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TWO FLUID EQUATIONS FOR RESISTIVE BALLOONING MODES IN AXISYMMETRIC TOROIDAL PLASMAS

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ABSTRACT

The equations governing linear stability of resistive ballooning modes are obtained from the full set of two-fluid equations, with diamagnetic, viscous and thermal conduction effects retained.

A cold ion model in which the full electron physics is retained is then developed further and the averaged equations valid in the resistive regime of ballooning space are derived.

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I. INTRODUCTION

Short wavelength modes driven by the pressure gradient are predicted by ideal mhd theory to be unstable in an axisymmetric toroidal plasma when the pressure gradient exceeds a critical value (DOBROTT, 1977; COPPI, 1977; CONNOR et al., 1978; CONNOR et al., 1979 and CHANCE et al., 1979). Such modes have very fast growth rates and might be expected to establish a definite limit to the β which can be confined in a tokamak equilibrium.

With the addition of a small but finite resistance to the mhd equations, resistive dissipation becomes important in a narrow layer and theory predicts that a similar, but weaker, short wavelength ballooning instability should occur whatever the local pressure gradient (CHANCE et al., 1979; GLASSER et al., 1979; LEE et al., 1979; and BATEMAN et al., 1978). It has been suggested (CARRERAS et al., 1983) that such resistive ballooning modes might provide an explanation of the anomalously large electron thermal transport in some Tokamaks.

The weakness of resistive instability theory lies in the fact that, unlike ideal instabilities, they are extremely sensitive to any other small transport effects which might compete with resistivity in the equations. In practice many effects are of comparable or greater magnitude, but the importance of resistivity is that it modifies the Ohm's law in such a way as to permit changes of magnetic topology and therefore to permit a new class of instabilities. Other terms in Ohm's law, such as the Hall terms, may be of larger magnitude, but do not break the 'frozen-in' property inherent in the ideal Ohm's law.

Thus the importance of the single-fluid resistive mhd model is that it demonstrates the existence of a new class of modes; its detailed predictions of stability boundaries for particular equilibria may however be in serious error. Examples of situations in which the single fluid resistive theory may give incorrect results are becoming more numerous. Short wavelength tearing modes (i.e. micro-tearing modes) are predicted to be stable in a single fluid theory whereas inclusion of a time dependent thermal force in Ohm's law (HASSAM, 1980; D'IPPOLITO, 1980; ROSENBERG et al., 1980; and GLADD et al., 1980) permits unstable modes driven by the electron temperature gradient. Gross tearing modes are predicted to be unstable (provided $\Delta' > 0$) (FURTH et al., 1963 and COPPI et al., 1966) yet recent theory shows that inclusion of parallel electron thermal conduction effects may completely stabilise these modes (DRAKE et al., 1983). Localised resistive interchange modes in a cylinder are predicted by single fluid resistive theory to be stable below a critical value of the pressure gradient (FINN and MANNHEIMER, 1982), yet kinetic theories incorporating parallel electron thermal conduction and diamagnetic terms predict instability (CORDEY et al., 1980). Finally it appears that the same thermal conduction mechanism which stabilises gross tearing modes in slab geometry may have a similar stabilising effect on resistive ballooning modes (SUNDARAM et al., 1984).

Thus if accurate stability analysis for resistive instabilities is required, either linearly or non-linearly, it must be based on a more realistic plasma model. Single fluid resistive theory is not a sufficiently realistic model.

The linear theory of both the ideal and the resistive ballooning instabilities involves an eikonal ansatz and a transformation of the linearised equations from the real space, poloidal angle to an extended angular variable y on an infinite domain (CONNOR et al. 1978). The problem (ideal or resistive) can then be reduced to the solution of a set of ordinary differential equations, which in the case of marginally stable ideal ballooning modes reduce to the familiar second order ballooning equation (DOBROTT et al., 1977 and CONNOR et al., 1978). The qualitatively new picture which appears in the resistive ballooning equations is the appearance of small terms (proportional to the resistivity), which nevertheless become significant (because of secular dependence of their coefficients) at large values of the independent variable y, and therefore control the asymptotic behaviour of the solutions. The resistive ballooning equations therefore, unlike their ideal mhd analogues, contain two lengths, a short length typical of the poloidal variation of equilibrium quantities, and a long, resistive length. This property is related to the two length scales of resistive modes such as the tearing mode or g-mode in slab (FURTH et al., 1963) or cylindrical geometry (COPPI et al., 1966), but the resistive scale of ballooning theory is longer than the equilibrium scale because of the Fourier-transform nature of the ballooning transformation, mentioned above.

For a Tokamak plasma of moderate temperature and high density the most realistic plasma model is the two-fluid model, where the electron and ion fluids obey separate evolution equations given, for example, by BRAGINSKII (1965). In this paper we establish the basic eigenvalue equations of linear theory for resistive ballooning modes using this complete two-fluid set of equations. A particularly simple subset - the cold ion equations - are then explored in more detail by exploiting the two-scale nature of the problem to develop 'averaged' equations valid at large values of the independent variable y. At large y, where resistive effects are significant, the equations contain rapidly oscillating coefficients (on the scale of the connection length) but the eigenfunctions are smooth in lowest order. A systematic expansion in y^{-1} generates equations for the lowest order eigenfunctions in which the coefficients are field line averages over the poloidal equilibrium cycle. These equations are related to the resistive layer equations of standard tearing instability theory. They provide a more tractable form for numerical solution since they do not contain widely disparate scales of variation. Their solutions must satisfy the usual vanishing boundary condition as $y \to \infty$, while at small values of y they must connect onto solutions of the 'inner' ideal equations as resistive terms become negligible. In principle these inner equations should also be solved numerically, for a specific equilibrium, with even or odd boundary conditions at $y \to 0$ according as modes of twisting or tearing parity are to be studied. Continuing such an inner solution to large values of y one would then identify the relative amplitudes of the various independent solutions and use these amplitudes as boundary conditions for the outer resistive region equations. Then as a given plasma parameter (e.g. the pressure gradient) is varied, both these amplitudes (and hence the boundary conditions) and the coefficients of the resistive equations will change and the eigenvalue will respond to both. In practice appropriate boundary conditions are often simulated by choosing amplitudes for the independent solutions which are typical, fixing them, and studying the dependence of the eigenvalue on various plasma parameters with these fixed boundary conditions.

II GEOMETRY AND COORDINATE SYSTEM

The geometry is that of an axisymmetric toroidal plasma of arbitrary aspect ratio and cross sectional shape. The coordinates used are the ψ, χ, ζ set in which ψ is the poloidal flux within a magnetic surface, χ is a poloidal angle-like variable and ζ is the toroidal angle. In terms of these coordinates the gradient operator is

$$\nabla = \underset{\sim}{e}_\psi RB_\chi \frac{\partial}{\partial\psi} + \underset{\sim}{e}_\chi \frac{1}{JB_\chi}\frac{\partial}{\partial\chi} + \underset{\sim}{e}_\zeta \frac{1}{R}\frac{\partial}{\partial\zeta} \tag{2.1}$$

where $\underset{\sim}{e}_\psi$, $\underset{\sim}{e}_\chi$, and $\underset{\sim}{e}_\zeta$ are unit vectors, R is the distance to the symmetry axis, B_χ is the poloidal magnetic field and J is the Jacobean

$$J = (\nabla\psi\cdot\nabla\zeta \times \nabla\chi)^{-1} \quad . \tag{2.2}$$

The equilibrium magnetic field is then given by

$$\underset{\sim}{B} = -\nabla\psi \times \nabla\zeta + I(\psi)\nabla\zeta \quad . \tag{2.3}$$

III THE TWO FLUID EQUATIONS

The equations governing the evolution of high n resistive ballooning modes are taken to be the two-fluid equations derived by BRAGINSKII (1965).

$$\frac{D_j N_j}{Dt} + N_j \nabla\cdot\underset{\sim}{V}_j = 0 \tag{3.1}$$

$$m_j N_j \frac{D_j \underset{\sim}{V}_j}{Dt} = -\nabla p_j - \nabla\cdot\underset{=}{\pi}_j + n_j e_j\left(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{V}_j \times \underset{\sim}{B}\right) + \underset{\sim}{R}_j \tag{3.2}$$

$$\frac{3}{2} N_j \frac{D_j T_j}{Dt} + p_j \nabla\cdot \underset{\sim}{V}_j = -\nabla\cdot \underset{\sim}{q}_j + Q_j - \underset{=}{\pi}_j : \nabla \underset{\sim}{V}_j \qquad (3.3)$$

and $\frac{D_j}{Dt} \equiv \frac{\partial}{\partial t} + \underset{\sim}{V}_j \cdot \nabla$. Together with the electromagnetic equations,

$$\text{curl } \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{j} \qquad (3.4)$$

$$\text{curl } \underset{\sim}{E} = -\frac{1}{c} \frac{\partial \underset{\sim}{B}}{\partial t} \qquad (3.5)$$

and the quasi neutrality condition $N_i = N_e$, these equations give a complete description for a plasma in the collisional regime.

