CLM-P720

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Preprint

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A. FERREIRA
M. R. O'BRIEN
D. F. H. START

CULHAM LABORATORY
Abingdon Oxfordshire

1984

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THE ROLE OF ELECTRON-ELECTRON COLLISIONS IN THE NEOCLASSICAL THEORY OF ECRH CURRENT DRIVE

A Ferreira*, M R O'Brien and D F H Start
Culham Laboratory Abingdon, Oxon, OX14 3DB, UK
(Euratom/UKAEA Fusion Association)

*Supported by the Comissão Nacional de Energia Nuclear of Brasil

ABSTRACT

The effect of electron trapping on ECRH driven currents in tokamak magnetic field configurations has been calculated using the full Fokker-Planck collision operator to take account of the electron-electron collisions. Relativistic effects are also included in the electron cyclotron resonance condition. The treatment is linear, two-dimensional in velocity space and applies to large aspect ratio tokamaks. For suprathermal, non-relativistic resonant electrons the inclusion of electron-electron collisions is found to substantially reduce the effect of trapping compared with predictions based on the Lorentz gas model. For thermal electrons the fractional reduction in current is similar to that obtained when electron self-collisions are neglected. When relativistic effects become important, trapping affects the current on the low field side of the resonance considerably less than the current driven on the high field side.

(Submitted for publication in Plasma Physics)

April 84

1. INTRODUCTION

The use of electron cyclotron waves to drive the plasma current in a tokamak relies on selectively heating electrons flowing in one direction along the magnetic field lines. In principle this can be achieved by choosing a wave frequency such that the Doppler shift tunes the wave frequency to the gyrofrequency of the electrons to be heated. Current drive results from the reduced frequency of collisions between the heated electrons and the thermal ions as was first discussed by Fisch and Boozer[1] in 1981. In the three years following the publication of that paper the method has been demonstrated experimentally in the Culham Levitron[2] and TOSCA[3] devices and almost every aspect has received some theoretical treatment, albeit to varying levels of sophistication. One such aspect, namely the effect of electron trapping in the inhomogeneous magnetic field of a tokamak, is the subject of the present paper.

The most obvious result of trapping is a decrease in the current due to the reduction in the number of current carrying electrons. Secondly the electron cyclotron wave tends to drive the passing resonant electrons into the trapped region of velocity space. As first discussed by Ohkawa[4], this generates an imbalance in the number of passing electrons circulating clockwise around the torus compared with the number flowing anticlockwise and thus generates a current which tends to cancel the Fisch-Boozer current. Thirdly there is an effect on the current due to frictional drag on the passing electrons by the trapped electrons[5]. Qualitatively therefore, the current is expected to be strongly influenced by neoclassical effects and to be a sensitive function of the tokamak aspect ratio, with the sensitivity diminishing as the parallel velocity of the resonant electrons is increased from thermal to suprathermal.

These features were first quantified by Chan et al[6] and by Cordey et al[7] on the basis of linearized Fokker-Planck calculations in the

Lorentz gas approximation. A particularly interesting result of Chan's calculation is the prediction of current reversal for sufficiently small aspect ratios for which the Ohkawa component of the current exceeds the Fisch-Boozer component. Chan also studied the influence of electron self collisions through the use of a model operator and predicted an enhancement of the trapping effects compared with the results of the Lorentz gas model. A more recent calculation[8] has investigated the consequences of poloidally localizing the ECRH power absorption on each flux surface, as would happen experimentally. The current drive efficiency is found to be least affected for power absorption on the high field side of a tokamak, because the number of trapped electrons is smaller on the inside than on the outside of each flux surface.

The above calculations did not treat electron-electron collisions fully and omitted relativistic corrections to the resonance condition. As first shown by Cairns et al[9] the relativisitic correction has a significant effect for quasi-perpendicular wave propagation even in low temperature plasmas. For the case of a uniform magnetic field, electron self-collisions can reduce the current drive efficiency from that predicted by the Lorentz gas model by up to a factor of six[10]. Therefore we include both elements in the present work which treats the neoclassical effects using a method developed from that of Rosenbluth, Hazeltine and Hinton[11 5] for ohmic currents in large aspect ratio tokamaks. The adaptation to ECRH current drive and, in particular, the analysis leading to the integro-differential equation which describes the neoclassical correction to the current have been presented in a previous paper[7]. For completeness the derivation of this equation, which is the starting point of the present calculations, is summarized briefly in section 2. The driving term in the integro-differential equation is generated from the complete electron distribution function for no trapping. In ref 7 the distribution function could only be obtained for the Lorentz gas model. In section 3 it is obtained in the form of a Legendre polynomial expansion with both electron-ion and electron-electron collisions included in the calculation. The reduction in current due to trapping is then obtained from a numerical solution of the equation derived in section 2. The results are expressed in terms of the total current density per unit absorbed power density, J/P_d, and are presented as a function of a parameter u_0 which is related to the parallel velocity of the resonant electrons and a parameter S which measures the relativistic correction to the resonace condition. In the non-relativistic limit S=0 and u_0 is the resonant electron parallel velocity normalized to the thermal velocity. The current drive efficiencies for S=0 are compared in section 3 with the recent non-relativistic results of Taguchi[12] which were reported while the present work was in progress.

The present theory does not allow for any dependence of the ECRH quasi-linear diffusion coefficient on the poloidal angle θ. At first sight this would appear to be a serious omission since ray tracing codes predict[13-14], and experiments indicate[3 15], very localized power deposition on each flux surface cut by the resonance layer. However we show in section 4 that such a localization in large aspect ratio tokamaks will only have a significant effect when the parallel velocity of the resonant electrons is comparable with, or less than the thermal velocity.

