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Abstract

We discuss RF heating of three classes of tokamak flux surface which intersect the electron cyclotron resonance heating region: rational-q, near rational-q, and irrational-q. On irrational-q surfaces, the degree of heating is the same for all electrons. On and near rational-q surfaces, heating is localised to closed helical bands of electrons. These hot bands support local enhancements of current density, and act as current-carrying filaments within the plasma. The poloidal magnetic field associated with these currents has X-points. The use of ECRH as a current drive mechanism will enhance these effects, as will an increase in the size of tokamak major radius.

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1. Introductory Results

Previous discussion (CAIRNS and LASHMORE-DAVIES, 1984) of the flux surface averaging of the velocity space diffusion coefficient under ECH conditions has considered the case of flux surfaces with irrational values of q . Here q denotes as usual the number of turns that a field line makes in the toroidal direction, per completed turn in the poloidal direction. In this paper, we consider the case of flux surfaces at or near rational values of q . We shall show that in this case, a restricted range of particles have a large value of the diffusivity, while other particles have a low value, relative to the value of the diffusivity on irrational-q surfaces which are well away from low mode number rational-q surfaces. We shall attempt to quantify these remarks, and to examine some possible consequences.

In order to bring out the role of rational-q values as clearly as possible, we shall initially adopt a somewhat simplistic view of the electron heating. The region of space where electrons are in resonance with the beam is determined by the intersection of the cross-section of the beam with the vertical plane $\Omega_e = \omega_{RF} =$ constant . Here Ω_e is the local electron cyclotron frequency, determined by the strength of the predominantly toroidal magnetic field. This resonant surface intersects many flux surfaces (see Fig. 1). We shall assume that each electron remains on its flux surface for a finite time τ_0 before it is scattered or drifts significantly away from it. In this time, the electron executes a total of $N_0 = \tau_0/\tau_1$ turns in the toroidal direction about the axis of symmetry of the tokamak, where τ_1 is the time taken for a single such turn. At each pass through the resonant region, we shall assume the electron to undergo an impulsive increase in its velocity $v_\perp$ perpendicular to the magnetic field. Within the constraints of this simplified model, we shall assume this to be the sole effect of the resonant wave-particle interaction. The degree of heating of a given electron is proportional to the number of passes N_{res} through the resonance region that the electron makes:

$$\langle \Delta v_\perp^2 \rangle = \sigma N_{res} \tag{1.1}$$

The constant of proportionality σ follows from the physics of the wave-particle interaction, and we refer to CAIRNS and LASHMORE-DAVIES (1984) for the details of its calculation. N_{res} is some fraction of the total number of turns N_0, and its calculation as a function of q is the main topic of this paper.

On a given magnetic surface, the resonant region is a horizontal line parallel to the magnetic axis, whose length is approximately equal to the horizontal width a of the incident beam (see Fig. 2). This line is a section of a circle extending right round the outside of the flux surface. We shall find it useful to refer to this entire line as the return line in the discussion which follows.

Consider first a magnetic surface which has an irrational value of q, well separated from any low mode number rational value. In the time τ_1 taken for one toroidal circulation, a fraction $1/q$ of the total number of electrons N_{es} on the surface will cross the return line. Of these, a fraction $a/2\pi R$ will cross the return line in the resonant region; here R is the distance from the axis of symmetry of the tokamak to the return line. So the number of resonant crossings in time τ_1 is

$$N_{tot} = \frac{a}{2\pi R} \frac{1}{q} N_{es} \tag{1.2}$$

Each electron has a lifetime τ_0 on the surface. The number of crossings of the resonant region on an irrational-q surface by a single electron in its lifetime τ_0 is therefore

$$N_{res}(irr) = \frac{1}{q} \frac{a}{2\pi R} \frac{\tau_0}{\tau_1} \tag{1.3}$$

This result applies uniformly to all electrons on the irrational surface. We follow CAIRNS and LASHMORE-DAVIES (1984) by defining an effective particle diffusion coefficient

$$D = \langle \Delta v_\perp^2 \rangle / \tau_0 \tag{1.4}$$

Combining (1.1), (1.3), and (1.4) we have

$$D_{irr} = \frac{1}{q} \frac{a}{2\pi R} \frac{\sigma}{\tau_1} \tag{1.5}$$

This result is the same as Eq. (4) of Ref. 1, except that in the latter the q-dependence is neglected.

