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TOROIDAL MINIMUM-B AND RELATED MHD EQUILIBRIA

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Abstract

Minimum-B mirror systems provide plasma stability but are subject to loss through the magnetic mirrors. Such losses are absent in systems with toroidal magnetic surfaces but it is well known that vacuum magnetic fields with toroidal minimum-B flux surfaces do not exist. Despite this, we have constructed a set of finite-β toroidal minimum-B equilibria (in which toroidal magnetic surfaces coincide with constant-B contours and in which B increases and p decreases with distance from the magnetic axis). These are the only possible axisymmetric minimum-B toroidal equilibria and are compatible with the vacuum field result because they have no low-β limit. They are members of a family of equilibria all of which correspond to a single set of magnetic surfaces. This set of magnetic surfaces is unique and is determined by a novel approach to the Grad-Shafranov equation. In these configurations all guiding centre drifts lie in the magnetic surface, so they have no neoclassically enhanced transport and they are free of all trapped particle effects. Unfortunately, however, they do not share the intrinsic stability properties of their mirror counterparts.

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1. Introduction

Minimum-B mirrors, or magnetic wells, are well-known systems for magnetic confinement of plasma[1]. They utilise configurations in which the contours of constant field strength form a set of nested surfaces with B increasing, and plasma pressure decreasing, towards the exterior. These properties provide stability but the field lines are open so that only the mirror effect limits the loss along the field. Such losses are absent in toroidal systems in which the magnetic field lines lie on closed toroidal flux surfaces. However, it is well-known that there are no vacuum or force-free magnetic fields in which the toroidal flux surfaces are also minimum-B surfaces[2]. This might suggest that toroidal minimum-B plasma equilibria do not exist - but this is not so, and in this paper we construct a set of finite-β toroidal minimum-B equilibria. These are axisymmetric scalar-pressure equilibria with finite aspect-ratio in which the toroidal magnetic surfaces coincide with the toroidal constant-B surfaces and in which B increases and p decreases everywhere with distance from the magnetic axis, with $p = p(B)$. They do not conflict with the vacuum field result because they exist only at high-β and have no low-β limit.

The existence of these axisymmetric minimum-B equilibria is closely related to the question, discussed in reference 3, of whether the shape of toroidal magnetic surfaces uniquely determines the current distribution. In a cylindrical coordinate system R, ϕ, Z the magnetic field can be written

$$\underset{\sim}{B} = \frac{\underset{\sim}{n} \times \nabla\psi}{R} + \frac{f(\psi)}{R} \underset{\sim}{e}_{\phi} \tag{1}$$

and $\psi(R,Z)$ satisfies the equation

$$\Delta^{+}(\psi) \equiv R^{2}\nabla\cdot\left(\frac{1}{R^{2}} \nabla\psi\right) = -\mu_{0}R^{2}\, p'(\psi) - ff'(\psi) \tag{2}$$

where $p(\psi)$ is the plasma pressure. If a different plasma equilibrium, defined by $F(R,Z)$, has identical flux surfaces there must be a functional relation $F = F(\psi)$ so that

$$\Delta^{+}F = \frac{dF}{d\psi}\, \Delta^{+}\psi + \frac{d^{2}F}{d\psi^{2}} \left|\nabla\psi\right|^{2} \quad . \tag{3}$$

But if F is indeed an equilibrium, the right hand side of this expression must have the same form as the r.h.s. of Eq. (2). This is the case if, and only if,

$$\left|\nabla\psi\right|^{2} = \alpha(\psi) + \beta(\psi)\, R^{2} \quad . \tag{4}$$

Hence, if any solution of Eq. (2) can be found which also satisfies (4), then there will be a whole family of equilibria, with different pressure and current profiles, which have exactly the same flux surfaces. We shall refer to these as degenerate equilibria.

Since the total magnetic field for degenerate equilibria is

$$B^2 = \left(\left|\nabla\psi\right|^2 + f^2\right)/R^2 = \beta(\psi) + \frac{\alpha(\psi) + f^2(\psi)}{R^2} \tag{5}$$

we see that if toroidal minimum-B equilibria (in which B must be constant over the flux surface ψ) exist, then they are members of a degenerate family in which $(\alpha + f^2) = 0$. In the next two sections we will show that there is only one configuration of flux surfaces which is associated with degenerate equilibria; that these degenerate equilibria include a set in which B is constant over the flux surface; and that some members of this set have the full minimum-B properties.