2. NEOCLASSICAL THEORY

The present calculations of the trapped electron correction to ECRH driven current are based on the 'banana' regime analysis for ohmic currents of Rosenbluth et al[11] and Hazeltine et al[5]. That analysis was modified by Cordey et al[7] for the present driving term which, in Legendre polynomial representation, contains terms of higher order than $P_1(\xi)$, where ξ is the cosine of the electron pitch angle. The treatment is steady state, perturbative and applies to large aspect ratio tokamaks; terms of higher order than $\varepsilon^{\frac{1}{2}}$ (ε=r/R) are neglected[11]. Thus the electron distribution fuction is written as

$$f_e = F_{me} + \tilde{f}_e \tag{1}$$

where F_{me} is the Maxwellian distribution and $\tilde{f}_e$ is the perturbation. The linearized Fokker-Planck equation for ECRH in toroidal geometry is given by;

$$\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\{v_\perp D\frac{\partial F_{me}}{\partial v_\perp}\} + C_{ei}(f_e^-,F_{mi}) + C_{ee}(f_e',F_{me}) + C_{ee}(F_{me},f_e^-) = \frac{v_\parallel}{r}\frac{B_\theta}{B_\phi}\frac{\partial f_e^-}{\partial\theta} \tag{2}$$

The first term on the left hand side is the quasi-linear diffusion operator for electron cyclotron waves, C_{ei} is the electron-ion Coulomb collision operator, and C_{ee} is the electron- electron collision term. The electron parallel and perpendicular velocity components are denoted by $v_\parallel$ and $v_\perp$ respectively, r is the minor radius and θ is the poloidal angle. The poloidal and toroidal magnetic fields are taken to have the form $B_\theta=B_{\theta 0}(r)[1+\varepsilon\cos\theta]^{-1}$ and $B_\phi=B_{\phi 0}[1+\varepsilon\cos\theta]^{-1}$ respectively.

The quasi-linear diffusion coefficient for X-mode fundamental absorption is given by:

$$D = D_0\delta[\frac{\omega}{k_\parallel} - \frac{\Omega}{k_\parallel}(1 - \frac{v^2}{2c^2})-v_\parallel] \tag{3}$$

The delta function represents the electron cyclotron resonance condition for mildly relativistic electrons (terms of order v^4/c^4 are neglected) in which ω is the wave frequency, $k_\parallel$ is the component of the wave vector parallel to the magnetic field and v is the electron velocity .

The parameter D_0 is given by [16]:

$$D_0 = \frac{\pi e^2}{4k_\parallel m_e^2}\left|E^- + \frac{k_\perp v_\parallel}{\omega}E_\parallel\right|^2 \tag{4}$$

where E^- is the electric field component which rotates in the same direction as the electron gyromotion and m_e is the electron mass. For X-mode fundamental absorption the second term in eq.4 is of relative order v/c and is neglected so that D_0 becomes independent of velocity space variables. The form of D_0 given by eq.4 is strictly only appropriate to homogeneous systems. In tokamaks both the variation of E^- around a flux surface and the effect of the rotational transform serve to make the diffusion coefficient depend on poloidal angle[17,18].

However in Section 4 we show that for large aspect ratio tokamaks and suprathermal resonant electrons such a dependence of D_0 on θ has only a minor effect on current drive efficiency and can be omitted for the purposes of the present calculations.

Although in this paper we treat only the X-mode fundamental we expect the results to apply qualitatively to both O-mode and X-mode and also to the second and higher harmonics since the current drive efficiencies for no trapping are not particularly sensitive to mode type or harmonic number [7 19].

The perturbation of the electron distribution function, to order $\varepsilon^{\frac{1}{2}}$ can be written[5 7].

$$f_e^- = f_{eo} + F_{me} \sum_n \left[h_n^o(v,\theta) + f_n^*(v,\theta)\right] P_n(\xi) \tag{5}$$

where f_{eo} is the perturbed distribution function for no trapped electrons. The term in brackets is the neoclassical correction of which the h_n^o coefficients contribute mostly in the trapped region of velocity space and the f_n^* are generated by collisions between the trapped and passing electrons. In a previous paper[7] it was found that the sum of the coefficients h_1^o and f_1^*, which is required for calculating the current, satisfies the following integro-differential equation;

$$b_1'' + P(x)b_1' + Q(x)b_1 - \frac{16}{3\pi^{\frac{1}{2}}\Lambda}\left[xI_3(x) - 1.2xI_5(x) - x^4(1 - 1.2x^2)\left(I_0(x) - I_0(\infty)\right)\right] = R(x) \tag{6}$$

in which

$$b_1 = \langle h_1^0 \rangle + \langle f_1^* \rangle \tag{7}$$

$$I_n(x) = \int_0^x b_1 e^{-y^2} y^n dy \tag{8}$$

$$P = -x^{-1} - 2x + 2x^2E'/\Lambda \tag{9}$$

$$Q = x^{-2} - 2(Z+E-2x^3E')/\Lambda \tag{10}$$

The variable x is the electron velocity normalized to the thermal velocity ($x=v/v_e$), which is defined in terms of the electron temperature T_e by $v_e = (2T_e/m_e)^{\frac{1}{2}}$ The function Λ is related to the error function E by $\Lambda = E-xE'$, Z is the effective plasma ionic charge and the triangular brackets denote an average over poloidal angle. The term R(x) is given by

$$R(x) = \frac{1}{\Lambda x^2}\left[a_1(x) - \sum_{n_{odd}} a_n(x)\, H_n(\varepsilon)\right]\left[xE' + (2x^2-1)E + 2Zx^2\right] \tag{11}$$

where the functions $H_n(\varepsilon)$ are given in Ref.7 and $a_n(x)$ are the coefficients in the Legendre polynomial expansion of the perturbed distribution function for no trapping

$$f_{eo} = F_{me} \sum_n a_n(x)\, P_n(\xi). \tag{12}$$

The total current density averaged over a flux surface is given by [7]

$$J = -\frac{4ev_e n_e}{3\pi^{\frac{1}{2}}} \int_0^\infty (a_1 + b_1)x^3 e^{-x^2} dx + O(\varepsilon) \tag{13}$$

This far we have summarised previous work in order to give the basis for the present calculations. The new contribution of these calculations is to determine all the odd order coefficients $a_n(x)$ required to generate the driving term R(x) and then to solve eq (6) for the neoclassical correction. The calculation of these a_n coefficients is carried out in the next section.