The transport coefficients have their classical values (BRAGINSKII, 1965), and the following approximations have been made:

(i) In the electron momentum equation, or Ohms law, electron inertia and the electron stress tensor are neglected. Neglect of electron inertia relative to resistivity corresponds to the inequality $\omega << \nu_e$, an approximation which has already been made in the derivation of the transport coefficients.

(ii) In the two thermal equations Ohmic heating and electron ion equilibration have been neglected. The former approximation corresponds to the inequality $\omega\tau_E >> 1$ with τ_E the energy confinement time, and the latter to the inequality $\omega >> \nu_{ie}$. It is then consistent to consider situations in which $T_i \neq T_e$.

The equations contain the electro-thermal terms, but the small time dependent thermal force terms (HASSAM, 1980), which are responsible in a fluid model for the micro-tearing instability (HASSAM, 1980 and ROSENBERG et al., 1980) have not been added to the Braginskii equations.

In practice it is convenient to replace the electron continuity equation, making use of the quasi-neutral condition, by $\nabla \cdot \underset{\sim}{J} = 0$ and to work with the fluid equation of motion rather than the ion momentum equation. A major simplification also follows from carrying out the linear analysis in the stationary ion frame of reference. In this frame the equilibrium electric field is non-zero, and is given by

$$\frac{\underset{\sim}{E}_0 \times \underset{\sim}{B}}{B^2} = - \frac{\underset{\sim}{B} \times \nabla P_i}{NeB^2} \tag{3.6}$$

The equations that we will finally obtain have a somewhat unfamiliar appearance in this frame, so we transform back to the frame of reference in which $\underset{\sim}{E}_0 = 0$, before presenting the final eigenmode equations.

Now, introducing the ballooning transformation and the eikonal ansatz for short wavelength perturbations, so that

$$\delta \underset{\sim}{E} = \sum_{m=-\infty}^{\infty} e^{2\pi i m \chi/\chi_0} \int_{-\infty}^{\infty} e^{-2\pi i m y/\chi_0} \, e^{i\ell S(\zeta, y, \psi)} \, \underset{\sim}{e}(y) e^{-i\omega t} dy \tag{3.7}$$

with $\chi_0 = \oint d\chi$ and the eikonal given by

$$S = \left[\zeta - \int^{y} \nu dy + \int^{\psi} k(\psi) d\psi\right] \tag{3.8}$$

with $\nu \equiv IJ/R^2$, we linearise the two fluid equations to obtain

$$N\nabla \cdot \underset{\sim}{v} = i\omega n - \underset{\sim}{v} \cdot \nabla N \tag{3.9}$$

$$\nabla \cdot \underset{\sim}{j} = 0 \tag{3.10}$$

$$i\omega\rho\underset{\sim}{v} = \nabla p + \nabla \cdot \underline{\underline{\pi}}_i - \frac{1}{c} \underset{\sim}{j} \times \underset{\sim}{B} - \frac{1}{c} \underset{\sim}{J} \times \underset{\sim}{b} \tag{3.11}$$

$$\begin{aligned} &\underset{\sim}{e} + n\nabla P_i/(N^2 e) + \underset{\sim}{v} \times \underset{\sim}{B} + \left(\nabla p_e - \underset{\sim}{j} \times \underset{\sim}{B}/c - \underset{\sim}{J} \times \underset{\sim}{b}/c\right)/Ne \\ &- 0.71\, \underset{\sim}{B}\left[(\underset{\sim}{b} \cdot \nabla) T_e + (\underset{\sim}{B} \cdot \nabla) t_e\right]/B^2 = \underline{\underline{\eta}} \cdot \underset{\sim}{j} \end{aligned} \tag{3.12}$$

$$\frac{3}{2} N(\underset{\sim}{v}\cdot\nabla)T_i - \frac{3}{2} i\omega N t_i + P_i(\nabla\cdot\underset{\sim}{v}) + \nabla\cdot\underset{\sim}{q}_i = 0 \qquad (3.13)$$

$$-\frac{3}{2} i\omega N t_e + P_e(\nabla\cdot\underset{\sim}{v}) + \nabla\cdot\underset{\sim}{q}_e - \frac{3}{2}\underset{\sim}{j}\cdot\nabla T_e/e + \frac{3}{2} N\underset{\sim}{v}\cdot\nabla T_e - \frac{3}{2}\underset{\sim}{J}\cdot\nabla t_e/e$$

$$+ \frac{P_e}{N^2 e}\underset{\sim}{J}\cdot\nabla n + \frac{P_e}{N^2 e}\underset{\sim}{j}\cdot\nabla N + 0.71\frac{J_\parallel}{e}\left[\frac{\underset{\sim}{b}}{B}\cdot\nabla T_e + \frac{\underset{\sim}{B}\cdot\nabla t_e}{B}\right] = 0 \qquad (3.14)$$

$$\frac{4\pi}{c}\underset{\sim}{j} = -\underset{\sim}{k}\times(\underset{\sim}{k}\times\underset{\sim}{a}) \qquad (3.15)$$

In equations (3.9)-(3.15) the lower case symbols n, p_j, $\underset{\sim}{j}$, $\underset{\sim}{v}$, $\underset{\sim}{b}$, $\underline{\underline{\pi}}_i$, t_j, $\underset{\sim}{e}$, $\underset{\sim}{a}$, $\underset{\sim}{q}_j$, denote the perturbed density, pressure (with $p = p_i + p_e$), current, fluid velocity, magnetic field, ion stress tensor, temperature, electric field, vector potential, and heat flux. $\rho = Nm_i$ is the equilibrium mass density, and

$$\underline{\underline{\eta}} = \eta_\perp\left(\underline{\underline{I}} - \underset{\sim\sim}{BB}/B^2\right) + \eta_\parallel \underset{\sim\sim}{BB}/B^2 \qquad (3.16)$$

is the unperturbed resistivity tensor. Terms arising from a perturbation of $\underline{\underline{\eta}}$ (the rippling mode driving terms) have been neglected in equation (3.12) compared to $\underline{\underline{\eta}}\cdot\underset{\sim}{j}$ since they are of order ℓ^{-1} smaller for the short wavelength modes considered here. In equation (3.15) the wavenumber $\underset{\sim}{k} \equiv \ell\nabla S$.

These equations are now simplified as follows. The perpendicular components of the equation of motion (3.11) and Ohms law (3.12) are used to obtain expressions for $\underset{\sim}{j}_\perp$ and $\underset{\sim}{v}_\perp$. These are

$$\underset{\sim}{j}_\perp = c(\underset{\sim}{f}\times\underset{\sim}{B})/B^2 - i\omega\frac{\rho c^2}{B^2}\left[\underset{\sim}{e}'_\perp - \underset{\sim}{f}_\perp/Ne\right] \qquad (3.17)$$

$$\underset{\sim}{v}_\perp = c(\underset{\sim}{e}'\times\underset{\sim}{B})/B^2 + c\underset{\sim}{f}\times\underset{\sim}{B}/(NeB^2)$$
$$- \frac{i\omega\rho c^2}{(NeB)^2}\underset{\sim}{f}_\perp\left(1 + i\eta_\perp N^2e^2/\rho\omega\right) - \frac{i\omega\rho c^2}{NeB^2}\underset{\sim}{E}_\perp \qquad (3.18)$$

where

$$\underset{\sim}{e}' = \underset{\sim}{e} - \frac{1}{c}\underset{\sim}{J} \times \underset{\sim}{b}/Ne + n\nabla P_i/(N^2 e) + \nabla p_e/Ne \tag{3.19}$$

$$\underset{\sim}{f} = \frac{1}{c}\underset{\sim}{J} \times \underset{\sim}{b} - \nabla p - \nabla\cdot\underline{\underline{\pi}}_i \tag{3.20}$$

and terms of order ν_{ie}/ω_{ci} and ω/ω_{ci} have been neglected ($\omega_{ci} \equiv$ ion-cyclotron frequency, $\nu_{ie} \equiv$ ion-electron collision frequency). The $\eta_\perp$ term in (3.18) has been retained although it is $O(\nu_{ie}/\omega)$ since it appears in the single fluid equations.

In lowest order $\underset{\sim}{v}_\perp$ is given by

$$\underset{\sim}{v}_{\perp 0} = - i\ell c \frac{\nabla S \times \underset{\sim}{B}}{B^2} (\phi + p_i/Ne) \tag{3.21}$$

with ϕ the perturbed electrostatic potential.