2. Construction of Degenerate Equilibria

In this section we seek solutions of equation (2) which have the "degeneracy" property (4). This is an unusual problem. Usually one solves Eq. (2) in a given boundary for known functions $p(\psi)$ and $f(\psi)$ (or something equivalent such as $q(\psi)$ the safety factor). In the present problem the boundary and the functions $p(\psi)$ and $f(\psi)$, as well as $\alpha(\psi)$ and $\beta(\psi)$, all have to be determined - given only that the eventual solution to equation (2) must take a particular form.

We write

$$\nabla\psi = R^2 g(\psi, R)\underset{\sim}{e} \tag{6}$$

where $\underset{\sim}{e}$ is a unit vector $\left(e_R, 0, e_Z\right)$ and

$$g^2 = \left(\alpha(\psi) + \beta(\psi)R^2\right)/R^4 \quad . \tag{7}$$

Then the Grad-Shafranov equation (2) can be reduced to

$$\frac{\partial}{\partial R}\left(Rge_R\right)\Big|_{\psi} = R\,L(\psi) + \frac{1}{R}\,M(\psi) \tag{8}$$

where

$$L(\psi) \equiv -\left(p' + \beta'/2\right) \quad , \qquad M(\psi) \equiv -\left(ff' + \alpha'/2\right) \quad . \tag{9}$$

Therefore

$$ge_R = \frac{R}{2}\,L(\psi) + \frac{\log R}{R}\,M(\psi) + \frac{1}{R}\,C(\psi) \quad , \tag{10}$$

which defines a single surface in terms of the three parameters L, M, C. If we introduce R_0 as the radius at which $e_R = 0$ (ie where the surface is tangential to the R axis) we have

$$e_R = h(\psi,R)/g(\psi,R) \qquad \text{and} \qquad e_z = (1 - h^2/g^2)^{1/2} \tag{11}$$

with

$$h(\psi,R) = \frac{L(\psi)}{2}\,\frac{\left(R^2 - R_0^2\right)}{R} + \frac{M(\psi)}{R}\,\log R/R_0 \tag{12}$$

and

$$g(\psi,R) \;=\; \left(\alpha(\psi) + \beta(\psi)R^2\right)^{1/2}/R^2 \quad . \tag{13}$$

Several conditions have yet to be imposed. One of these is that the magnetic surface defined by (10) should be a smooth closed curve - but it is convenient to defer consideration of this until other conditions have been dealt with. These stem from the requirement that

$$\nabla \mathrm{x}\left(R^2 g\underset{\sim}{e}\right) \;=\; 0 \quad . \tag{14}$$

Using (11) for e_R and e_Z, introducing $(R,\ \psi)$ as independent variables and carrying out some manipulation, this leads to the condition

$$\frac{R^3 h}{2}\frac{\partial g^2}{\partial\psi} - R^3 g^2\frac{\partial h}{\partial\psi} + \frac{R}{2}\frac{\partial g^2}{\partial R} - \frac{R}{2}\frac{\partial h^2}{\partial R} + 2\left(g^2 - h^2\right) \;=\; 0 \quad . \tag{15}$$

At this point it is convenient to introduce new quantities

$$P \;\equiv\; \alpha/L^2\ , \qquad Q \;\equiv\; \beta/L^2 \qquad \text{and} \quad X \;\equiv\; R_0^2 \quad . \tag{16}$$

Then, when g and h are introduced into Eq. (15), one sees that it is satisfied if, and only if, $M = 0$ and

$$\text{(i)} \qquad \frac{dP}{d\lambda} = \frac{4Q^2}{P + XQ} - X$$

$$\text{(ii)} \qquad \frac{dQ}{d\lambda} = 3 \qquad (17)$$

$$\text{(iii)} \qquad \frac{dX}{d\lambda} = \frac{-2Q}{P + XQ}$$

where $\quad d\lambda = d\psi/L(\psi)$.

We must now consider the surface closure condition mentioned earlier. From (10), using $M = 0$, the equation for a single magnetic surface becomes

$$\frac{dZ}{dx} = \frac{(x - X)}{4\left[P + Qx - x(x - X)^2/4\right]^{1/2}} \qquad (18)$$

where $x = R^2$. Then the closure condition can be written

$$F\left(P/X^3,\ Q/X^2\right) \equiv \int \frac{(s - 1)ds}{\left[P/X^3 + sQ/X^2 - s(s - 1)^2/4\right]^{1/2}} = 0 \qquad (19)$$

where the integration is between the two largest roots of the denominator. The integral can be expressed in terms of complete elliptic integrals (see appendix).

