

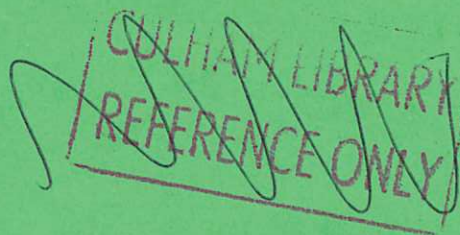
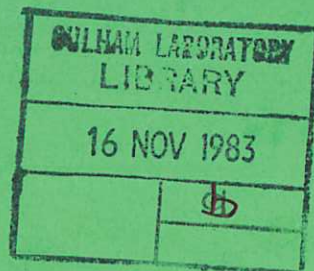


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# ELECTROSTATIC ENHANCEMENT OF MIRROR CONFINEMENT

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# ELECTROSTATIC ENHANCEMENT OF MIRROR CONFINEMENT

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## Abstract

The basic requirements for the electrostatic enhancement of magnetic mirrors, as in Tandem-mirror and thermal barrier designs, are examined. This discussion leads to a simple, intuitively convincing, picture of the operation of such systems and resolves the question of the self-consistency of simultaneous electrostatic confinement of electron and ion populations.

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The adiabatic confinement of plasma ions and electrons in a magnetic mirror has been extensively studied, both theoretically, and experimentally, and is well understood. Unfortunately, collisional scattering into the loss cones in velocity space is such that, even in the absence of any instability problems, ion-confinement is barely, if at all, adequate for the needs of fusion power. The "tandem-mirror" /1,2/ (T.M.) and "thermal-barrier" /3/ (T.B.) concepts, in which electrostatic effects are used to enhance confinement of thermonuclear ions, are therefore crucial for the development of magnetic-mirror fusion devices.

Unfortunately, these schemes are complex, and it is difficult to establish clearly the principles of operation and the self-consistency of the various potentials with each other, with the particle distributions and with the particle losses. In this note, therefore, we examine the basic requirements for setting-up self-consistent electrostatic potentials to enhance confinement in a magnetic mirror system. In addition to clarifying the general concepts this discussion leads us to a conceptually simple scheme for electrostatically enhanced confinement, including a thermal barrier, in which self consistency can be demonstrated and the potentials readily estimated. Although intended purely for heuristic purposes this simple scheme may have practical applications.

In a simple magnetic mirror, collisions scatter electrons more rapidly than ions and would be less well confined were it not for the net positive charge which develops to retain them. A simple mirror therefore acquires a positive 'ambipolar' potential  $\phi$ , typically somewhat larger than the electron temperature  $T_e$ . The basic idea behind the tandem-mirror is to use this potential, created in a small "plug" mirror machine, to electrostatically block the escape of ions from a larger central solenoid containing the bulk of the thermonuclear plasma. The magnetic field distribution is shown in fig. 1 and the potential distribution is shown in fig. 2. Thermonuclear ions in the central cell are confined mainly by the electrostatic barrier  $\phi_i$  created in the plug or ends-cell; electrons are confined mainly by the overall potential  $\phi_e$ .

Because the magnitude of  $\phi_i$  is related to the electron temperature  $T_e$  it would be advantageous to heat the electrons, but to heat all of them would

be uneconomic, hence a further refinement, the thermal barrier, is introduced. This requires an additional dip  $\phi_b$  in the potential (fig. 2) which acts as a barrier to isolate electrons in the plug from those in the central cell, so that the former can be heated without unacceptable energy loss to the latter. A method proposed /3,4/ for forming this additional potential barrier is the creation of a local depression in the magnetic field so as to reduce the density of ions streaming through it; simultaneously the accumulation of trapped ions within the potential dip would have to be prevented by some form of ion pump-out. The potential dip could be supplemented by creating a population of high energy electrons magnetically confined in its vicinity.