Expressions (3.17) and (3.18) are now used to calculate $\underset{\sim}{\nabla}\cdot\underset{\sim}{j}_\perp$ and $\nabla\cdot\underset{\sim}{v}_\perp$ with care being taken to retain contributions from small terms in $\underset{\sim}{j}_\perp$ and $\underset{\sim}{v}_\perp$ since $\underset{\sim}{k}_\perp\cdot\underset{\sim}{j}_\perp = \underset{\sim}{k}_\perp\cdot\underset{\sim}{v}_\perp \equiv 0$ in lowest order. Explicit evaluation gives

$$\begin{aligned}\nabla\cdot\underset{\sim}{j}_\perp = \; & Ne\Big\{-i\omega b_i \left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) + 2i \frac{p}{P_i}\,\omega_B + i\left(4\pi p - b_\parallel B\right)\frac{\omega_{*p}}{B^2} \\ & + \frac{c}{Ne}\,\nabla\cdot\Big[\frac{\underset{\sim}{B} \times \nabla\cdot\underline{\underline{\pi}}_i}{B^2}\Big]\Big\}\end{aligned} \tag{3.22}$$

$$\begin{aligned}\nabla\cdot\underset{\sim}{v}_\perp = \; & -i\omega b_i \left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) + i\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left[\omega_\rho + \omega_B\right] + i\omega\,\frac{b_\parallel}{B} \\ & + \eta_\perp k_\perp^2 c^2\,\frac{p}{B^2} + i\omega_{*i}\left[\frac{t_i}{T_i} - \eta_i\,\frac{n}{N}\right] + \nabla\cdot\Big[\frac{\underset{\sim}{B} \times \nabla\cdot\underline{\underline{\pi}}_i}{NeB^2}\Big]\end{aligned} \tag{3.23}$$

where $b_i = k_\perp^2 T_i/(m_i \omega_{ci}^2)$, $\eta_j = \frac{d\ell nT_j}{d\ell nN}$ and the frequencies ω_{*i} , ω_{*p} , ω_ρ and ω_B are defined as follows:

$$\omega_{*i} = (\underset{\sim}{k}_\perp \times \underset{\sim}{B}\cdot\nabla N)\,\frac{cT_i}{NeB^2} \tag{3.24a}$$

$$\omega_{*p} = \frac{c(\underset{\sim}{k}_\perp \times \underset{\sim}{B}\cdot\nabla P)}{NeB^2} \tag{3.24b}$$

$$\omega_\rho = \left[\underset{\sim}{k} \times \underset{\sim}{B}\cdot\nabla\left(4\pi P + \frac{1}{2}B^2\right)\right]\frac{cT_i}{eB^4} \tag{3.24c}$$

$$\omega_B = (\underset{\sim}{k} \times \underset{\sim}{B}\cdot\nabla B)\,\frac{cT_i}{eB^3} \tag{3.24d}$$

with e the proton charge.

From these definitions it follows that

$$\omega_\rho = \omega_B + \frac{\beta_i}{2}\,\omega_{*p} \tag{3.25}$$

with $\beta_j = 8\pi P_j/B^2$, and also that

$$\frac{\underset{\sim}{V}_\perp\cdot\nabla N}{N} = -i\omega_{*i}\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) \tag{3.26a}$$

$$\frac{\underset{\sim}{V}_\perp\cdot\nabla B}{B} = -i\omega_B\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) \tag{3.26b}$$

$$\underset{\sim}{V}_\perp\cdot\underset{\sim}{\rho} = -i\omega_\rho\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) \tag{3.26c}$$

where $\underset{\sim}{\rho} = \frac{1}{B^2}\,\nabla_\perp\left(4\pi P + \frac{1}{2}B^2\right)$ is the field line curvature.

Equations (3.22) and (3.23) are now used in conjunction with the ion continuity equation (3.9) and equation (3.10), together with the results

$$\nabla \cdot \underset{\sim}{j}_{\parallel} = \nabla \cdot \frac{ck_{\perp}^2 \underset{\sim}{a}_{\parallel}}{4\pi} \equiv \underset{\sim}{B} \cdot \nabla \left(\frac{ck_{\perp}^2 a_{\parallel}}{4\pi B}\right) \tag{3.27}$$

and

$$\nabla \cdot \underset{\sim}{v}_{\parallel} \equiv \underset{\sim}{B} \cdot \nabla \left(\frac{v_{\parallel}}{B}\right) \tag{3.28}$$

to give

$$\begin{aligned} \underset{\sim}{B} \cdot \nabla \left(\frac{ck_{\perp}^2 a_{\parallel}}{4\pi NeB}\right) = {} & i\omega b_i \left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) - \frac{p_i}{P_i} \left(\omega_B + \omega_\rho\right) \\ & + ib_{\parallel} B \frac{\omega_{*p}}{B^2} - \frac{1}{Ne} \nabla \cdot \left(\frac{\underset{\sim}{B} \times \nabla \cdot \underline{\underline{\pi}}_i}{B^2}\right) \end{aligned} \tag{3.29}$$

$$\begin{aligned} \underset{\sim}{B} \cdot \nabla \left(\frac{v_{\parallel}}{B}\right) = {} & i \left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left(\omega b_i - \omega_\rho - \omega_B + \omega_{*i}\right) - i\omega \frac{b_{\parallel}}{B} - \frac{\eta_{\perp} k_{\perp}^2 c^2 p}{B^2} \\ & + i\omega \frac{n}{N} - i\omega_{*i} \left(\frac{t_i}{T_i} - \eta_i \frac{n}{N}\right) - \nabla \cdot \left(\frac{\underset{\sim}{B} \times \nabla \cdot \underline{\underline{\pi}}_i}{NeB^2}\right) + i\omega b_i \frac{p}{P_i} . \end{aligned} \tag{3.30}$$

Noting that $\underset{\sim}{B} \cdot \nabla \equiv \frac{1}{J} \frac{\partial}{\partial y}$, equations (3.29) and (3.30) are seen to be a pair of first order differential equations in the longitudinal ballooning variable y, involving the perturbed quantities $a_{\parallel}$, $v_{\parallel}$, ϕ, n, t_e, and t_i. Two more first order differential equations are provided by the parallel components of the equation of motion (3.11) and the Ohms law (3.12). These are

$$\underset{\sim}{B} \cdot \nabla p = i\omega\rho B v_{\parallel} - i\left(\frac{Nea_{\parallel}}{c}\right) B\omega_{*p} - \underset{\sim}{B} \cdot \left(\nabla \cdot \underline{\underline{\pi}}_i\right) \tag{3.31}$$

$$\underset{\sim}{B}\cdot\nabla\left[Ne\phi - 1.71Nt_e - nT_e\right] = iNe\frac{a_\parallel}{c}B\left[\omega + \omega_{*p} - 0.71\omega_{*e}\eta_e + \frac{i\eta_\parallel k_\perp^2c^2}{4\pi}\right] \qquad (3.32)$$

where $\omega_{*e} \equiv -\frac{T_e}{T_i}\omega_{*i}$. Finally, the pair of thermal equations (3.13) and (3.14) provide two (second-order) differential equations for t_i and t_e.

Before presenting these we return to equations (3.29)-(3.32) to examine the contribution from the ion stress tensor in these equations. First, we must eliminate the magnetic compression $b_\parallel$ in favour of the perturbed pressure p. Returning to equation (3.11) and taking the scalar product with $i\underset{\sim}{k}$, one obtains

$$-k_\perp^2\left(p + \frac{b_\parallel B}{4\pi}\right) + i\underset{\sim}{k}\cdot\left(\nabla\cdot\underline{\underline{\pi}}_i\right) = 0 \qquad (3.33)$$

Thus ion-viscous and gyro-viscous effects appear in equations (3.29)-(3.31) in the terms $\underset{\sim}{k}\cdot\left(\nabla\cdot\underline{\underline{\pi}}_i\right)$, $\underset{\sim}{B}\cdot\left(\nabla\cdot\underline{\underline{\pi}}_i\right)$, $\frac{1}{Ne}\nabla\cdot\left[\frac{\underset{\sim}{B}\times\nabla\cdot\underline{\underline{\pi}}_i}{B^2}\right]$, and $\nabla\cdot\left[\frac{\underset{\sim}{B}\times\nabla\cdot\underline{\underline{\pi}}_i}{NeB^2}\right]$. After some laborious analysis these may be evaluated to obtain:

$$\frac{1}{Ne}\nabla\cdot\left(\frac{\underset{\sim}{B}\times\nabla\cdot\underline{\underline{\pi}}_i}{B^2}\right) = \frac{b_i}{2}\left\{i\omega\frac{n}{N} + i\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left[-\omega_{*i}\left(1 + 2\eta_i\right) + \omega_\rho + 3\omega_B\right]\right\}$$
$$+ \frac{\underset{\sim}{B}\cdot\nabla}{2B}\left(b_i v_\parallel\right) - \frac{b_i v_\parallel}{B^2}\underset{\sim}{B}\cdot\nabla B + 0.3\nu_i b_i^2\frac{e\phi}{T_i} + \frac{iW}{P_i}\left(2\omega_\rho - \omega_B\right) \qquad (3.34)$$

$$\nabla\cdot\left(\frac{\underset{\sim}{B}\times\nabla\cdot\underline{\underline{\pi}}_i}{NeB^2}\right) = \frac{1}{Ne}\nabla\cdot\left(\frac{\underset{\sim}{B}\times\nabla\cdot\underline{\underline{\pi}}}{B^2}\right) - \frac{iW}{P_i}\omega_{*i}$$
$$- i\frac{b_i}{2}\omega_{*i}\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) \qquad (3.35)$$

