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A TRIPLE WAVE RESONANCE MODEL FOR THE EMISSION FROM TOKAMAKS OF NARROW-BAND BURSTS OF RADIATION AT THE PLASMA FREQUENCY

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Abstract

When the distribution of electron velocities parallel to the magnetic field has a tail with a bump, a linear triple wave resonance instability can cause the growth of electromagnetic waves at ω_p . The bandwidth of the radiation depends on the ratio of the number of electrons in the tail to the total number of electrons; the smaller this ratio, the narrower the bandwidth. The instability occurs only when the mean tail velocity takes a specific value. The radiation thus gives an indication of tail structure.

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1. INTRODUCTION

Intense, narrow-band rapidly fluctuating bursts of radiation at the plasma frequency ω_p are emitted under certain conditions by tokamak plasmas [1-4]. There is considerable theoretical interest in this phenomenon [1-8], which is distinct from the less intense, broadband, steady ω_p emission [1-9]. The intense narrow-band bursts of radiation are usually observed in the low-density regime. It is therefore natural to seek an explanation in terms of the behaviour of an extended tail in the distribution of electron velocities parallel to the magnetic field. The model suggested here is based on the assumption that the tail distribution has a bump. Bumps can be generated in tokamak plasma tail distributions by at least two distinct mechanisms. Firstly, there is the diffusion in velocity space [10-12] arising from the anomalous Doppler instability of the tail [13]. This indicates that a bump-in-tail distribution is the state to which a flat or slowly monotonically decreasing tail would relax at the termination of current drive. A radiation mechanism based on a bump-in-tail distribution is therefore in keeping with the experimental observation [4] that the presence of an energetic tail is a necessary condition for the occurrence of radiation bursts at ω_p, but that such bursts do not occur during the current drive phase in which the tail is initially drawn out. We note however that the second mechanism for bump-in-tail production, the slideaway regime described by Coppi and coworkers [14], does operate under certain low-density current drive conditions.

Our theory is based on the fact that a bump in the tail can support wave modes additional to those supported by the bulk thermal plasma. Instability can occur for excitations of frequency ω and wavenumber $\underline{k}$

such that a wave supported by the bump is in resonance with a wave supported by the bulk plasma [15-17]. Here, we consider a triple wave resonance which does not appear to have been examined previously. The three waves concerned are the electrostatic modes supported by the bulk plasma and by the tail, and the right-hand circularly polarised Whistler-electron cyclotron branch supported by the bulk plasma. This resonance is possible only in plasmas for which $\omega_p < \Omega$, where Ω is the electron gyrofrequency. We shall show that under these resonance conditions, instability can occur. The growth rate γ of the instability falls off rapidly as the angle θ of inclination of $\underline{k}$ to the magnetic field increases from zero. The excited wave has a significant right-hand circularly polarised component for small but finite values of θ. Balancing these effects, the instability represents a mechanism for generating narrow-band bursts of electromagnetic radiation at the plasma frequency. The exact bandwidth and frequency are found to be sensitive to the values of the tail parameters.

The role of a related linear resonance has already been examined for the case of unmagnetised plasma [18], and relativistic effects were found to be significant. For magnetised plasma, however, coupling between electrostatic and electromagnetic modes need not rely on relativistic effects[19]. Similarly, relativistic effects make no essential alteration to the theory presented here, and are omitted for clarity. They are included, for completeness, in Ref. 20.

2. BASIC EQUATIONS

Let us consider the contribution of a bump in the tail of the parallel electron velocity distribution to the dielectric tensor. We shall neglect thermal corrections arising from the finite spread of velocities in the bump. This is equivalent to treating the bump in the tail as a beam. Let this beam contain a fraction $\xi \ll 1$ of the total number of electrons, and have a velocity v_o which is several times the the thermal velocity. For this distribution, Maxwell's equations take the form $\underline{\underline{D}}\cdot\underline{E} = 0$, where $\underline{E} = (E_R, E_L, E_z)$ and

$$\underline{\underline{D}} = \begin{vmatrix} (\varepsilon_R - n_z^2) - n_\perp^2/2 & n_\perp^2/2 & n_\perp n_z/\sqrt{2} \\ n_\perp^2/2 & (\varepsilon_L - n_z^2) - n_\perp^2/2 & n_\perp n_z/\sqrt{2} \\ n_\perp n_z/\sqrt{2} & n_\perp n_z/\sqrt{2} & \varepsilon_3 - \chi_3 - n_\perp^2 \end{vmatrix} . \tag{1}$$