Note that $L(\psi)$ does not explicitly appear in the closure condition or equations (17) and can be chosen arbitrarily. This reflects the degeneracy of the plasma equilibria we are seeking. However, the magnetic surfaces themselves are unique, up to a scale transformation $X \to \mu X$, $P \to \mu^3 P$, $Q \to \mu^2 Q$, $\lambda \to \mu\lambda$, which merely magnifies and re-labels the surfaces.

We now turn to the calculation of these magnetic surfaces. This proceeds as follows. We first use the scale invariance to set $X = 1$ at the magnetic axis. Then the closure condition shows (see appendix) that near the magnetic axis $P \to 0$, $Q \to 0$ with $P/Q = -1/3$. Using these as starting values, $P(\lambda)$, $Q(\lambda)$ and $X(\lambda)$ are then computed from the differential equations (17). We set $\lambda = 0$ on the axis so that $Q(\lambda) = 3\lambda$. The computed values for $P(\lambda)$ and $X(\lambda)$ are shown in Figs. 1 and 2. [A very good approximation is $P(\lambda) = \lambda(10\lambda - 1)$ and $X(\lambda) = 1 - 3\lambda$.] Once $P(\lambda)$, $Q(\lambda)$ and $X(\lambda)$ are known the individual magnetic surfaces are computed from (18). In this way the complete configuration is built-up surface by surface. The result is shown in Fig. 3. [It is implicit in this method, (see appendix), that if the initial surface is closed so are all others.]

The construction of the degenerate magnetic configuration can be summarised as follows. We first obtain an equation satisfied by a <u>single</u> magnetic surface; this contains three unknown parameters. The condition that this single surface be part of an equilibrium imposes constraints on these three parameters in the form of differential equations and the condition that the surfaces be closed provides the initial values for these differential equations. Their solution then provides the value of the three parameters on each magnetic surface. Note that the equilibrium is obtained without at any time needing to solve a partial differential equation.

3. Properties of the Equilibria

The configuration shown in Fig. 3 is unique, apart from a scale factor. It is remarkable in three respects. First, it is the only toroidal configuration which corresponds to more than one plasma equilibrium. (It is thus the only exception to the argument given by Christiansen and Taylor[3] that the current distribution is determined by the shape of the magnetic surfaces. In this regard it is for toroidal equilibria what a configuration of concentric circles is for cylindrical equilibria.) Second, it is the only axisymmetric toroidal configuration which may correspond to equilibria in which constant-B surfaces and flux surfaces can coincide. Third, it is the only such axisymmetric toroidal configuration which may be a minimum-B equilibrium. We now show that there are, in fact, equilibria associated with the configuration in Fig. 3 which fulfill the last two conditions.

The equilibria are described by α , β , p , and f , which are given in terms of $P(\lambda)$, $Q(\lambda)$, as

$$\alpha(\lambda) = L^2(\lambda)P(\lambda)$$

$$\beta(\lambda) = L^2(\lambda)Q(\lambda)$$

$$\frac{\partial}{\partial\lambda} p(\lambda) = - L^2(\lambda) - \frac{\partial}{\partial\lambda}\left(\frac{L^2 Q}{2}\right) \tag{20}$$

$$\frac{\partial}{\partial\lambda}(f^2 + \alpha) = 0 \quad .$$

We see that the general equilibrium corresponding to Fig. 3 involves one free function $L(\lambda)$ and two constants of integration. If the flux and constant-B surfaces are to coincide we must have $(f^2 + \alpha) = 0$ which requires only that one of the integration constants be set to zero. The other can be used to set the pressure to zero on any chosen surface. Then the arbitrary function $L(\psi)$ still allows a choice of pressure profile. Consequently there is a family of toroidal equilibria in which flux surfaces and B = constant surfaces coincide, so that $p = p(B)$. These equilibria have the following features.