$$\underset{\sim}{B}\cdot\left(\nabla\cdot\underline{\underline{\pi}}_i\right) = 1.2\rho v_{\parallel} b_i \nu_i B - i\rho v_{\parallel}\left[\omega_{*i}\left(1+\eta_i\right) - \omega_B - 3\omega_\rho\right]B$$

$$+ b_i\,\underset{\sim}{B}\cdot\nabla\left(p_i + Ne\phi\right) + \frac{\left(p_i + Ne\phi\right)}{2B}\,\underset{\sim}{B}\cdot\underset{\sim}{\nabla}\left(Bb_i\right)$$

$$+ 3WB\,\underset{\sim}{B}\cdot\nabla\left(\frac{1}{B}\right) + 2\underset{\sim}{B}\cdot\nabla W\ . \qquad (3.36)$$

$$\frac{i\underset{\sim}{k}\cdot\left(\nabla\cdot\underline{\underline{\pi}}_i\right)}{k_\perp^2} = \frac{b_i P_i}{2}\left(\frac{p_i}{P_i} + \frac{e\phi}{T_i}\right) + W \qquad (3.37)$$

where

$$W = -\frac{0.96P_i}{\nu_i}\left[\frac{1}{B}\,\underset{\sim}{B}\cdot\nabla v_{\parallel} - \frac{1}{3}\,i\omega\,\frac{n}{N} + i\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left(\omega_\rho - \frac{1}{3}\,\omega_{*i}\right)\right]\ . \qquad (3.38)$$

In equations (3.34)-(3.37) the gyro-viscous terms are those which are independent of ν_i , the perpendicular viscosity appears in terms proportional to ν_i and the parallel viscosity in terms involving ν_i^{-1} .

Proceeding to the thermal equations, we evaluate $\nabla\cdot\underset{\sim}{q}_e$ and $\nabla\cdot\underset{\sim}{q}_i$ to obtain

$$\nabla\cdot\underset{\sim}{q}_e + \frac{0.71}{NeB}\,J_{\parallel}\left(\underset{\sim}{B}\cdot\nabla t_e - i\omega_{*e}\eta_e\,\frac{a_{\parallel}eB}{c}\right) = -K_{\parallel e}\,\underset{\sim}{B}\cdot\nabla\left[\frac{\underset{\sim}{B}\cdot\nabla t_e}{B^2} - i\omega_{*e}\eta_e\,\frac{ea_{\parallel}}{cB}\right.$$

$$\left. - 0.71\,\frac{T_e}{e}\,\underset{\sim}{B}\cdot\nabla\left(\frac{ck_\perp^2 a_{\parallel}}{4\pi B}\right)\right] + \frac{5}{2}\,iP_e\left[\frac{t_e}{T_i}\left(\omega_{*i} - \omega_\rho - \omega_B\right) + \omega_{*e}\eta_e\left(\frac{n}{N} + \frac{4\pi p}{B^2}\right)\right]$$

$$+ K_{\perp e}k_\perp^2\,t_e\ . \qquad (3.39)$$

Classical perpendicular electron thermal conduction has been retained in (3.39) despite the neglect of other comparably small terms (DRAKE,J.F. private communication). This has been done so that the effect of the observed anomalously large $K_{\perp e}$ can be studied with the equations developed in this paper.

$$\nabla\cdot\underset{\sim}{q}_i = -K_{\parallel i}\,\underset{\sim}{B}\cdot\underset{\sim}{\nabla}\Big[\frac{B\cdot\nabla t_i}{B^2} + i\omega_{*i}\eta_i\,\frac{ea_\parallel}{cB}\Big]$$

$$- \frac{5}{2}\,iP_i\Big[\frac{t_i}{T_i}\,(\omega_{*i} - \omega_\rho - \omega_B) - \omega_{*i}\eta_i\Big(\frac{n}{N} + \frac{4\pi p}{B^2}\Big)\Big] + K_{\perp i}k_\perp^2 t_i \qquad (3.40)$$

where $K_{\parallel e} = 3.2\,\dfrac{P_e}{m_e\nu_e}$, $K_{\perp e} = 4.7\,\dfrac{P_e\nu_e}{m_e\omega_{ce}^2}$, $K_{\parallel i} = 3.9\,\dfrac{P_i}{m_i\nu_i}$,

$K_{\perp i} = 2\,\dfrac{P_i\nu_i}{m_i\omega_{ci}^2}$, $\nu_e = \dfrac{4}{3}\,\dfrac{\sqrt{2\pi}\,N\lambda e^4}{\sqrt{m_e}\,(kT_e)^{3/2}}$ and $\nu_i = \dfrac{4}{3}\,\dfrac{\sqrt{\pi}\,N\lambda e^4}{\sqrt{m_i}\,(kT_i)^{3/2}}$.

Using these expressions the thermal equations may be written in the form

$$\frac{K_{\parallel e}}{N}\,\underset{\sim}{B}\cdot\nabla\Big[\frac{\underset{\sim}{B}\cdot\nabla t_e}{B^2T_e} - i\omega_{*e}\eta_e\,\frac{ea_\parallel}{cBT_e}\Big] + \frac{0.71}{Ne}\,\underset{\sim}{B}\cdot\nabla\,\Big(\frac{k_\perp^2\,ca_\parallel}{4\pi B}\Big) - \frac{K_{\perp e}}{N}\,k_\perp^2\,\frac{t_e}{T_e}$$

$$= -\frac{3}{2}\,i\,\frac{t_e}{T_e}\,\Big[\omega + \omega_{*i}(1 + \eta_i) + \frac{5}{3}\,\frac{T_e}{T_i}\,(\omega_\rho + \omega_B)\Big] + i\,\frac{n}{N}\,\big[\omega + \omega_{*i}(1 + \eta_i)\big]$$

$$- i\omega_{*e}\Big(1 - \frac{3}{2}\,\eta_e\Big)\,\frac{e\phi}{T_e} + \frac{5}{2}\,i\omega_{*e}\eta_e\,\frac{4\pi p}{B^2} \qquad (3.41)$$

$$\frac{K_{\parallel i}}{N}\,\underset{\sim}{B}\cdot\underset{\sim}{\nabla}\,\Big[\frac{\underset{\sim}{B}\cdot\underset{\sim}{\nabla}t_i}{B^2T_i} + i\omega_{*i}\eta_i\,\frac{ea_\parallel}{cBT_i}\Big] - \frac{K_{\perp i}}{N}\,k_\perp^2\,\frac{t_i}{T_i}$$

$$= -\frac{3}{2}\,i\,\frac{t_i}{T_i}\,\Big[\omega + \omega_{*i}(1 + \eta_i) - \frac{5}{3}\,(\omega_\rho + \omega_B)\Big] + i\,\frac{n}{N}\,\big[\omega + \omega_{*i}(1 + \eta_i)\big]$$

$$+ \frac{5}{2}\,i\omega_{*i}\eta_i\,\frac{4\pi p}{B^2} + i\omega_{*i}\Big(1 - \frac{3}{2}\,\eta_i\Big)\,\frac{e\phi}{T_i} \qquad (3.42)$$

In equations (3.29)-(3.32), (3.41) and (3.42) we have the complete description of the eighth order eigenvalue problem. Inserting the expressions (3.34)-(3.37) and using (3.33), we finally transform back to the frame in which the equilibrium electric field is zero. The set of six equations then becomes:

$$\underset{\sim}{B}\cdot\nabla p = i\rho B v_{\parallel}\left[\omega - \omega_B - 3\omega_\rho + 1.2i\nu_i b_i\right] - \frac{iNea_{\parallel}}{c} B\omega_{*p} - b_i\,\underset{\sim}{B}\cdot\nabla\left(p_i + Ne\phi\right)$$

$$- \frac{\left(p_i + Ne\phi\right)}{2B}\,\underset{\sim}{B}\cdot\nabla\left(Bb_i\right) + 3W\,\frac{\underset{\sim}{B}\cdot\nabla B}{B} - 2\underset{\sim}{B}\cdot\nabla W \qquad (3.43)$$

$$\underset{\sim}{B}\cdot\nabla\left[Ne\phi - 1.71Nt_e - nT_e\right] = \frac{iNea_{\parallel}}{c} B\left[\omega - \omega_{*e}\left(1 + 1.71\eta_e\right) + \frac{i\eta_{\parallel}k_{\perp}^2c^2}{4\pi}\right] \qquad (3.44)$$

$$\underset{\sim}{B}\cdot\nabla\left(\frac{k_{\perp}^2 ca_{\parallel}}{4\pi NeB}\right) = ib_i\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left[\omega - \omega_{*i}\left(1 + \eta_i\right)\right] - 2i\omega_\rho\,\frac{p}{P_i}$$

$$+ i\,\frac{b_i}{2}\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left[\omega_{*i}\left(1 + 2\eta_i\right) - \omega_\rho - 3\omega_B\right] - i\left[\omega - \omega_{*i}\left(1 + \eta_i\right)\right]\frac{n}{N}\,\frac{b_i}{2}$$