Here $E_{R,L} = (E_x \pm iE_y)/\sqrt{2}$ are the circularly polarised field amplitudes; $\varepsilon_{R,L} = \varepsilon_1 \pm \varepsilon_2$; ε_1 , ε_2 and ε_3 are the standard cold plasma dielectric tensor elements[21]; and $n_\perp$ and n_z are components of the refractive index relative to the magnetic field, which is oriented along the z-axis. The only tail contribution [15,16] to $\underline{\underline{D}}$ which is of significance for the resonance condition outlined above is $\chi_3 = \omega_b^2/(\omega - k_z v_0)^2$, where $\omega_b = \xi^{1/2}\omega_p$ is the plasma frequency associated with the tail. We shall restrict consideration to the case where $k_\perp$ is sufficiently small that quartic and higher order terms in $n_\perp$ may be neglected in the dispersion relation following from Eq. (1). Consider further the case where ω is simultaneously close to solutions of $\varepsilon_R - n_z^2 = 0$ and of $\varepsilon_3 - \chi_3 = 0$, but where $\varepsilon_L - n_z^2 \neq 0$. Then $E_R, E_z \neq 0$, $E_L = 0$, and Eq. (1) gives the dispersion relation

$$(\varepsilon_R - n_z^2)(\varepsilon_3 - \chi_3) = \frac{n_\perp^2 n_z^2}{2} , \text{ i.e.} \tag{2}$$

$$\left(1 - \frac{\omega_p^2}{\omega(\omega - \Omega)} - n_z^2\right)\left(1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2}{(\omega - k_z v_0)^2}\right) = \frac{n_\perp^2 n_z^2}{2} \quad . \tag{3}$$

Consider the triple resonance condition for Eq. (3):

$$\omega_p = k_z v_0 = \omega_1 \quad , \tag{4}$$

where ω_1 is the low-frequency or Whistler solution of $\varepsilon_R - n_z^2 = 0$. This resonance is possible only if $\omega_p < \Omega$. When Eq. (4) applies, Eq. (3) has roots $\omega = \omega_p + \delta\omega$, where $|\delta\omega| << \omega_p$ and

$$(\delta\omega)^3 - C_1\delta\omega - C_0 = 0 \quad , \tag{5}$$

$$C_0 = \frac{\xi\omega_p^3}{2} \quad , \quad C_1 = \frac{\omega_p^2}{4} n_\perp^2 n_z^2 P(x) \quad , \tag{6}$$

$$P(x) = (1 - x)^2/(2 - 3x + 2x^2) \quad . \tag{7}$$

Here $x = \omega_p/\Omega$, and $P(x)$ is a positive definite polynomial. Provided $k_\perp$ is small enough and ξ large enough that

$$R \equiv \frac{C_0^2/4}{C_1^3/27} = \frac{108}{[n^2{}_z P(x)]^3} \frac{\xi^2}{n_\perp^6} > 1 \quad , \tag{8}$$

there exists a pair of complex conjugate roots for Eq. (5), one of which gives rise to exponential wave growth. This root is

$$\delta\omega_1 = -\Delta + i\gamma \tag{9a}$$

$$\Delta = S_+ + S_- \quad , \quad \gamma = \frac{\sqrt{3}}{2}\,(S_+ - S_-) \tag{9b}$$

$$S_\pm = \left(\frac{C_0}{2}\right)^{1/3} \left[1 \pm (1 - 1/R)^{1/2}\right]^{1/3} \tag{9c}$$

where R is given by Eq. (8). In particular,

$$\gamma = \gamma_0\left\{\left[\frac{1}{2}\left(1 + [1 - 1/R]^{1/2}\right)\right]^{1/3} - \left[\frac{1}{2}\left(1 - [1 - 1/R]^{1/2}\right)\right]^{1/3}\right\} \tag{10a}$$

$$\gamma_0 = \frac{\sqrt{3}}{2^{4/3}}\,\xi^{1/3}\omega_p \quad . \tag{10b}$$

For ξ as small as 10^{-6}, $\gamma_0 \simeq 0.01\omega_p$, so that for R greater than a few times unity growth is very rapid. We note that γ_0 is the well-known growth rate for the beam plasma resonance instability for electrostatic waves in the absence of a resonant Whistler wave[15-17]. By Eq. (10a), in general $\gamma < \gamma_0$ but in the limit $R \gg 1$, γ tends to γ_0. Consider the energy density $U = \partial/\partial\omega\,\left[\omega E_i^* \varepsilon_{ij} E_j\right]$; for the wave at $\omega_p + \delta\omega$ under triple resonance conditions,

$$U = \left[1 + x^2/(1 - x)^2\right]\left|E_R\right|^2 + 2\left[1 + \xi\omega_p^3/(\delta\omega)^3\right]\left|E_z\right|^2 \quad . \tag{11}$$