The toroidal field is

$$B_\phi^2 = - L^2(\lambda)P(\lambda)/R^2 \quad . \tag{21}$$

The poloidal field is

$$B_p^2 = L^2(\lambda)\left(Q(\lambda) + P(\lambda)/R^2\right) \quad . \tag{22}$$

The total field is

$$B^2 = L^2(\lambda)Q(\lambda) \quad . \tag{23}$$

The toroidal current density is

$$j_\phi = -R\left(L + \frac{1}{2L}\frac{d}{d\lambda}(L^2Q)\right) - \frac{1}{2LR}\frac{d}{d\lambda}(L^2P) \tag{24}$$

and the plasma pressure is

$$p(\lambda) = p_0 - \frac{L^2Q}{2} - \int_0^\lambda L^2(\lambda')d\lambda' \quad . \tag{25}$$

Note that the field strength B^2 increases with distance from the magnetic axis when $\lambda L^2(\lambda)$ is an increasing function of λ so that some members of the family are full minimum-B equilibria. However, $(p + B^2/2)$ is always a decreasing function of λ so that minimum-B fields exist only for finite-β. Furthermore, although $L(\lambda)$ is arbitrary it must be finite and so B^2 vanishes on the magnetic axis. (In minimum-B mirrors it was required that B^2 did not vanish in order for the magnetic moment to be conserved.)

The properties of the equilibria given in Eqs. (20)-(25) depend on the choice of $L(\lambda)$ but some features are independent of L . These include the safety factor q , shown in Fig. 4, the ratio B_p/B_ϕ and $\mu \equiv \underset{\sim}{j}\cdot\underset{\sim}{B}/B^2$ given by

$$\mu = \frac{d}{d\lambda} \left|P\right|^{1/2} - \frac{5}{6\lambda} \left|P\right|^{1/2} \tag{26}$$

4. Stability

The importance of minimum-B mirror systems lies in their intrinsic stability. This stability is particularly evident when the pressure tensor takes the form $p_\perp = p_\perp(B)$, $p_\parallel = p_\parallel(B)$ and such mirror equilibria are stable under very weak restrictions indeed[1]. Toroidal minimum-B equilibria have the analogous property that $p = p(B)$ but they do not have the same automatic stability as their mirror counterparts.

The energy integral for anisotropic plasma can be expressed in the form[4]

$$\begin{aligned}
\delta W \;=\; & Q_\perp^2 \left(1 + \frac{p_\perp - p_\parallel}{B^2}\right) + Q_\parallel^2 \left(1 + \frac{2p_\perp + C}{B^2}\right) \\
& - j_\parallel \, \underset{\sim}{n} \cdot \underset{\sim}{Q}_\perp \times \underset{\sim}{\xi} \left(1 + \frac{p_\perp - p_\parallel}{B^2}\right) + q\left[\underset{\sim}{\xi} \cdot \nabla p_\parallel + (p_\perp - p_\parallel)s\right] \\
& - \left(\frac{2Q_\parallel}{B} + s\right)\left[\underset{\sim}{\xi} \cdot \nabla p_\perp - (2p_\perp + C)s\right] + \kappa^2
\end{aligned} \tag{27}$$

where $s = \dfrac{\underset{\sim}{\xi} \cdot \nabla B}{B}$, $q = -\underset{\sim}{\xi} \cdot (\underset{\sim}{n} \cdot \nabla)\underset{\sim}{n}$ and $\underset{\sim}{Q} = \nabla \times (\underset{\sim}{\xi} \times \underset{\sim}{B})$. The last term in δW is a contribution due to trapped particles and C is a moment of the distribution function. We also have the equilibrium relations

$$\frac{\partial p_\parallel}{\partial s} = \frac{\left(p_\parallel - p_\perp\right)}{B} \frac{\partial B}{\partial s}$$

$$\frac{\partial p_\perp}{\partial s} = \frac{\left(C + 2p_\perp\right)}{B} \frac{\partial B}{\partial s} \quad . \tag{28}$$

Consequently, for mirror equilibria with $p_\perp = p_\perp(B)$, $p_\parallel = p_\parallel(B)$, where the parallel current is zero, the energy integral reduces to

$$\delta W = \underset{\sim}{Q}_\perp^2 \left(1 + \frac{p_\perp - p_\parallel}{B^2}\right) + Q_\parallel^2 \left(1 + \frac{2p_\perp + C}{B^2}\right) + \kappa^2 \quad . \tag{29}$$

This is automatically positive for all disturbances provided only that the weak requirements (for mirror and firehose stability)

$$B^2 + p_\perp > p_\parallel \quad \text{and} \quad \frac{dp_\perp(B)}{dB} + B > 0 \tag{30}$$

are satisfied. Therefore no detailed stability analysis is necessary for these mirror minimum-B equilibria, even at finite-β.