$$- \frac{\underset{\sim}{B}\cdot\nabla}{2B}\left(b_i v_{\parallel}\right) + \frac{b_i v_{\parallel}}{B^2}\,\underset{\sim}{B}\cdot\nabla B - 0.3\nu_i b_i^2\,\frac{e\phi}{T_i}$$

$$+ i\,\frac{b_i}{2}\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left[\omega_\rho - \omega_B\right] + i\,\frac{\omega_{*p}}{B^2}\,W \qquad (3.45)$$

$$\underset{\sim}{B}\cdot\nabla\left(\frac{v_\parallel}{B}\right) = i(\omega_{*i} - \omega_B - \omega_\rho)\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right) + i(\omega - \omega_{*pi})\frac{n}{N} + i\frac{p}{P_i}\frac{\beta_i}{2}(\omega - \omega_{*pi})$$

$$- i\omega_{*i}\left(\frac{t_i}{T_i} - \eta_i\frac{n}{N}\right) + i\frac{W}{P_i}\left[\omega_{*i} - \omega_\rho - \frac{\beta_i}{2}(\omega + \omega_{*p} - \omega_{*pi})\right] - 0.3\nu_i b_i^2\frac{e\phi}{T_i}$$

$$+ ib_i\left[\left(\frac{e\phi}{T_i} + \frac{p_i}{P_i}\right)\left(\omega - \frac{1}{2}\omega_\rho - \frac{3}{2}\omega_B\right) - \frac{1}{2}\frac{n}{N}(\omega - \omega_{*pi})\right]$$

$$- \frac{1}{2}\underset{\sim}{B}\cdot\nabla\left(\frac{b_i v_\parallel}{B}\right) - \frac{1}{2}b_i v_\parallel \underset{\sim}{B}\cdot\underset{\sim}{\nabla}\left(\frac{1}{B}\right) - \frac{\eta_\perp k_\perp^2 c^2 p}{B^2} \qquad (3.46)$$

$$\frac{K_{\parallel e}}{N}\underset{\sim}{B}\cdot\nabla\left[\frac{\underset{\sim}{B}\cdot\nabla}{B^2}\left(\frac{t_e}{T_e}\right) - i\frac{\omega_{*e}\eta_e}{B}\frac{ea_\parallel}{cT_e}\right] + \frac{0.71}{Ne}\underset{\sim}{B}\cdot\nabla\left(\frac{k_\perp^2 c\, a_\parallel}{4\pi B}\right) - \frac{K_{\perp e}}{N}\frac{k_\perp^2 t_e}{T_e}$$

$$= -\frac{3}{2}i\frac{t_e}{T_e}\left[\omega + \frac{5}{3}\tau(\omega_\rho + \omega_B)\right] + i\frac{n}{N}\omega - i\omega_{*e}\left(1 - \frac{3}{2}\eta_e\right)\frac{e\phi}{T_e} + \frac{5}{2}i\omega_{*e}\eta_e\frac{4\pi p}{B^2} \qquad (3.47)$$

$$\frac{K_{\parallel i}}{N}\underset{\sim}{B}\cdot\nabla\left[\frac{\underset{\sim}{B}\cdot\nabla}{B^2}\left(\frac{t_i}{T_i}\right) + i\frac{\omega_{*i}\eta_i}{B}\frac{ea_\parallel}{cT_i}\right] - \frac{K_{\perp i}}{N}k_\perp^2\frac{t_i}{T_i}$$

$$= -\frac{3}{2}i\frac{t_i}{T_i}\left[\omega - \frac{5}{3}(\omega_\rho + \omega_B)\right] + i\omega\frac{n}{N} + \frac{5}{2}i\omega_{*i}\eta_i\frac{4\pi p}{B^2} + \omega_{*i}\left(1 - \frac{3}{2}\eta_i\right)\frac{e\phi}{T_i} \qquad (3.48)$$

IV THE COLD ION MODEL

The single-fluid mhd equations only give a satisfactory description of resistive instabilities for rather low temperature plasmas. On the other hand the complexity of the full set of two-fluid equations means that even linear stability analysis is a formidable calculation.

An intermediate model is provided by the cold ion equations. These are obtained from the full two fluid set, by neglecting the ion collisional viscosities and taking the limit $T_i/T_e \to 0$ in the remaining terms. This has the advantage of removing the finite ion larmor radius terms proportional to b_i while retaining all the electron physics without introducing any geometric approximations (such as large aspect ratio expansions). The equations also reduce from an eighth order differential set to a sixth order one. In addition they still contain the single fluid equations so that the standard results of resistive ballooning theory can be recovered.

The cold ion model equations in general geometry are given below (in the $\underset{\sim}{E}_0 = 0$ frame):

$$\frac{\underset{\sim}{B}\cdot\nabla p}{B} = i\omega\rho v_\parallel - i\omega_{*p}\left(\frac{Nea_\parallel}{c}\right) \tag{4.1}$$

$$\underset{\sim}{B}\cdot\nabla\left(\frac{v_\parallel}{B}\right) = i\omega\frac{n}{N} + i\frac{e\phi}{T_e}\left[\omega b_s - \omega_{*e} - 2\tau\omega_\rho\right] + \frac{i\beta}{2}\left(\omega\frac{p}{P} + \omega_{*p}\frac{e\phi}{T_e}\right) - \frac{\eta_\perp k_\perp^2 c^2}{B^2}p \tag{4.2}$$

$$\frac{\underset{\sim}{B}\cdot\nabla}{B}\left[\frac{e\phi}{T_e} - 1.71\frac{t_e}{T_e} - \frac{n}{N}\right] = i\frac{ea_\parallel}{cT_e}\left[\omega - \omega_{*e}\left(1 + 1.71\eta_e\right) + i\eta_\parallel\frac{k_\perp^2 c^2}{4\pi}\right] \tag{4.3}$$

$$\underset{\sim}{B}\cdot\nabla\left(\frac{ck_\perp^2 a_\parallel}{4\pi NeB}\right) = i\omega b_s\frac{e\phi}{T_e} - 2i\frac{p}{P}\left(\omega_\rho\tau\right) \tag{4.4}$$

$$\frac{K_{\parallel e}}{N}\, \underset{\sim}{B}\cdot\nabla\Big[\frac{1}{B^2}\, \underset{\sim}{B}\cdot\nabla\Big(\frac{t_e}{T_e}\Big) - i\,\frac{a_{\parallel} e}{cT_e}\,\frac{\omega_{*e}}{B}\,\eta_e\Big] + \frac{0.71}{Ne}\,\underset{\sim}{B}\cdot\nabla\Big(\frac{k_\perp^2 a_\parallel c}{4\pi B}\Big) - \frac{K_{\perp e}}{N}\,k_\perp^2\,\frac{t_e}{T_e}$$

$$= -\frac{3}{2}\, i\omega\,\frac{t_e}{T_e} - 5i\,\frac{t_e}{T_e}\,\tau\omega_\rho + i\,\frac{n}{N}\,\omega - i\omega_{*e}\Big(1 - \frac{3}{2}\,\eta_e\Big)\frac{e\phi}{T_e}$$

$$+ \frac{5}{4}\, i\beta\,\Big[\frac{p}{P}\,\omega_{*e}\eta_e - \frac{t_e}{T_e}\,\omega_{*e}(1+\eta_e)\Big] \qquad (4.5)$$

with $b_s = b_i\, T_e/T_i$ now permitted to be of order unity.

The resistive diffusion term has been retained in equation (4.2) so that the single fluid equations can be recovered. However, since the electron-ion equilibration has been neglected in equation (4.5), the mode frequency ω must satisfy $\omega >> \nu_{ie}$ for consistency. Consequently the diffusion term in (4.2) is necessarily small compared to ωb_s terms. Its retention is a device to enable us to make contact with single fluid theory. This limit is obtained by taking $\frac{\omega_*}{\omega} << 1$, $\frac{e\phi}{T} \sim \frac{\omega}{\omega_*}\frac{n}{N} \sim \frac{\omega}{\omega_*}\frac{t_e}{T_e}$, $\omega >> K_\parallel\left(\frac{B\cdot\nabla}{B}\right)^2$ when the temperature equation yields the solution

$$\frac{t_e}{T_e} = \frac{2}{3}\frac{n}{N} - \frac{\omega_{*e}}{\omega}\Big(\frac{2}{3} - \eta_e\Big)\frac{e\phi}{T_e} \quad . \qquad (4.6)$$

Using this expression to eliminate n/N in favour of p/P in equation (4.2), equations (4.1)-(4.5) now reduce to the single fluid equations, which can be written in the form

$$\frac{\underset{\sim}{B}\cdot\nabla p}{B} = i\omega\rho v_\parallel - i\omega_{*p}\Big(\frac{Nea_\parallel}{c}\Big) \qquad (4.7)$$

$$\underset{\sim}{B}\cdot\nabla\Big(\frac{v_\parallel}{B}\Big) = i\omega\Big(\frac{p}{P} + \frac{\omega_{*p}}{\omega}\,\frac{e\phi}{T_e}\Big)\Big(\frac{3}{5} + \frac{4\pi P}{B^2}\Big) - \frac{\eta_\perp k_\perp^2 c^2}{B^2}\, p - 2i\tau\omega_\rho\,\frac{e\phi}{T_e} \qquad (4.8)$$

$$\frac{\underset{\sim}{B}\cdot\nabla}{B}\left(\frac{e\phi}{T_e}\right) = \frac{iea_{\parallel}}{cT_e}\left(\omega + i\eta_{\parallel}\frac{k_{\perp}^2 c^2}{4\pi}\right) \tag{4.9}$$

$$\underset{\sim}{B}\cdot\nabla\left(\frac{ck_{\perp}^2 a_{\parallel}}{4\pi NeB}\right) = i\omega b_s \frac{e\phi}{T_e} - 2i\frac{p}{P}\left(\omega_\rho \tau\right) \quad . \tag{4.10}$$

The cold ion equations, like the single fluid equations, are valid in any axisymmetric torus with arbitrary β and at any value of ℓ^2/S . For small values of ℓ^2/S the solutions of the cold ion equations display two scale behaviour; that is solutions decay on a long resistive scale $\propto \left(\frac{\ell^2}{S}\right)^{-1/3}$, as well as varying on the scale of the connection length. We take advantage of this two scale behaviour to generate a set of averaged equations which are valid in the resistive region, i.e. large values of the independent variable y . Stability properties can then be determined by numerical solution of these simpler averaged equations subject to suitable boundary conditions at small values of y .

