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TOKAMAK β-LIMIT

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Abstract

The β-limit for stability against mhd modes is calculated for a large aspect ratio tokamak. The results are found to be similar to those calculated numerically for small aspect ratio. For ideal modes, the β-limit is found to be proportional to current, having the form $(28\varepsilon/q_a)$% with a maximum value of 14ε% at $q_a = 2$, ε being the inverse aspect ratio. A limit of 7ε% is found when tearing mode stability is also required.

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Introduction

In a large aspect ratio tokamak with $\beta \sim \varepsilon^2$, where ε is the inverse aspect-ratio, stability against internal kink and surface kink modes can readily be achieved in the regime $q_a > 2$ and $q_0 > 1$[1]. Stability against ideal mhd pressure driven modes is determined by the Mercier criterion, and for circular cross-sections the requirement is only that $q > 1$ everywhere across the plasma[2].

If β is increased to $\beta \sim \varepsilon$, kink modes still do not provide a direct constraint on β. However, stability now requires that the ballooning mode criterion[3] be satisfied and at high q_a the ballooning modes give the β-limit. This limit increases with decreasing q_a and the maximum β is obtained for a configuration which satisfies the ballooning mode criterion and has the lowest q_a allowing kink stability. The calculation of this maximum β is given below.

It is found that the configuration determined by this procedure has a very large current gradient within the plasma. This is not compatible with stability against tearing modes. A more realistic β-limit is therefore obtained by requiring tearing mode stability also. This extra constraint reduces the β-limit by a factor two.

Ideal Mhd

β is defined by

$$\beta = \frac{4\mu_0}{a^2B^2}\int_0^a prdr \qquad (1)$$

where p is the plasma pressure, B the toroidal magnetic field and a the plasma minor radius.

We shall maximise β subject to the constraint of ballooning mode stability using the stability diagram for circular flux surfaces[4] shown in figure 1. The coordinates of this diagram are

$$s = \frac{r}{q}\frac{dq}{dr} \qquad \alpha = -\frac{2\mu_0 R q^2}{B^2}\frac{dp}{dr}$$

where s is the shear and α measures the pressure gradient. We shall not be concerned here with the so-called second region of stability which generally is unstable to kink modes.

In order to impose the stability constraint on dp/dr on the integral (1) for β we integrate by parts to obtain

$$\beta = \frac{2\mu_0}{a^2B^2}\int_0^a \left(-\frac{dp}{dr}\right) r^2 dr \quad . \tag{2}$$

This expression is to be maximised with the constraint that dp/dr is given by the stability boundary of figure 1. However, before proceeding

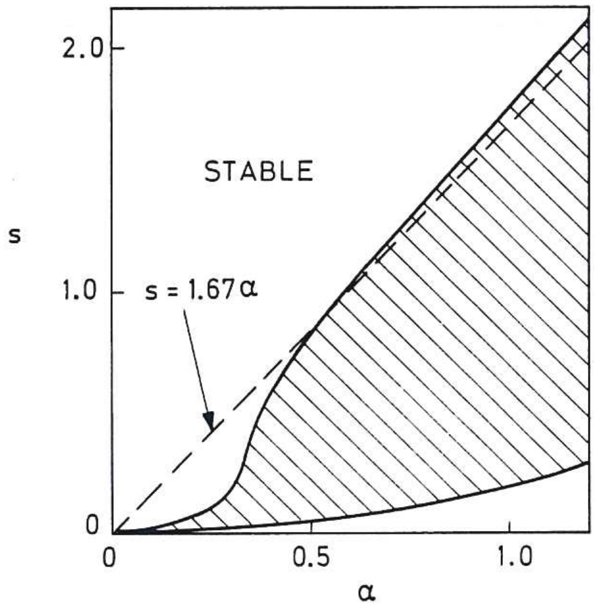

Fig. 1 Ballooning mode stability diagram.

to the complete optimisation it is instructive to carry out a simplified calculation using an approximation to the stability boundary. We shall find that this gives a value of β which is quite close to the accurate value.

We choose the straight line approximation shown in figure 1

$$s = 1.67\alpha \quad .$$

This gives

$$-\frac{dp}{dr} = 0.3\,\frac{B^2}{\mu_0 R}\,\frac{r}{q^3}\,\frac{dq}{dr}$$

and equation (2) then leads to

$$\beta_m = -0.3\,\frac{1}{Ra^2}\int_0^a \frac{d}{dr}\left(\frac{1}{q^2}\right) r^3 dr \quad . \tag{3}$$

For a given value of q_a and with q_0 limited to unity by the Mercier criterion, the integrand is maximised for non-negative current profiles by allowing all of the change in q to occur at the maximum possible radius. This is achieved by the q and current profiles shown in figure 2.