Let us examine these suggestions more carefully. A crucial factor, not brought out in the published accounts, which is essential for understanding electrostatic enhancement is the following. The aim is to confine the bulk of the electrons and ions by electrostatic potentials. If this is achieved they will acquire something close to a Maxwell-Boltzmann distribution, so the charge density due to the main population will be

$$\rho_{es} = n_i \exp(-\phi/T_i) - n_e \exp(+\phi/T_e) \quad (1)$$

which decreases monotonically with  $\phi$ . Then in Poisson's equation, assuming potential variations along the field are much more rapid than across it, we have

$$\frac{d^2\phi}{dz^2} = -\rho_{es}(\phi) + \rho_m$$

where  $\rho_m$  is the contribution from non-electrostatically confined species (such as the energetic plug-ions). Suppose that in the central solenoid where the potential is sensibly uniform, both  $\rho_{es}$  and  $\rho_m$  are negligible. Then if we measure  $\phi$  from its value in the centre, the monotonicity of  $\rho_{es}$  means that to create a positive potential barrier ( $d^2\phi/dz^2 < 0$ ;  $\phi > 0$ ) we must have  $\rho_m < 0$ . Similarly to create a negative potential barrier ( $d^2\phi/dz^2 > 0$ ;  $\phi < 0$ ) we must have  $\rho_m > 0$ . Thus, conceptually, a positive potential barrier is created by a component of magnetically trapped ions,

(as in the basic TM design), and a negative potential barrier is created by magnetically trapped electrons. Viewed in this light the operation of the TM-TB system becomes obvious and convincing. The magnetically trapped species act as fixed charged "grids" which are necessary for the creation of the desired potentials. The electrostatically confined species, whether they be ions or electrons, can only reduce the barriers created by the magnetically confined species. This conclusion is not surprising (since Earnshaw's theorem states that pure electrostatic confinement is not possible) but armed with this picture of their operation one can envisage simple forms of electrostatically enhanced magnetic mirrors.

These schemes involve only a single magnetic mirror at each end of a central zone, Fig 3. To electrostatically plug the ion losses in such a mirror requires only a population of high-energy "sloshing" ions sufficiently peaked in pitch angle that they form large positive density peaks near their reflection points. In the light of our earlier discussion the thermal barrier requires the positioning of negative charge just inboard of the positive charge forming the electrostatic plug. This can be obtained from a similar high-energy sloshing-electron population reflected from the magnetic field just inside the sloshing-ion reflection point. The magnetically restrained charge densities are then as shown in fig 4, and the corresponding potential has the form shown in fig 5. We now introduce into this potential (i) the main population of thermonuclear ions, electrostatically confined by the barrier  $\phi_i$ , (ii) the main electron population, electrostatically confined by the barrier  $\phi_e$ , (iii) a population of hot electrons, electrostatically confined in the plug potential  $(\phi_i + \phi_b)$ . These are shown in fig 6. As our earlier comments indicated, the presence of these populations tends to reduce the potential barriers but does not remove them. We now turn to the crucial matters of demonstrating that the desired potentials can be self-consistently maintained and of determining the various barrier heights.

The system described contains five particle populations:

- (a) The magnetically restrained sloshing-ions  $n_{is}$
- (b) the magnetically restrained sloshing-electrons  $n_{es}$
- (c) the main ion population, electrostatically confined,  $n_{ic}$



- (d) the main electron population, electrostatically confined,  $n_{ec}$
- (e) the hot-electron population, electrostatically confined,  $n_{ep}$

There are three significant potential barrier heights to be determined;

- (f) the ion retaining barrier  $\phi_i$
- (g) the electron retaining barrier  $\phi_e$
- (h) the plug barrier  $\phi_b$ .

The populations (a) and (b) are under external control, through the processes creating the sloshing species (e.g. ion-injection and ECRH) and we will take the peak sloshing densities  $n_{is}$  and  $n_{es}$  to be given. The thermonuclear ion density is also under control, by e.g. the rate of gas or plasma feed, and we can take the ion central density  $n_{ic}$  as also given. This leaves five quantities which must be self-consistently determined; the plug hot-electron density  $n_{ep}$ , the centre electron density  $n_{ec}$  and the three potentials  $\phi_i$ ,  $\phi_e$ ,  $\phi_b$ .

For heuristic purposes we may assume that the sloshing populations are sufficiently energetic that they are entirely confined by the magnetic field and essentially collisionless. We may also assume that near their respective reflection points the density of each sloshing species much exceeds that of the similar electrostatically confined species (which are repelled from these regions). Then charge neutrality in the plug gives the relation

$$n_{ep} = n_{is} .$$

Moreover, we may assume the sloshing densities are negligible in the centre-zone so that charge neutrality there gives

$$n_{ec} = n_{ic} ,$$

leaving only the three potentials to be determined in terms of externally controlled quantities.