3. THE ELECTRON DISTRIBUTION f_{eo}

For the case of a uniform magnetic field the linearized Fokker-Planck equation is given by eq(2) with $f_e^- = f_{eo}$ and $\partial f_e^-/\partial\theta = 0$. We substitute the Legendre representation of f_{eo} given by eq (12) into eq (2). The collision terms C_{ee} and C_{ei} may be derived from the Rosenbluth potentials[20], and given the form of their kernel, the expansion in Legendre polynomials is most easily accomplished using the addition theorem for spherical harmonics. The final result is a set of uncoupled equations for the velocity-dependent coefficients $a_n(x)$ of the form:

$$a_n''(x) + P(x)a_n'(x) + Q_n(x)a_n(x) + S_n(x) = T_n(x) \tag{14}$$

The differential terms are obtained entirely from $C_{ee}(f_{eo}, F_{me})$ and the integral term $S_n(x)$ from $C_{ee}(F_{me}, f_{eo})$; all three collision terms contribute to $Q_n(x)$. The driving term $T_n(x)$ results from the quasi-linear diffusion operator. Specifically these terms are as follows:

$$Q_n(x) = n(n+1)\left(\frac{1}{2x^2} - 1 - \frac{Z}{\Lambda} - \frac{xE'}{\Lambda}\right) + \frac{4x^3E'}{\Lambda}, \tag{15}$$

$$\begin{aligned} S_n(x) = &-\frac{4x^2}{2n+1}\left[1 + \frac{n(n-1)}{2n-1}x^2\right]\frac{1}{x^n\Lambda}\, L_{n,n+2}(x) \\ &+ \frac{4x^3}{2n+1}\left[\frac{(n+1)(n+2)}{2n+3}x^2 - 1\right]\frac{x^n}{\Lambda}\, L^*_{n,1-n}(x) \\ &- \frac{4x^3\, n(n-1)}{(2n+1)(2n-1)}\frac{x^n}{\Lambda}\, L^*_{n,3-n}(x) \\ &+ \frac{4x^2(n+1)(n+2)}{(2n+1)(2n+3)}\frac{1}{x^n\Lambda}\, L_{n,n+4}(x) \end{aligned} \tag{16}$$

$$L_{n,\ell}(x) = \frac{4}{\pi^{1/2}}\int_0^x a_n(t)e^{-t^2}t^\ell dt \tag{17}$$

$$L^*_{n,\ell}(x) = \frac{4}{\pi^{1/2}}\int_x^\infty a_n(t)e^{-t^2}t^\ell dt \tag{18}$$

$$T_n(x) = \frac{4(2n+1)D}{v_e^3\ \nu_0\Lambda}\ x_0x\{\ \frac{x}{x_0}(x_0^2+1-x^2)P_n(\frac{x_0}{x}) + [(x^2-x_0^2)P_n'(\frac{x_0}{x})$$

$$- 2xx_0P_n(\frac{x_0}{x})]\ \frac{S}{x_0}\}\{H(x-\alpha_-) - H(x-\alpha_+)\} \qquad (19)$$

where $S = \Omega v_e/(2k_\parallel c^2)$, $x_0 = u_0 + x^2S$, $u_0 = (\omega-\Omega)/k_\parallel v_e$, $\alpha_\pm = \left|(1\pm\sqrt{1-4u_0S})/2S\right|$, H is the Heaviside function, $\nu_0 = e^4n_e\ell n\lambda/4\pi\varepsilon_0^2v_e^3m_e^2$, n_e is the electron density and $\ell n\lambda$ is the Coulomb logarithm. The parameter S relates to the strength of the relativistic correction to the ECRH resonance condition. In the non-relativistic limit, S=0, the resonance condition is represented by the straight line in velocity space $v_\parallel = u_0v_e$. The effect of the relativistic mass increase is to convert this line into a semi-circle centred at the point $[v_\perp/v_e=0, v_\parallel/v_e=(2S)^{-1}]$ and having a radius $(1-4u_0S)^{\frac{1}{2}}/2S$ in $v_\parallel/v_e$ and $v_\perp/v_e$ velocity space[10]. Examples of resonance lines for various values of S and u_0 are given in fig 1 and ref 10. The limits α_- and α_+ are the minimum and maximum values respectively of the electron speed, normalized to v_e, for which electrons can be in resonance.

