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# ONE DIMENSIONAL CALCULATIONS OF TWO-PHASE MIXING FLOWS



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# ONE DIMENSIONAL CALCULATIONS OF TWO-PHASE MIXING FLOWS

by

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Abstract

A new algorithm for the solution of two-phase fluid flow problems is presented. The algorithm consistently ensures that the evolved volume fractions are all positive and sum to unity, even in the presence of large spatial and temporal gradients of volume fraction. The momentum equations are then solved using an iterative pressure correction method. Example calculations for buoyancy driven flows, with and without phase changes, are presented.

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## 1. INTRODUCTION

The numerical calculation of multiphase fluid flow is now of considerable importance in the process and nuclear industries [1]. The presence of more than one phase neccessitates the treatment of inter-phase transfers of momentum and energy. A change of phase may also occur, as in the case of the flow of water and steam.

The present work is motivated by the need to understand the behaviour of a hot fluid which mixes with a volatile cold fluid; a situation which occurs in certain industrial processes, submarine volcanisms and which could arise in a hypothetical reactor accident when molten core material pours into liquid coolant. It is of interest to be able to predict the evolution of the vapour volume fraction, together with the transient velocities of the hot liquid and the liquid and vapour phases of the volatile liquid [2]. We have decided to develop a code to model this situation in several stages; accordingly in this paper we describe a numerical algorithm developed for calculating transient one-dimensional, two-phase flow with a prescribed rate of phase change.

In the problem of interest all the flow velocities are small compared to the sound speeds, so it is a good approximation to assume that all the fluids are incompressible. This avoids the need to specify an equation of state for each material and is computationally cheaper. This approach leads to the problem of determining the dynamic pressure and consistently ensuring that the evolved volume fractions are all positive and sum to unity. An algorithm has been developed to allow the governing equations to be advanced in time in such a manner that the above requirements are satisfied, even in the presence of large source terms.

The code has been used to examine two test problems. The first is the settling of an initially homogeneous mixture of water and steam (without phase change) in a closed vessel, due to the action of buoyancy.

In the second, the behaviour of a pool of water, which is transformed into steam at a specified rate in an open vessel, is examined.

In section 2 we describe the mathematical formulation of the problem and in section 3 the solution algorithm is described. In section 4 we describe the results obtained from the test problems and in section 5 we make some concluding remarks.

## 2. MATHEMATICAL FORMULATION

For definiteness we consider a liquid-vapour system contained within a circular cylindrical vessel of radius R and height H. All liquid phase properties (density, volume fraction etc) will be denoted by the suffix L while the suffix V will designate the vapour phase properties. The vertical acceleration due to gravity is denoted by g. SI units are used throughout. Of the many forms of two-phase flow equations given in the literature [3,4], we choose the following set. Considering only the variation in the vertical direction (y) and the time (t), the dependent variables are: $\alpha_L(y,t)$, $\alpha_V(y,t)$ (volume fractions of liquid and vapour respectively), $V_L(y,t)$, $V_V(y,t)$ (vertical liquid and vapour velocities) and $p(y,t)$ (the common pressure field). The densities $\rho_L$ and $\rho_V$ are taken as specified constants (incompressibility approximation). This is reasonable provided $V_L$ and $V_V$ are small compared to the acoustic velocities. The dynamics of the above system are assumed to be governed by the following system of nonlinear equations.

$$\frac{\partial}{\partial t}(\rho_L \alpha_L) + \frac{\partial}{\partial y}(\rho_L \alpha_L V_L) = \dot{m}_L \tag{2.1}$$

$$\frac{\partial}{\partial t}(\rho_V \alpha_V) + \frac{\partial}{\partial y}(\rho_V \alpha_V V_V) = \dot{m}_V \tag{2.2}$$

$$\frac{\partial}{\partial t}(\rho_L \alpha_L V_L) + \frac{\partial}{\partial y}(\rho_L \alpha_L V_L^2) = -\alpha_L \frac{\partial p}{\partial y} - \rho_L \alpha_L g + F_{LV} + F_{L\dot{m}} \tag{2.3}$$

$$\frac{\partial}{\partial t}(\rho_V \alpha_V V_V) + \frac{\partial}{\partial y}(\rho_V \alpha_V V_V^2) = -\alpha_V \frac{\partial p}{\partial y} - \rho_V \alpha_V g + F_{VL} + F_{V\dot{m}} \tag{2.4}$$

$$\alpha_L + \alpha_V = 1 \tag{2.5}$$

The above equations must be solved given initial data on $\alpha_L$, $\alpha_V$, $V_L$ and $V_V$ and the endpoint conditions: $V_L(0,t) = V_V(0,t) = V_L(H,t) = V_V(H,t) = 0$ (impermeable boundaries). The inter-phase drag forces $F_{LV}$, $F_{VL}$ satisfy Newton's third law $F_{LV} + F_{VL} = 0$. They are in general very complicated functions of $\alpha_L$, $\alpha_V$ and $V_L - V_V$. For example, $F_{LV}$ has the form,

