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MAGNETIC TURBULENCE, ERGODICITY AND TEST-PARTICLE TRANSPORT IN TOKAMAKS

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Abstract

An investigation of very low frequency $\left(\omega \ll \frac{v_{thi}}{R}\right)$ magnetic fluctuations in a two-fluid model is carried out. This shows that such modes do not make a significant contribution to cross field transport. Test-particle diffusion due to an ensemble of quasi-static fluctuations is discussed. It is found that there are significant differences in the low collisionality limit between periodic and infinite systems. Necessary conditions for field line ergodicity due to equilibrium constraints are derived. A novel asymptotic magnetic surface construction is used to derive bounds on test-particle diffusion, where all relevant time-scales are taken into account.

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1. INTRODUCTION

It is generally accepted that electron energy transport in present tokamaks is not in accordance with neoclassical/classical theory. This fact, together with the non-observance of the bootstrap current (BICKERTON, 1972) and the failure to describe impurity transport correctly (see for example TFR GROUP, 1982), have led to doubts concerning the validity of neoclassical theory. On the other hand, the ion transport is approximately neoclassical and Spitzer resistivity appears to be correct in the toroidal direction, at least. Interestingly, recent investigations of various auxiliary heating methods in Stellarators have demonstrated a marked improvement in confinement properties as the ohmic current is reduced. Indeed, for HELIOTRON-E (IIYOSHI, A et al 1982) operated with zero net current it has been claimed that both ions and electrons behave neoclassically.

The present theoretical understanding of the above phenomena is clearly very unsatisfactory. An adequate theory must at least offer a correct qualitative description, as well as being self-consistent within its own terms of reference. Furthermore, it should be able to account for effects due to particle and energy sources. In the present paper, however, we shall restrict our discussion to ohmically heated tokamaks with prespecified sources.

There are two possible ways in which a plasma theorist can react to the above situation. First, that the neoclassical theory has been incorrectly derived from kinetic theory. Given that certain aspects of the theory are approximately substantiated by experiment, then at least part of the present analysis would appear to be valid. Second, and this is the view which we have followed in previous papers (THYAGARAJA et al, 1980, HAAS et al, 1981), that neoclassical theory is essentially correct for the instantaneous state of the plasma, but turbulence must be invoked in order to interpret the observed radial electron thermal conduction as derived from the averaged two-fluid equations of motion. As we have noted previously (THYAGARAJA et al, 1980, HAAS et al, 1981) in order to obtain the effective radial electron thermal conduction, $\kappa_{\perp e}$ it is necessary to have an appropriate form for the parallel electron thermal conduction coefficient, $K_{\parallel e}$. While the classical form (BRAGINSKII, 1965) is suitable for highly collisional plasmas, it is inadequate for conditions where the mean free path is large compared to the dimensions of a tokamak. Since the methodology of neoclassical theory does not allow the determination of parallel transport coefficients, a first-principles derivation of these is strictly necessary, but to our knowledge this has never been carried through. In our previous work (HAAS et al, 1981) we introduced a heuristic form which leads to values of $K_{\perp e}$ in qualitative agreement with several experiments including ALCATOR (COOK et al, 1982). Recent ECRH studies on TOSCA (ALCOCK, et al 1981) have indicated $K_{\parallel e}$ to be of order one hundredth of the classical value, and on substituting the appropriate TOSCA parameters into our heuristic form, we find agreement to within a factor of two. In support of a turbulence interpretation it has recently been suggested (MOLVIG et al, 1981) that low frequency turbulence could destroy the bootstrap current; these authors remark that the level of turbulence required is too low to influence the Ware Pinch effect.

The objective of the present paper is to use the two-fluid equations to explore further the inter relationship between anomalous thermal transport and the concepts of magnetic turbulence and ergodicity. In section 2 we consider whether low-frequency modes can lead to an enhancement of thermal transport. We find that only frequencies greater than or of order $\frac{v_{thi}}{R}$ can produce a significant enhancement. This implies, that very low frequency phenomena do not significantly contribute to anomalous thermal conduction, or for that matter, particle diffusion. As we have commented before (HAAS et al, 1981) while it is possible that time-dependent magnetic fields are ergodic, our derivation of $K_{\perp e}$ makes no explicit reference to this concept. For the static case the magnetic fluctuations are time independent 'ripples' on the mean fields. It is well-known that these ripples modify the standard neoclassical formulae. Hence when we refer to neoclassical theory it is with the assumption that this effect has already been incorporated. Current theories of anomalous transport are based on the idea that an ensemble of ripple fields, which in general are expected to be ergodic, lead to enhanced transport across the mean surfaces.

In section 3 we consider an ensemble of quasi-static small amplitude field fluctuations in a periodic system such as a tokamak. The test-particle diffusion in such a model is discussed and certain ambiguities of interpretation are noted. In section 4 we first derive some necessary conditions for the occurrence of ergodic regions in steady-state tokamaks. Secondly, we discuss a novel asymptotic magnetic surface construction and consider its consequences for test-particle transport across mean-surfaces. Section 5 contains a discussion of several specific points arising from our paper. Finally, section 6 presents our conclusions.

2. VERY LOW FREQUENCY MODES AND ENHANCED TRANSPORT

Using the two-fluid equations, we have previously (HAAS et al, 1981) presented a somewhat simplified turbulence interpretation of anomalous transport. Following this work, we assume a cylindrical model for a tokamak of minor radius a and periodicity length $2\pi R$; we further assume that all perturbed quantities are of the form

$$\delta f = \delta\hat{f}(r) \exp \left[i(\omega t + m\theta + \frac{nz}{R}) \right] \tag{1}$$

when for convenience the m, n, ω dependence of the amplitude is suppressed.* Mean or unperturbed quantities are denoted by the suffix zero. As before we assume the mean quantities, n_o, T_{eo}, T_{io}, p_{eo} etc, to be constant along the unperturbed field lines, $\underline{B}_o$. Unlike our earlier work we now take full account of the electron motion. Our immediate aim is to determine whether or not very low frequency modes can lead to significant enhancement of electron thermal conduction. By very low, we mean those frequencies for which the ion inertia is negligible compared with the pressure gradient term in the ion equation of motion. For typical tokamak conditions this implies that

*Note the present notation is different from that of our earlier papers. We no longer use the explicit parameter ε. Thus in terms of our old notation, Δf and δf denote the same quantities, namely the actual perturbation in position space. The corresponding Fourier amplitude is denoted with a carat as in Eq.(1).

the frequency has to satisfy the inequality $\omega/2\pi << \frac{v_{thi}}{2\pi R} \sim 30$ kHz.