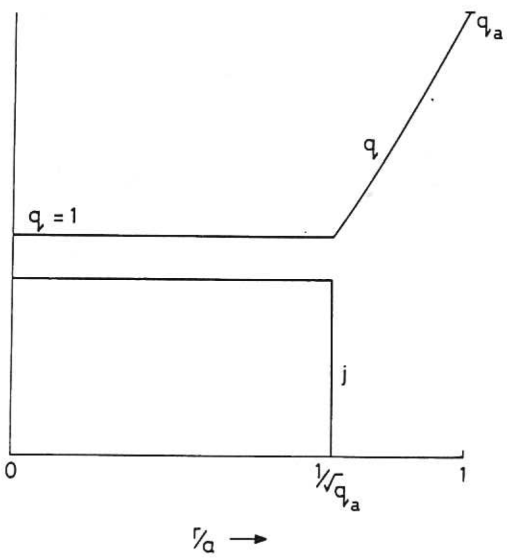

Fig.2 Current and q profiles for maximum β using the approximation $s = 1.67\alpha$.

Thus, substituting $q = q_a \left(\frac{r}{a}\right)^2$ over $\frac{a}{\sqrt{q_a}} < r < a$ into equation (3) gives

$$\beta_m = 1.2 \frac{a^2}{Rq_a^{\ 2}} \int_{a/\sqrt{q_a}}^{a} \frac{1}{r^2}\, dr$$

so that

$$\beta_m = 1.2 \frac{\varepsilon}{q_a^{\ 2}} \left(q_a^{\ 1/2} - 1\right) . \qquad (4)$$

The form of $\beta_m(q_a)$ is shown in figure 3. It is seen that over a considerable part of the range the curve is well represented by the linear approximation

$$\%\beta_m = 28 \frac{\varepsilon}{q_a}$$

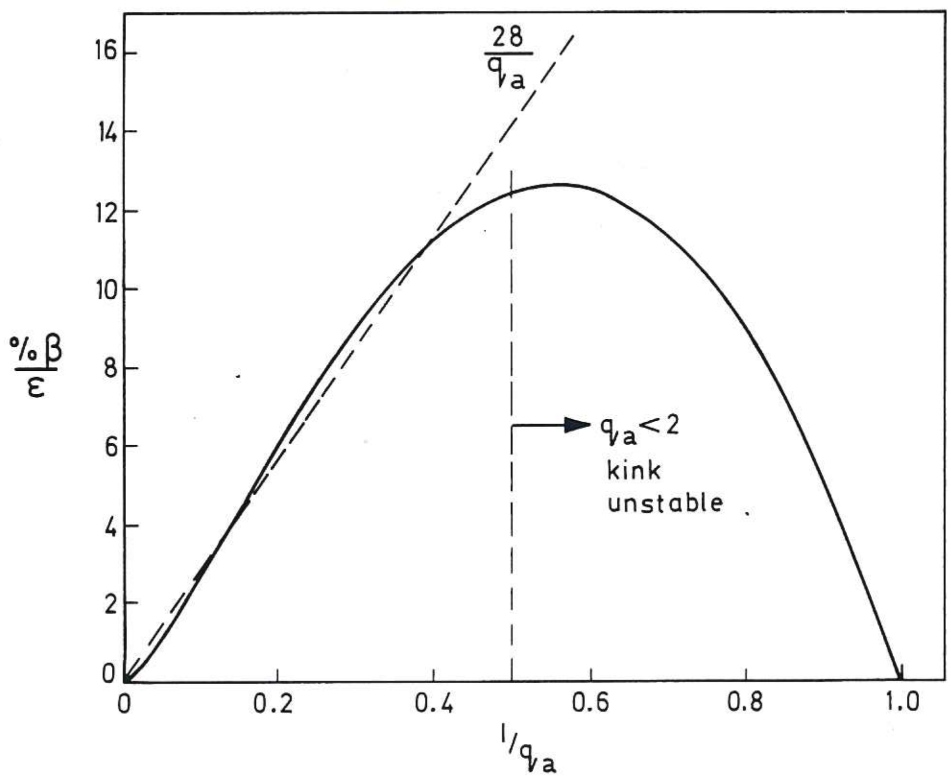

Fig.3 Maximum β as a function of q_a using the approximation $s = 1.67\alpha$.

which is similar to the form given by Sykes et al[5] and Troyon et al[6], who derived the empirical formulae $\%\beta_m = 22\ \varepsilon/q_a$ and $\%\beta_m = 14\ \varepsilon/q_a$ respectively, for ranges of toroidal equilibria with prescribed q-profiles.