$$F_{LV} = C_{LV} \frac{\rho_L \alpha_L}{\tau_{LV}} (V_V - V_L) \tag{2.6}$$

where the non-dimensional coefficient $C_{LV}$ and the momentum relaxation time $\tau_{LV}$ have no generally valid forms but have to be determined either from experimental correlations or from theoretical considerations involving 'micro physics' of the inter-facial forces. The evaporation reaction forces $F_{L\dot{m}}$, $F_{V\dot{m}}$ are taken to have the forms $F_{L\dot{m}} \equiv \dot{m}_L V_L$ and $F_{V\dot{m}} \equiv \dot{m}_V V_L$. The mass sources $\dot{m}_L$, $\dot{m}_V$ depend on the physical situations of interest. For example, if L and V are different substances which do not chemically react and cannot be transformed into each other, $\dot{m}_L$ and $\dot{m}_V$ are equal to zero in the absence of external mass sources. If L and V refer to water and steam respectively, $\dot{m}_L + \dot{m}_V = 0$ and $\dot{m}_L$ depends on the thermodynamic properties of the fluid. To calculate $\dot{m}_L$, an enthalpy equation is required together with appropriate phase-transformation relations (e.g. the Clausius-Clapeyron equation). In general, the specification of the sources $\dot{m}_L$, $\dot{m}_V$, $F_{LV}$ and $F_{VL}$ is a problem of constitutive relations. It has little to do with the numerical algorithms involved in solving Eqns. (2.1) to (2.5). We shall concentrate on the latter aspect in this paper.

It is important to note that Eq.(2.5) is different in character from the others. It is not an evolution equation for p, but a constraint. The equation for p is of elliptic type (unlike the others which are

hyperbolic). It can be derived from (2.1), (2.2) and (2.5) and takes the form,

$$\frac{\partial}{\partial y}\left(\alpha_L V_L + \alpha_V V_V\right) = \left(\frac{\dot{m}_L}{\rho_L} + \frac{\dot{m}_V}{\rho_V}\right) \qquad (2.7)$$

It will be seen that (2.7) effectively determines p, in (2.3) and (2.4). The $\alpha$'s calculated using (2.1), (2.2) (2.3), (2.4) and (2.7) are non-negative and satisfy (2.5) for properly defined sources. Thus the condition $V_L = V_V = 0$ at $y = H$ is appropriate if $\dot{m}_L, \dot{m}_V$ satisfy $\int_0^H \left(\frac{\dot{m}_L}{\rho_L} + \frac{\dot{m}_V}{\rho_V}\right) dy = 0$. Otherwise, the flow rates at y=H must be consistently prescribed. We give an example of this later.

Turbulent stresses are not explicitly accounted for in equations (2.3) and (2.4). Provided the drag-terms are obtained from experimental correlations they will include turbulent effects. It would thus be double counting to include two-phase analogues of the Reynolds stresses.

## 3. NUMERICAL ANALYSIS OF THE MODEL

Before we proceed to a description of the numerical methods employed in this study, we state our guiding principles; we adopt a practical, engineering approach to the computations as opposed to a mathematically rigorous error/stability analysis. The cardinal requirement of the algorithm is that it should produce physically consistent and meaningful solutions to physically well-posed problems. Thus high order accuracy (for given mesh sizes) is less relevant than numerical stability, qualitative consistency and acceptable convergence properties of the algorithm. This standpoint is motivated by the fact that there are large uncertainties in the physical specification of the momentum sources and there is little value in obtaining highly accurate solutions of approximate equations. A second, and no less important, requirement of the code is flexibility. By this we mean that the same scheme with no

modifications other than the obvious changes of parameters must be able to handle a wide variety of flow and source specifications. In view of these a priori criteria, we formulate below a first order (in $\frac{\Delta y}{H}$, $\Delta t$), semi-implicit, iterative finite-difference scheme to solve the problem posed in Section 2.

Figure 1 shows a typical equally spaced grid arrangement of 5 "continuity cells". The 'nodal points' are located at $y_j = j\Delta y - \frac{\Delta y}{2}$, where j = 1,5. ($5\Delta y = H$). The quantities $\alpha_L$, $\alpha_V$ and p are 'stored' at these points and are denoted $\alpha_L(j,n\Delta t)$ or simply $\alpha^n_{Lj}$. $n\Delta t$ (n = 0, ..) denotes the time from t = 0. We also need to have an index for iterations at a particular time $n\Delta t$ at a point $y_j$. We use $\nu$ for this purpose. The velocities $V_L$ and $V_V$ are stored at the 'staggered' locations $y^*_j = j\Delta y, j = 0,5$. Note that $V^n_{LO} = V^n_{Lj=5} = V^n_{VO} = V^n_{Vj=5} = 0$, and are not explicitly calculated by virtue of the boundary conditions.

In Figure 1 ABCD represents a typical continuity cell at j = 3. Quantities pertaining to the adjacent cell j = 4 will be denoted by '+' (also in reference to the cell boundary AB). Analogously, quantities pertaining to CD are denoted by '-'. It is useful to observe that $V^n_{Lj}$ is not stored at $y_j$; but at $y^*_j$. If $\Delta y = \frac{H}{N}$, there are N continuity cells and $p_j$ is a N-vector while there are only N-1 solved velocities.