Neglecting inertia and resistivity, the electron motion in the θ and z directions is governed by the equations

$$0 = -en_o \left(\delta\hat{E}_\theta - \frac{\delta\hat{v}_{er}}{c} B_{oz} + \frac{v_{eoz}}{c} \delta\hat{B}_r\right) - \frac{im}{r} \delta\hat{p}_e \tag{2}$$

$$0 = -en_o \left(\delta\hat{E}_z + \frac{\delta\hat{v}_{er}}{c} B_{o\theta} - \frac{v_{eo\theta}}{c} \delta\hat{B}_r\right) - \frac{in}{R} \delta\hat{p}_e, \tag{3}$$

where the poloidal and 'toroidal' electric fields are related through the radial component of Faraday's law,

$$\frac{im}{r} \delta\hat{E}_z - \frac{in}{R} \delta\hat{E}_\theta = -\frac{i\omega}{c} \delta\hat{B}_r. \tag{4}$$

From these equations it is simple to derive

$$\delta\hat{v}_{er} = \frac{\Omega_e}{k_\parallel} \cdot \frac{\delta\hat{B}_r}{B_o} \tag{5}$$

$$\text{where } \Omega_e = \omega + \underline{k}.\underline{v}_{eo} \text{ and } k_\parallel \equiv \frac{\underline{k}.\underline{B}_o}{B_o} = \frac{m}{r}\frac{B_{o\theta}}{B_o} + \frac{n}{R}\frac{B_{oz}}{B_o} \tag{6}$$

A similar procedure for the ions yields the result

$$\delta\hat{v}_{ir} = \frac{\Omega_i}{k_\parallel} \frac{\delta\hat{B}_r}{B_o} \tag{7}$$

where $\Omega_i = \omega + \underline{k}.\underline{v}_{io}$. Henceforth for convenience, we omit the carat over linearised quantities. Using Equation (7), the ion continuity equation can be written as

$$i\,\frac{\delta n}{n_o} + \frac{1}{k_{\shortparallel} n_o}\,\frac{dn_o}{dr}\,\frac{\delta B_r}{B_o} = 0, \tag{8}$$

where the term $\nabla.\underline{\delta v}_i$ has been neglected; this approximation is valid for the frequencies of interest. In fact, Eq.(8) implies

$$(\underline{B}_o + \underline{\delta B}).\nabla(n_o + \delta n) = 0\ (\delta B\ \delta n), \tag{9}$$

that is, the density is constant to first-order along the instantaneous magnetic field lines. Using Eq. (7) and taking cognisance of $\nabla.\underline{\delta v}_i = 0$, the linearised ion energy balance equation becomes

$$\left(i\,\frac{\delta T_i}{T_{io}} + \frac{1}{k_{\shortparallel} T_{io}}\,\frac{dT_{io}}{dr}\,\frac{\delta B_r}{B_o}\right)\left({}^3/_2\ \Omega_i - ik_{\shortparallel}{}^2\,\frac{K_{\shortparallel i}}{n_o}\right) \simeq 0, \tag{10}$$

where all other 'fluttered' contributions such as equipartition, $K_{\perp i}$ etc, are very much smaller. It follows that

$$(\underline{B}_o + \underline{\delta}B).\nabla(T_{io} + \delta T_i) \simeq 0\ (\delta B\ \delta T_i), \tag{11}$$

and hence the ion temperature is also constant to first order along the instantaneous field lines. Combining Eqs.(9) and (11) with the total momentum balance we find,

$$(\underline{B}o + \underline{\delta}B).\nabla(T_{eo} + \delta T_e) = 0\ (\delta B\ \delta T_e), \tag{12}$$

where we have again neglected ion inertia. Thus the electron temperature is also constant to first-order along the instantaneous magnetic field.

Given the above result we now calculate the enhancement. By averaging over the electron energy balance equation we have previously shown (HAAS et al, 1981) the effective perpendicular electron thermal conductivity to be

$$K_{\perp e} = \bar{K}_{\perp e} + K_{\shortparallel e}\,\Gamma_e, \tag{13}$$

where the form factor Γ_e is given by

$$\Gamma_e = \left\langle \left(\frac{\delta B_r}{B_o} \right)^2 \right\rangle + \left\langle \frac{\delta B_r}{B_o} \Delta_{\parallel} \, (\delta T_e) \right\rangle \left(\frac{dT_{eo}}{dr} \right)^{-1} \tag{14}$$

and $\Delta_{\parallel}$ denotes differentiation along the unperturbed field. The coefficient $\bar{K}_{\perp e}$ is taken to be neoclassical, while the form for $K_{\parallel e}$ is unimportant for the present purpose. Using Eq. (12) we deduce that $\Gamma_e \sim \max \left[0 \left(\omega^2 \left(\frac{\bar{K}_{e\parallel}}{n_o R^2} \right)^{-2} \right), 0 \left(\frac{\bar{K}_{e\perp}}{K_{e\parallel}} \right)^2 \right]$. Thus for our model, we conclude that very low frequency modes cannot enhance electron thermal conduction over and above the neoclassical value.

We note that combining Eq. (8) with the electron continuity equation implies $\nabla . \underline{\delta v}_e \simeq 0$. This suggests that we can reverse our line of argument. Thus if we consider frequencies and fluctuation levels such that $|\nabla . \underline{\delta v}_e| \ll \omega \left|\frac{\delta n}{n_o}\right|$, then Eq. (8) immediately follows from Eq. (5) and the electron continuity equation. Consequently, an equation for δT_e exactly analogous to Eq. (10) can be directly derived from the electron energy balance. Once again we deduce Γ_e to be negligible without however, having to neglect ion inertia. Inclusion of electron-ion resistive drag in the above analysis makes only a minor correction comparable in size with the classical $\bar{K}_{\perp e}$. Although thermal forces can contribute significantly, provided $\underline{\nabla . \delta v_e}$ is negligible in the electron energy balance, then Eq. (12) is again appropriate; hence the enhancement is unimportant.