For given values of S and u_0, eq(14) was solved numerically for values of n up to some cut-off value n_{max} using the two-point boundary value code described by Cordey et al[21]. For example a value n_{max}=17 gave an error of less than 1% in the current for u_0=2. The solutions were then used along with the $H_n(\varepsilon)$ to form R(x) by means of eq(11). The two-point boundary value code was used once more to solve eq(6) and obtain $b_1(x)$. The electron current was obtained from $a_1(x)$ and $b_1(x)$ using eq(13). The calculation was repeated for several values of n_{max} to check for convergence. The results are expressed as the usual current drive efficiency, namely the current density J divided by the deposited power density P_d. The latter is given by

$$P_d = \tfrac{1}{2}m_e\int \frac{v^2}{v_\perp}\frac{\partial}{\partial v_\perp}\{D_0v_\perp\delta[\ \frac{\omega}{k_\parallel} - \frac{\Omega}{k_\parallel}(1-\frac{v^2}{2c^2})-v_\parallel]\ \frac{\partial F_{me}}{\partial v_\perp}\}d^3v \qquad (20)$$

which reduces to

$$P_d = \frac{4D_o n_e m_e}{\pi^{\frac{1}{2}} v_e} \int_{\alpha_-}^{\alpha_+} x^3(x^2-x_0{}^2-1 + 2x_0S)e^{-x^2}dx \tag{21}$$

The integral can be evaluated analytically giving[10]

$$P_d = \frac{2D_o n_e m_e}{\pi^{\frac{1}{2}} v_e} [-S^2K_3 + (1-2u_0S+2S^2)K_2 - (1+u_0^2-2u_0S)K_1] \tag{22}$$

where $K_n = \int_{\alpha_-^2}^{\alpha_+^2} y^n e^{-y}\, dy$ and can be obtained from the recurrence relation

$$K_n = nK_{n-1} + \alpha_-^{2n}\exp(-\alpha_-^2) - \alpha_+^{2n}\exp(-\alpha_+^2) \tag{23}$$

The current drive efficiencies $\bar{J}/P_d$ are expressed in dimensionless units by normalizing the current density to $-n_e e v_e$ and P_d to $n_e m_e v_e^2 \nu_0$ in the standard way[22].

The results for the non-relativistic limit (S=0) are shown in fig 2 where the current drive efficiency $\bar{J}/P_d$ is plotted against u_0^2 for Z=1 and for values of the inverse aspect ratio ε=0, and ε=0.1. These results can be compared with those of Taguchi[12] who also based his calculations on the formalism of Hazeltine and Hinton but obtained the current by a technique which is substantially different from and somewhat more elegant than that used in the present paper. Taguchi's method is similar to that given earlier by Antonsen and Chiu[23] and uses the self adjoint property of the collision operator to show that the Spitzer Harm solution is the Green's function for the current produced by any type of driving mechanism. The excellent agreement between the present results in the non-relativistic limit and those of Taguchi (see Fig.1 of ref.12) bears witness to the high precision of both methods.

Also shown in Fig.2 are similar curves for the Lorentz gas model which, for large values of u_0 and ε=0, predicts efficiencies higher by a factor (Z+5)/Z than those obtained with electron-electron collisions included in the calculation[1,7,9] For ease of comparison, therefore, the Lorentz gas efficiencies have been divided by $(Z+5)/Z$ in fig 2. It can be seen from fig 2 that for suprathermal resonant electrons (large values of u_0) the inclusion of electron-electron collisions in the calculation has a large effect on the fractional reduction in current due to trapping. For example the Lorentz model predicts a factor of two reduction for $u_0=4$ whereas the full calculation predicts only a 17% reduction. In the case of resonant velocities close to thermal the two calculations predict approximately the same reduction in current which is substantially greater than that of either model for suprathermal electrons.

The result of including the relativisitic correction to the resonance condition can be seen in fig 3 for the cases of $u_0=3$ and $u_0=-3$. We define $k_\parallel$ to be positive when directed along $\underline{B}$ and negative when against $\underline{B}$ so that $u_0 = 3$ corresponds to waves travelling along $\underline{B}$ and being absorbed on the low field side of the resonance. The value $u_0=-3$ refers to the same waves being absorbed on the high field side. In fig 3 the current drive efficiency is plotted against S for Z=1 and for $\varepsilon=0$ and $\varepsilon=0.1$. Again the Lozentz gas model results are divided by the factor $(Z+5)/Z$. On the low field side the effect of the trapping reduces as S increases which is not too surprising since the relativistic correction causes the resonance line in velocity space to curve away from the trapped particle region as shown in fig 1. The Lorentz gas model overestimates the trapped electron correction and this overestimate becomes greater as S increases. Note that on the low field side a cut-off exists such that the resonance condition cannot be satisfied for $S>(4u_0)^{-1}$. This cut-off occurs at the point where the mass increase separates the gyrofrequency from the wave frequency to such an extent that only those electrons with $v_\parallel=v$ are resonant. Further increase in S would require the parallel velocity of the resonant electrons to exceed their speed.

On the high field side ($u_0=-3$) the effect of trapping is increased as S increases. Qualitatively this is because the mass increase tends to curve the resonance line towards the trapped particle region (see fig 1) for negative values of u_0. For large values of S the fractional reduction in current predicted by the Lorentz gas model agrees more closely with that given by the full calculation.

4. THE EFFECT OF LOCALISED POWER ABSORPTION

The preceding analysis assumes that the quasi-linear diffusion coefficient D_0 in eq(2), does not vary with poloidal angle θ. In practice, however, microwave power is injected into tokamaks from directional antennae so that the wave intensity is far from uniform around a flux surface. The question also arises as to whether the present results could be used in conjunction with ray tracing codes to obtain current density profiles. In this case the power absorption from each ray is strongly localised in θ at the point where the ray intersects the flux surface[8]. The effect of such a localized power absorption has been studied in the Lorentz gas approximation in ref 8 for the case of a non-relativistic resonance condition. Following the analysis in ref 8 but using the mildly relativistic resonance condition, that is by using the diffusion operator

$$\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\{D_0 v_\perp \delta[\frac{\omega}{k_\parallel} - \frac{\Omega}{k_\parallel}(1-\frac{v^2}{2c^2})-v_\parallel]\frac{\partial F_{me}}{\partial v_\perp}\}\delta(\theta-\theta_a) \tag{24}$$

we obtain the following expression for the current density

$$\bar{J}(\varepsilon,\theta_a) = -\frac{em_e n_e D_0}{\pi^{\frac{1}{2}} T_e v_0 Z}(1+\varepsilon\cos\theta_a)\{\int_{\alpha_-^2}^{\alpha_+^2} w^2(w-w_0-1+2Sw_0^{\frac{1}{2}})e^{-w}dw\int_{\lambda_0}^{1/B_{max}}\frac{d\lambda}{<(1-\lambda B)^{\frac{1}{2}}/B>}$$

$$- B_a^{-1}\int_{\alpha_-^2}^{\alpha_+^2}\frac{2Sww_0^{\frac{1}{2}}(w-w_0)e^{-w}}{<(1-\lambda_0 B)^{\frac{1}{2}}/B>}dw\} \tag{25}$$