Consider now a rational-q surface, $q = m/n$. An electron which starts in the resonant region, returns to it after a time $m\tau_1$. In this time, it has crossed the return line n times. It follows that a fraction

$$f_1(\text{rat}) = \frac{na}{2\pi R} \tag{1.6}$$

of the electrons on the rational surface make a total of

$$N_{\text{res }1}(\text{rat}) = \frac{\tau_0}{m\tau_1} \tag{1.7}$$

returns to the resonant region each, in the course of the lifetime τ_0 on the surface. For this small localised group of electrons, we see on comparing (1.7) with (1.3) that the rate of heating is much greater than on nearby irrational-q surfaces. The remaining fraction

$$f_2(\text{rat}) = 1 - f_1(\text{rat}) \tag{1.8}$$

never pass through the resonant region, and are not heated at all:

$$N_{\text{res }2}(\text{rat}) = 0 \tag{1.9}$$

There are thus two distinct electron populations on a rational surface subject to ECH. We note that by (1.6)-(1.9) and (1.3)

$$\sum_{i=1}^{2} f_i(\text{rat})\, N_{\text{res }i}(\text{rat}) = N_{\text{res}}(\text{irr}) \tag{1.10}$$

It follows that the total number of electron crossings of the resonant region on a given surface is independent of whether q for that surface is rational or irrational. It is the distribution of crossings among electrons that differs. On rational surfaces, strong heating occurs on narrow closed helical windings, outside which there is no heating. On irrational surfaces, heating occurs at a lower rate for any single electron, but is uniformly distributed over all electrons on the surface. Specifically, using (1.1), (1.4), and (1.7),

$$D_{res\ 1} = \frac{1}{m}\frac{\sigma}{\tau_1} \quad , \quad D_{res\ 2} = 0 \qquad (1.11a,b)$$

to be compared with (1.5).

2. The Transition Region

Consider now a flux surface with a value of q close to a rational value:

$$q = \frac{m}{n} + \varepsilon \quad , \quad \varepsilon \ll q \qquad (2.1)$$

An electron starting at a given position on the return line returns to a point on the return line very close to the initial position after n complete poloidal turns. In this time it has completed $(m + n\varepsilon)$ toroidal turns. It has therefore advanced a distance

$$\Delta x = n\varepsilon \cdot 2\pi R \tag{2.2}$$

along the return line (see Fig. 3). There is room for only

$$N_\Delta = \frac{a}{\Delta x} = \frac{a}{2\pi R n}\frac{1}{\varepsilon} \tag{2.3}$$

such steps within the length a of the resonant region. When ε is so close to zero that $N_\Delta > N_{res\ 1}(rat)$, the maximum number of returns to the heated region remains $N_{res\ 1}(rat)$. The maximum number of returns is still limited by the lifetime τ_o, as on the rational surface itself (see Fig.4). However, electrons initially near the edge of the heated region leave it before the maximum number of returns has been made, as we discuss below. Once ε is large enough that

$$N_\Delta < N_{res\ 1}(rat) \tag{2.4}$$

which requires

$$\varepsilon > \varepsilon_c \equiv \frac{a}{2\pi R} q \frac{\tau_1}{\tau_0} \quad , \tag{2.5}$$

all population 1 electrons leave the resonant region before the lifetime τ_0 on the surface expires. They therefore undergo less heating than would occur on the rational surface. Once ε is so large that

$$N_\Delta = N_{res}(irr) \tag{2.6}$$

which requires

$$\varepsilon = \varepsilon_T \equiv \frac{q}{n}\frac{\tau_1}{\tau_0} , \qquad (2.7)$$

the degree of heating is the same as on an irrational surface. The condition (2.7) represents a good criterion for the merging of the rational-q and irrational-q regimes (see Fig. 4). Note that by (1.3), (2.3), and (2.7) we have $N_\Delta = (\varepsilon_T/\varepsilon)N_{res}(irr)$; and that by (1.7), (2.3), and (2.5) we have $N_\Delta = (\varepsilon_c/\varepsilon)N_{res\ 1}$.

As ε increases from zero, so the number of electrons exposed to heating increases. Electrons with return line crossing points initially outside the heating region move into it by taking steps of length Δx, at the same rate as electrons with return line crossing points initially within the heating region step out of it. It is useful to define a length

$$L = \frac{\tau_0}{m\tau_1}\Delta x \qquad (2.8)$$

which is the total distance covered on the return line by the set of crossing points generated by a single electron in its lifetime τ_0 on the surface. There are n such sets of crossing points for a given electron (see Fig. 5a). We note firstly that when ε is large enough that the condition

$$L = L_T = 2\pi R/n \qquad (2.9)$$

is satisfied, the set of crossing points generated by a single electron is distributed uniformly all round the return line. The condition $L = L_T$ thus reproduces the irrational-q regime (see Fig. 5b). Combining (2.2), (2.8) and (2.9), we see that $L = L_T$ occurs at the same value $\varepsilon = \varepsilon_T$ at which (2.6) is satisfied. Thus the criteria (2.6) and (2.9) for the merging of the rational-q regime with the irrational-q regime are equivalent.