The kink stability of the 'top-hat' current profiles was analysed by Shafranov[10]. For $m \geqslant 2$, stability is achieved if $(r_0/a)^2 < (m - 1)/m$ where r_0 is the radius of the current channel, here $a/\sqrt{q_a}$. For the worst case of $m = 2$, the condition for kink stability is therefore $q_a > 2$.

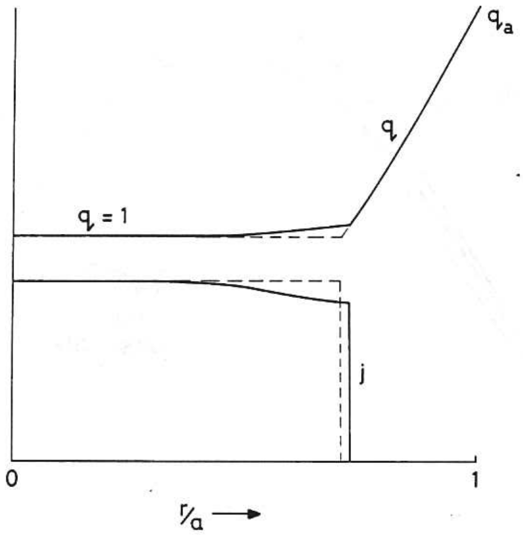

Fig.4 Current and q profiles for maximum β.

The complete optimisation of β using the accurate marginal stability curve of figure 1 has been carried out by a numerical calculation. The optimum current and q profiles are shown in figure 4 and the resulting β-limit is shown in figure 5. It is seen that the earlier approximate calculation gives results which are quite close to the accurate values and that, for $q_a > 2$, β_m is well fitted by

$$\%\beta_m = 28\ \frac{\varepsilon}{q_a}$$

Increasing the current, and hence reducing q_a, leads to a maximum of β_m at $q_a = 1.65$. This is beyond the stability limit for kink modes: thus the overall maximum β is obtained taking $q_a = 2$ in figure 5 and is found to be

$$\%\beta_{max} = 14\ \varepsilon$$

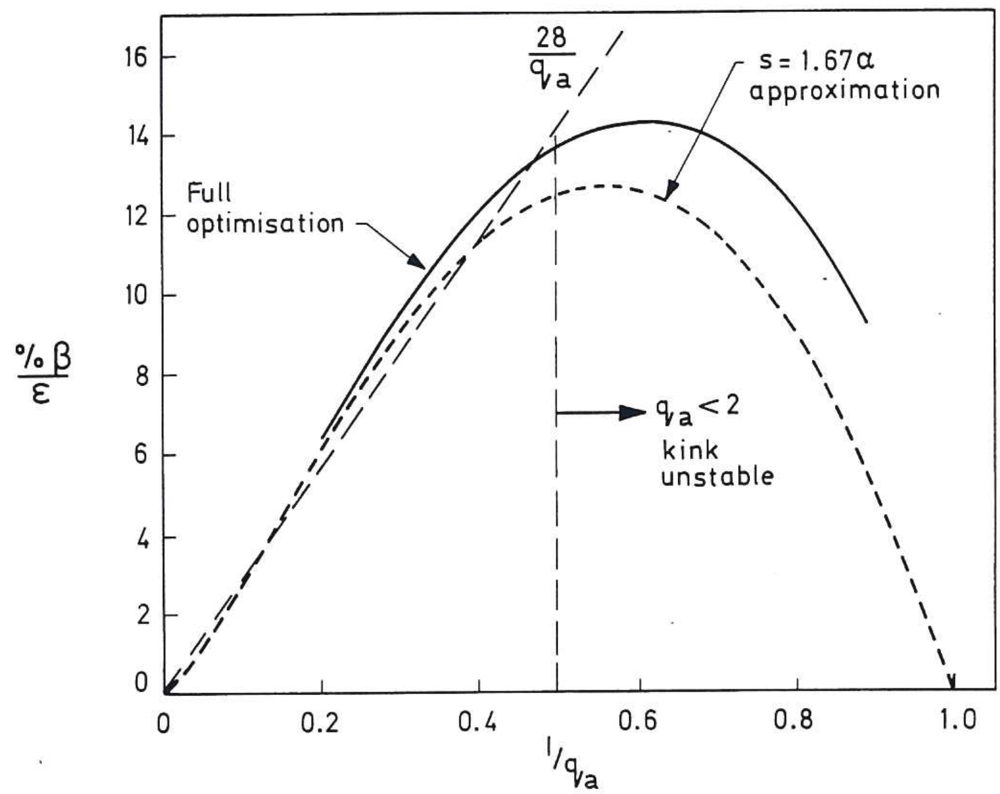

Fig.5 Maximum β as a function of q_a.