On the other hand, the corresponding energy principle for scalar-pressure plasma (obtained from (27) by setting $p_\perp = p_\parallel$, $(2C + p_\perp) = 0$, and dropping the trapped particle contribution) is

$$\delta W = \underset{\sim}{Q}_\perp^2 + Q_\parallel^2 - j_\parallel\left(\underset{\sim}{n}\cdot\underset{\sim}{Q}_\perp \times \underset{\sim}{\xi}\right) + \underset{\sim}{\xi}\cdot\nabla p\left(q - s - 2Q_\parallel/B\right) \tag{31}$$

This has to be minimised over the two component vector $\underset{\sim}{\xi} = \underset{\sim}{\xi}_\perp$ but it is permissible to carry out one of the minimisations over $Q_{\parallel}$. Then

$$\delta W = \underset{\approx}{Q}_{\perp}^{2} - j_{\parallel}\left(\underset{\sim}{n}\cdot\underset{\approx}{Q}_{\perp} \times \underset{\sim}{\xi}\right) + 2\left(\underset{\sim}{\xi}\cdot\nabla p\right)\left(\underset{\sim}{\xi}\cdot\nabla(p + B^2/2)\right) \tag{32}$$

where we have introduced the explicit form for s and expressed q as $-\underset{\sim}{\varepsilon}\cdot\nabla(p + B^2/2)/B^2$.

It can be seen from this that with scalar pressure, even though $p = p(B)$, potential sources of instability associated with both ∇p and with $j_{\parallel}$ remain. Consequently, unlike the mirror case, no general conclusion about stability follows from the fact that $p = p(B)$. Each equilibrium must be examined in detail though it can be shown (see appendix) that all are unstable on axis by the Mercier/Suydam criterion. One reason (in addition to the presence of $j_{\parallel}$) why the stability of mirror equilibria with $p_{\perp} = p_{\perp}(B)$, $p_{\parallel} = p_{\parallel}(B)$ does not carry over to scalar pressure can be seen from Eq. (28). With $p_{\parallel} = p_{\parallel}(B)$ this implies

$$\frac{dp_{\parallel}(B)}{dB} = \frac{p_{\perp} - p_{\parallel}}{B}$$

and the scalar pressure limit of this is p = constant !

5. Conclusions

Contrary to the impression created by the fact that minimum-B toroidal vacuum and force-free fields cannot exist, there are toroidal minimum-B equilibria (whose magnetic surfaces coincide with toroidal constant-B surfaces and in which B increases and p decreases with distance from the magnetic axis). They do not contradict the vacuum field result because they exist only at high-β and have no low-β limit.

These minimum-B equilibria are members of a family of equilibria which are degenerate in that they all correspond to a single configuration of magnetic surfaces. We have constructed this configuration, which is unique apart from a scale factor and is shown in Fig. 3, using a novel solution of the Grad-Shafranov equation in which the magnetic surfaces are calculated individually. Each surface is defined by an ordinary differential equation containing three parameters; these parameters themselves satisfy differential equations representing the fact that each surface is part of a global equilibrium and the initial values for these equations are determined by the requirement that the surfaces are closed. Although they all have the unique configuration of surfaces shown in Fig. 3, the corresponding equilibria contain a single arbitrary function by means of which different pressure profiles can be generated. However, some features, such as the q-profile and $\mu = \underset{\sim}{j} \cdot \underset{\sim}{B}/B^2$ are universal and apply to all the equilibria.

These minimum-B toroidal equilibria do not fit easily into the conventional classification of toroidal confinement systems. They differ from tokamaks in that $B_p \gtrsim B_\phi$ and from the toroidal pinch in that $q(\psi)$ is increasing with minor radius rather than decreasing, and from both in that B and q are zero on the magnetic axis. Their most significant feature is simply that flux-surfaces and constant-B surfaces coincide. This means that plasma pressure is a function of field strength only

$(p = p(B))$ and all guiding centre drifts lie in the magnetic surface, (i.e. the equilibria are omnigeneous[5]). Consequently, they are not subject to any neoclassically enhanced transport and are free of all trapped particles and any instabilities or anomalous transport they may cause. However, although they have the property that $p = p(B)$ they do not possess the automatic stability of their mirror counterparts. Each case requires individual analysis but all are unstable on axis by the Mercier-Suydam criterion. We cannot therefore claim any unique stability for these equilibria as we could for the corresponding mirror equilibria. Nevertheless, their freedom from neoclassical and trapped particle effects might make them interesting if examples with gross stability could be found. In this regard, the instability to localised modes on axis need not be catastrophic - any more than it is in tokamaks with $q < 1$ or in pinches.