In the thermal barrier (which attracts ions and repels electrons) we may assume that only the thermonuclear ion density and the sloshing electrons are important, then

$$n_{ic} \exp(\phi_b/T_{ic}) = n_{es} \quad (2)$$

so determining

$$\phi_b = T_{ic} \log(n_{es}/n_{ic}) \quad (3)$$

The remaining potentials are determined by collisional processes. First we require that collisional end-loss of ions and electrons be equal, hence (see appendix)

$$\frac{T_{ic} n_{ic} \exp(-\phi_i/T_{ic})}{\tau_{ic} \phi_i \log(B_p/B_c)} \sim \frac{T_{ec} n_{ec} \exp(-\phi_e/T_{ec})}{\tau_{ec} \phi_e \log(B_i/B_e)} \quad (4)$$

(where  $\tau$  is the usual  $90^\circ$  scattering time  $\sim n^{-1} m^{1/2} T^{3/2}$ ). Neglecting some unimportant logarithmic factors this gives,

$$\frac{\phi_e}{T_{ec}} \sim \frac{\phi_i}{T_{ic}} + \frac{1}{2} \log \left( \frac{M T_{ic}^3}{m T_{ec}^3} \right) \quad (5)$$

The second important collisional process is the exchange between the centre cell and plug electrons. From the plug side the exchange is over a barrier  $(\phi_i + \phi_b)$  at temperature  $T_{ep}$ , while from the central cell side it is over a barrier  $\phi_b$  at temperature  $T_{ec}$ . Balancing the two contributions (see appendix) gives the condition

$$\frac{T_{ep}}{\tau_{ep}} \exp[-(\phi_i + \phi_b)/T_{ep}] \sim \frac{T_{ec}}{\tau_{ec}} \exp[-\phi_b/T_{ec}] \quad (6)$$

so that

$$\frac{\phi_i + \phi_b}{T_{ep}} \sim \frac{\phi_b}{T_{ec}} + \log \left( \frac{n_{ep} T_{ec}^{1/2}}{n_{ec} T_{ep}^{1/2}} \right) \quad (7)$$

Equations (5) and (7) determine the potentials  $\phi_i$  and  $\phi_e$  in terms of externally controlled quantities. The values obtained are of course only rough estimates but the argument serves to make clear that the potentials and densities are determined and are self consistent with each other and with particle scattering. The potential barriers are:

$$\frac{\phi_b}{T_{ic}} \sim \log \left\{ \frac{n_{es}}{n_{ic}} \right\} \quad (8)$$

(this restrains the bulk electrons from entering the high temperature plug)

$$\frac{\phi_i}{T_{ic}} \sim \left\{ \frac{T_{ep}}{T_{ec}} - 1 \right\} \log \left\{ \frac{n_{es}}{n_{ic}} \right\} + \frac{T_{ep}}{T_{ic}} \log \left\{ \frac{n_{is}}{n_{ic}} \frac{T_{ec}^{1/2}}{T_{ep}^{1/2}} \right\} \quad (9)$$

(this barrier confines thermonuclear ions to the central-cell)

$$\frac{\phi_e}{T_{ec}} \sim \left\{ \frac{T_{ep}}{T_{ec}} - 1 \right\} \log \left\{ \frac{n_{es}}{n_{ic}} \right\} + \frac{T_{ep}}{T_{ic}} \log \left\{ \frac{n_{is}}{n_{ic}} \frac{T_{ec}^{1/2}}{T_{ep}^{1/2}} \right\} + \frac{1}{2} \log \left\{ - \frac{M}{m} \frac{T_{ic}^3}{T_{ec}^3} \right\} \quad (10)$$

(this barrier confines the bulk electrons to the system)

$$\frac{\phi_i + \phi_b}{T_{ep}} \sim \frac{T_{ic}}{T_{ec}} \log \left\{ \frac{n_{es}}{n_{ic}} \right\} + \log \left\{ \frac{n_{is}}{n_{ic}} \frac{T_{ec}^{1/2}}{T_{ep}^{1/2}} \right\} \quad (11)$$

(this barrier restrains hot plug-electrons from entering the centre cell.)

These estimates show that the machine can indeed operate in the manner described if  $n_{ic} < n_{es}$ . Note that the ion-retaining potential barrier is enhanced by the ratio of plug electron temperature to the bulk-electron

temperature as intended. The sloshing-electron density required for operation could be reduced if ions trapped in the potential  $\phi_b$  could economically be pumped out. In accord with our interpretation such pumping lessens the undesirable effect of the ions but it cannot replace the magnetically restrained sloshing electrons.