Suppose we have in store $\alpha_L(j,n)$, $\alpha_V(j,n)$ $V_L(j,n)$, $V_V(j,n)$ and $\dot{m}_L$, $\dot{m}_V$ at (j,n). The following explicit scheme is used to calculate $\alpha_L(j,n+1)$, $\alpha_V(j,n+1)$. The scheme is first order accurate and is specially designed to ensure that $\alpha_L(j,n+1)$, $\alpha_V(j,n+1)$ satisfy (2.5). Furthermore, it ensures that $0 \leqslant \alpha_L(j,n+1) \leqslant 1$, and $0 \leqslant \alpha_V(j,n+1) \leqslant 1$.

We integrate equation (2.2) over a continuity cell such as ABCD (treating $\rho_V$ to be constant).

$$\{\alpha_V(j,n+1) - \alpha_V(j,n)\}\frac{\Delta y}{\Delta t} + [\alpha_V V_V]^+ - [\alpha_V V_V]^- = \dot{m}_{Vj}\frac{\Delta y}{\rho_v} \qquad (3.1)$$

Since the scheme is explicit,

$$[\alpha_V V_V]^+ = \{\theta_+ \alpha_V(j,n) + (1-\theta_+)\alpha_V(j+1,n)\}V_V(j,n)$$

$$[\alpha_V V_V]^- = \{\theta_- \alpha_V(j-1,n) + (1-\theta_-)\alpha_V(j,n)\}V_V(j-1,n)$$

bearing in mind that $V_V(j,n)$ is located on the boundary AB. The interpolation factors $\theta_+$, $\theta_-$ are chosen to imply "upwind" or "donor-cell" differencing.

$$\text{Thus } \theta_+ = 1 \text{ if } V_V(j,n) \geq 0$$
$$= 0 \text{ if } V_V(j,n) < 0 \qquad (3.2)$$

With these definitions (3.1) can be rearranged and put in the following general form:

$$\Phi_j^{n+1} = A_j \Phi_j^n + A_j^- \Phi_{j-1}^n + A_j^+ \Phi_{j+1}^n + S_j \qquad (3.3)$$

$$S_j \equiv \dot{m}_{Vj}\frac{\Delta t}{\rho_v}$$

$$A_j \equiv 1 - \theta_+(j)\, V_V(j,n)\frac{\Delta t}{\Delta y} + (1-\theta_-(j))\, V_V(j-1,n)\frac{\Delta t}{\Delta y}$$

$$A_j^+ \equiv -(1-\theta_+(j))V_V(j,n)\frac{\Delta t}{\Delta y}$$

$$A_J^- \equiv \theta_-(j)\, V_V(j-1,n)\frac{\Delta t}{\Delta y}.$$

$$\Phi_j^n \equiv \alpha_V(j,n\Delta t).$$

We observe that the upwind differencing rule ensures that $A_j^+$, $A_j^-$ are always non-negative. Observe also that by definition $\theta_+(j) = \theta_-(j+1)$.

From this we obtain the identity

$$A_j + A^-_{j+1} + A^+_{j-1} = 1 \qquad (3.4)$$

In order to find conditions which ensure the physical consistency of the scheme, we consider the important special case $S_j \equiv 0$. We shall call an arbitrary column vector $\{x_i\}(i=1,N)$ admissible, if $0 \leq x_i$ and $\sum_{i=1}^{N} x_i = K$, where K is a fixed constant. Clearly $\Phi^n$ and $\Phi^{n+1}$ have to be admissible vectors if $S_j \equiv 0$, for all j and n, with the same K. Consider an arbitrary linear transformation L on the set of admissible vectors, defined by the square matrix $\{L_{ij}\}$. We seek general necessary and sufficient conditions on L such that Lx will be an admissible vector given that x is. The elements of the column vector y = Lx are given by,

$$y_i = \sum_{j=1}^{N} L_{ij}\, x_j .$$

Now, $0 \leq y_i$ for all i given that x is admissible is clearly true if $0 \leq L_{ij}$ for all i and j. Furthermore, $\sum_{i=1}^{N} y_i = \sum_{j=1}^{N} x_j$, if $\sum_{i=1}^{N} L_{ij} = 1$ for all j. If the matrix elements $L_{ij}$ satisfy the conditions $0 \leq L_{ij}$, $\sum_{i=1}^{N} L_{ij} = 1$ for all i,j respectively, we shall call L a positive, faithful transformation, defined on the set of admissible vectors. It is not difficult to show that the conditions enumerated are both necessary and sufficient for L to map the set of admissible vectors into itself. Evidently a sufficient condition for the tridiagonal matrix defined by $A_j$, $A^+_j$, $A^-_j$ to be a positive, faithful transformtion is the "Courant" condition,

$$\frac{\Delta y}{\Delta t} > 2 \max_{j=1,N} \left| V_V(j,n) \right| \qquad (3.5)$$

If this is satisfied, the $A_j$ are all positive and (3.4) implies that $\Phi^{n+1}$ is admissible. Physically, the Courant condition means that the 'numerical domain of dependence' of Eq.(3.1) includes the 'analytic domain of dependence' of the hyperbolic equation (2.1). We have shown above that this automatically ensures that $(A_j, A^+_j, A^-_j)$ is positive, faithful. Consequently it is plain that Eq.(3.1) and Eq.(2.1) share the

same conservation properties in the absence of sources. Clearly, we also require the $\alpha$'s to be less than or equal to unity. This can only be achieved if the velocities satisfy the constraint (2.7), which in turn implies equation (2.5). Provided the V's used in equations (3.1) and (3.3) are consistent in this sense, the $\alpha$'s implied by equation (3.3) will always be less than or equal to unity.

