$ estimated from the above formula is smaller than the 'observed' value by a factor of 10^3.

An interesting feature of the incoherent modes is the relationship between the absolute values of the density and magnetic field fluctuations shown in Fig.1; the experimental results do not suggest a simple algebraic relationshop between $\frac{\delta n}{n}$ and $\frac{\delta B_r}{B_o}$. This experimental fact, however, can be simply and directly interpreted through Eq. (35). Thus we consider a mode with given m, n, ω and formally integrate Eq. (35) to obtain

$$\Delta_n = -\frac{\hat{\xi}}{G} - \frac{1}{rG} \exp\left(\int_o^r \frac{H}{G}\, dr\right) \int_o^r r\hat{\xi}\, \frac{H}{G} \exp\left(-\int_o^r \frac{H}{G}\, dr\right) dr \tag{83}$$

where G and H are given by Eqs. (34) and (36) respectively. Noting that

$$\Delta_n \equiv i\,\frac{\hat{\delta n}}{n_o} + \frac{n_o'}{n_o}\,\hat{\xi}$$

and
$$\frac{\hat{\delta B}_r}{B_o} = k_{\parallel}\hat{\xi},$$

the above equation expresses $\frac{\hat{\delta n}}{n_o}$ in terms of $\frac{\hat{\delta B}_r}{B_o}$. We remark that in deriving this relation we make use of the boundary condition $\frac{\hat{\delta n}}{n_o} \rightarrow 0$ as $r \rightarrow 0$. This conditions is consistent with the experiment observations. If we take $|\frac{\hat{\delta B}_r}{B_o}|$ to have the radial variation shown in Fig. 1, then Eq. (83) yields a profile for $|\frac{\hat{\delta n}}{n_o}|$ which is consistent both in trend and magnitude with the experimental curve. This result pertains to the range of m, n, ω typical of MACROTOR. Thus, summing over all modes, we are able to interpret the observed non-local relation between the magnetic and density fluctuations.

By considering the functions G and H we can give a simple qualitative explanation of the behaviour of $\frac{\hat{\delta n}}{n_o}$. First, we note that these functions depend only weakly on m, n and ω; their behaviour with r (as $r \rightarrow a$) is virtually independent of these parameters. To illustrate this we assume a Gaussian temperature profile for T_{oe} and a parabolic density profile for n_o. It is easy to see that $\omega^* \frac{p_o}{p_o'} \sim \frac{1}{r}$ as $r \rightarrow 0$. This implies that $rG \rightarrow$ constant and H is finite as $r \rightarrow 0$. Thus irrespective of the behaviour of $\hat{\xi}$ (provided it remains finite), Eq. (83) shows $\frac{\hat{\delta n}}{n_o} \rightarrow 0$ as $r \rightarrow 0$; furthermore, as $r \rightarrow a$ then G behaves like $\frac{\omega^*}{\omega}\frac{p_o}{p_o'}$ with a multiplying factor of O(1). The form of $\frac{\omega^*}{\omega}\frac{p_o}{p_o'}$ as $r \rightarrow a$ is given by Eq. (29), namely,

$$\frac{\omega^*}{\omega}\frac{p_o}{p_o'} = + \frac{cT_o}{eB_o}\frac{1}{\omega}\left(-\frac{m}{a}\frac{B_{oz}}{B_o} + \frac{n}{R}\frac{B_{o\theta}(a)}{B_o}\right) = + \frac{cT_o}{eB_o}\frac{1}{\omega}\frac{n}{R}\frac{B_{o\theta}(a)}{B_o}\left(1 - \frac{m}{n}\,q(a)\right).$$

Clearly since $T_o \to 0$ as $r \to a$ then $|G|$ tends to zero irrespective of m, n, ω. It is easy to verify that in the same limit $|\hat{H}|$ remains finite. This means from Eq. (83) that even for $\hat{\xi} \to 0$ as $r \to a$, $|\frac{\delta n}{n_o}|$ rises with r and can have large values towards the boundary. Thus our theory is qualitatively capable of accounting for this feature of the experiment. Numerical calculation also suggest that Eq. (83) predicts density fluctuation of approximately the correct size (up to 30% at the edge for the given $\frac{\delta B_r}{B_o} \sim 10^{-4}$). It is interesting to note that the Q_{eff} calculated using experimental profiles and the deduced $\frac{\hat{\delta n}}{n_o}$ profile falls short of the heat flux needed to explain the experimental electron energy confinement time of 1 m.sec by a factor of 10^3. Thus suggesting on the basis of our model that incoherent modes alone cannot account for the experimentally observed transport.