Acknowledgements

We are grateful for useful discussions with B. J. Braams (who aroused our interest in whether there were exceptions to the Christiansen-Taylor argument), and with R. J. Hastie. We are also grateful to T. J. Martin who carried out the necessary numerical integrations and to A. Sykes who checked our final results against a standard Grad-Shafranov integration code.

Note

After this work was completed it was brought to our attention that many of the properties of these equilibria had been derived some time ago by D Palumbo [A]. We would like to apologise to Professor Palumbo for being unaware of his work on this topic.

[A] D Palumbo, Il Nuovo Cimento, X 53B, 507 (1968).

Appendix

A. The surface closure condition

As explained in the main text, a single magnetic surface is described by the equation

$$\frac{dZ}{dx} = \frac{x - X}{4\left[P + Qx - x(x - X)^2/4\right]^{1/2}} \tag{A1}$$

which contains three parameters $P(\psi)$, $Q(\psi)$ and $X(\psi)$. These parameters themselves must satisfy the differential equations (17). However, in addition, one must ensure that the surface generated by (A1) is a smooth closed curve. This requires that the three roots s_0, s_1, s_2 of the denominator be real and

$$\int_{s_1}^{1} + \int_{1}^{s_2} \frac{(s - 1)ds}{\left[P/X^2 + sQ/X^3 - s(s - 1)^2/4\right]^{1/2}} = 0 \tag{A2}$$

where s_1 and s_2 are the two largest roots. [The curve is parallel to the Z-axis at $s = s_1$ and $s = s_2$ and parallel to the R-axis at $s = 1$.] This closure condition can be expressed in terms of complete elliptic integrals of the first and second kind as

$$E(a) + ab\, K(b) = 0 \tag{A3}$$

where $a = (s_2 - s_1)/(s_2 - s_0)$ and $b = (s_0 - 1)/(s_2 - s_1)$.

At the magnetic axis s_1 and s_2 are coincident and the roots become 0, 1, 1. This occurs for $P/X^3 \to 0$, $Q/X^2 \to 0$. In the neighbourhood of the axis, when P/X^3 and Q/X^2 are small, the roots are

$$s_0 \simeq \frac{4P}{X^3} , \qquad s_1, s_2 = 1 - \frac{2P}{X^3} \pm 2 \left(\frac{P}{X^3} + \frac{Q}{X^2}\right)^{1/2} . \tag{A4}$$

In this limit the integrals in (A2) or the elliptic functions in (A3) can readily be evaluated to give the closure condition in the vicinity of the axis as

$$\frac{Q}{X^2} = \frac{-3P}{X^3} . \tag{A5}$$

It can be verified that this is compatible, in the appropriate limit, with the differential equations (17). We have, in fact, verified (with the aid of MACSYMA) that the <u>full</u> closure condition (A3) is consistent with these differential equations. This means that if one ensures that the initial values of P, Q, X correspond to a closed surface then closure is ensured for all other surfaces. Closure, although crucial to the calculation, is required only to determine initial values for the differential equations (17); thereafter it follows automatically.

B. Behaviour near magnetic axis

If, near the magnetic axis, we introduce local cylindrical coordinates (ρ, θ, s) then $\lambda \simeq \rho^2/2$ and the equilibrium is given by

$$B_s = \frac{\rho}{\sqrt{2}} L(\rho) \quad , \quad B_\theta = \sqrt{2}\, \rho\, L(\rho) \quad ,$$

$$-\frac{1}{\rho}\frac{dp}{d\rho} = \frac{5}{2} L^2(\rho) + \frac{3\rho}{4}\frac{dL^2}{d\rho} \quad , \quad q(\rho) = \rho/\sqrt{2}\, R \tag{B1}$$

where $L(\rho)$ is at our disposal. If $L = 1$ then the Suydam/Mercier stability criterion is clearly violated on axis. Strictly speaking, j_θ would be discontinuous at $\rho = 0$ when $L = 1$ and such equilibria can only be considered as a limiting form - in the same way as other idealised equilibria with discontinuous current density. The current density is continuous if $L \sim \rho^s$ with $s > 0$, but the Suydam/Mercier criterion is still not satisfied.

References

[1] J. B. Taylor, Plasma Physics, IAEA Vienna 1965, pp 449-484.

[2] J. D. Jukes, Plasma Physics 6, 84 (1964).

[3] J. P. Christiansen and J. B. Taylor, Nucl. Fusion 22, 111 (1982).

[4] R. J. Hastie and J. B. Taylor, Phys. Fluids 8, 323 (1965).