If the mass source is non zero, $\Delta t$ is further restricted by accuracy requirements. For many purposes it is convenient to replace $A_j$, $A_j^+$, $A_j^-$ by $C_j$, $C_j^+$, $C_j^-$ which define a positive faithful matrix <u>independently</u> of the sizes of $\Delta y$ and $\Delta t$ relative to $V_V(j,n)$(i.e. even if the Courant condition (3.5) is <u>not</u> satisfied).

Thus we put,
$$C_{j-1}^+ \equiv A_{j-1}^+/(1+A_{j-1}^+ + A_{j+1}^-)$$
$$C_{j+1}^- \equiv A_{j+1}^-/(1+A_{j-1}^+ + A_{j+1}^-) \qquad (3.6)$$
$$C_j \equiv 1 - C_{j-1}^+ - C_{j+1}^-$$

Clearly, in the limit $\Delta t \to 0$ for fixed $\Delta y$ (or more generally for $\Delta t$, $\Delta y \to 0$ with $\frac{\Delta y}{\Delta t} >> \text{Max } |V_V|$), the C's tend to the A's and one obtains usual numerical consistency with the differential equation. However, by construction the C's always define a positive faithful transformation, regardless of $\Delta y$, $\Delta t$ and $V_V$. Having advanced the $\alpha'_V$s using (3.3) (or equivalent) we calculate $\alpha_{Lj}^{n+1}$ from (2.5).

It remains to obtain $V_L$, $V_V$ and $p$ by solving finite-difference forms of (2.3), (2.4) and (2.7). Finite-differencing these equations is straight forward provided we recall that 'velocity cells' are staggered and all convective terms must be upwind differenced. It is also important to treat the inter-phase drag forces $F_{LV}$, $F_{VL}$ and the buoyancy terms carefully. We first define a 'reduced pressure' $\bar{p}$ by the equation,

$$p = \bar{p} + \int_y^H g(\alpha_L \rho_L + \alpha_V \rho_V) dy \qquad (3.7)$$

In terms of $\bar{p}$ we have

$$- g\alpha_L \rho_L - \alpha_L \frac{\partial p}{\partial y} = - \alpha_L \frac{\partial \bar{p}}{\partial y} + g\alpha_L{}^2 \rho_L - g\alpha_L \rho_L + g\alpha_V \rho_V \alpha_L$$

$$= - \alpha_L \frac{\partial \bar{p}}{\partial y} - g\alpha_L \alpha_V(\rho_L - \rho_V) \qquad (3.8)$$

$$- g\alpha_V \rho_V - \alpha_V \frac{\partial p}{\partial y} = - \alpha_V \frac{\partial \bar{p}}{\partial y} + g\alpha_L \alpha_V(\rho_L - \rho_V) \qquad (3.9)$$

Since $F_{LV} = C_{LV} \rho_L \alpha_L (V_V - V_L)/\tau_{LV}$, this term is treated implicitly in finite-differencing (2.3), (2.4). It is easy to verify that these equations can be written in the form

$$\begin{aligned} V_{Vj}^{n+1} &= B_j^+ V_{Vj+1}^{n+1} + B_j^- V_{Vj-1}^{n+1} + S_j^V \\ V_{Lj}^{n+1} &= D_j^+ V_{Lj+1}^{n+1} + D_j^- V_{Lj-1}^{n+1} + S_j^L \end{aligned} \qquad (3.10)$$

The tridiagonal matrix elements $B_j^+$ etc. and sources $S_j^V$, $S_j^L$ involve $\alpha$'s, V's (at n), p's (at n+1) and are given in Appendix 1. These must be solved iteratively to obtain $V_{Lj}^{n+1}$, $V_{Vj}^{n+1}$, $p_j^{n+1}$ consistent with the finite-difference form of (2.7).

This solution procedure uses a form of Newton's method. Suppose that we know the $\nu^{th}$ iterate $V_{Lj}^{n+1,\nu}$, $p_j^{n+1,\nu}$ $V_{Vj}^{n+1,\nu}$. We obtain the $\nu+1^{st}$ iterate as follows:

(i) Using $p_j^{n+1,\nu}$ in the right hand side of (3.10) and $V_{Lj}^{n+1,\nu}$, $V_{Vj}^{n+1,\nu}$

in evaluating $B_j$'s, $D_j$'s and $S_j^V$'s, $S_j^L$'s we obtain a system of linear algebraic equations for $V_{Lj}^{n+1,\nu+1}$ $V_{Vj}^{n+1,\nu+1}$.