Zweben et al do not give the level of the coherent field fluctuations except to state that they are somewhat larger than the incoherent amplitudes. However, assuming $\frac{\hat{\delta B}_r}{B_o} \sim 2.0 \times 10^{-3}$ $(m = 2,\ n = 1)$ for frequencies of order 7kHz we find a value for Q_{eff} which leads to a confinement time of a few millisecs. This result can be checked using Eqs. (55) and (56). Thus taking $\chi_{\parallel e} \simeq v_{the}^2 \tau_e$ we have $\chi_{\parallel e} \sim 2 \times 10^{11}\ cm^2\ sec^{-1}$; at 7 kHz, $\Gamma_e \sim 1$ and therefore $\chi_{\perp e} \sim 8 \times 10^5\ cm\ sec^{-1}$ leading to $\tau_{eE} \sim 3$ m secs. It is of interest to note that the linear (2,1) tearing mode eigenfunction can be made compatible with our two-fluid equations (Thyagaraja and Haas, 1983) by requiring the real frequency ω to be chosen so that the electron and ion velocities are finite at the resonant point. Thus referring to Eqs. (27) and (28), this condition amounts to setting $\Omega = 0$, which is also equivalent to $\omega = -\omega^*$ at the resonant point. Taking a density profile proportional to $(1 - r^2/a^2)^{2/3}$ and a temperature proportional to $(1 - r^2/a^2)^{4/3}$, we calculate the j_z profile assuming Spitzer resistivity to be proportional to $(1 - r^2/a^2)^2$. The experimental value of q at the edge is $q(a) = 3.0$. Taking the plasma current to be 50 kA, we obtain a q-profile which has $q(o) = 1.0$. This specification of equilibrium is compatible with the MACROTOR observations. The (2,1) mode is linearly unstable and the resonant point occurs at $r \approx 0.8a$. It is of interest to note that at this point ω^* calculated from the equilibrium profiles (Eq. (29)) is 6.94 kHz, very close to the observed value of ω of order 7.0 kHz. We must mention two caveats to our estimate of the transport due to the (2,1) mode. The assumption that the finite amplitude profile has the same radial variation as the linearised cylindrical eigenfunction is questionable, although possibly close to the truth. More importantly, MACROTOR being a tight aspect-ratio machine, the linear (2,1) mode is toroidally and possibly non-linearly coupled to many other low m,n modes. Ignoring these couplings means that our estimates can be no more than qualitative. Of course the full answer depends on all the other contributions to Q_{eff} and cannot properly be assessed as a function of r unless $\delta\hat{B}_r/B_o$ for all the low m, n modes is known both in amplitude and phase as a function of r. This must await further experimental observations or calculations based on non-linear theory.

A new feature of our present calculation, as opposed to our earlier work (Haas et al. 1981), is the fact that both electron and ion parallel thermal conduction are possible phase-shift mechanisms. That ion parallel thermal conduction can influence electron transport is a consequence of quasi-neutrality. It is of interest to ask which of these effects is the more important. In a qualitative form this question is answered in the table. Thus we investigate four cases appropriate to MACROTOR; we consider an $m = 2$, $n = 1$ mode at 7 kHz with a representative $\frac{\delta B_r}{B_o}$ profile. At a particular radius $r = 3{\cdot}5$ cm we calculate the various fluxes indicated in the table. The four cases differ only in the values chosen for $\chi_{\parallel e}$ and $\chi_{\parallel i}$. In case (1) $\chi_{\parallel e}$ and $\chi_{\parallel i}$ are calculated using the formulae $\chi_{\parallel e} = \frac{\pi^2 R^2}{\tau_e}$ and $\chi_{\parallel i} = \frac{\pi^2 R^2}{\tau_i}$. In case (2) $\chi_{\parallel e}$ is arbitrarily taken to be 10^3 larger than in case (1), with $\chi_{\parallel i}$ unaltered. The table shows that this reduces the electron fluxes ($<\delta u_{er} \frac{\delta n}{n_o}>$, Q_e^{cond}, Q_e, $Q_{e\ eff}$) by approximately a factor of two. The ion fluxes are virtually unaffected. Case (3) is the same as case (2) except that $\chi_{\parallel i}$ is 10^2 times larger. The electron fluxes are apparently increased, as are the ion fluxes. However, case (4) shows that a further ten-fold increase of $\chi_{\parallel i}$ leads to a drastic reduction of all fluxes. These runs show that the ion parallel thermal conduction is at least as important, if not more important, than electron parallel thermal conduction in producing phase-shifts of low-frequency, low amplitudes modes. Case (3) also suggests that there is an optimal value of $\chi_{\parallel i}$ at which the given mode would lead to the maximum transport.