[5] L. S. Hall and B. McNamara. Phys. Fluids 18, 552 (1975).

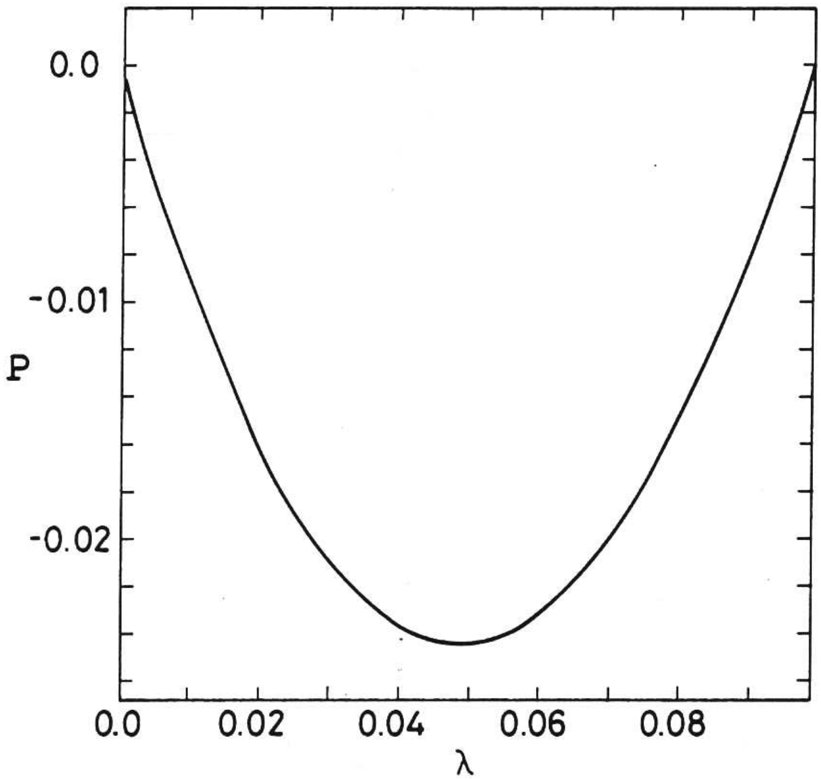

Fig. 1 The function $P(\lambda)$

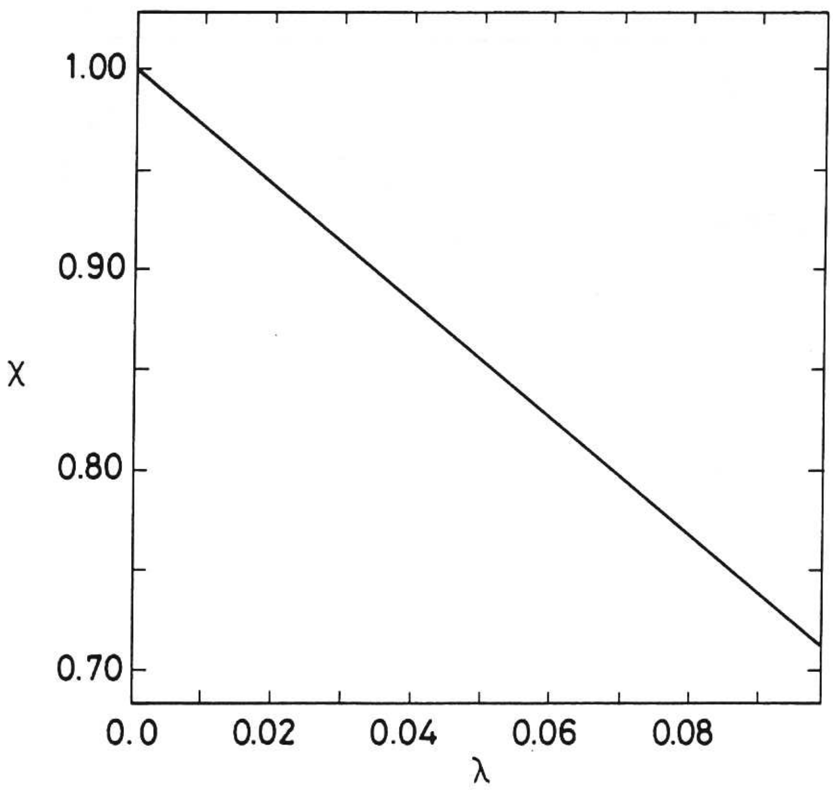

Fig. 2 The function $X(\lambda)$

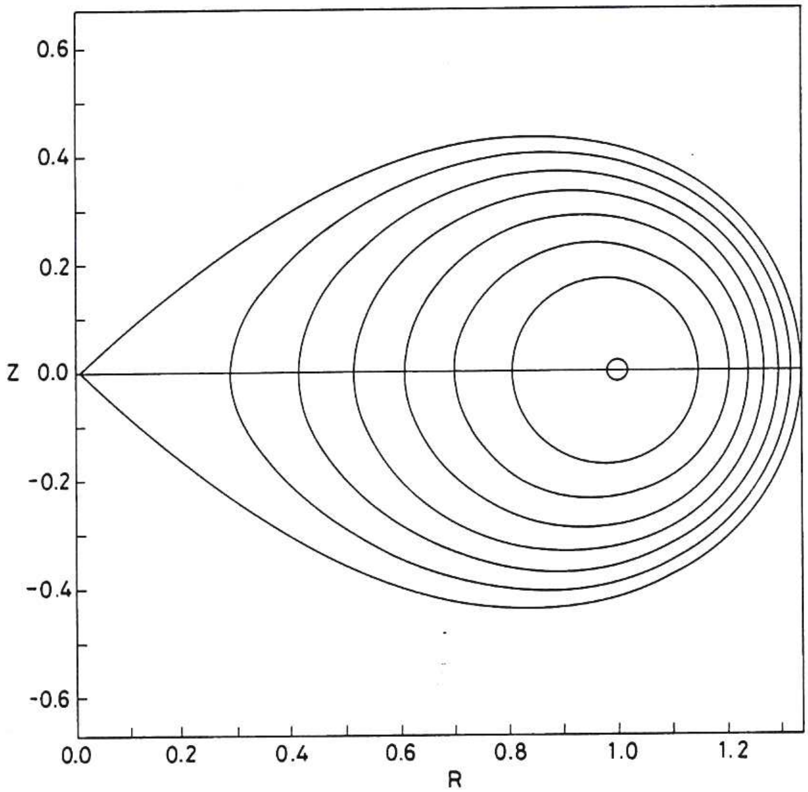