(ii) We solve these linear algebraic equations by the tridiagonal matrix algorithm. Owing to upwind differencing and a proper treatment of the inter-phase drag we have,

$0 \leqslant B_j^+ \leqslant 1$, $0 \leqslant D_j^+ \leqslant 1$, $0 \leqslant B_j^- \leqslant 1$, $0 \leqslant D_j^- \leqslant 1$. Furthermore, $B_j^+ + B_j^- \leqslant 1$. Thus these matrices are non-singular, since they are positive and diagonally dominant.

(iii) We obtain $p_j^{n+1,\nu+1}$ by writing

$$p_j^{n+1,\nu+1} = p_j^{n+1,\nu} + \delta p_j$$

Substitution in (3.10) gives (assuming $\delta p_j$ is 'small') in a 'diagonal' approximation,

$$\delta V_{Vj}^{n+1} = \frac{\partial S_j^V}{\partial p}^{n+1,\nu} \left(\delta p_j - \delta p_{j+1}\right)$$

(iv) Substituting $V_{Vj}^{n+1,\nu+1} + \delta V_{Vj}^{n+1}$ etc. in the finite difference form of (2.7) gives a tridiagonal matrix system of the same form as (3.10) for the vector $\{\delta p_j\}$. The sources $S_j^p$ are simply the continuity 'errors'

$$S_j^P \equiv \left[\alpha_L(V_L + \delta V_L) + \alpha_V(V_V + \delta V_V)\right]_j^{+n+1}$$
$$- \left[\alpha_L(V_L + \delta V_L) + \alpha_V(V_V + \delta V_V)\right]_j^{-n+1} - \Delta y \left(\frac{\dot{m}_L}{\rho_L} + \frac{\dot{m}_V}{\rho_V}\right)_j \qquad (3.11)$$

The solution gives $\delta p_j$, in turn leading to an estimate for $p_j^{n+1,\nu+1}$.

The iteration is carried out until the continuity errors $S_j^P$ are sufficiently reduced and the $\delta p_j$ are negligible in comparison with $p_j^{n+1,\nu}$. This completes the solution at $t = n\Delta t$. The procedure is continued for as long as desired. This 'relaxation method' for obtaining the solution of the momentum equations together with a self-consistent pressure was originated for incompressible hydrodynamics by Spalding and co-workers (e.g. [5]) and used by one of us [6] for transonic turbomachinery calculations. The experience with the present algorithm indicates it to be a feasible technique for solving multiphase flow problems provided the $\alpha$'s are obtained using the method of positive faithful transformations described earlier.

## 4. TEST CASES

We briefly describe two model problems/test cases which have been solved.

Consider a cylindrical vessel of height H (3m) and radius R (1m) with impermeable walls, resting on one of its flat faces. We assume gravity to act vertically downwards. At $t = 0$, the vessel is filled with a mixture of liquid (density $\rho_L = 980$ kg/m$^3$) and a vapour ($\rho_V = 0.6$ kg/m$^3$). The initial conditions are: $\alpha_L = \alpha_V = \frac{1}{2}$, $0 \leq y \leq H$. $V_L = V_V = 0$. The pressure distribution is hydrostatic. It is assumed in the first test case that $\dot{m}_L = \dot{m}_V = 0$, implying that there is no phase transformation or mass sources. Thus the total mass of the liquid and the vapour must be <u>separately</u> conserved. The interphase drag force $F_{VL}$ is assumed to be given by the following law:

$$F_{VL} = \frac{\rho_V \alpha_V}{\tau_{VL}} (V_L - V_V), \qquad (4.1)$$

where

$$\frac{1}{\tau_{VL}} = a + b\left|V_L - V_V\right|, \quad a = 1.0, \; b = 150. \qquad (4.2)$$

$F_{LV}$ of course equals $-F_{VL}$. This law is an ansatz, adopted for this example, which may be appropriate to certain types of liquid-vapour mixtures. The evolution of the system is easy to understand physically. It is a strongly damped, nonlinear settling problem as the heavier liquid falls to the bottom of the vessel while expelling the lighter vapour to the top. A typical time scale for this is provided by $\sqrt{\frac{2H}{g}} \approx 0.8$ sec, though of course the drag slows down the actual process.

Figure 2 shows the profiles of the liquid volume fraction $\alpha_L$ as a function of $y$ at several times $t$. In this simulation, $\Delta y = \frac{H}{20}$ m. while $\Delta t = 5 \times 10^{-4}$ sec. Typical liquid velocities are $\sim \sqrt{gH}$. The profiles are produced at 0.2 sec intervals. After 1 second, the final steady state is nearly attained. The conservation of total mass (of liquid and vapour) is accurate to within the roundoff error. Typically 3 to 5 iterations per time step are needed to solve the momentum and pressure equations to reduce a meansquare, nondimensional estimate of local continuity error to less than 1%.

In the second test case, we consider a vessel initially filled with liquid up to y=H/2, the rest of the plenum being filled with vapour. At t=0, the velocities everywhere are assumed to be zero. We now consider $\dot{m}_V = \rho_V \alpha_L \lambda(t)$, $\dot{m}_L = -\dot{m}_V$ where the vapour production rate $\lambda(t)$ is of the form

$$\lambda(t) = \lambda_o (1 - e^{-t/\tau}). \tag{4.3}$$

As an example, we take $\lambda_o = 30.0 \text{sec}^{-1}$ and $\tau = 10^{-2}$ sec. At $y=H$, we set $V_L(H,t) = V_V(H,t)$, which is equivalent to full momentum equilibration and obtain $V_L(H,t)$ from the relation,

$$V_L(H,t) = \int_o^H \alpha_L(y,t) \left(1 - \frac{\rho_V}{\rho_L}\right) dy \; \lambda(t) \tag{4.4}$$

in its appropriate finite-difference form.