We now summarise the physics of the above analysis. At low frequencies both the ion and electron fluids essentially move with the time-dependent field lines; this is the content of Eqs. (5) and (7). For these frequencies the ion motion is incompressible and the inertia negligible, and hence as shown, the density and ion temperature are constant along field lines (Eqs. (9) and (11)). Through Eq. (9) the electron continuity equation shows the electron fluid motion to be incompressible. Total momentum balance combined with these results leads to the electron temperature also being constant along field lines. Under these circumstances no significant mean energy transport can occur. For an enhancement there must be electron temperature variation along field lines, and this requires that work be done on the electron fluid. Our argument shows that the neglect of ion inertia and incompressibility of the ion fluid are equivalent. While the incompressibility of the ion fluid implies that of the electron fluid the converse is not necessarily true, except at low frequencies. The above considerations also lead to Γ_i being small. Furthermore, since δv_{er} amd δv_{ir} are 90^o out of phase with the density fluctuations, the particle fluxes,

$$\left\langle \delta v_{er} \frac{\delta n}{n_o} \right\rangle \text{ and } \left\langle \delta v_{ir} \frac{\delta n}{n_o} \right\rangle$$

must vanish. Similarly the convective energy fluxes are negligible.

It follows that if a significant turbulent enhancement of cross-field transport is to occur, fluctuations of sufficiently high frequency must be involved. In particular, very low frequency modes cannot be expected to directly contribute to either anomalous energy or particle transport. In principle it is still possible that these modes can drive high frequency turbulence, and therefore, may indirectly lead to enhanced transport.

The above argument neglects the work done by the viscous stresses in the electron fluid, which generally make a small contribution to the phase as do other dissipative processes, for example, electron-ion equilibration. As an example we consider the very interesting results obtained by KADOMTSEV and POGUTSE (1979). These authors consider a single-fluid energy balance equation in which only parallel and perpendicular electron thermal conduction is taken into account. This model can easily be reworked in our two-fluid framework with essentially the same results. The main point is that while we neglect perpendicular thermal conduction in our proof that low frequency modes do not significantly enhance transport, KADOMTSEV and POGUTSE consider the effect of classical/neoclassical perpendicular conduction as a phase-shift creating mechanism even in quasistatic conditions. Actually, as we show in our appendix, the formulae of KADOMTSEV and POGUTSE (see their Eqs. (21) and (22)) have non-negligible contributions only at the resonant points. As we observe in the next section, such expressions are apparently incapable of interpreting the experimental anomalies. This justifies our neglect of dissipative processes with rates small compared to parallel thermal conduction as effective phase-shift mechanisms.

3. STATIC STOCHASTIC FIELDS AND ANOMALOUS TRANSPORT

We now examine a very interesting alternative magnetic turbulence interpretation of anomalous transport put forward by RECHESTER and ROSENBLUTH (1978), and discussed by KROMMES et al (1982) and other authors. This is possibly the best known of all magnetic turbulence models of cross-field electron heat transport. The RECHESTER and ROSENBLUTH formula for $K_{\perp e}$ is based on the following assumptions:

(1) The electron heat transport can be correctly derived by treating the electron guiding centres as test particles and ignoring all collective effects.

(2) The magnetic field in a tokamak can be statistically represented for the purpose of calculating the average cross-field transport as a sum of a mean magnetic field and random static magnetic fluctuations. The mean magnetic field is assumed to produce a set of nested closed magnetic surfaces and the fluctuations are small in amplitude compared with the mean.

(3) The magnetic fields satisfy no other constraint but $\nabla.\underline{B} = 0$; in particular, the motion of the electrons is not related to the evolution of the fields. In this sense the theory is purely kinematic in that the electron diffusion across the mean surfaces is due to field line wandering.

(4) RECHESTER and ROSENBLUTH recognise that their formula does not give the correct particle diffusion coefficient since the result for ions and electrons would be different; this would therefore violate the overall charge neutrality of the plasma. They assume however, that it does give the correct energy loss rate for the electrons.

(5) They derive formulae both in the collisional ($\tau_e v_{the}/qR \ll 1$) and in the so-called collisionless limit. In the latter limit the thermal diffusivity $\bar{K}_{\perp e}/n$ is independent of collision time and therefore of density.

The above assumptions imply according to RECHESTER and ROSENBLUTH a thermal diffusivity $\chi_{\perp st}$ given by

$$\chi_{\perp st}(r) = \pi R v_{the} \sum_m \int dn \frac{|b_{mn}(r)|^2}{B_z^2} \delta\left(\frac{m-nq(r)}{q(r)}\right) \tag{15}$$

where q(r) is the safety-factor. This formula is derived from the field line diffusion calculation given by ROSENBLUTH et al (1966). It may also be written in the equivalent form

$$\chi_{\perp st} = \pi R v_{the} \sum_m \sum_n \frac{|b_{mn}(r)|^2}{B_z^2} \delta_{m,nq} \equiv \pi R v_{the} \sum_m \frac{|b_{mm/q}(r)|^2}{B_z^2} \tag{16}$$

where the Kronecker $\delta_{m,nq} = 1$ when $m = nq$ and $= 0$ otherwise.

Before we go on to consider the consequences of Eq.(16) it is useful to clarify the physical meaning of $\frac{|b_{mn}(r)|^2}{B_z^2}$. An important statistical property of the ensemble is the correlation function,

$$F(r,\theta,z) = \frac{1}{B_z^2} \langle B_r(r,\theta' z') B_r(r,\theta' + \theta, z' + z) \rangle, \tag{17}$$

where the average is over the ensemble of field fluctuations, and stationarity has been assumed. Now $F(r,\theta,z)$ is experimentally measurable in a tokamak and must be periodic in θ,z (cylindrical model with periodicity length $2\pi R$). It then follows from the well-known Wiener-Khintchine theorem that,

$$F(r,\theta,z) = \sum_m \sum_n \frac{1}{B_z^2} |b_{mn}(r)|^2 e^{i(m\theta + \frac{nz}{R})} . \tag{18}$$

We now examine whether Eq.(16) can be used to explain anomalous transport in tokamaks. Eq.(18) tell us $\frac{|b_{mm/q}(r)|^2}{B_z^2}$ is strictly defined in a periodic system only when $^m/_q$ is integer. Since q(r) is generally non-constant in a tokamak we encounter a difficulty: thus when g is irrational, which it is for almost every r, $\frac{|b_{mm/q}(r)|^2}{B_z^2}$ is apparently zero from Eq.(18). If this is accepted, then $\chi_{\perp st}$ makes a contribution to the total thermal conductivity apparently only at rational points (which although infinite in number is a set of measure zero). It follows that the temperature profile calculated using $\chi_{\perp st}$ will be flattened at rational points, but otherwise identical to that obtained using $\chi_{\perp e}{}^{NC}$ (neoclassical). The periodicity of F(r,θ,z) similarly raises difficulties over the interpretation of $\int dn$ in Eq (15).