Next, consider the way in which the heating is distributed among the different electrons on a given surface. Let the resonant region cover the interval $(x_0 , x_0 + a)$ on the return line. All electrons with initial crossing points in the interval $(x_0 - L , x_0 + a)$ are subject to some degree of heating. This is repeated with n-fold symmetry around the return line. For $\varepsilon < \varepsilon_c$ (or equivalently, $L < a$), N_Δ exceeds $N_{res\ 1}(rat)$. The corresponding distribution of N_{res} as a function of the initial position of the crossing point on the return line is given in Fig. 6a. For $\varepsilon_T > \varepsilon > \varepsilon_c$ (or equivalently, $L_T > L > a$) we refer to Fig. 6b. Note how the maximum value of N_{res} has fallen (see Fig. 4), while the number of electrons undergoing heating has increased. This corresponds to a broadening in the poloidal direction of the closed helical band with enhanced diffusivity D , while the value of D diminishes, as $\varepsilon = q - m/n$ increases from zero. For $\varepsilon > \varepsilon_T$ (or equivalently, $L > L_T$) the regime approximates to the uniform irrational-q heating conditions described at the beginning of the preceding section.

The total number of crossings of the resonant region remains the same in all these regimes. From Fig. 6a, we see that for $\varepsilon < \varepsilon_c$ this number is

$$n\left(\frac{a}{2\pi R}\right) N_{es} N_{res\ 1}(rat) = \left(\frac{a}{2\pi R}\right) \frac{\tau_0}{q\tau_1} N_{es} \qquad (2.10)$$

where we have used (1.7) and N_{es} is as usual the total number of electrons on the flux surface. From Fig. 6b, we see that for $\varepsilon_T > \varepsilon > \varepsilon_c$, the total number of crossings is given by

$$n\left(\frac{L}{2\pi R}\right) N_{es} \left(\frac{a}{\Delta x}\right) = \left(\frac{a}{2\pi R}\right) \frac{\tau_0}{q\tau_1} N_{es} \qquad (2.11)$$

where we have used (2.8). For the irrational-q regime well away from rational-q values, the total number of crossings of the resonant regions is

$$N_{es} N_{res}(irr) = \left(\frac{a}{2\pi R}\right) \frac{\tau_0}{q\tau_1} N_{es} \qquad (2.12)$$

where we have used (1.3). The results (2.10)-(2.12) are identical, and represent a generalisation of our previous result (1.10).

3. Discussion

The formula (1.7) for $N_{res\ 1}(rat)$ gives the number of returns to the resonant region on a rational surface with $q = m/n$ which are made by one of the small fraction of electrons on that surface whose orbits pass through the resonant region. These electrons form a closed helical band on the flux surface. The other electrons on the rational-q surface are not heated at all. The formula (1.3) for $N_{res}(irr)$ gives the number of returns to the resonant region made by each electron on an irrational-q surface which is well-separated from a low mode number rational-q surface in the sense discussed in Section 2, which is examined further below. It is important to note that by (1.3) and (1.7), the value of the ratio

$$\frac{N_{res\ 1}(rat)}{N_{res}(irr)} = \frac{2\pi R}{a}\frac{q}{m} = \frac{2\pi R}{a}\frac{1}{n} \qquad (3.1)$$

can be very large. The horizontal width a of the gyrotron beam is of the order of a few cm. The major radius of a typical tokamak is of the order of 100 cm, and so therefore is the distance R from the centre line to the return line. Hence at present

$$\frac{N_{res\ 1}(rat)}{N_{res}(irr)} \simeq \frac{100}{n} \qquad (3.2)$$

As tokamaks increase in size, so will the value of the ratio (3.1). We note that this ratio can also be increased in size by reducing the beam width a. It follows from (3.1) and (3.2) that the heating of an integer-q (i.e., $n = 1$) flux surface will generate a closed helical band of electrons which are subjected to heating which is approximately 100 times stronger than that of neighbouring electrons on irrational-q surfaces. Because the value of $(2\pi R/a)$ is so large, a wide range of values of m/n give appreciable excess localised heating. Using (3.2), we see for example that between $q = 3/2$ and $q = 2$, the value of $N_{res\ 1}(rat)/\ N_{res}(irr)$

exceeds 10 on eleven rational surfaces ($q = 5/3, 7/4, 8/5, 9/5, 11/6, 11/7, 12/7, 13/7, 13/8, 15/8, 14/9$). In addition, using (2.7), these eleven rational surfaces can be calculated to have some effect over a significant fraction $\simeq 6.6\ \tau_1/\tau_0$ of the interval (3/2, 2) considered. The effect of rational-q values appears to be strong and widespread. We note that the most promising method of observing such a spatially localised group of hot electrons is via their enhanced electron cyclotron emission.