Here $x = \omega_p/\Omega$ as usual. Growth in amplitude of the wave is consistent with energy conservation because of the negative energy contribution from the electrostatic branch for the unstable root $\delta\omega = \delta\omega_1$ given by Eq. (9). However, by Eq. (11), the contribution of the Whistler component to the energy is always positive. The presence of this additional positive energy wave in resonance reduces the growth rate γ from the value γ_0 which applies when this wave is not present. This aspect of resonance instability is considered in a related context in Ref. [17].

Let us turn to the polarisation and direction of propagation of the excited wave. By Eq. (1), since $E_L = 0$, the polarisation ratio

$$P_R \equiv \left(\frac{E_R}{E_z}\right)^2 = \frac{\varepsilon_3 - \chi_3}{\varepsilon_R - n_z^2} = \frac{[n_\perp n_z P(x)]^2}{2} \left(\frac{\omega_p}{\delta\omega}\right)^2 \tag{12}$$

where Eqs. (5)-(7) have been used. The angle of inclination of $\underline{k}$ to the magnetic field is $\theta = \tan^{-1}(k_\perp/k_z)$, which by Eq. (8) is

$$\theta = \tan^{-1}\left[\left\{\frac{\xi^2}{R} \frac{108}{[n_z^4 P(x)]^3}\right\}^{1/6}\right] . \tag{13}$$

Note that for $\omega \simeq \omega_p$ and $n_z^2 \simeq \varepsilon_R$, $n_z^2 = (1 - x)^{-1}$. The most rapidly growing modes correspond to values of R which are large compared to unity, by Eq. (10a). In this limit, Eqs. (8), (9) and (12) give

$$|P_R| = 2.16\, P(x) \left(\frac{1}{R}\right)^{1/3} . \tag{14}$$

In Table 1 the characteristics of the excited modes parametrised by R are given for the case $x = 1/3$ with the two stated values of ξ. The amplitudes of the excited modes depend exponentially on the value of γ. Waves with γ/γ_0 significantly less than unity will not arise in the excited spectrum. For the cases considered in Table 1, the excited spectrum corresponds to values of $R \gtrsim 20$. As R increases, $|P_R|$ decreases. For $R \gtrsim 1000$, the fraction of energy in the right-hand circularly polarised mode is negligible. We conclude that significant radiation in this mode is limited to the parameter range $20 \lesssim R \lesssim 1000$. The radiation is limited to a narrow range of small values of θ , which depends on the value of ξ . The bandwidth of the radiation follows from the real part Δ of $\delta\omega_1$, see Eq. (9), and is given by $|\Delta(R = 20) - \Delta(R = 1000)|/\omega_p$ in Table 2.

The bandwidth is narrow, of order 0.1% for the parameter values considered, and sensitive to the tail density ξ . It is centred at a frequency of order $0.99\omega_p$ for the case $\xi = 10^{-6}$. In general, as ξ decreases, the radiation bandwidth becomes narrower and the frequency of the radiation draws closer to ω_p . Very low values of ξ , however, correspond to low growth rates by Eq. (10), so that there is a limit on the degree of narrowness obtainable.

3. DISCUSSION

We wish to emphasise firstly that the bandwidth and exact frequency of the observed radiation offer an indication of the value of the fractional density of tail electrons. Secondly, the instability can occur only when the triple resonance condition Eq. (4) is satisfied at some point in the centre of the plasma, where the high-energy tail is concentrated. It follows from Eq. (4) that the mean tail velocity v_0 must satisfy

$$v_0 = v_{res} \equiv [1 - \omega_{po}/\Omega_o]^{1/2} c , \qquad (15)$$

where subscript zero refers to values at the centre of the plasma. If $v_0 \neq v_{res}$, there is no electromagnetic instability. Conversely, according to the model suggested here, whenever narrow-band bursts of radiation at ω_p are observed, v_0 must momentarily take the value v_{res}. The radiation bursts thus give indications of both tail parameters ξ and v_0. This information has significance for the efficiency of radio frequency plasma heating and current drive schemes.