6. Conclusions

Using the two-fluid equations we have derived radial mean energy equations for both the electrons and the ions. We are able to express the corresponding heat fluxes and turbulent sources in terms of correlations between the various fluctuating quantities. The general properties of tokamak equilibria ensure that these systems are ambipolar in the mean. For this to be self-consistently satisfied, however, we have to include second-order radial velocities in the mean-state (equilibrium) for both electrons and ions; this procedure is typical of turbulence. Furthermore, these velocities play an important role in interpreting the 'phenomenological' simulations of discharges for which particle sources are negligible (Coppi and Sharky, 1981).

To evaluate the correlations involved in the fluxes and sources we have used linear relations, but with parallel thermal conductivities included as phase-producing mechanisms. We have found the electron heat flux (conduction plus convection) to be equal to the energy carried by the nominal electron particle flux. Consideration of the momentum transport leads to a mechanism for disruption which is different from that usually proposed. We have given an argument as to why turbulence does not influence the toroidal resistivity in tokamaks.

Finally, we have applied our analysis to an interpretation of MACROTOR. For this device classical (Braginskii) forms for the parallel electron and ion thermal conductivities are appropriate. We have assumed the radial magnetic fluctuation profile to be similar to that of experiment and find the density fluctuation profile to rise monotonically towards the boundary, as observed. Furthermore, if we assume the coherent (7 kHz) modes to have amplitudes typical of tearing modes ($\delta B_r/B_o \sim 10^{-3}$), then we deduce an enery confinement time consistent with experiment. Interestingly, the parallel ion thermal conduction proves to be a more significant phase-producing mechanism than parallel electron thermal conduction.

These qualitative results suggest that a two-fluid turbulence interpretation of transport is plausible. However, thorough vindication or otherwise must await a full numerical simulation.

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TABLE 1

CASE	$\chi_{\parallel e}$ $cm^2 sec^{-1}$	$\chi_{\parallel i}$ $cm^2\ sec^{-1}$	$\langle \delta u_{er} \frac{\delta n}{n_o} \rangle$ cm sec^{-1}	Q_e^{cond} ergs $cm^{-2}\ sec^{-1}$	Q_i^{cond} ergs $cm^{-2}\ sec^{-1}$	Q_e ergs $cm^{-2}\ sec^{-1}$	$Q_{e\ eff}$ ergs $cm^{-2}\ sec^{-1}$	Q_i ergs $cm^{-2}\ sec^{-1}$	$Q_{i\ eff}$ ergs $cm^{-2}\ sec^{-1}$
1	1.8×10^{10}	4.0×10^{8}	1463.0	4.1×10^{6}	5.8×10^{5}	1.1×10^{6}	2.7×10^{6}	6.0×10^{5}	6.2×10^{5}
2	1.8×10^{13}	4.0×10^{8}	557.0	4.3×10^{5}	6.0×10^{5}	4.3×10^{5}	6.5×10^{5}	6.5×10^{5}	6.4×10^{5}
3	1.8×10^{13}	4.0×10^{10}	823.0	6.5×10^{5}	8.7×10^{5}	1.3×10^{6}	9.5×10^{5}	9.5×10^{5}	9.8×10^{5}
4	1.8×10^{13}	4.0×10^{11}	86.0	7.5×10^{4}	9.0×10^{4}	1.3×10^{4}	1.4×10^{4}	9.7×10^{4}	1.0×10^{5}