Fig. 3 The Unique Magnetic Surfaces of Degenerate Equilibria

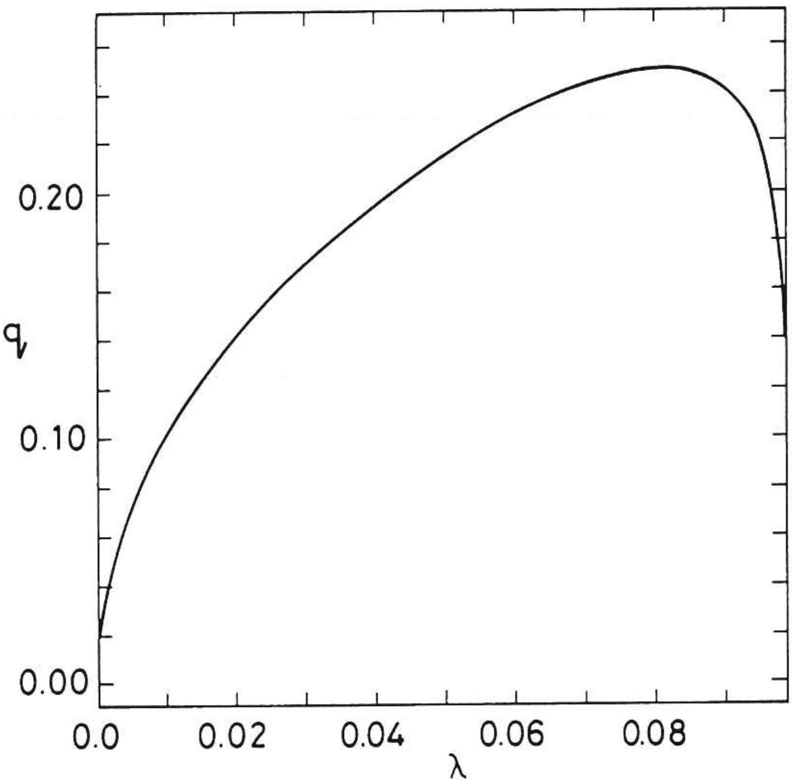

Fig. 4 The function q(λ)

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ANALYSIS OF INTERNAL MAGNETIC FLUCTUATIONS IN THE HBTX1A REVERSED FIELD PINCH

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