When electrons are heated, their electrical conductivity increases as $T_e^{3/2}$. The hot closed helical band on a rational flux surface therefore represents a localised region of strongly enhanced conductivity, relative to the level of conductivity on a heated irrational-q surface. The cooler, unheated regions of the rational-q surface have a conductivity which is lower than that of a heated irrational-q surface. Suppose firstly that the ECRH is operating in its pure heating mode, that is with a symmetric power spectrum in $k_\parallel$. Then in an Ohmic discharge, the loop voltage will drive excess current in the hot closed helical band on the rational-q surface, which thus becomes in effect an internal stellarator current winding. Note that relative to the current sustained by the heated irrational-q surfaces, the cooler out-of-resonance regions of the rational-q surface support a reverse current. These effects extend for a finite distance away from the rational-q surface, as discussed in Section 2. If the ECRH is used to drive current using a peaked asymmetric power spectrum in $k_\parallel$ (FISCH and BOOZER, 1980) the effects will be enhanced.

These currents support a magnetic field $\underline{B}_c$ which must be added to that already in existence. The poloidal component of $\underline{B}_c$ near a rational-q surface is shown in Fig. 7. Because of the opposing current flows, the occurrence of X-points in the poloidal component of $\underline{B}_c$ appears unavoidable on rational-q heated surfaces. In particular, for a tokamak discharge in which the current is supported entirely by ECRH, the poloidal magnetic field will be determined exclusively by $\underline{B}_c$ and will therefore in general display X-points. This is a significant difference from the case of Ohmic discharges, which may affect confinement and MHD stability.

4. Conclusions

We have discussed radio-frequency heating of the three classes of tokamak flux surface which intersect the region of electron cyclotron resonance $\omega_{RF} = \Omega_e$: rational-q, close to rational-q, and irrational-q . On irrational-q surfaces, the degree of heating is the same for all electrons. On and near rational-q surfaces, heating is localised to closed helical bands of electrons. We have shown how these regimes merge: as q moves away from a rational value, so the closed helical band broadens in the poloidal direction, and the degree of heating of each electron within the band diminishes. The hot closed helical bands support local enhancements of current density, and act as current-carrying filaments within the plasma. The poloidal magnetic field associated with these currents has X-points. As shown in the example in Section 3, the distribution of rational values of q for which these effects are significant is so dense that it will prove difficult to avoid them in experiments. The use of ECRH as a current drive mechanism, and an increase in the size of tokamak major radius, will both cause the effects outlined above to become more pronounced.

References

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FISCH, N. J., and BOOZER, A. H. (1980) Phys. Rev. Lett. 45, 720.

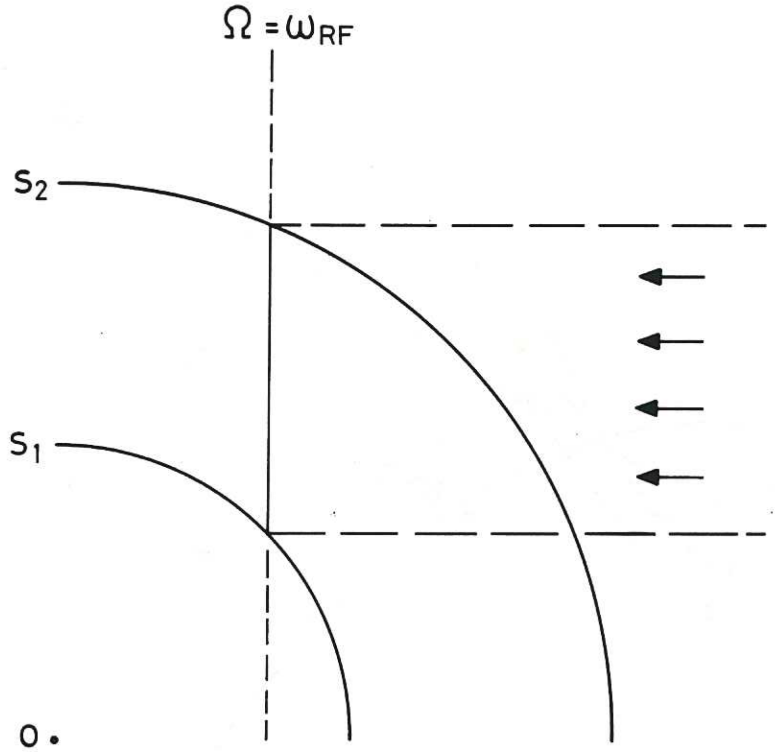

Fig. 1 Poloidal cross-section of irradiated plasma. RF heating is incident on the resonant surface $\Omega = \omega_{RF}$ from the right. S_1 and S_2 are respectively the innermost and outermost flux surfaces that intersect the heated region. O denotes the magnetic axis.