In eq (25) θ_a is the poloidal angle at which wave absorption occurs, $w=v^2/v_e^2$, $\lambda=v_\perp^2/(v^2B)$, $\lambda_0=(v^2-v_0^2)/(v^2B_a)$, $v_0=\frac{\omega}{k_\parallel}-\frac{\Omega}{k_\parallel}(1-\frac{v^2}{2c^2})=u_0v_e+Sv^2/v_e$, $w_0=v_0^2/v_e^2$ and $B_a=B(\theta=\theta_a)$.

Formally the integration over w is between the limits α_-^2 and α_+^2. However the condition $\lambda_0 < 1/B_{max}$, which limits the integration to the passing particles, implies further restrictions on w. This condition can be written

$$(1-w_0/w) \leqslant B_a/B_m \tag{26}$$

Noting that $w_0=(u_0+Sw)^2$ we find that eq(26) leads to the inequalities

$$w \leqslant \frac{1}{2S^2}\left\{1-\frac{B_a}{B_m}-\frac{2u_0S}{v_e}-\left[\left(1-\frac{B_a}{B_m}\right)\left(1-\frac{B_a}{B_m}-\frac{4u_0S}{v_e}\right)\right]^{\frac{1}{2}}\right\} \tag{27}$$

$$w \geqslant \frac{1}{2S^2}\left\{1-\frac{B_a}{B_m}-\frac{2u_0S}{v_e}+\left[\left(1-\frac{B_a}{B_m}\right)\left(1-\frac{B_a}{B_m}-\frac{4u_0S}{v_e}\right)\right]^{\frac{1}{2}}\right\} \tag{28}$$

Real values of w given by the equalities in eqs(27) and (28) correspond to the points at which the resonance line cuts the trapped particle boundary. Imaginary values correspond to the case where the resonance semi-circle misses the trapped region altogether. In the zero trapping limit, $\varepsilon=0$, equation (25) reduces to the expression given by Start et al[10] for the case of a Lorentz gas in a uniform magnetic field, namely

$$J(\varepsilon=0) = \frac{em_en_eD}{\pi^{\frac{1}{2}}T_e\,\nu_0Z}\int_{\alpha_-^2}^{\alpha_+^2} w^{3/2}w_0^{\frac{1}{2}}\left[w-w_0-1+2Sw_0^{\frac{1}{2}}-S(w-w_0)w_0^{-\frac{1}{2}}\right]e^{-w}dw \tag{29}$$

Equation (25) has been evaluated numerically as a function of ε for absorption angles $\theta_a=0°,90°$ and $135°$. Results for values of the parameters (u_0,S) = (3,0), (3,0.06) and (-3,0.2) are shown in fig 4 where the current density is normalised to the current in a uniform field, $J(\varepsilon=0)$, as given by eq (29). For absorption on the low field side, $(u_0,S)=(3,0.06)$, and in the non relativistic limit, (u_0,S) = (3,0), where the resonant electrons have large parallel velocities, the dependence of the current on θ_a is small, at least for values of ε less than 0.2. However in the case of absorption on the high field side the variation with θ_a becomes significant as S increases and the resonance line curves towards lower parallel velocities. For $(u_0,S)=(-3,0.02)$, and $\varepsilon=0.1$ the current varies by approximately $\pm$ 30% about the value for $\theta_a=90°$. In this particular case the sensitivity to the absorption angle is expected to be reduced somewhat by the inclusion of the electron-electron collisions since, as shown in fig 3, their presence produces less reduction in current due to trapping than is predicted by the Lorentz gas model. However for values S>0.2 the effect of electron-electron collisions on the fractional reduction in current is minimal and so the dependence on the poloidal angle of absorption will be significant.

5. SUMMARY

The trapped electron correction to the current driven by X-mode electron cyclotron waves has been calculated for large aspect ratio tokamaks using the full Fokker-Planck operator to describe the electron collisions. The calculations also include the mildly relativistic ECRH resonance condition. For suprathermal electrons the fractional reduction in current due to trapping is substantially less than that predicted by the Lorentz gas model. On the other hand both models give approximately the same percentage reduction for resonant electrons with parallel velocities close to thermal. The relativistic effects in the resonance condition tend to reduce the trapped particle correction to the current in the case of waves absorbed on the low field side. This is due to the fact that the resonance line curves away from the trapped particle region of velocity space as the electron mass increases. Conversely, on the high field side the trapping effect is increased since the resonance becomes curved towards the trapped electron region. The validity of the calculations when the power is absorbed locally on each flux surface has been tested. For suprathermal electrons, and for the large aspect ratios appropriate to the full Fokker-Planck calculation, the current is found to be almost independent of where the power is absorbed. For a given aspect ratio, however, the current becomes increasingly sensitive to such localisation as the resonant electron velocity is reduced. Future development of electron trapping calculations towards smaller aspect ratio tokamaks will require a full bounce average treatment of the quasi-linear diffusion operator. In addition, as the microwave power injected into present day experiments is increased, nonlinear effects will have to be taken into account. For example using Fielding's[17] formula for D (eq 4 of ref.17) and the expression given by Fidone et al[24] for the ratio of E^- for the X-mode fundamental to the total rf electric field we estimate that nonlinear effects would become apparent ($2D/\nu_0 v_e^2 \sim 0.2$) in a tokamak like DITE ($n_e = 3\times10^{13}$ cm^{-3}, T_e = 1 keV, R = 1.17 m, r=0.1m, $\sin\chi_r = \frac{1}{2}$) for an input rf power of 1.6 MW at 60 GHz. This estimate of the threshold at which nonlinear effects become important will be increased somewhat by finite beam width effects which spread the resonance in velocity space and which become noticeable when the particles pass out of the beam before they experience the magnetic field variation due to the rotational transform[18].