V AVERAGING THE COLD ION EQUATIONS

In this section we outline the procedure for exploiting the two length scales to obtain a set of averaged equations from the cold ion model (equations (4.1)-(4.5)).

The secularly increasing quantity $Z = \left[\int^y \frac{\partial \nu}{\partial \psi} dy - k(\psi)\right]$ is introduced as a second, long scale independent variable, so that the derivative $\underset{\sim}{B}\cdot\nabla = \frac{1}{J}\frac{d}{dy}$ becomes

$$\underset{\sim}{B}\cdot\nabla \to \frac{1}{J}\left(\frac{\partial}{\partial y} + \nu'\frac{\partial}{\partial Z}\right) \tag{5.1}$$

where the prime denotes a derivative with respect to the poloidal flux ψ . In terms of Z , the perpendicular wavenumber $k_\perp$ is given by

$$k_\perp^2 = \ell^2\left|\nabla S\right|^2 = \ell^2\left[\lambda B^2 Z^2 + 1/\lambda\right] \tag{5.2}$$

where $\lambda = R^2B_\chi^2/B^2$. The quantity ω_ρ also displays secular behaviour, and is given by

$$\omega_\rho \tau = \frac{c\ell}{2}\frac{P}{e}\left[K_1 + K_0 \frac{Z}{J}\frac{\partial}{\partial\chi}\left(\frac{1}{B^2}\right)\right] \qquad (5.3)$$

where $K_0 = -RB_\zeta$ and $K_1 = -\frac{2}{B}\frac{\partial}{\partial\psi}\left(4\pi P + \frac{1}{2}B^2\right)$.

Averaged equations can now be generated by expanding the solutions in powers of $1/Z$, and treating b_s and $\eta k_\perp^2 c^2/4\pi\omega$ as being of order unity. Thus with $p = p_0 + \frac{1}{Z}p_1 + \ldots.$, $\phi = \phi_0 + \frac{1}{Z}\phi_1 + \ldots.$ etc, equation (4.1) for example yields

$$\frac{\partial p_0}{\partial y} = 0 \qquad (5.4)$$

$$\frac{1}{J}\left(\frac{1}{Z}\frac{\partial p_1}{\partial y} + \nu'\frac{dp_0}{dZ}\right) = i\omega\rho B^2 u_0 - i\omega_{*p} Nea \qquad (5.5)$$

where we have introduced the notation $u = v_\parallel/B$, and $a = a_\parallel B/c$.

Now annihilating the term involving $p_1(y,Z)$ by integrating $\oint Jdy$ over one equilibrium period, one obtains from (5.5)

$$\langle\nu'\rangle \frac{dp_0}{dZ} = i\omega\rho \langle JB^2u_0\rangle - i\omega_{*p}\langle Ja_0\rangle\, Ne \qquad (5.6)$$

where $\langle X\rangle = \oint X dy$.

Turning to Ohms law, equation (4.3), we obtain in the lowest order

$$\frac{\partial}{\partial y}\left[Ne\phi_0 - 1.71t_0 - n_0\right] = 0 \qquad (5.7)$$

Taken in conjunction with equation (5.4) and the similar result for t_0 which emerges from the thermal equation, equation (5.7) has the solution:

$$\frac{\partial\phi_0}{\partial y} = \frac{\partial t_0}{\partial y} = \frac{\partial n_0}{\partial y} = 0 \tag{5.8}$$

The first order Ohms law

$$\frac{1}{ZJ}\frac{\partial}{\partial y}\left[Ne\phi_1 - 1.71t_1 - n_1\right] + \frac{\nu'}{J}\frac{d}{dZ}\left[Ne\phi_0 - 1.71t_0 - n_0\right]$$

$$= iNea_0\left[\omega - \omega_{*e}\left(1 + 1.71\eta_e\right) + i\eta_{\parallel}k_{\perp}^2c^2/4\pi\right] \tag{5.9}$$

now provides an equation for the first order quantity $(Ne\phi_1 - 1.71t_1 - n_1)$, and, on annihilating this term by integration over one period in y, the averaged Ohms law is obtained:

$$\langle\nu'\rangle\frac{d}{dZ}\left[Ne\phi_0 - 1.71t_0 - n_0\right] = iNe\left[\omega - \omega_{*e}\left(1 + 1.71\eta_e\right)\right]\langle Ja_0\rangle$$

$$- \eta_{\parallel}\ell^2Z^2\langle Ja_0\lambda B\rangle Ne\,\frac{c^2}{4\pi}\ . \tag{5.10}$$

In lowest order the continuity equation (4.2) takes the form

$$\frac{\partial}{\partial y}u_0 = ci\ell\phi_0K_0Z\frac{\partial}{\partial y}\left(\frac{1}{B^2}\right) \tag{5.11}$$

determining u_0 up to an arbitrary function of Z, as

$$u_0 = \frac{ci\ell\phi_0K_0Z}{B^2} + \bar{u}(Z) \tag{5.12}$$

In next order an equation involving $\left(\frac{\partial u_1}{\partial y} + \nu'\frac{\partial u_0}{\partial Z}\right)$ is obtained, and on annihilating u_1 by integrating over one period in y, and substituting (5.12) for u_0 the following first order equation for $\bar{u}(Z)$ is obtained.

$$\langle\nu'\rangle \frac{d\bar{u}}{dz} + ci\ell K_0 \langle\frac{\nu'}{B^2}\rangle \frac{d}{dZ}\left(Z\phi_0\right) + ci\ell K_0\langle\frac{1}{B^2}\frac{\partial\phi_1}{\partial y}\rangle - i\omega p_0 \langle J\,\frac{B^2 + 4\pi P}{B^2 P}\rangle$$

$$- ci\ell\phi_0\Big[\langle JK_0\rangle + \langle J\Big[\frac{1}{N}\frac{dN}{d\psi} + \frac{\beta}{2}\frac{1}{P}\frac{dP}{d\psi}\Big]\rangle + \omega\ell Z^2 \frac{m_i c}{e}\langle J\lambda\rangle\Big] + i\omega\frac{t}{T}\langle J\rangle$$

$$+ \eta c_I^2\, \ell^2 Z^2 p_0 \langle J\lambda\rangle \;=\; 0 \; . \qquad (5.13)$$

To completely develop this equation we must first construct $\langle\frac{1}{B^2}\frac{\partial\phi_1}{\partial y}\rangle$ in terms of zero order quantities ϕ_0, p_0 etc. The first order Ohms law (5.9) contains $\frac{\partial\phi_1}{\partial y}$, but only in combination with t_1 and n_1 in the form $\frac{\partial}{\partial y}\left(Ne\phi_1 - 1.71t_1 - n_1\right)$. However, the first order momentum equation (5.5) contains $\frac{\partial}{\partial y} p_1$, and the first order thermal equation (see equation (5.19) later) will yield an expression for $\frac{\partial t_1}{\partial y}$. Taken together these three equations enable us to form the averaged quantity $\langle\frac{1}{B^2}\frac{\partial\phi_1}{\partial y}\rangle$ [and also $\langle\frac{1}{B^2}\frac{\partial t_1}{\partial y}\rangle$ and $\langle\frac{1}{B^2}\frac{\partial n_1}{\partial y}\rangle$ which are required below]. Secondly equation (5.6) provides an explicit expression for $\bar{u}(Z)$ (in terms of p_0, ϕ_0 and a_0) which is substituted into (5.13) to produce a second order differential equation involving, for example, d^2p_0/dZ^2.