The test-particle conductivity due to magnetic fluctuations can be expected to have the general form

$$\chi_{\perp e} = \pi R V_{the} \sum_m \sum_n \frac{|b_{mn}(r)|^2}{B_z^2} \; G\left(\frac{m-nq(r)}{\epsilon_{mn}}\right) \qquad (19)$$

where G replaces $\delta_{m,nq}$ in Eq.(16). We have derived a formula of this type for time dependent fluctuations (see Eq.(50)). In the next section we obtain an estimate for test-particle diffusion, Eq.(45) which also leads to a specific form for G and the widths ϵ_{mn}. Our principal aim is to draw attention to the fact that in Eq.(16) some reinterpretation of $\delta_{m,nq}$ is necessary. Furthermore, the comparison with experiment is sensitive to this. In particular, it is not permissible to replace $\delta_{m,nq}$ by unity without some justification.

4. RELEVANCE OF ERGODICITY FOR ANOMALOUS TRANSPORT IN TOKAMAKS

In the previous section we discussed steady stochastic magnetic fields in tokamaks. There are two possibilities for the topology of such fields; either there exists a set of nested closed magnetic surfaces which are slight deformations of the mean surfaces, or there are regions of finite volume in which the field lines wander ergodically. In the former case temperatures and densities will in general be constant on the magnetic surfaces and as we have shown, our model suggests that there is no enhancement. It is generally assumed that it is the second alternative that prevails. We now examine whether this alternative is in fact compatible with the equilibrium conditions (pressure balance and ohm's law) which any plasma must satisfy in a steady state.

(a) CONSTRAINTS ON ERGODICITY IN STEADY-STATE PLASMAS

The arguments we give below are quite general and apply even in toroidal geometry as long as there exists a set of mean magnetic surfaces. Since the static field perturbations are small compared to the mean, the total volume of the ergodic regions must be a fraction of the total plasma volume (MOSER, 1978). Any satisfactory theory must account for the observed transport in the non-ergodic as well as in the ergodic regions. Leaving aside this point for the moment, we consider a mean magnetic surface ψ_o in a toroidal plasma. For simplicity we assume that the field lines within the plasma volume bounded by the surfaces $\psi_o+\Delta\psi$ and $\psi_o-\Delta\psi$ wander ergodically and fill this region. Actually of course, the argument applies to any ergodic set of positive measure within the plasma volume. In a steady state the density and temperature of the ions and electrons in the plasma are definite functions of position which are in principle experimentally measurable. First we consider the case in which there are no plasma flows within the ergodic region. Under steady-state conditions in an ergodic region, the total transport is effectively determined by the parallel transport coefficients. This implies that in these regions, the gradients of temperature and density are very small, and hence the pressure must be uniform. It follows from the momentum equations that the current must be force free.*

There are two possibilities to be considered. In the first, the density in the ergodic region is non-zero. It follows, that in general, Ohm's law cannot be satisfied since $\underline{j}$ must be parallel to $\underline{B}$ and the latter is not parallel to the applied $\underline{E}$. The second possibility is that the density in the ergodic region is zero. In this case, the ergodic region is actually a vacuum and $\underline{E} = \eta \underline{j}$ is not required. However this implies the presence of cavities or filaments, so that density and temperature distribution would not in general be monotonic decreasing functions of ψ. While in principle such equilibria may be possible, we do not believe that there is any experimental support for such phenomena.

The argument given above applies only to equilibria in which there are strictly no flows. This is a very restrictive assumption. In general flow velocities small compared to ion sound speed must be taken into account. We now analyse this case in terms of the single-fluid formalism, that is resistive MHD with $T_i = T_e$.

As before, the temperature and density in the ergodic region must be uniform. The flows being small compared with the ion sound speed, they do not affect the momentum balance. They do, however, play an important role in Ohm's law. From the fact that p is uniform we deduce that

$$\underline{j} = \alpha\ (\underline{r})\ \underline{B}. \tag{20}$$

Since $\nabla.\underline{j} = 0$ and the field lines are ergodic, then α is constant in this region. Since density and temperatures are uniform, the Spitzer resistivity (taken isotropic for simplicity) is also uniform. If the

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*Indeed by definition the pressure must be uniform in an ergodic volume since otherwise the field lines would lie on surfaces. More generally, in an ergodic volume the equation $\underline{B}.\nabla S = 0$ cannot have a non-trivial single-valued function S as a solution.

applied electric field is denoted by $\underline{E}_o$ and the plasma electrostatic potential by ϕ, the flow velocity $\underline{v}$ in the ergodic region must satisfy

$$\frac{\underline{v} \times \underline{B}}{c} = -\underline{E}_o - \nabla\phi + \eta\alpha\,\underline{B} \tag{21}$$

and an equation of continuity which we provisionally take to be

$$\nabla.\underline{v} = 0 \tag{22}$$

The condition that Eq.(21) has a solution $\underline{v}$ is

$$\underline{B}.\nabla\phi = -\underline{B}.\underline{E}_o + \eta\,\alpha\,B^2 \tag{23}$$

Eq.(23) has a solution ϕ in the ergodic region (volume V) provided

$$\underset{L \to \infty}{\text{Limit}} \frac{\int_o^L ds\, \frac{\underline{B}.\underline{E}_o}{B}}{\int_o^L ds\, B} = \eta\,\alpha, \tag{24}$$

where the integral is taken along an ergodic field line. Having calculated ϕ from Eq. (23), $\underline{v}$ is obtained as follows. We write

$$\underline{v} = \underline{v}_\perp + \lambda\underline{B}, \tag{25}$$

where $\underline{v}_\perp$ is given by

$$\underline{v}_\perp = \frac{c}{B^2}\,(\underline{E}_o + \nabla\phi) \times \underline{B} \tag{26}$$

and λ is determined from the equation

$$\underline{B}.\nabla\lambda = -\nabla.\underline{v}_\perp. \tag{27}$$

We note that B and α are themselves solutions of the eigenvalue problem

$$\nabla \times \underline{B} = \frac{4\pi}{c} \alpha \underline{B}, \tag{28}$$

in V with $\underline{n}.\underline{B} = 0$ at the boundary. It follows that $\underline{\nabla}.\underline{v}_{\perp}$ cannot identically vanish in V, since this would constitute an additional constraint on $\underline{B}$, incompatible in general with Eq. (28). In making this statement we explicitly use the assumption that the region is ergodic; thus $\underline{B}$ has no special symmetries. It then follows from Eq.(27) that

$$\underset{L \to \infty}{\text{Limit}} \frac{1}{L} \int_{o}^{L} \frac{1}{B} \nabla.\underline{v}_{\perp} \, ds = 0 \tag{29}$$

Eqs.(24) and (29), together with Eq.(28) are necessary conditions which apparently overdetermine the problem. This suggests that the assumed ergodicity of the $\underline{B}$ field is impossible in the steady-state, even if flows are allowed. Even if an ergodic solution of these could be found, $\frac{\partial}{\partial s} (\lambda \underline{B})$ is not generally zero. This implies via the parallel momentum equation that pressure is non-uniform in the ergodic region.