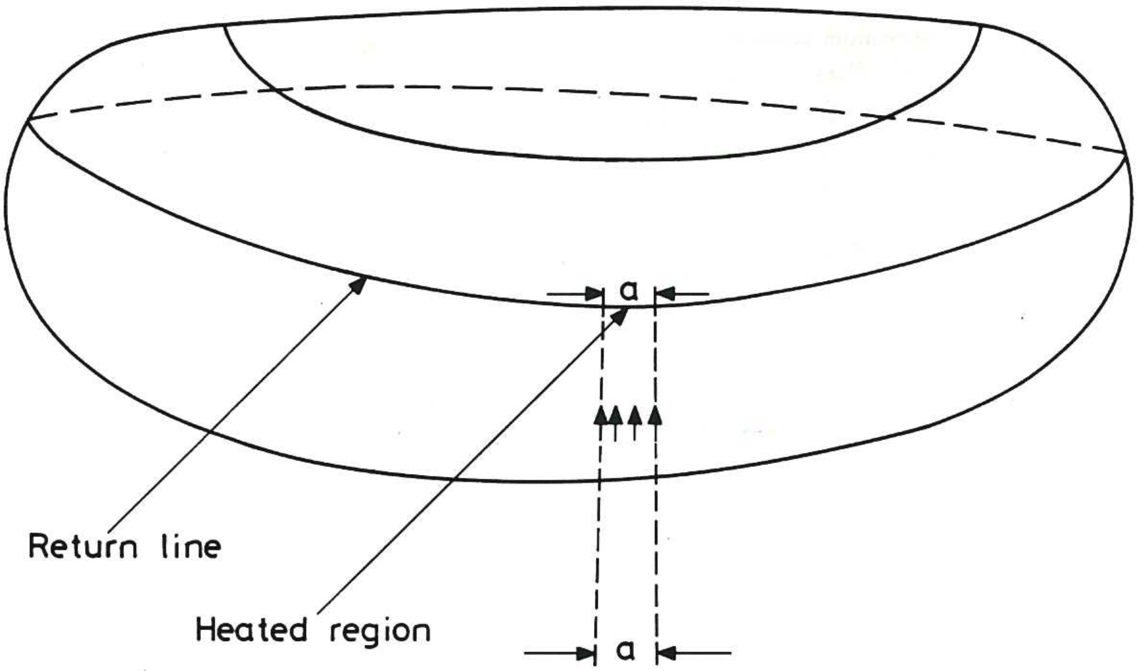

Fig. 2 A single toroidal flux surface showing the incident RF radiation, heated region, and return line.