Appendix

In this appendix we give the steps involved in deriving our final form for the ion continuity equation, Eq. (35). First, we multiply Eqs. (17) and (18) by in/R and - im/r respectively, and add; eliminating the electric field components through Eq. (23) then leads to Eq, (27) for the radial electron velocity perturbation. Making use of the equilibrium equation, Eq. (8), and substituting for $\delta\hat{u}_{er}$ in Eqs. (17) and (18), we derive expressions for the poloidal and longitudial fluctuating electric field components. Thus

$$\delta\hat{E}_\theta = -\frac{1}{en_o}\frac{m}{r}\left(i\delta\hat{p}_e + \frac{dp_o}{dr}\hat{\xi}\right) + \frac{\omega}{c}B_{oz}\hat{\xi} \qquad \text{(A1)}$$

$$\delta\hat{E}_z = -\frac{1}{en_o}\frac{n}{R}\left(i\delta\hat{p}_e + \frac{dp_o}{dr}\hat{\xi}\right) - \frac{\omega}{c}B_{o\theta}\hat{\xi}\ , \qquad \text{(A2)}$$

where $\hat{\xi} = \frac{1}{k_{\|}}\frac{\delta B_r}{B_o}$. The calculation of $\delta\hat{E}_r$ on the other hand, involves the ion motion. Substituting for $\delta\hat{E}_\theta$ in Eq (11) we obtain

$$\delta\hat{u}_{i\theta} = \frac{1}{m_i n_o i\omega}\left\{-\frac{im}{r}\delta\hat{p} - \frac{m}{r}\frac{dp_o}{dr}\hat{\xi} - \frac{en_o}{c}B_{oz}\left(\delta\hat{u}_{ir} - \omega\hat{\xi}\right)\right\} \qquad \text{(A3)}$$

It follows that Eq (12) gives

$$\delta\hat{u}_{iz} = \frac{1}{m_i n_o i\omega}\left\{-\frac{in}{R}\delta\hat{p} - \frac{n}{R}\frac{dp_o}{dr}\hat{\xi} + \frac{en_o}{c}B_{o\theta}\left(\delta\hat{u}_{ir} - \omega\hat{\xi}\right)\right\}. \qquad \text{(A4)}$$

Multiplying Eqs (24) and (25) by $\frac{n}{R}$ and $-\frac{m}{r}$ respectively, and adding, we can now derive a form for $\delta\hat{E}_r$. Upon using Eqs. (21), (A1) and (A2), $\delta\hat{E}_r$ can be expressed as

$$\delta\hat{E}_r = -\frac{1}{\frac{n^2}{R^2}+\frac{m^2}{r^2}}\left\{-\left(\frac{n^2}{R^2}+\frac{m^2}{r^2}\right)\frac{d}{dr}\left(i\frac{\delta\hat{p}_e}{en_o} + \frac{1}{en_o}\frac{dp_o}{dr}\hat{\xi}\right) + \frac{4\pi en_o\omega}{c^2}\left(\delta\hat{u}_{ir} - \omega\hat{\xi}\right)\right.$$

$$\left. + \frac{2}{r}B_o\frac{n\omega}{cR}\hat{\xi} - \frac{1}{r}\frac{d}{dr}(r\hat{\xi})\frac{\omega}{c}\left(\frac{n\,B_{o\theta}}{R} - \frac{m}{r}B_{oz}\right)\right\}. \qquad \text{(A5)}$$

Whereas $\delta\hat{E}_\theta$ and $\delta\hat{E}_z$ have been expressed entirely in terms of electron pressure and magnetic perturbations, $\delta\hat{E}_r$ also involves the radial ion velocity perturbation. Thus it is now necessary to obtain an appropriate form for $\delta\hat{u}_{er}$.