If one dispensed with the thermal barrier one would have an even simpler electrostatically-enhanced mirror, requiring only sloshing-ions for its operation. In such a system the barrier heights are determined by charge neutrality in the centre cell,  $n_{ec} = n_{ic}$ , by charge neutrality in the ion barrier

$$n_{ec} \exp(\phi_i/T_{ec}) = n_{is} \quad (12)$$

and by the equality between collisional loss of ions over the barrier  $\phi_i$  and collisional loss of electrons over the barrier  $\phi_e$  (equation (4)). This gives the barrier heights for ions and electrons (there is, of course no  $\phi_b$ ) as:

$$\frac{\phi_i}{T_{ic}} \sim \frac{T_{ec}}{T_{ic}} \log \left( \frac{n_{is}}{n_{ic}} \right) \quad (13)$$

$$\frac{\phi_e}{T_{ec}} \sim \frac{T_{ec}}{T_{ic}} \log \left( \frac{n_{is}}{n_{ic}} \right) + \frac{1}{2} \log \left( \frac{M}{m} \frac{T_{ic}^3}{T_{ec}^3} \right) . \quad (14)$$

As anticipated, in this elementary system the ion barrier height is determined by the bulk-electron temperature and is therefore lower than in the thermal barrier system.

Although these simple models are intended only for heuristic purposes, they could have certain advantages. These include: reduced radial transport because of the simplified magnetic configuration; improved micro-instability properties because the loss cones are partially filled by the sloshing populations; greater flexibility for exploiting minimum-B fields to avoid ballooning-interchange and trapped-particle instabilities; potential for exploiting maximum-J property /5,6/ of sloshing distribution.



It is also possible that the energy cost of producing sloshing electrons may be less than that of pumping out ions. Of course there are counteracting disadvantages in the simple model and it is not our intention to discuss the relative advantages of different systems - which are determined by engineering and power-balance considerations.

#### Acknowledgement

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## Appendix Collisional processes

The calculation of collisional losses over a combined magnetic mirror and electrostatic potential barrier has been given by Pastukhov /7/. For a mirror ratio  $R$  and potential height  $\phi > T$ , he finds the loss rate as

$$\frac{1}{n} \frac{dn}{dt} = \frac{2}{\pi^{1/2}} \frac{1}{\tau} \frac{2R}{2R+1} \frac{T}{\phi} \frac{\exp(-\phi/T)}{\log(4R+2)} \quad (A1)$$

where  $\tau$  is the usual collision time ( $\tau = n^{-1} m^{1/2} T^{3/2} / 2^{1/2} \pi e^4 \log \Lambda$ ). We have used a simplified version of (A1) which is more than adequate for our purposes.

The collisional exchange between centre-cell and plug electrons has been calculated in reference /8/. However, it can be estimated as follows. The rate at which plug electrons get across the barrier ( $\phi_i + \phi_b$ ) to join the centre-cell population is given by the Pastukhov formula, setting  $R \sim 1$ , as

$$S_1 \sim \frac{n_{ep}}{\tau_{ep}} \frac{T_{ep}}{(\phi_i + \phi_b)} \exp[-(\phi_i + \phi_b)/T_{ep}] \quad (A2)$$

The rate at which electrons from the centre-cell join the plug population cannot be obtained directly from the Pastukov formula. [Most particles from the plug which overcome the barrier join the central-cell population but particles from the centre-cell which cross the barrier do not typically join the plug population.] However suppose we were to allow the centre cell population to come to equilibrium in the plug potential; then the rate at which such electrons escaped from the plug would be given by the Pastukov formula, and would be

$$S = \frac{n_*}{\tau_*} \frac{T_{ec}}{(\phi_i + \phi_b)} \exp[-(\phi_i + \phi_b)/T_{ec}] \quad (A3)$$

The density  $n_*$  would be  $n_{ec} \exp(\phi_i/T_{ec})$  and  $n_* \tau_* = n_{ec} \tau_{ec}$ . In this hypothetical equilibrium  $S$  would, by detailed balance, also be the rate at

which centre-cell electrons join the plug population. In the actual situation this rate is multiplied by the ratio of actual to hypothetical densities in the plug, ie by  $n_{ep}/n_*$ , so that

$$S_2 \sim \frac{n_{ep} T_{ec} \exp(-\phi_b/T_{ec})}{\tau_{ec} (\phi_i + \phi_b)} . \quad (A4)$$

Equating this to  $S_1$  gives the formula used in the text.