The results are shown in Figure 3. It is interesting to note that as the liquid turns into vapour, it is dragged upwards and a more or less homogeneous (in y) mixture forms which transforms in about 0.5 sec almost entirely into vapour. There is a little liquid left at the bottom of the vessel which becomes vapour more slowly. As before, the continuity errors are very small and the solution appears to be physically reasonable.

## 5. CONCLUSIONS

A numerical scheme has been developed for the calculation of one dimensional transient two phase flow. An algorithm has been developed to allow the governing equations to be advanced in time in such a manner that the volume fraction of each component is accurately calculated even in the presence of large spatial and temporal gradients of volume fraction. The momentum equations are solved using a pressure correction method based on Newton's method. Example calculations are presented to illustrate the application of the procedure to realistic problems.

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## Appendix 1: The coefficients in the finite differenced momentum equations

In this Appendix we give the coefficients for the finite differenced momentum equations. For the steam momentum equation we define $\tau = \tau_{LV}/C_{LV}$ and $\lambda$ as in equation 4.3 so that:

$$B_j^+ = -\ 1/2\ \Delta t(1-\Theta_+)\ \rho_V\ \alpha_{Vj+1}^{n+1/2}\ V_{Vj+1/2}^{n+1/2}\ /\ B_j^o \qquad \text{(A1)}$$

$$B_j^- = 1/2\ \Delta t\ \Theta_-\ \rho_V\ \alpha_{Vj}^{n+1/2} V_{Vj-1/2}^{n+1/2}\ /\ B_j^o \qquad \text{(A2)}$$

$$\begin{aligned} S_j^V = \{&\Delta y\, \rho_V \alpha_{Vj+1/2}^{n}\ V_{Vj}^{n} - B_{j+1}^- V_{Vj+1}^{n} \\ &- B_{j+1}^+ V_{V\ j-1}^{n} + B_j^+\ V_{V\ j+1}^{n} + B_j^-\ V_{Vj-1}^{n} \\ &- \alpha_{Vj+1/2}^{n+1/2}\ \Delta t(\bar{p}_{j+1}^{n+1} - \bar{p}_j^{n+1}) \\ &+ (\rho_L - \rho_V) g\ \alpha_{V\ j+1/2}^{n+1/2}\ (1 - \alpha_{Vj+1/2}^{n+1/2})\ \Delta y\ \Delta t \\ &+ \Delta y\ \Delta t\ \rho_V\ \alpha_{V\ j+1/2}^{n+1/2}\ V_{L\ j}^{n+1/2}\ /\ \tau_{j+1/2}^{n} \\ &+ \Delta y\, \Delta t\ \rho_V (1-\alpha_{Vj+1/2}^{n+1})\ V_{L\ j}^{n+1}\ \lambda^{n+1}\}/B_j^o \end{aligned} \qquad \text{(A3)}$$

$$B_j^o = \Delta y\, \rho_V \alpha_{Vj+1/2}^{n+1} + B_{j+1}^- + B_{j-1}^+ + \rho_V \Delta y\ \Delta t\ \alpha_{Vj+1/2}^{n+1/2}\ /\ \tau_{j+1/2}^{n} \qquad \text{(A4)}$$

The coefficients for the finite differenced momentum equation for the water phase can be obtained by permuting the suffices V and L in the above equations and modifying the term due to vaporisation. The final term in equation (A3) should be removed and the term $\rho_V \Delta y \Delta t \alpha^{n+1/2}_{Lj+1/2} \lambda^{n+1}$ should be added to the central coefficient $B^o_j$.

The form of the evaporation reaction force term is not symmetric because we only consider the situation where water transforms into steam. Thus since the evaporation reaction force is proportional to $V_L$ this term is treated implicitly in the water phase momentum equation and explicitly in the vapour phase momentum equation. The extension of the scheme to incorporate other forms for this force, as well as the inter-phase drag law, is straightforward.