We have assumed in the above that the incompressibility condition is valid. This suggests that particle sources or slow time-evolution is a necessary requirement for ergodicity.

(b) EXISTENCE AND IMPLICATIONS OF ASYMPTOTIC MAGNETIC SURFACES IN TOKAMAKS

We consider a cylindrical model of the tokamak. The principal features of this model are the following. The vacuum toroidal field is taken to be B_o and uniform throughout the minor radius. The mean current density is denoted by $j_{oz}(r)$. This produces a poloidal field $B_{o\theta}(r)$. We assume that $q(r) = \frac{rB_{oz}}{RB_{o\theta}}$ is a monotonic increasing function of r. In addition to this mean field which clearly lies on magnetic surfaces r = const, we assume that a perturbation $\Delta\underline{B}(r,\theta,z)$ is imposed on the system. For tokamaks $\Delta\underline{B}$ has to be periodic in θ, and z and must satisfy some smallness conditions which we now formulate. Experiment indicates that typically $\left|\frac{\Delta B_r}{B_{oz}}\right| \lesssim 10^{-3}$. Actually since density fluctuations in tokamaks are of order a few percent then $\left|\frac{\Delta j}{j_{oz}}\right| \lesssim 10^{-2}$. Taken together, these experimental limits imply that m, n are of order 100 or less. A similar limitation on m, n can also be obtained from the consideration that the shortest wavelength of the magnetic field perturbation must be of order the ion larmor radius. These considerations suggest that $\Delta\underline{B}$ in tokamaks is effectively a finite Fourier series in θ and z.

We now investigate whether $\underline{B} + \Delta\underline{B}$ lies on magnetic surfaces. We do this by a construction which exploits the assumed smallness of the perturbation. Consider the equation

$$(\underline{B}_o + \Delta\underline{B}) . \nabla S = 0 \qquad (30)$$

where S is a function of r, θ and z which is required to be periodic in θ and z. It is obvious that $S_o(r)$ (arbitrary) is a solution to leading order. However, these are the mean surfaces. We wish to construct a solution which will satisfy the equation to first-order. This is done as follows. We set

$$S = S_o + S_1$$

and hence S_1 satisfies

$$\underline{B}_o.\nabla S_1 + \Delta B_r \frac{dS_o}{dr} = 0 \qquad (31)$$

i.e $$\left(B_{oz}\frac{\partial}{\partial z} + \frac{B_{o\Theta}}{r}\frac{\partial}{\partial\Theta}\right) S_1 + \Delta B_r \frac{dS_o}{dr} = 0, \tag{32}$$

where $$\frac{\Delta B_r}{B_{oz}} = \varepsilon \sum_{-M,-N}^{M,N} b_{mn}(r) \cos\left(m\theta + \frac{nz}{R}\right), \tag{33}$$

with M,N denoting the maximum values of m,n respectively ($b_{oo}(r) \equiv o$)

In the above, ε is the perturbation parameter. Strictly sine terms should be included but do not alter the argument in principle. The $b_{mn}(r)$ are well behaved functions of r. Writing

$$S_1 = \epsilon \sum_{-M,}^{M,} \sum_{-N}^{N} \sigma_{mn}(r) \sin\left(m\theta + \frac{nz}{R}\right) \tag{34}$$

we find σ_{mn} satisfy the relations

$$\sigma_{mn} = -\frac{dS_o}{dr} \frac{b_{mn}(r) B_{oz}}{m\frac{B_{o\theta}}{r} + n\frac{B_{oz}}{R}}$$

$$= \frac{-RdS_o}{dr} \frac{q(r) b_{mn}(r)}{m + nq(r)} \tag{35}$$

This solution appears to suggest that S_1 is well defined and small compared with S_o which is so far arbitrary, except at a finite number of resonant points, $r = r_{mn}$ where $m + nq(r_{mn}) = 0$. We now exploit the arbitrariness of S_o to show that S_1 can actually be well-defined at these resonance points. For this purpose consider the following function of r

$$\phi(r) = \sum_{-M,}^{M,} \sum_{-N}^{N} \frac{b_{mn}^2(r_{mn})}{(m + nq(r))^2} \tag{36}$$

It should be stressed that $\phi(r)$ is a well-defined non-negative function of r for all values of r other than resonant radii. It should also be stressed that $\phi(r)$ can be defined even if M and N tend to infinity, in which case, $\phi(r)$ is a non-negative function of r almost everywhere (ARNOLD and AVEZ 1968). We now define S_o through the differential equation

$$\frac{dS_o}{dr} = \exp\left[-\phi(r)\right] \tag{37}$$

Since $\phi(r)$ is non-negative and $o < \exp(-\phi(r)) \leq 1$ then $S_o(r) = \int_o^r e^{-\phi(u)}du$ is well defined and a monotone increasing function. With this choice σ_{mn} clearly vanishes and has finite derivatives of arbitrary order at all points. However, the behaviour of S_{1/S_o} as a function of ϵ must be investigated near resonant points even though it is zero at resonant points. To see this, it is sufficient to restrict attention to a particular resonant point r_{mn}. If we consider σ_{mn} in the neighbourhood of r_{mn}, putting $r - r_{mn} = x$, we get

$$\sigma_{mn}(x) \sim \frac{R\, e^{-\phi(r_{mn}+x)} q\,(r_{mn})\, b_{mn}\,(r_{mn})}{nq'\,(r_{mn})\, x} \tag{38}$$

Now $$\phi\,(r_{mn} + x) \sim \left(\frac{b_{mn}(r_{mn})}{nq'(r_{mn})x}\right)^2 + \sum_{-M}^{M}\sum_{-N}^{N}{}_{m'\neq m,\; n'\neq n} \frac{b^2_{m'n'}(r_{mn})}{\left(m' + n'q(r_{mn})\right)^2} \tag{39}$$

Thus, $$\sigma_{mn}\,(x) \sim \frac{C}{x}\exp\left(-\frac{D}{x^2}\right)$$

The maximum of σ_{mn} occurs when $x \sim D^{\frac{1}{2}}$. But this implies* that the maximum of $\frac{S_1}{S_o}$ in the neighbourhood of r_{mn} is $O\,(\varepsilon)$. This applies to all resonances.