We derive $\delta\hat{u}_{ir}$ from Eq (10) by substituting from Eqs (A3), (A4) and (A5). After considerable algebra we can write $\delta\hat{u}_{ir}$ in the form

$$\delta\hat{u}_{ir} = \omega\left(1 + \frac{\omega}{\omega_{ci}} M\right)\hat{\xi} + D^{-1}\left\{\frac{p_o}{p_o'}\,\omega^*\Delta p + \frac{i\omega}{\omega_{ci}}\frac{c\,T_o}{e\,B_o}\left(\frac{T_o'}{T_o}\Delta_p + \Delta_p'\right)\right.$$

$$\left. + \frac{i\omega}{\omega_{ci}}\frac{c\,T_o}{e\,B_o}\left(-\frac{\delta\hat{p}}{p_o}\frac{n_o'}{n_o} + \frac{\delta\hat{n}}{n_o}\frac{p_{oi}'}{p_o}\right) - \frac{\omega}{k^2\omega_{ci}}\left(\frac{n\,B_{o\theta}}{R\,B_o} - \frac{m}{r}\frac{B_{oz}}{B_o}\right)\frac{1}{r}\frac{d}{dr}(r\,\omega\,\hat{\xi})\right\} \tag{A6}$$

where

$$\left.\begin{aligned} D &\equiv 1 - \frac{\omega^2}{\omega_{ci}^2}\left(1 + \frac{\omega_{pi}^2}{k^2c^2}\right), \\ \omega_{ci} &\equiv \frac{e\,B_o}{m_i\,c}, \\ \omega_{pi}^2 &\equiv \frac{4\pi n_o e^2}{m_i}, \\ M &\equiv D^{-1}\left(\frac{\omega}{\omega_{ci}} + \frac{2\,B_{o\theta}}{B_o}\frac{n}{k^2\,rR}\right) \end{aligned}\right\} \tag{A7}$$

and ω^* is defined by Eq (29). To keep the analysis within bounds we restrict ourselves to frequencies such that $\omega \ll \omega_{ci}$. For present tokamaks this implies that $D \sim 1{\cdot}0$ and $M \ll 1$. Thus $\delta\hat{u}_{ir}$ can be greatly simplified, and we obtain

$$\delta\hat{u}_{ir} = \omega\hat{\xi} + \frac{p_o}{p_o'}\,\omega^*\,\Delta_p + 0\left(\frac{\omega}{\omega_{ci}}\right), \tag{A8}$$

which is the result quoted in Eq (28).

Using Eq (8), the poloidal and longitudinal components of the ion velocity perturbation given in Eqs (A3) and (A4) can be put into the simpler forms

$$\delta\hat{u}_{i\theta} = \frac{ip_o \Delta_p}{m_i n_o \omega} \left\{ \frac{m}{r} + \frac{e n_o B_{oz}}{c} \frac{\omega^*}{p_o'} \right\} \tag{A9}$$

$$\delta\hat{u}_{iz} = \frac{i p_o \Delta_p}{m_i n_o \omega} \left\{ \frac{n}{R} - \frac{e n B_{o\theta}}{c} \frac{\omega^*}{p_o'} \right\} \tag{A10}$$

It follows straightforwardly that

$$\nabla \cdot \underline{\delta \hat{u}}_i = \frac{1}{r} \frac{d}{dr} \left(r \left[\omega \hat{\xi} + \frac{p_o}{p_o'} \omega^* \Delta_p \right] \right) - \frac{k_\parallel^2 v_{th}^2}{\omega} \Delta_p \tag{A11}$$

where $v_{th}^2 = \frac{T_o}{m_i}$. Our final form for the ion continuity equation, Eq (35), now follows, the mathematics being simplified by the use of the quantities defined in Eq. (26). As mentioned previously, a second equation relating $\hat{\xi}$ and Δ_p can be derived. This is the radial component of the total momentum balance and can be obtained from the equations of Section 3 ; the details of the derivation have been described in an earlier paper (Thyagaraja and Haas (1983)).