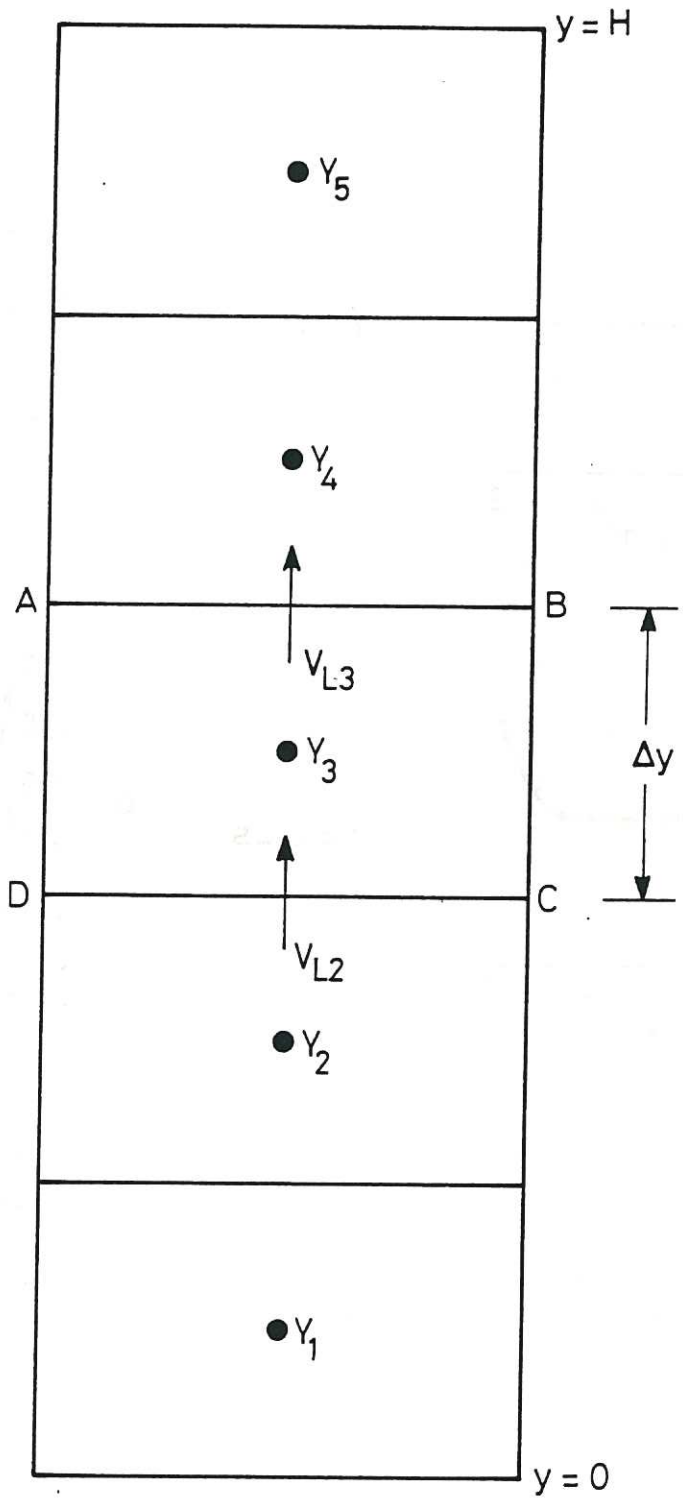


Fig .1 . Typical grid arrangement

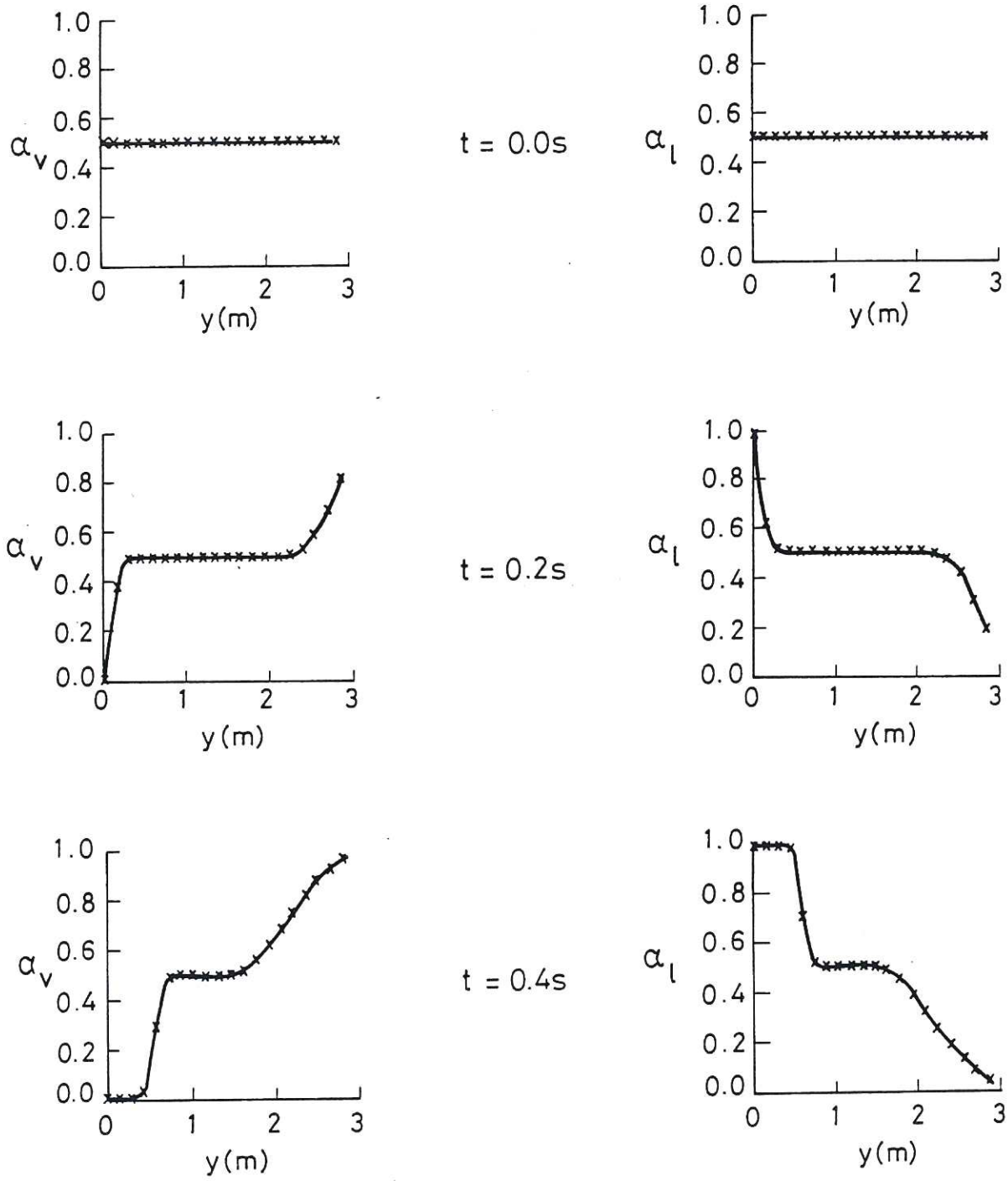