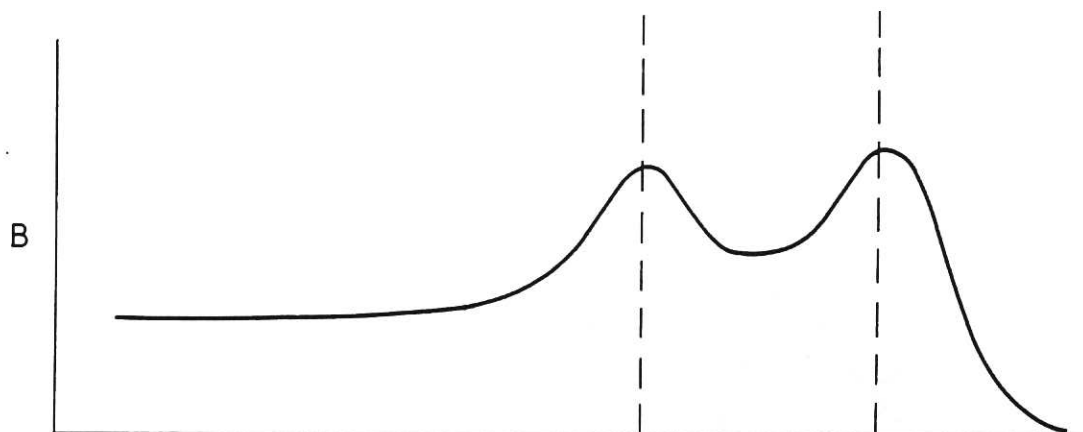


Fig.1 Magnetic field in tandem-mirror.

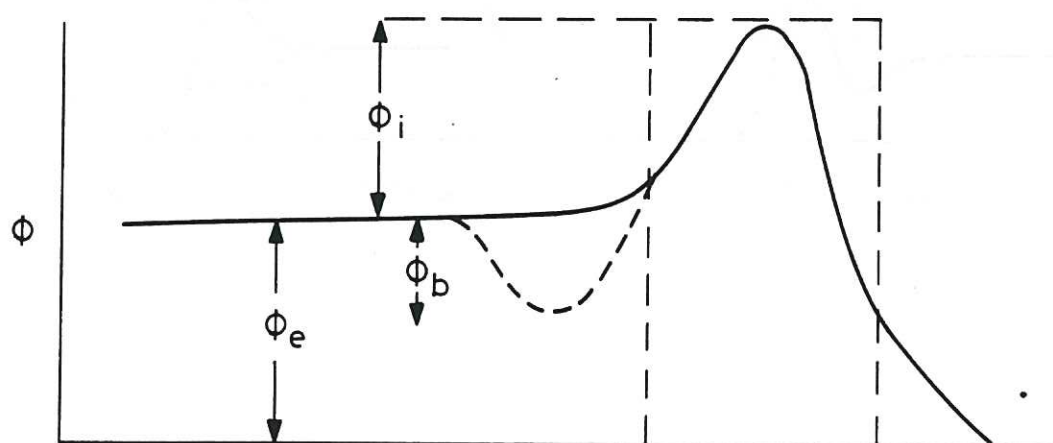


Fig.2 Potential in tandem-mirror.

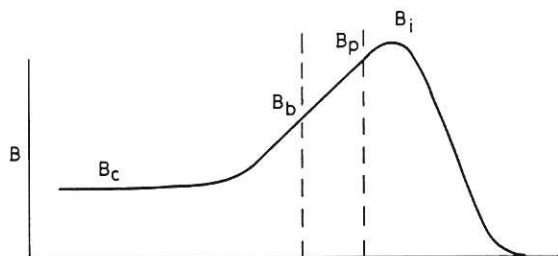


Fig.3 Magnetic field.

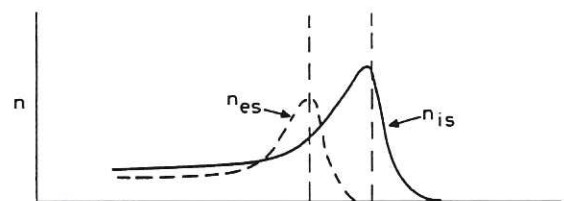


Fig.4 "Sloshing" populations.

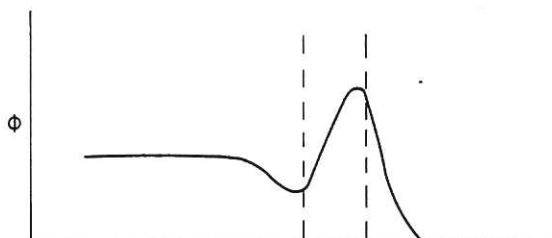


Fig.5 Potential.