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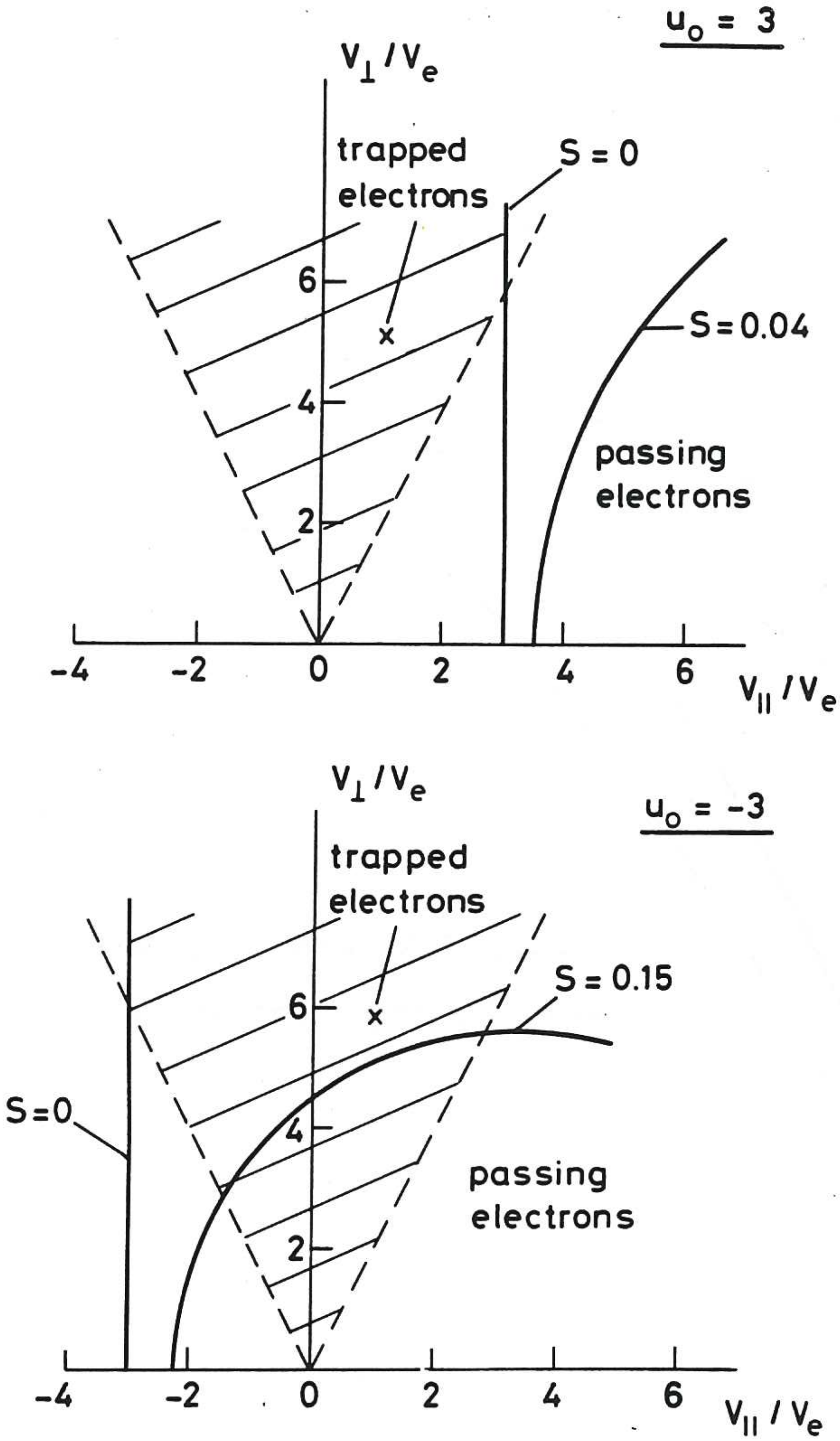

Fig. 1 Electron cyclotron resonance lines in velocity space for $u_0 = 3$ and $u_0 = -3$ and for several values of S. The shaded area shows the trapped particle region for $\epsilon = 0.1$ and $\theta = 0^0$.