The treatment of the vorticity equation (4.4) is similar to that of the momentum equation discussed above. In lowest order this equation has the form

$$\frac{\partial}{\partial y}\left(\frac{c^2}{4\pi}\,\ell^2 Z^2\lambda a_0\right) \;=\; ci\ell K_0 Z\,\frac{\partial}{\partial y}\left(\frac{1}{B^2}\right) \qquad (5.14)$$

with solution

$$\frac{c^2}{4\pi}\,\ell^2 Z^2\lambda a_0 \;=\; \frac{ci\ell K_0 Z}{B^2} + \bar{A}(Z) \qquad (5.15)$$

Annihilating the quantity a_1 in the first order equation by appropriate averaging in y generates the following averaged equation:

$$\langle\nu'\rangle \frac{d}{dZ}\bar{A} + \langle\frac{\nu'}{B^2}\rangle ci\ell K_0 \frac{d}{dZ}\left(Zp_0\right) + ci\ell K_0 \langle\frac{1}{B^2}\frac{\partial p_1}{\partial y}\rangle - ci\ell p_0 \langle JK_1\rangle$$

$$- i\omega\rho\ell^2 Z^2 c^2 \langle J\lambda\rangle \phi_0 = 0 \tag{5.16}$$

Now $\langle\frac{1}{B^2}\frac{\partial p_1}{\partial y}\rangle$ can be obtained as discussed above and the averaged Ohms law (5.10) provides an explicit expression for $\bar{A}(Z)$ (after using (5.15)) in terms of ϕ_0, t_0 and n_0 so that (5.16) becomes a second order differential equation containing, for example, $\frac{d^2}{dZ^2}\phi_0$.

The third, and final averaged equation comes from the thermal equation. In lowest order, we assume parallel thermal conduction over the connection length to be the dominant process, so that

$$\frac{\partial}{\partial y}\left(\frac{1}{JB^2}\frac{\partial t_0}{\partial y}\right) = 0 \tag{5.17}$$

with solution $t_0 = t_0(Z)$. In first order the thermal equation yields

$$\frac{K_\parallel}{N}\frac{1}{J}\frac{\partial}{\partial y}\left[\frac{1}{Z}\frac{1}{JB^2}\frac{\partial t_1}{\partial y} + \frac{\nu'}{JB^2}\frac{dt_0}{dZ} + i\omega_{*e}\eta_e\frac{ea_0 N}{B^2}\right] + 0.71\frac{T_e}{e}\frac{1}{J}\frac{\partial}{\partial y}\left[\frac{c^2\ell^2 Z^2\lambda a_0}{4\pi}\right]$$

$$= \frac{5}{2} ic \frac{T_e}{e}\ell K_0 Z \frac{t_0}{J}\frac{\partial}{\partial y}\left(\frac{1}{B^2}\right) \tag{5.18}$$

where the parallel thermal conductivity $K_\parallel = 3.2 NT_e/m_e\nu_e$. Equation (5.18) may be integrated once to give

$$\frac{K_{\parallel}}{N}\left\{\frac{1}{Z}\frac{1}{JB^2}\frac{\partial t_1}{\partial y} + \frac{\nu'}{JB^2}\frac{dt_0}{dZ} + i\omega_{*e}\eta_e\frac{ea_0N}{B^2}\right\} + 0.71\frac{T_e}{e}\frac{c^2\ell^2Z^2\lambda a_0}{4\pi}$$

$$- \frac{5}{2}\,cit_0\frac{T_e}{e}\frac{K_0\ell Z}{B^2} = \bar{c}(Z) \quad . \tag{5.19}$$

$\bar{c}(Z)$ is an arbitrary function of Z which may be evaluated immediately in terms of a_0 and t_0 by annihilating the t_1 term in equation (5.19) to give

$$\bar{c}(Z)\langle JB^2\rangle = \frac{K_{\parallel}}{N}\left\{\langle\nu'\rangle\frac{dt_0}{dZ} + i\omega_{*e}\eta_e eN\langle Ja_0\rangle\right\} + 0.71\frac{T_e}{e}\frac{c^2\ell^2Z^2}{4\pi}\langle JB^2\lambda a_0\rangle$$

$$- \frac{5}{2}\,ic\ell t_0K_0Z\frac{T_e}{e}\langle J\rangle \tag{5.20}$$

Equations (5.19) and (5.20) now give the explicit expression for the first order quantity $\partial t_1/\partial y$ alluded to in the discussion below Eq (5.13).

The required average of the thermal equation is obtained in second order. This equation contains t_2 and a_1 in addition to t_1 and zero order quantities. The t_2 and a_1 terms may be simultaneously annihilated to obtain the following averaged equation.

$$\frac{K_\parallel}{N}\frac{d}{dZ}\left\langle\frac{\nu'}{JB^2}\frac{\partial}{\partial y}\left(\frac{t_1}{Z}\right)\right\rangle + \frac{K_\parallel}{N}\frac{d}{dZ}\left\langle\frac{\nu' a_0}{B^2}\right\rangle i\omega_{*e}\eta_e Ne + 0.71\,\frac{T_e}{e}\frac{d}{dZ}\left[\frac{c^2\ell^2 Z^2\langle\nu'\lambda a_0\rangle}{4\pi}\right]$$

$$+ \frac{K_\parallel}{N}\left\langle\frac{\nu'^2}{JB^2}\right\rangle\frac{d^2 t_0}{dZ^2} + i\omega_{*e}\left(1 - \frac{3}{2}\eta_e\right)Ne\phi_0\langle J\rangle - ip_0\left\langle J\left[\omega + \frac{5}{4}\beta\omega_{*e}\eta_e\right]\right\rangle$$

$$- \frac{5}{2}\, i\ell c\,\frac{T_e}{e}\, t_0\langle JK_1\rangle + \frac{5}{2}\, it_0\langle J\left[\omega + \frac{\beta}{2}\,\omega_{*e}\left(1 + \eta_e\right)\right]$$

$$+ \frac{5}{2}\, i\,\frac{T_e}{e}\, c\ell K_0\left\langle\frac{1}{B^2}\frac{\partial t_1}{\partial y}\right\rangle - \ell^2 Z^2 t_0\left\langle\frac{JB^2\lambda K_\perp}{N}\right\rangle = 0 \qquad (5.21)$$

Expressions for t_1 (from (5.19) and (5.20)) and a_0 (from (5.15) and (5.10)) are now substituted into (5.21) to give the final averaged thermal equation involving only the quantities t_0 , n_0 and ϕ_0 . Together with equation (5.13) (in which ϕ_1 and $\bar{u}$ have been eliminated) and (5.15) (in which p_1 and $\bar{A}$ have been eliminated) this completes the sixth order differential set of averaged equations.

After considerable algebra these equations may be obtained in the following form:

$$\frac{d}{dX}\left(\frac{X^2}{\Gamma_1}\frac{d\tilde{\phi}}{dX}\right) + \frac{HX}{\Gamma_1}\frac{d}{dX}\left(\tilde{\phi} - p\right) + p\left(D - \frac{H(H+1)}{\Gamma_1} - HX\frac{d}{dX}\left(\frac{1}{\Gamma_1}\right)\right)$$

$$- X^2 QQ_1\left[\tilde{\phi} - \frac{i\omega_*}{Q_1}\left(1 + \eta_e\right)(p + \alpha_1 t)\right] = 0\ . \qquad (5.22)$$

$$\frac{d}{dX}\left[\frac{dp}{dX} - \frac{1}{\Gamma_1}\frac{d\tilde{\phi}}{dX}\right] - \frac{H}{Q_1}\frac{d}{dX}\left(\frac{Xp}{\Gamma_1}\right) - Q^2 KH\,\frac{X}{\Gamma_1}\frac{d\tilde{\phi}}{dX} + Q^2 t\left[G_1 + \frac{5}{2}\frac{\alpha_1\alpha_2}{\alpha_3}\frac{X^2}{Q}\frac{M_1}{M}\right]$$

$$- Q^2 p\left[G_1 + KF + KH^2\left(1 - \frac{1}{\Gamma_1}\right) + \frac{X^2}{Q}\left\{\alpha_2\left(1 - \frac{M_1}{M}\right) + \left(1 + \frac{\alpha_2\alpha_1^2}{\alpha_3}\right)\frac{M_1}{M}\right\}\right]$$

$$- \left[\frac{\alpha_2 Q^2}{\nu}X^2 + iQ\omega_*\left(1 + \eta_e\right)\left(KE + \frac{G_1}{1 + \eta_e}\right)\right]\left[p + \alpha_1 t + \frac{iQ_1}{\omega_*\left(1 + \eta_e\right)}\tilde{\phi}\right] = 0$$

$$(5.23)$$

$$\alpha_3 \left\{\frac{d^2t}{dX^2} - \frac{\eta_e}{1+\eta_e}\frac{d}{dX}\left(\frac{\Gamma_2}{\Gamma_1}\frac{d\tilde{\phi}}{dX}\right) - \frac{\eta_e}{1+\eta_e}\frac{H}{Q_1}\frac{d}{dX}\left(\frac{Xp}{\Gamma_1}\right)\right\}$$

$$+ i\nu\omega_*(1+\eta_e)\ KH\ \left[\alpha_1 \frac{d}{dx}\left(\frac{Xp}{\Gamma_1}\right) - \frac{5}{2}\frac{\eta_e}{1+\eta_e}\frac{X}{\Gamma_1}\frac{d\tilde{\phi}}{dX}\right]$$

$$+ \frac{5}{2}\,t\nu\,\left[i\omega_*(1+\eta_e)\ KE - QG_1 - \frac{5}{2}\frac{\alpha_2}{\alpha_3}\frac{M_1}{M}X^2 - \frac{2}{5}\alpha_2\alpha_4\left(1-\frac{M_1}{M}\right)X^2\right]$$

$$- p\nu\,\left[\frac{5}{2}\,i\omega_*\eta_e K\left(F + H^2 - H^2/\Gamma_1\right) - QG_1 - \frac{5}{2}\frac{\alpha_1\alpha_2}{\alpha_3}\frac{M_1}{M}X^2\right]$$

$$+ i\nu\omega_*\left(1-\frac{3}{2}\eta_e\right)G_1\left[p + \alpha_1 t + \frac{iQ_1}{\omega_*(1+\eta_e)}\tilde{\phi}\right] = 0 \qquad (5.24)$$