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*The above estimates for $\sigma_{mn}(x)$ is considered here for the "generic case" when $b_{mn}(r_{mn}) \neq 0$. If $b_{mn}(r_{mn})$ were zero as some theories require, then $\sigma_{mn}(x)$ vanishes and hence $S_{1/S_o} \sim O(\varepsilon)$ anyway.

The above procedure can obviously be continued to any desired order in ε. In second and higher orders, secondary resonances not contained in $\phi(r)$ are created and these are controlled as before by suitable additions to ϕ. The form of S_o ensures that the resulting expansion is not affected by the resonant denominators. Ultimately, it is only the smoothness of $q(r)$ and $b_{mn}(r)$ which determines the size of the successive terms; we do not know if the series converges. It is not possible to conclude from Poincaré's well-known result (see WHITTAKER, 1965) on the non-existence of analytic integrals for Hamiltonian systems, that the series for S is divergent. This is because Poincaré's proof applies only to analytic S, whereas our S_o, although infinitely differentiable, is not Taylor expandable. Thus we have only established the existence of surfaces in an asymptotic sense. The magnetic field lines must be confined to these surfaces provided we consider only finite lengths along them. Thus let us consider the function $S = S_o + S_1$. A test particle moving along a field line with velocity v_{the} (taken constant for simplicity) satisfies the equation

$$\frac{d\underline{r}}{dt} = v_{the} \frac{\underline{B}}{B_o} . \tag{40}$$

It then follows that

$$\frac{d}{dt}\left(S_o + S_1\right) = v_{the} \frac{\underline{\underline{B}}}{B_o} . \nabla\left(S_o + S_1\right) = O(\varepsilon^2). \tag{41}$$

This equation implies that significant changes in $S_o(r)$, and therefore of r, occur only on time-scales of order $\frac{R}{v_{the}\,\varepsilon^2}$. The latter is a measure of particle confinement time in the absence of collisions. For $\varepsilon \sim 10^{-3}$, the time is of order 1 sec for typical conditions, which is much larger than the observed confinement times of about 30 millisecs.

We must emphasise two important points. First, our construction refers only to the motion of particles for time-scales over which $S_o + S_1$ is an approximate invariant. Thus it <u>does not</u> shed any light on whether a particle followed for an <u>infinitely</u> long time explores a finite volume ergodically. Secondly, it <u>does</u> state that for sufficiently small times (for any given level of ε), irrespective of the topology of the field lines, the maximum radial excursion of the particle with respect to its initial position is limited. In our view confinement relates to particle excursions on time-scales fixed by experiment. This means that the topology of the trajectories is important only on those time-scales.

We now show how the surface construction can be used to estimate a test-particle diffusion coefficient. It is important to note that a test-particle stays on a field line for a time τ_e (collision time). Since $\tau_e \ll qR/v_{the}\,\varepsilon$, for example in PLT ($n \sim 5\times 10^{13}\,cm^{-3}$, $T_{oe} \sim 1$ keV, $R \sim 150$ cms, $a \sim 30$ cms, $q \sim 2.0$ and $\varepsilon \le 10^{-3}$) $\tau_e \sim 6\times 10^{-6}$ sec and $qR/v_{the}\,\varepsilon \sim 3\times 10^{-4}$ sec, then $S_o + S_1$ is certainly a good constant of the motion for $t \le \tau_e$. This fact enables us to estimate $\langle(\Delta r)^2\rangle$.

From Eq. (40)

$$\frac{dS_o}{dt} = -\,v_{the}\,\frac{\underline{\underline{B}}}{B_o}\cdot\nabla S_1 + O(\varepsilon^2). \tag{42}$$

Integrating from $t = o$ to $t = \tau_e$, the change in S_o is given by

$$\Delta S_o = \varepsilon v_{the}\,\frac{dS_o}{dr}\sum_m\sum_n b_{mn}(r)\left\{\frac{\sin\left(\frac{v_{the}}{qr}(m+nq)\tau_e\right)}{\frac{v_{the}}{qR}(m+nq)}\right\} + O\left(\frac{\varepsilon^2 v_{the}\tau_e}{qR}\right) \tag{43}$$

Ensemble averaging with respect to the magnetic fluctuations, Eq.(43) implies

$$\langle(\Delta r)^2\rangle = \varepsilon^2 q^2 R^2 \sum_m\sum_n \langle b^2_{mn}(r)\rangle \left\{\frac{\sin^2\left(\frac{v_{the}}{qR}(m+nq)\tau_e\right)}{(m+nq)^2}\right\} + O\left(\frac{\varepsilon^4 v^2_{the}\tau^2_e}{q^2R^2}\right). \tag{44}$$

Intuition suggests that the associated test-particle diffusion coefficient is given by

$$D_\perp \sim \frac{1}{\tau_e}\,\langle(\Delta r)^2\rangle = \frac{\varepsilon^2 q^2 R^2}{\tau_e}\sum_m\sum_n \frac{\langle b^2_{mn}(r)\rangle}{(m+nq)^2}\,\sin^2\left(\frac{v_{the}}{qR}(m+nq)\tau_e\right). \tag{45}$$

The above formula has a number of interesting features which we now discuss. We note that the formula is derived for a periodic system and takes account of the two relevant timescales $\left(\text{ie } \tau_e \text{ and } \frac{qR}{v_{the}}\right)$. It is independent of the choice of S_o, as is required by physical considerations. Furthermore, the formula only requires the specification of the spectral functions $\langle b^2_{mn}(r)\rangle$, and is otherwise independent of the nature of the modes.

It is of interest to note that the formula contains the Knudsen number $Kn = \frac{v_{the}\tau_e}{qR}$ explicitly.