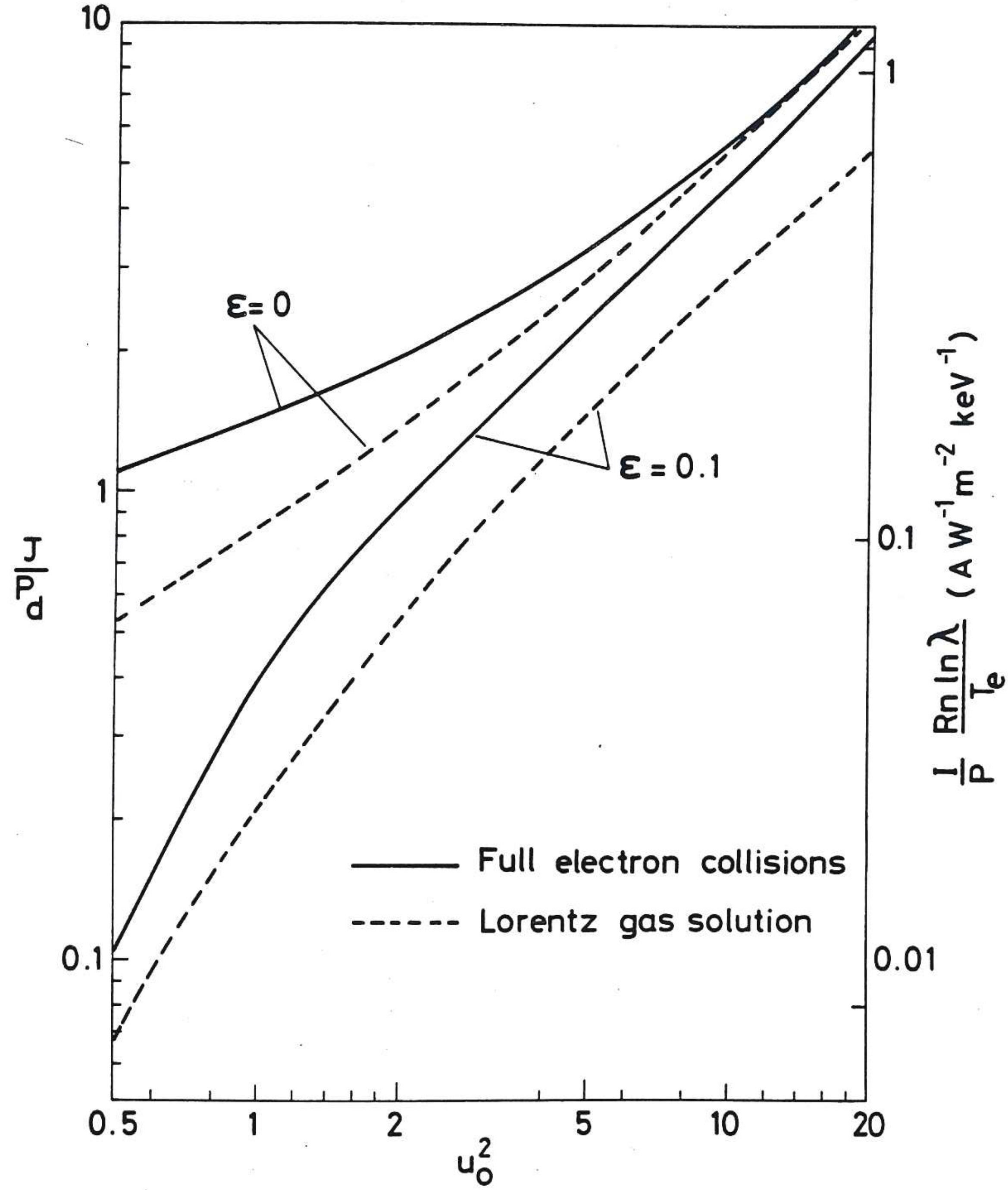

Fig.2 Current drive efficiency $\overline{J}/P_d$ versus u_0^2 in the non-relativistic limit for values of the inverse aspect ratio $\epsilon = 0$ and $\epsilon = 0.1$. The full curves are the results with e–é collisions taken into account. The dotted curves are the results for the Lorentz gas case divided by the factor (Z+5)/Z. The scale on the right-hand side enables the ratio of current to power in amps/watt to be obtained. The ratio I/P is the current flowing in the annular volume between adjacent flux surfaces of radii r and r+dr divided by the power absorbed in this volume for those electrons lying on the resonance semi-circle in velocity space defined by u_0 and S.