In these equations the quantities D , E , F, H , K , M and M_1 are field line averaged quantities equivalent to those defined by GLASSER et al. (1975). The quantity G_1 is the same as the quantity G in that reference but with the adiabatic index γ replaced by unity. The definitions are:

$$E = \frac{4\pi p'\langle J/\lambda\rangle}{\langle\nu'\rangle^2}\left\{\langle JK_1\rangle - \langle\nu' K_0/B^2\rangle + \frac{\langle JK_0\rangle\langle\nu'\rangle}{\langle JB^2\rangle} + 4\pi p'\langle J/B^2\rangle\right\}$$

$$F = \frac{(4\pi p')^2\langle J/\lambda\rangle}{\langle\nu'\rangle^2}\left\{\langle JK_0{}^2/\lambda B^4\rangle - \frac{\langle JK_0/\lambda B^2\rangle^2}{\langle J/\lambda\rangle} + \langle J/B^2\rangle\right\}$$

$$H = \frac{4\pi p'\langle J/\lambda\rangle}{\langle\nu'\rangle}\left[\frac{\langle JK_0/\lambda B^2\rangle}{\langle J/\lambda\rangle} - \frac{\langle JK_0\rangle}{\langle JB^2\rangle}\right]$$

$$M = \frac{\langle J/\lambda\rangle}{\langle J\rangle^2}\left\{\langle J\lambda\rangle + \langle JK_0{}^2/B^2\rangle - \frac{\langle JK_0\rangle^2}{\langle JB^2\rangle}\right\}$$

$$M_1 = \frac{\langle J/\lambda\rangle}{\langle J\rangle^2}\left\{\langle JK_0{}^2/B^2\rangle - \frac{\langle JK_0\rangle^2}{\langle JB^2\rangle}\right\}$$

$$K = \langle JB^2\rangle\langle\nu'\rangle^2/\left\{M\langle J/\lambda\rangle(4\pi p')^2\langle J\rangle^2\right\}$$

$$G_1 = \frac{\langle JB^2\rangle}{4\pi PM\langle J\rangle}$$

$$D = H^2 + F - E \ .$$

The independent variable X is defined by $X = Z/Z_r$ where

$$Z_r^3 = 4\pi\langle\nu'\rangle\langle J/\lambda\rangle/\{(4\pi\rho M)^{1/2}\langle J\rangle\langle JB^2\rangle\eta_{\parallel}\ell^2c^2\} \quad .$$

The eigenvalue $Q \equiv -i\omega/\omega_r$ with

$$\omega_r^3 = \eta_{\parallel}c^2\ell^2\langle JB^2\rangle\langle\nu'\rangle^2/\{16\pi^2\rho M\langle J/\lambda\rangle\langle J\rangle^2\}$$

and the remaining quantities are defined by

$$\omega_* = \omega_{*e}/\omega_r$$

$$\nu = \frac{m_e}{m_i}\frac{\nu_e}{\omega_r}$$

$$Q_1 = Q + i\omega_*\big(1 + \eta_e(1 + \alpha_1)\big)$$

$$\Gamma_1 = 1 + X^2/Q_1 \quad , \quad \Gamma_2 = 1 + i\,\frac{\alpha_1\alpha_2}{\alpha_3\omega_*\eta_e}\,X^2$$

while α_1 , α_2 , α_3 and α_4 are constants with $\alpha_1 = 0.71$ (arising from the thermal force terms), $\alpha_2 = \eta_{\perp}/\eta_{\parallel} = 1.95$, $\alpha_3 = 3.2$ (arising from parallel electron thermal conduction) and $\alpha_4 = 4.7$ (arising from classical perpendicular electron thermal conduction). Finally the new dependent variable $\tilde{\phi}$ is defined by

$$\tilde{\phi} = \frac{i\omega_*(1 + \eta_e)}{Q_1}\left[Ne\phi_0 - p_0 - \alpha_1 t_0\right] \quad .$$

The averaged cold-ion equations (5.22)-(5.24) no longer reduce simply to the averaged single fluid resistive equations when $\omega_*/\omega \to 0$ and $\nu \gg 1$. This is because electron thermal conduction has already been assumed to be dominant on the scale of the connection length, and taking $\nu \to \infty$ no longer recovers the adiabatic limit (even when perpendicular thermal conduction is dropped). In the limit $\omega_*/\omega \to 0$, $\nu >> 1$ equations (5.22)-(5.24) reduce to the form:

$$\frac{d}{dX}\left(\frac{X^2}{\Gamma}\frac{d\tilde{\phi}}{dX}\right) + \frac{HX}{\Gamma}\frac{d}{dX}(\tilde{\phi} - p) + p\left(D - H\frac{(H+1)}{\Gamma} - HX\frac{d}{dx}\left(\frac{1}{\Gamma}\right)\right)$$

$$- X^2Q^2\tilde{\phi} = 0 \tag{5.25}$$

$$\frac{d}{dX}\left[\frac{dp}{dX} - \frac{1}{\Gamma}\frac{d\tilde{\phi}}{dX}\right] - \frac{H}{Q}\frac{d}{dX}\left(\frac{Xp}{\Gamma}\right) - Q^2KH\frac{X}{\Gamma}\frac{d\tilde{\phi}}{dX} + Q^2t\left[G_1 + \frac{5}{2}\frac{\alpha_1\alpha_2}{\alpha_3}\frac{M_1}{M}\frac{X^2}{Q}\right]$$

$$- Q^2p\left[G_1 + KF + KH^2\left(1 - \frac{1}{\Gamma}\right) + \frac{X^2}{Q}\left[\alpha_2\left(1 - \frac{M_1}{M}\right) + \left(1 + \frac{\alpha_2\alpha_1^2}{\alpha_3}\right)\frac{M_1}{M}\right]\right.$$

$$\left. + Q^2\tilde{\phi}\left(KE + \frac{G_1}{1 + \eta_e}\right)\right. = 0 \tag{5.26}$$

$$t\left[\frac{5}{2}\left(G_1 + \frac{5}{2}\frac{\alpha_2}{\alpha_3}\frac{M_1}{M}\frac{X^2}{Q}\right) + \alpha_2\alpha_4\left(1 - \frac{M_1}{M}\right)\frac{X^2}{Q}\right] = p\left[G_1 + \frac{5}{2}\frac{\alpha_1\alpha_2}{\alpha_3}\frac{M_1}{M}\frac{X^2}{Q}\right]$$

$$- \tilde{\phi}\,\frac{1 - \frac{3}{2}\eta_e}{1 + \eta_e}\,G_1 \tag{5.27}$$

where $\Gamma \equiv 1 + X^2/Q$.

These equations reduce to the standard averaged single fluid resistive equations when $\eta_\perp/\eta_\parallel \to 1$, $M_1 \to 0$ and $\alpha_4 \to 0$, i.e. resistivity is isotropic, fast parallel thermal conduction effects appearing in M_1 are neglected and perpendicular thermal conduction, labelled by α_4 , is ignored.

VI SUMMARY

The main object of this paper has been the derivation of the equations governing resistive ballooning modes from a complete two fluid model. These equations are obtained in section III and presented as equations (3.43)-(3.48), with the appropriate expressions involving the ion stress tensor given in equations (3.34)-(3.38).

Resistive instabilities display two length scales, and this fact may be exploited to derive simpler averaged equations which apply in the resistive regime. These equations contain some simple field line average quantities (averages over the poloidal angle) and are much more suitable for numerical solution than the dual scale ballooning equations. Their solutions must satisfy the usual boundary conditions at large values of the independent variable, and must also match to solutions of the non-resistive inner region equations at small values of the independent variable.

In section V we have outlined the derivation of these averaged equations for a cold-ion plasma model. In this model collisional ion viscosities are neglected and $T_i \ll T_e$ is assumed, so that ion diamagnetic effects may be neglected. The model retains the full electron physics and charge separation due to the ion polarisation drift. The relevant equations are (5.22)-(5.24).

Finally it is shown how these equations are related to the averaged equations of single-fluid resistive theory.

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