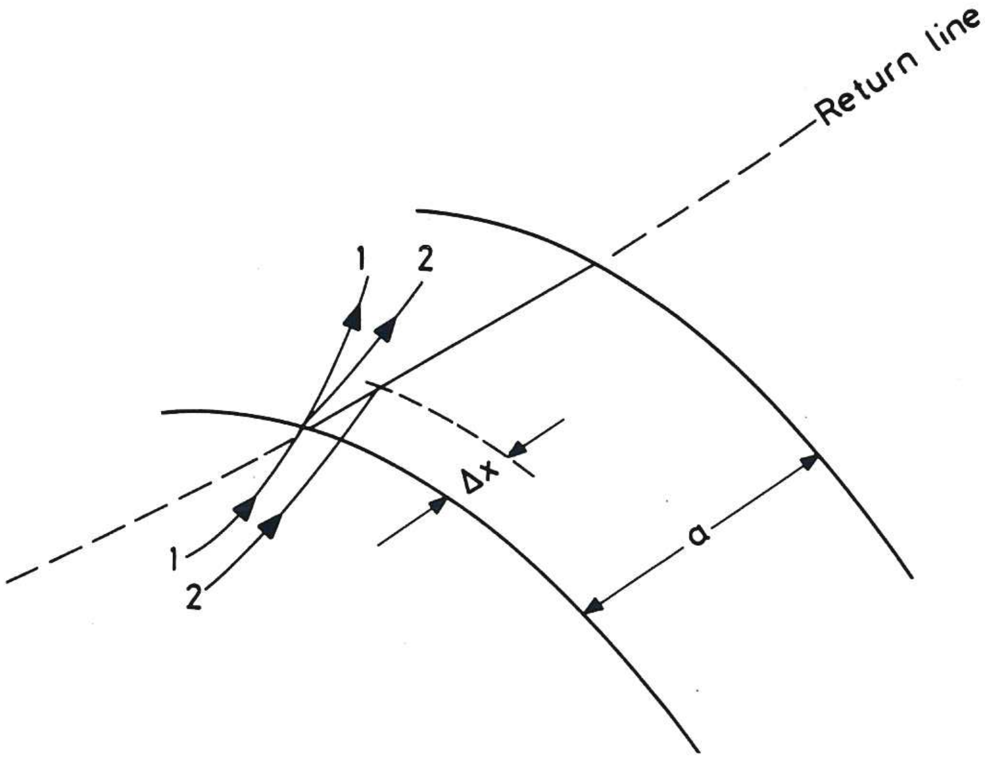

Fig. 3 Two classes of electron path on a heated flux surface. Both paths originate at the extreme left-hand edge of the heated region.
Type 1: on a rational-q surface, the path closes on itself after a finite number of toroidal circulations.
Type 2: on an irrational-q surface near a rational-q surface, the path crosses the return line in the heated region at a distance Δx to the right of its first crossing.

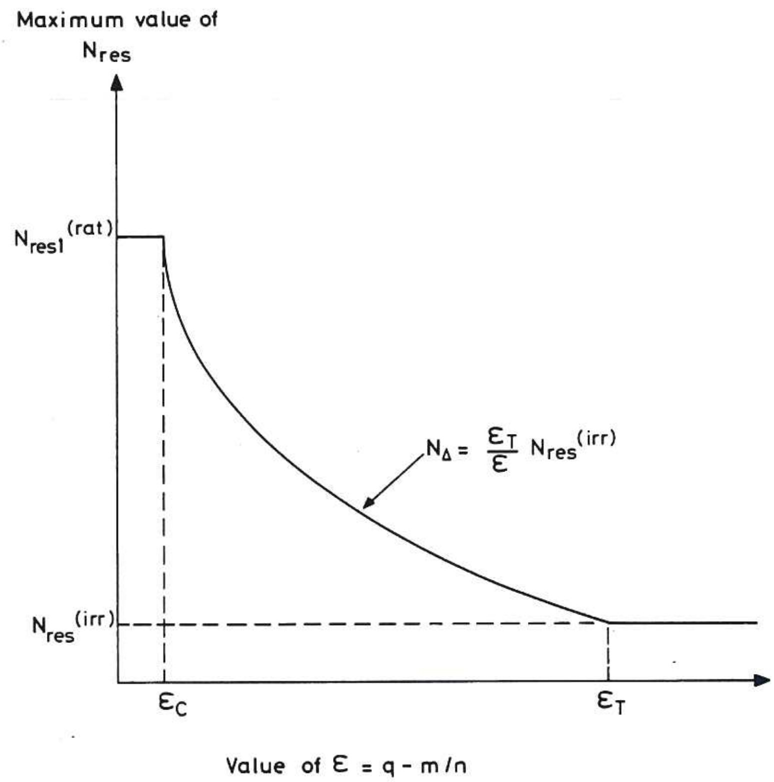

Fig. 4 Maximum value of the number of returns N_{res} made by an electron to the heated region as a function of the difference ϵ between the value of q on the surface and the nearest low mode number rational value of q.

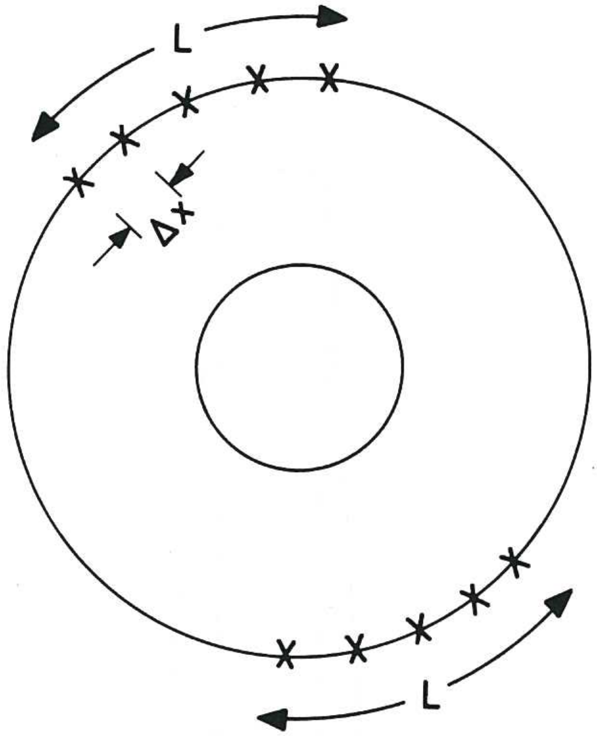

Fig. 5a Return line viewed from above for q = 3/2 + ϵ. X denotes crossing point generated during the first few toroidal circulations by a given electron. The associated distance L is shown.