It is remarkable that both the linear dependence of β on current and the maximum value of β found here are similar to those derived for JET by Wesson[7]. If the JET aspect ratio is taken to be $3/(1.25 \times 2)^{1/2} = 1.9$ the maximum β predicted by the present analysis is 7.4% compared to the value $\simeq$ 8% found in the full calculations.

Tearing Modes

The current profile corresponding to the q profile of figure 2 has a constant current for $r < \frac{a}{\sqrt{q_a}}$ and zero current outside this region. It represents, of course, only an idealisation of the practical situation. Furthermore, this current profile would be unstable to tearing modes.

A more realistic β-limit may therefore be obtained by taking a current profile for which the current has been maximised and q_a minimised with the constraint of tearing mode stability. Such a profile was given by Glasser et al[8] and is shown in figure 6.

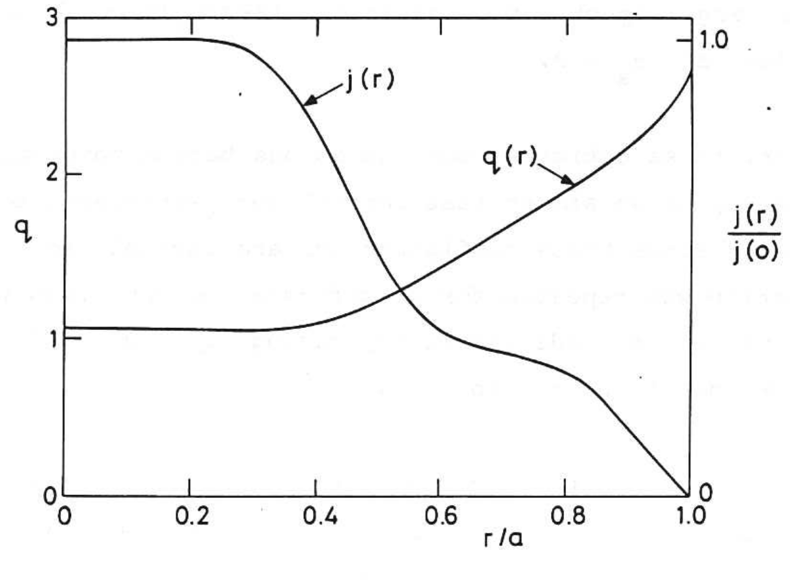

Fig.6 Current and q profiles stable to tearing modes.

If the q profile is approximated by

$$
\begin{aligned}
q &= 1 && 0 < r/a < 0.43 \\
&= 2.34\ r/a && 0.43 < r/a < 0.90 \\
&= 2.6(r/a)^2 && 0.90 < r/a < 1
\end{aligned}
$$

and these values are substituted in equation (3), the resulting β-limit is $7.1\varepsilon(\%)$. A numerical calculation using the s/α of figure 1 can be made as in the ideal mhd case but yields almost the same value, $7.14\varepsilon\%$. Hence the maximum β with tearing, kink and ballooning mode stability is essentially

$$\%\beta_{max} = 7\varepsilon$$

Discussion

The β-limit for tokamaks has been calculated using the large aspect ratio approximation. For ideal modes, the results are similar in form to those obtained for smaller aspect ratios using numerical codes: in particular the same linear dependence on current and aspect ratio is found. The large aspect ratio result is $(28\varepsilon/q_a)\%$ which has a maximum value of 14ε at $q_a = 2$.

However, these optimised configurations have a toroidal current profile made up of an almost flat central core surrounded by a region of zero current. Since these configurations are unstable to tearing modes the calculation was repeated for a configuration optimised with the constraint of tearing mode stability, having $q_a = 2.6$. It is then found that the β-limit is halved to $7\varepsilon\%$.

In this analysis it was assumed that there was no stabilising effect from a conducting wall. If a perfectly conducting wall is placed at the plasma boundary, free boundary kink modes are removed. This then allows high β stability against ideal modes to be achieved in the so-called second region of stability[9]. Again, however, the stability of tearing modes must be taken into account.

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