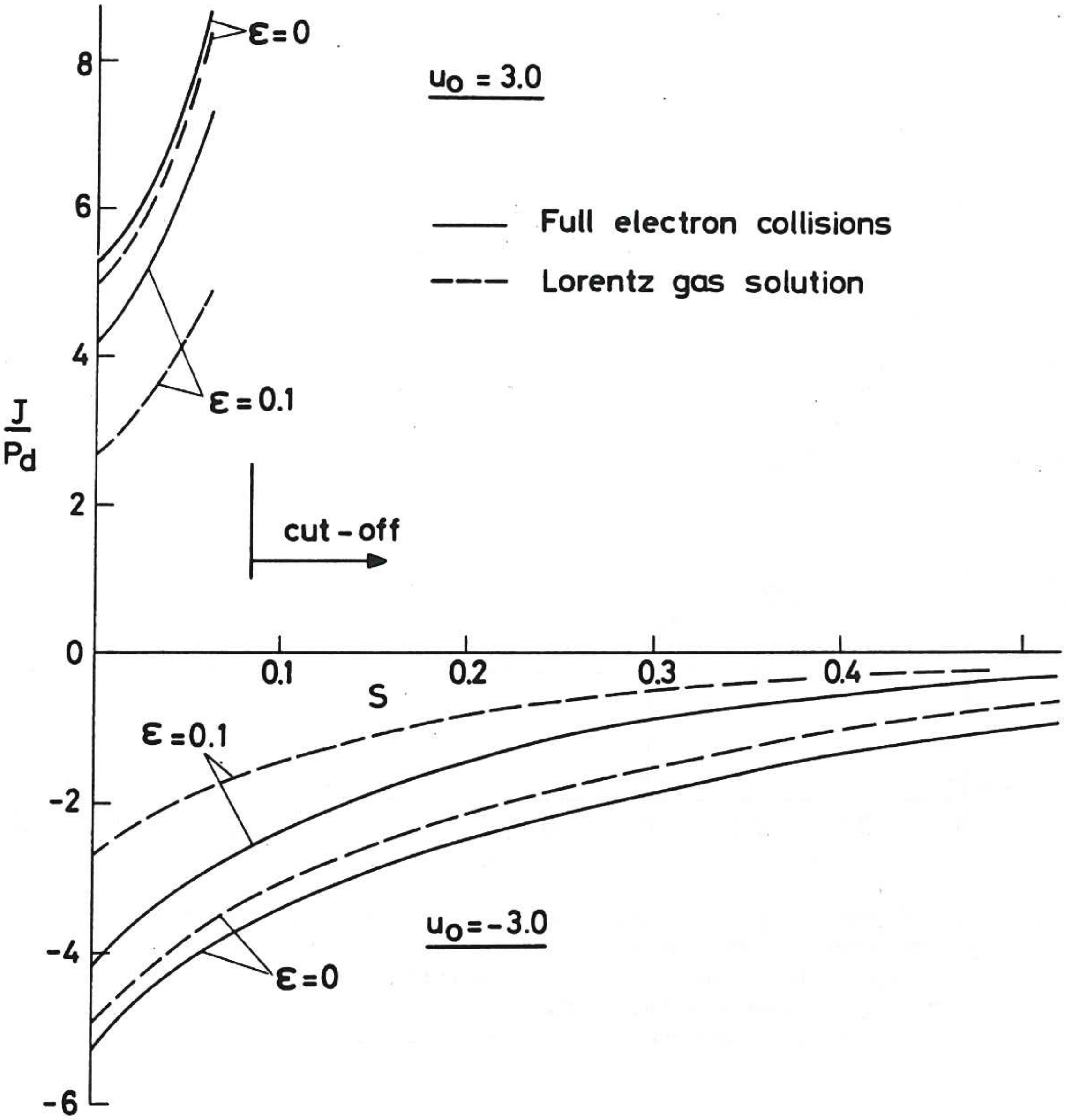

Fig.3 Current drive efficiency versus the relativistic resonance condition parameter S for u_0 = 3 (absorption on the low-field side of the resonance) and u_0 = −3 (absorption on the high-field side). Values are given for $\epsilon = 0$ and $\epsilon = 0.1$. The solid curves refer to the full Fokker-Planck treatment and the dotted lines refer to the renormalised Lorentz gas model results.

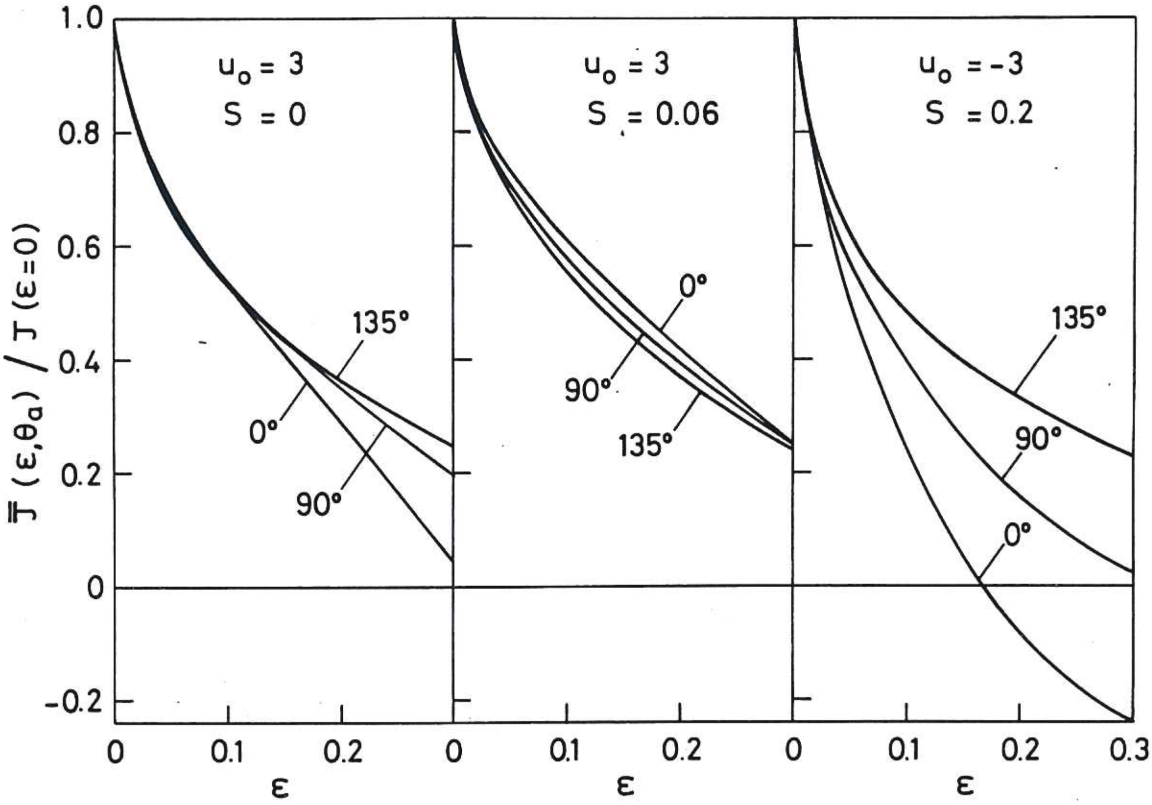

Fig. 4 Current density with trapping normalised to the current density for no trapping plotting against ϵ for several values of the poloidal angle of absorption. All the curves are for $|u_0| = 3$. The left hand set refers to the non-relativistic limit (S = 0). The centre set refers to absorption on the low-field side by weakly relativistic electrons (S = 0.06). The right hand set is for absorption on the high-field side for S = 0.2.