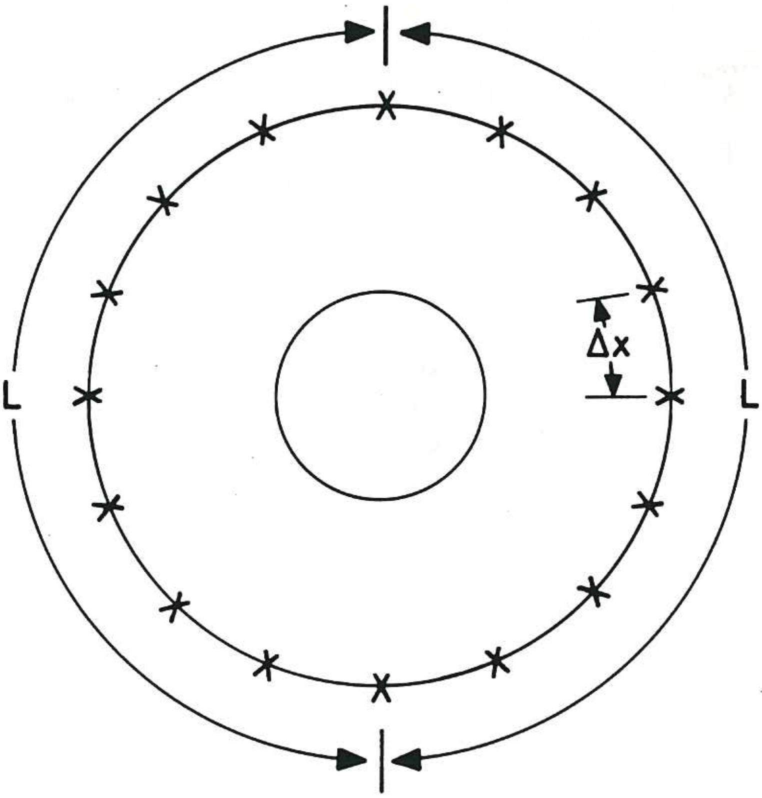

Fig. 5b Return line viewed from above for q = 3/2 + ϵ_T. X denotes crossing points and the associated distance L is seen to cover the return line.

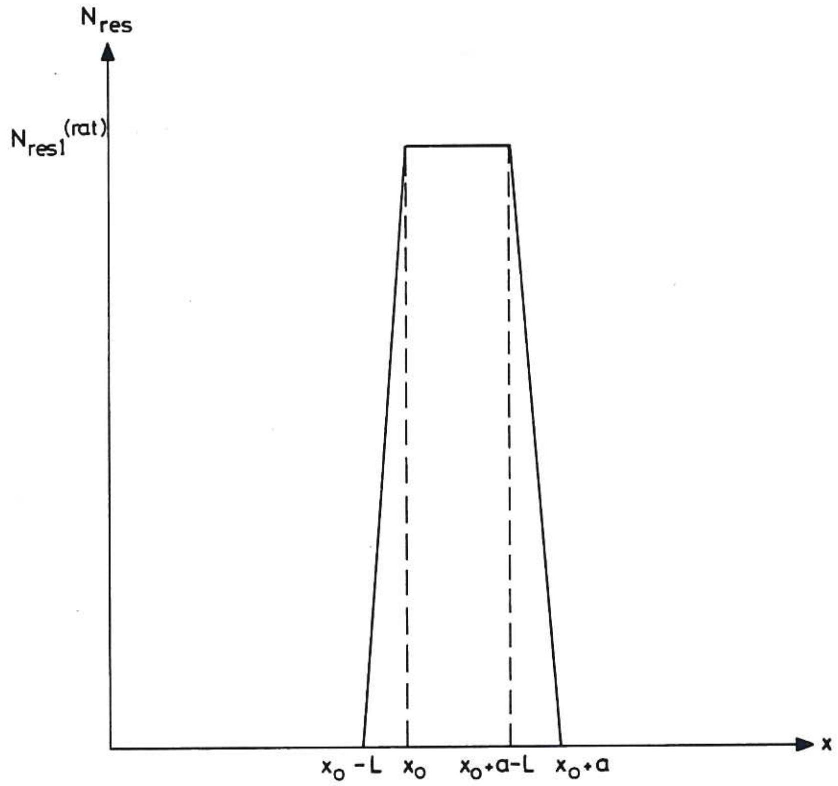

Fig. 6a N_{res} versus position on return line of initial electron crossing point for the case $L < a$ (or equivalently $\epsilon < \epsilon_c$).