We first consider the limit of high collisionality (ie $Kn \ll 1$). In this limit we find

$$D_\perp = \varepsilon^2 v_{the}^2 \tau_e \sum_m \sum_n \langle b_{mn}^2(r) \rangle$$

$$= \chi_{\parallel}(\text{Braginski}) \left\langle \left(\frac{\Delta B_r}{B_o} \right)^2 \right\rangle, \tag{46}$$

which of course approaches zero at $\tau_e \to 0$. This form has been proposed by CALLEN (1977). Next we consider the range $qR/v_{the} \ll \tau_e \ll \frac{qR}{v_{the}\varepsilon}$, and obtain

$$D_\perp = \varepsilon^2 \pi R v_{the} \sum_m \sum_n \langle b_{mn}^2(r) \rangle \, \delta\left(\frac{m + nq}{q} \right). \tag{47}$$

Eq. (47) is obtained from Eq. (45) using the well-known results

$$\lim_{\alpha \to \infty} \frac{\sin^2 \alpha x}{\alpha x^2} = \pi \delta(x)$$

and

$$a\,\delta(x) = \delta\left(\frac{x}{a}\right).$$

A physically more transparent form is obtained by noting that for large Kn

$$\frac{q^2R^2}{\tau_e} \frac{\sin^2\left(\frac{v_{the}\tau_e}{qR}(m + nq)\right)}{(m + nq)^2} \to v_{the}^2 \tau_e \quad \text{if } m + nq = 0 .$$

$$\to 0 \quad \text{if } m + nq \neq 0 .$$

This results in

$$D_\perp = \varepsilon^2 \chi_{\parallel e}(\text{Braginskii}) \sum_m \sum_n \langle b_{mn}^2(r) \rangle \, \delta_{m,-nq} .$$

As we show in our appendix this is exactly the result obtained by a renormalised Kadomtsev-Pogutse approach (Eq. (A5)). Eq. (47) is very similar to that given in Eq. (15), except for the fact that it is appropriate to a periodic system. A similar argument indicates that at any fixed value of Kn , the effects of high m , n modes are localised about the resonant points.

Eq.(45) also suggests an upper bound to test-particle diffusion given by,

$$D_\perp^{max} = \frac{\varepsilon^2 q^2 R^2}{\tau_e} \sum_m \sum_n \frac{\langle b_{mn}^2(r)\rangle}{(m+nq)^2} . \tag{48}$$

Although in general $D_\perp^{max}$ is singular at the resonant points, it leads to a definite prediction for the temperature profile if interpreted as a thermal diffusivity. It can be used to give a lower bound for the central electron temperature for given heat sources and $\langle b_{mn}^2(r)\rangle$. It is of interest to note that $D_\perp^{max}$ involves $\frac{q^2R^2}{\tau_e}$ which is of the same order as our Knudsen corrected parallel thermal diffusivity, $\frac{\pi^2R^2}{\tau_e}$. Finally, we remark that any profile calculated from $D_\perp^{max}$ with a suitably smooth source leads to a temperature profile which has all the characteristics of $S_o(r)$ and could indeed have been used in its place in our surface construction.

We noted earlier the importance of distinguishing between a periodic and an infinite system in discussing test-particle diffusion. This shows up even more clearly with regard to our surface construction. For example, in an infinite system the function $\phi(r)$ would not be well-defined and consequently S_o and $D_\perp^{max}$ do not exist. By the same token, Eq.(47) would lead to a well-defined $D_\perp$ independent of τ_e since the sum over n would be replaced by an integral and $\langle b^2_{m\frac{m}{q}}(r)\rangle$ is a smooth function of r. The fact that one has a diffusion coefficient independent of collisions does not lead to a paradox since in an infinite reversible system diffusion (ie irreversible behaviour) is possible.

5. DISCUSSION

In this section a number of specific points arising from our interpretation of anomalous transport are discussed. We observe that any turbulence interpretation of anomalous transport should take account of both electrons and ions. Thus single-fluid theories are not satisfactory. Next we remark that collisional processes (eg. parallel thermal conduction) play an important role even in the so-called low collisionality regime. This follows from the fact that in the total absence of all collisions, charged particles are permanently attached to field lines and cannot on the average leave a mean magnetic surface. Thus any cross-field transport must depend on the basic collision frequencies. In this respect there is an essential difference between topologically closed configurations (tokamak, stellarators) and systems with ends. In a system with ends, irreversible behaviour (that is, particle or energy transport) can occur through boundary interactions. For closed systems with definite magnetic surfaces, boundary conditions alone cannot lead to cross-field transport. Collisional terms in the equations are therefore essential.

Our calculation of $K_{\perp e}$ depends crucially on three basic features. First, the parallel electron energy transport is essentially diffusive. Second, $K_{\parallel e}$ is of order $\frac{1}{4} n_o \frac{(2\pi R)^2}{\tau_e}$ in the appropriate low-collisional regime. Finally the magnetic fluctuation spectrum is taken to be given. The frequencies of interest are between 10 and 500 kHz, and the amplitude levels are between 10^{-3} and 10^{-4}.

Next we discuss the relationship between our formula for $K_{\perp e}$, Eq.(13), and the result of RECHESTER and ROSENBLUTH, Eq.(16). We define

$$\chi_{\perp e} = \frac{K_{\perp e}}{n_o} \simeq \frac{K_{\parallel e}\Gamma_e}{n_o} \qquad (49)$$

where for our simple model (THYAGARAJA, HAAS, COOK, 1980)

$$\Gamma_e = \sum_m \sum_n \int_{-\infty}^{\infty} d\omega\, C^2_{mn}\,(r,\omega) \left[\frac{(\omega - \Omega_{mn})^2}{(\omega - \Omega_{mn})^2 + 4/9\,(\bar{\omega}_{\parallel e}\,\xi^2_{mn})^2} \right] \qquad (50)$$

with

$$\left.\begin{aligned} \bar{\omega}_{\parallel e} &= \frac{K_{\parallel e}}{n_o a^2} \\ \Omega_{mn} &= -\left(\frac{m}{r}\, U_{oe\theta} + \frac{n}{R}\, U_{oez} \right) \\ \xi_{mn} &= \frac{ma}{r}\, \frac{B_{o\theta}}{B_o} + n\,\frac{a}{R}\,\frac{B_{oz}}{B_o} \end{aligned}\right\} \qquad (51)$$

In Eqs.(50) and (51) we consider the limit $\frac{|\omega - \Omega_{mn}|}{\bar{\omega}_{e\parallel}} \equiv \alpha \to 0$ for all m, n such that C^2_{mn} makes a substantial contribution to the sum in Eq.(50). It follows that

$$\lim_{\alpha \to 0} \chi_{\perp e} = \pi a^2 \sum_m \sum_n \delta\left(\frac{2}{3}\,\xi^2_{mn}\,(r)\right) \int_{-\infty}^{\infty} d\omega\, C^2_{mn}\,(r,\omega)\,|\omega - \Omega_{mn}| \qquad (52)$$

It is interesting to note the structural similarity between $\chi_{\perp e}$ and $\chi_{\perp st}$ given by Eq.(16). Setting,

$$|\hat{\omega}_{mn}| \left\langle \left(\frac{\delta B}{B_o}\right)^2 \right\rangle_{mn} = \int_{-\infty}^{\infty} d\omega\, C^2_{mn}\,(r,\omega)\,|\omega - \Omega_{mn}| \qquad (53)$$

$\chi_{\perp e}$ is seen (in the limit $\alpha \to 0$) to be a sum of resonant contributions of the dense set of surfaces m, n. Unlike $\chi_{\perp st}$, $\chi_{\perp e}$ depends on the effective Doppler-shifted mode frequency $|\hat{\omega}_{mn}|$ defined by Eq.(53). The collision time disappears since we are assuming that $\frac{(2\pi R)^2}{a^2 \tau_e} \gg |\hat{\omega}_{mn}|$.