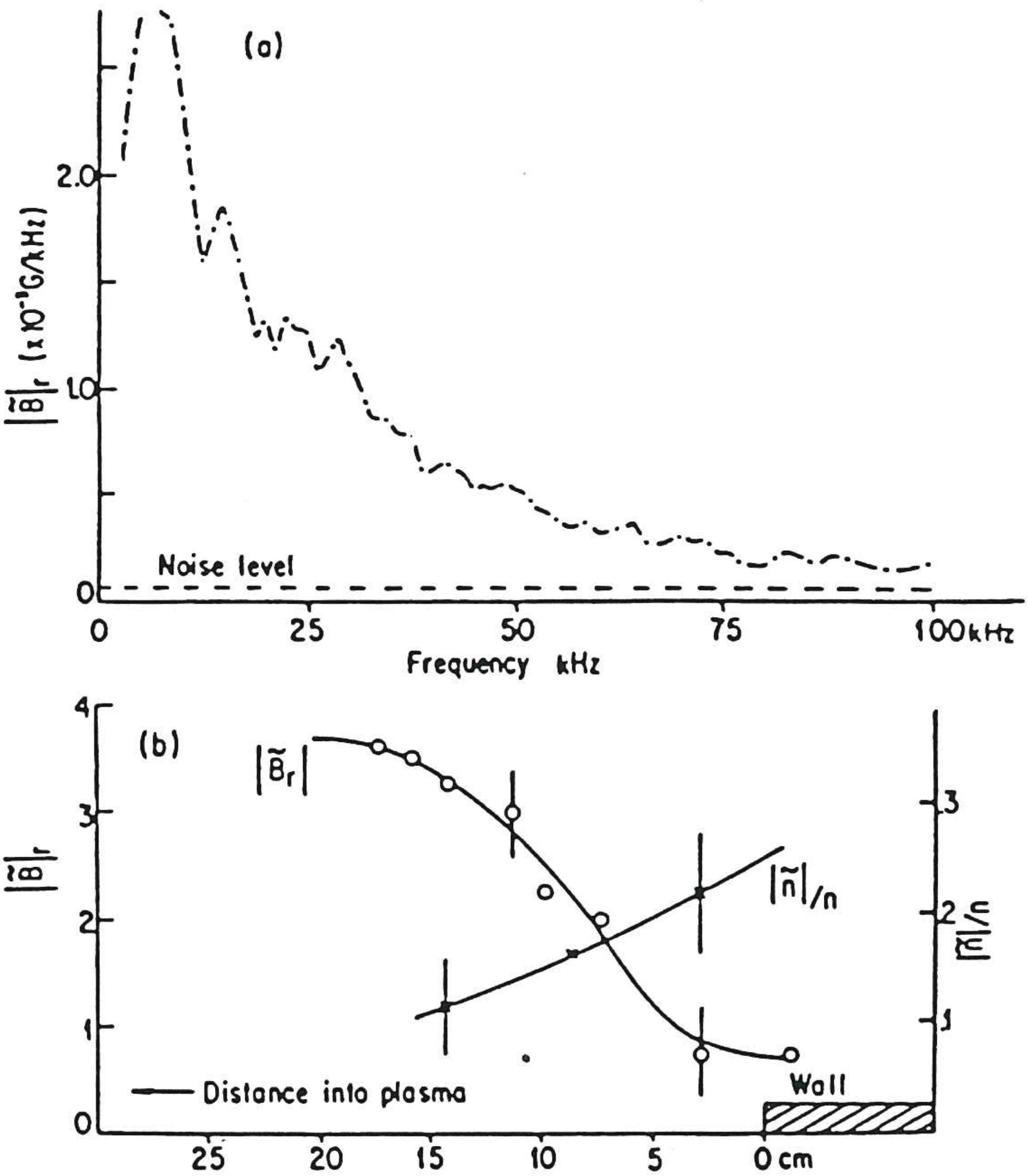

Fig.1 (a) Spectrum of $\tilde{B}_r$ taken at 10 cm into the plasma during the steady state of a typical discharge. (b) The variation of $|\tilde{B}_r|$ vs coil position for $f \simeq 25$ kHz. The spectrum shape remains fairly constant over this range of coil positions. Also in (b) is $|\tilde{n}|_{rms}/n$ profile as measured in the ion saturation current of a Langmuir probe. The ion-saturation-current profile itself looks similar to the profile of $|\tilde{B}_r|$. (Zweben et al 1979).