We wish to make two additional points which are specific to tokamak geometry. Firstly, the wave is excited with a value of $n_{z0}^2 = (1 - \omega_{p0}/\Omega_0)^{-1} > 1$. An electromagnetic wave propagating in a vacuum satisfies $n_z^2 + n_\perp^2 = 1$. Consequently a wave with $n_z^2 > 1$ at the plasma edge cannot leave the plasma, since this corresponds to imaginary $n_\perp$ in a vacuum. The quantity $n_z r$, where r is the distance from the central axis of symmetry, is a conserved quantity insofar as the tokamak geometry has azimuthal symmetry. As the wave propagates outwards from the centre of the plasma, r increases and n_z diminishes. Since ω_{p0}/Ω_0 is small in low density plasmas, the initial value n_{z0} of n_z does not greatly exceed unity, and at the plasma edge n_z can take a value less than unity except in large aspect ratio tokamaks. The wave is then able to propagate into the vacuum. Secondly, we expect the electromagnetic radiation at ω_p to be right-hand circularly polarised on emission. However, there are obstacles to the observation and interpretation of any polarisation signal[22]. The radiation is emitted from the centre of the plasma, all round the magnetic axis, and propagates in a direction nearly tangential to the magnetic field. Multiple reflections off the conducting wall of the chamber will occur, destroying polarisation information, before the radiation is received at the observation ports, which are in general oriented perpendicular to the wall. The signal received will therefore display no significant polarisation.

A theory of the narrow band radiation bursts at ω_p has recently been proposed by Hutchinson and coworkers [4,8]. Cavity modes, spatially localised to the centre of the plasma, are calculated for the case of periodic cylindrical geometry and a parabolic radial electron density profile. The modes have $\omega \approx \omega_p$, and are analysed in the regime $\omega_p/\Omega \ll 1$. When the electron velocity tail distribution has a positive slope, the inverse Landau damping wave particle resonance can cause such modes to grow. This results in an electromagnetic field, localised to the plasma centre, with $\omega \approx \omega_p$. The field at the edge is negligible for typical parameter values, so that scattering from unknown density perturbations must be invoked to explain the observed free space radiation.

The theory we suggest here has much in common with that of Hutchinson and coworkers. We both rely on a beam-driven excitation mechanism, but differ in the use of wave-particle or wave-wave resonance. A further distinction is in the choice of geometry. The major differences are a consequence of the fact that we develop our theory in terms of the linear modes of a cold plasma model. This approach enables us to obtain the relationships between the wavenumber, growth rate, polarisation, and bandwidth of the unstable modes, and the mean velocity and density of the tail. Furthermore, we only require that $\omega_p / \Omega < 1$. Our interpretation in terms of cold plasma linear modes leads us to consider a possible escape mechanism for the electromagnetic wave, which involves a quantity

$n_z r$ which is conserved in toroidal geometry.

We conclude that narrow-band bursts of radiation at ω_p may be explicable in terms of tail distributions with a bump. The basic excitation mechanism proposed is a linear triple wave resonance instability. The occurrence of the bursts gives an instantaneous determination of the mean velocity v_0. Determination of the bandwidth of the radiation gives an indication of the fractional tail density ξ. In general, the smaller ξ, the narrower the bandwidth.

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R	γ/γ_0	$\lvert P_R \rvert$	$\theta°$ $\xi = 3 \times 10^{-5}$	$\theta°$ $\xi = 10^{-6}$
5	0.61	0.46	3.3	1.05
10	0.70	0.36	2.9	0.94
20	0.76	0.29	2.6	0.84
40	0.81	0.23	2.3	0.74
100	0.86	0.17	2.0	0.64
1000	0.94	0.08	1.4	0.44
3000	0.96	0.05	1.1	0.36

Table 1: Growth rate, polarisation, and propagation angle of excited waves

R	Δ/ω_p $\xi = 3 \times 10^{-5}$	Δ/ω_p $\xi = 10^{-6}$
20	3.03×10^{-2}	0.97×10^{-2}
1000	2.66×10^{-2}	0.86×10^{-2}
Bandwidth	3.7×10^{-3}	1.1×10^{-3}

Table 2: Frequency shift and bandwidth of excited waves