However, $\chi_{\perp e}$ defined in this limit by Eq.(52) is zero almost everywhere, and according to our arguments in section 3 makes no effective contribution to cross-field transport. This line of reasoning also predicts that the high m, n do not contribute to $\chi_{\perp e}$ provided $|\hat{\omega}_{mn}| << \omega_{e\parallel}\xi^2_{mn}$, as previously noted (HAAS, THYAGARAJA, COOK, 1981). It is very important to remark that this last conclusion relies heavily on the assumption that parallel energy transport is diffusive. This assumption is strictly valid only in the collisional regime (Kn << 1). The present status of our assumptions regarding parallel transport in tokamaks needs to be clarified. Both theoretical and experimental investigations are called for. The theory must establish whether parallel transport at low collisionality is or is not diffusive.

In our approach to transport due to an ensemble of magnetic fluctuations, we do not need to refer explicitly to the ergodicity, or otherwise, of field lines. Many current approaches appear to be based on CHIRIKOV'S (1979) well-known criterion for the occurrence of ergodicity due to island overlap. As we have noted before, there is no actual conflict between the conclusions deduced from our surface construction and field line ergodicity. It is of interest to note, however, that CHIRIKOV himself (loc. cit. p.308) remarks on the limitations of the criterion with respect to certain numerical investigations. We also remark that the considerations of KAM theory do not have any direct relevance to our surface construction. This is because the KAM estimates of ergodic and non-ergodic regions (ARNOLD and AVEZ, 1968, and MOSER, 1978) do not apply to non-analytic functions like $S_o(r)$.

It is important to recognise that our formula for $D_\perp$ (Eq. (45)) refers to a statistical ensemble of magnetic fluctuations. Hence, it does not apply to the case of a fixed (non-random) perturbation, which can give rise to a deformed surface. In this case the $b_{mn}(r)$ are deterministic qualities and the problem is no longer a random walk; it follows that the transport across such surfaces can only be classical.

6. CONCLUSIONS

We have investigated very low frequency magnetic fluctuations within our two fluid transport model and find that they do not contribute significantly to cross field transport. Further, we have discussed test-particle diffusion due to an ensemble of quasi-static magnetic fluctuations and note a significant difference between periodic (such as tokamaks) and infinite systems. Making use of a novel magnetic surface construction, we estimate upper bounds to test-particle transport in a periodic system, taking account of collisions.

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APPENDIX

KADOMTSEV and POGUTSE derive (in our notation) the following formula for $\chi_{\perp e}$ $\left(= \frac{K_{\perp e}}{n_o}\right)$. Thus

$$\chi_{\perp e} = \frac{1}{2} \sum_{m,n} \int \frac{dk_\perp \, \bar{\chi}_{\perp e} \, k_\perp^2 \left(\frac{\delta B_r}{B_o} (m,n,k_\perp^2)\right)^2}{k_{\parallel}^2 + \frac{\bar{\chi}_{\perp e}}{\chi_{\parallel e}} k_\perp^2} \tag{A1}$$

In the above equation $\bar{\chi}_{\perp e}$ is the classical/neoclassical thermal conduction. The field fluctuation $\frac{\delta B_r}{B_o}$ at a radius r is a function of m,n and a perpendicular wave number $k_\perp$. KADOMTSEV and POGUTSE show that Eq.(A1) can be written in the form

$$\chi_{\perp e} = \frac{\pi}{2} \sqrt{\chi_{\parallel e} \, \bar{\chi}_{\perp e}} \sum_{m,n} \int dk_\perp \, \delta(k_\parallel) \left(\frac{\delta B_r}{B_o}\right)^2 \tag{A2}$$

Eq.(A2) is exactly the same structure as the formula given by RECHESTER and ROSENBLUTH (see Eq.(15)). Thus as our arguments in section 3 show such a $\chi_{\perp e}$ cannot lead to anomalous transport.

Actually the same conclusion can be derived from Eq.(A1) from an even more general point of view. Thus we can argue that the $\bar{\chi}_{\perp e}$ in Eq.(A1) should actually be set to $\chi_{\perp e}$ thus deriving the following "renormalised" equation

$$\frac{\chi_{\perp e}}{\chi_{\parallel e}} \equiv \gamma = \frac{1}{2} \sum_{m,n} \int \frac{dk_\perp \, \gamma k_\perp^2 \left(\frac{\delta B_r}{B_o}\right)^2}{k_\parallel^2 + \gamma k_\perp^2} \tag{A3}$$

This equation can be rewritten as

$$\gamma + \frac{1}{2} \sum_{m,n} \int \frac{dk_\perp \, k_\parallel^2 \left(\frac{\delta B_r}{B_o}\right)^2}{k_\parallel^2 + \gamma k_\perp^2} = \frac{1}{2} \sum_{m,n} \int dk_\perp \left(\frac{\delta B_r}{B_o}\right)^2 \tag{A4}$$

It is easily seen that all points r such that $k_{\parallel} \neq 0$ for any m,n, the only solution of Eq. (A4) is $\Upsilon = 0$. Thus in fact it is easy to verify that as a function of r the solution of Eq.(A4) can be written as

$$\gamma(r) = \frac{1}{2} \sum_{m,n} \int dk_{\perp} \left(\frac{\delta B_r}{B_o}\right)^2 \delta(k_{\parallel}) \qquad \text{(A5)}$$

where $\delta(k_{\parallel})$ is unity if $k_{\parallel} = 0$ and vanishes otherwise. It is important to note that this renormalised calculation does not lead to a "resonance broadening" of the Kronecker - δ function as might have been expected. Again Eq.(A5) leads to a $\chi_{\perp e}$ which is non-zero only at resonant points. Thus we conclude that the renormalised Kadomtsev-Pogutse theory predicts a $\chi_{\perp e}$ due to field fluctuations identical in form with RECHESTER and ROSENBLUTH's expression and with our own, Eq.(52) in a suitable limit.