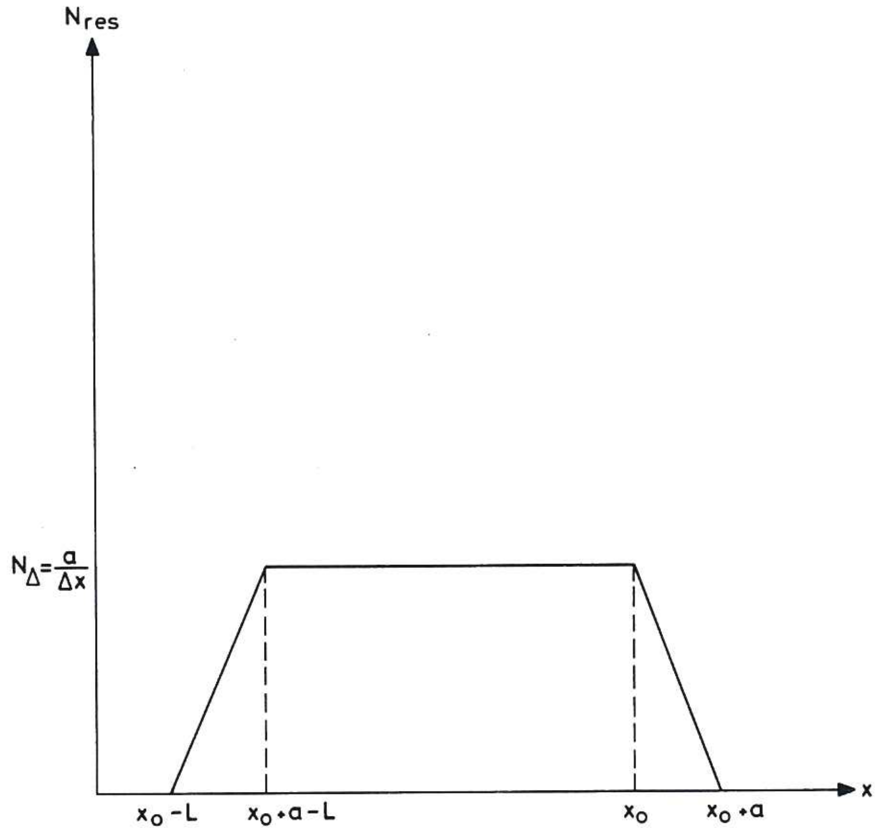

Fig. 6b N_{res} versus position on return line of initial electron crossing point for the case $2\pi R/n > L > a$ (or equivalently $\epsilon_T > \epsilon > \epsilon_c$).

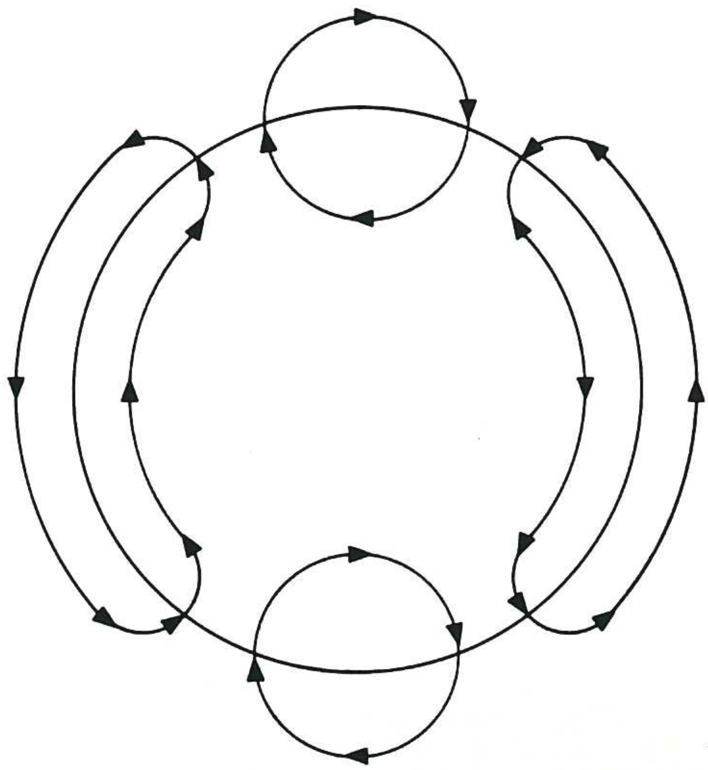

Fig.7 Poloidal cross-section of a heated rational surface q = 3/2 showing the perturbation to the poloidal magnetic field produced when current is driven. The hot helical band (top and bottom) supports a strong localised current in the forward direction. Relative to the current driven on neighbouring heated irrational-q surfaces, the unheated regions of the q = 3/2 surface support a diffuse backward current.