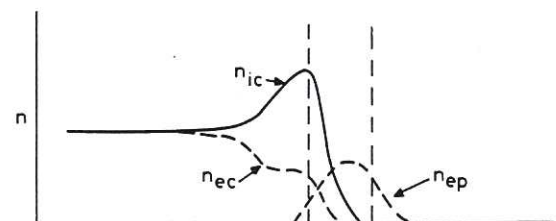
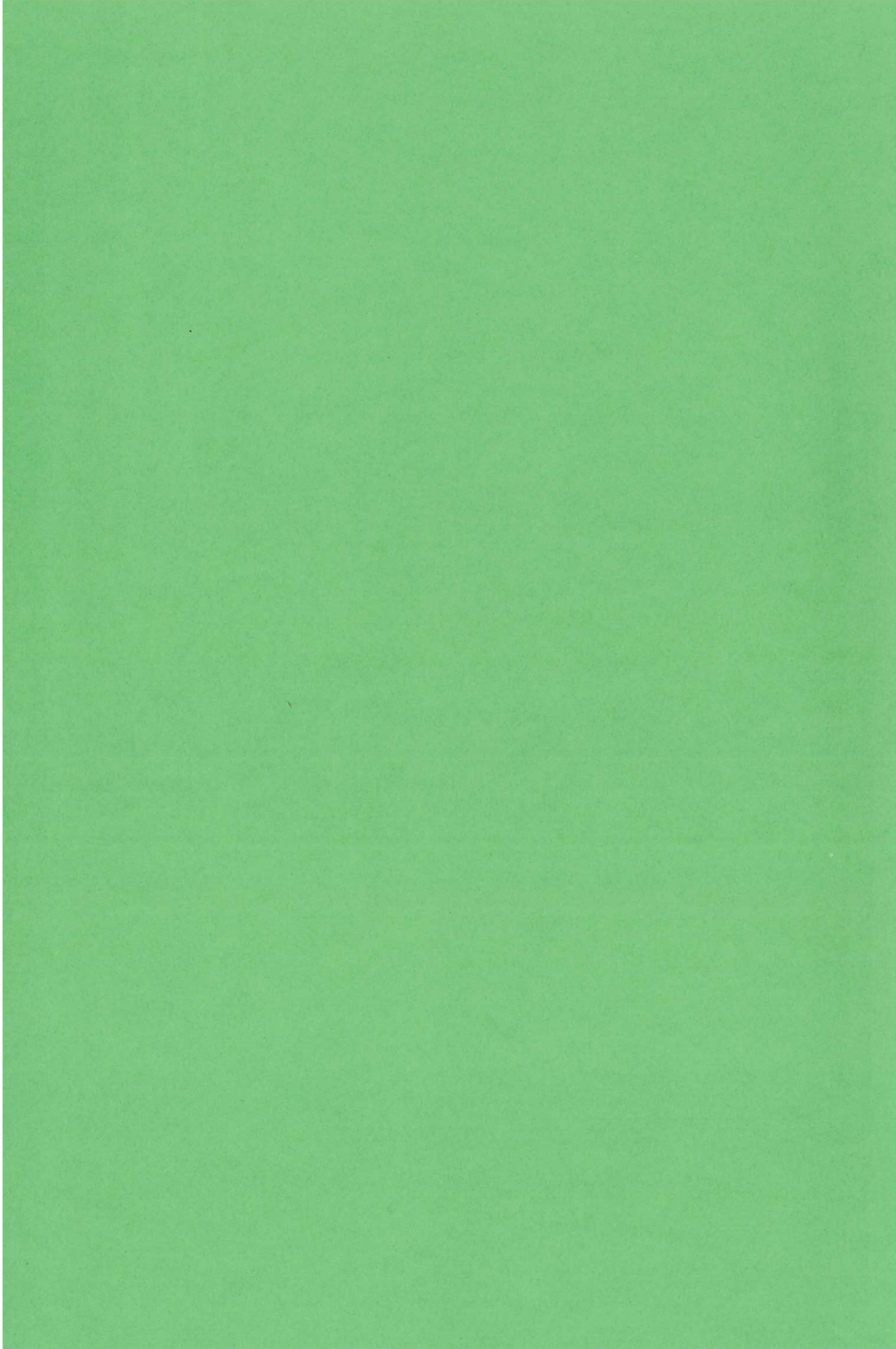


Fig.6 Electrostatically confined populations.





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