Fig. 2a. Settling problem

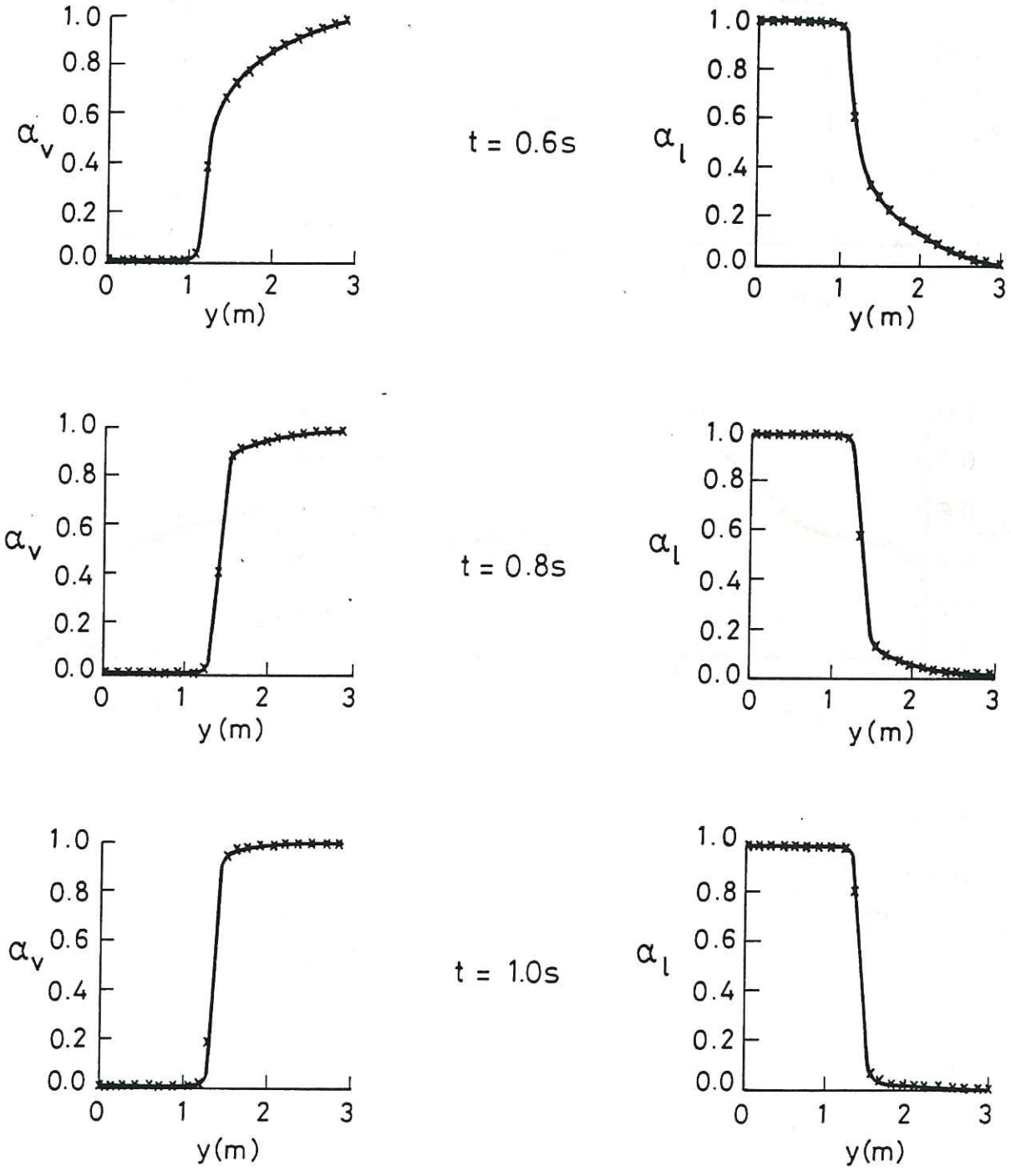


Fig. 2b. Settling problem

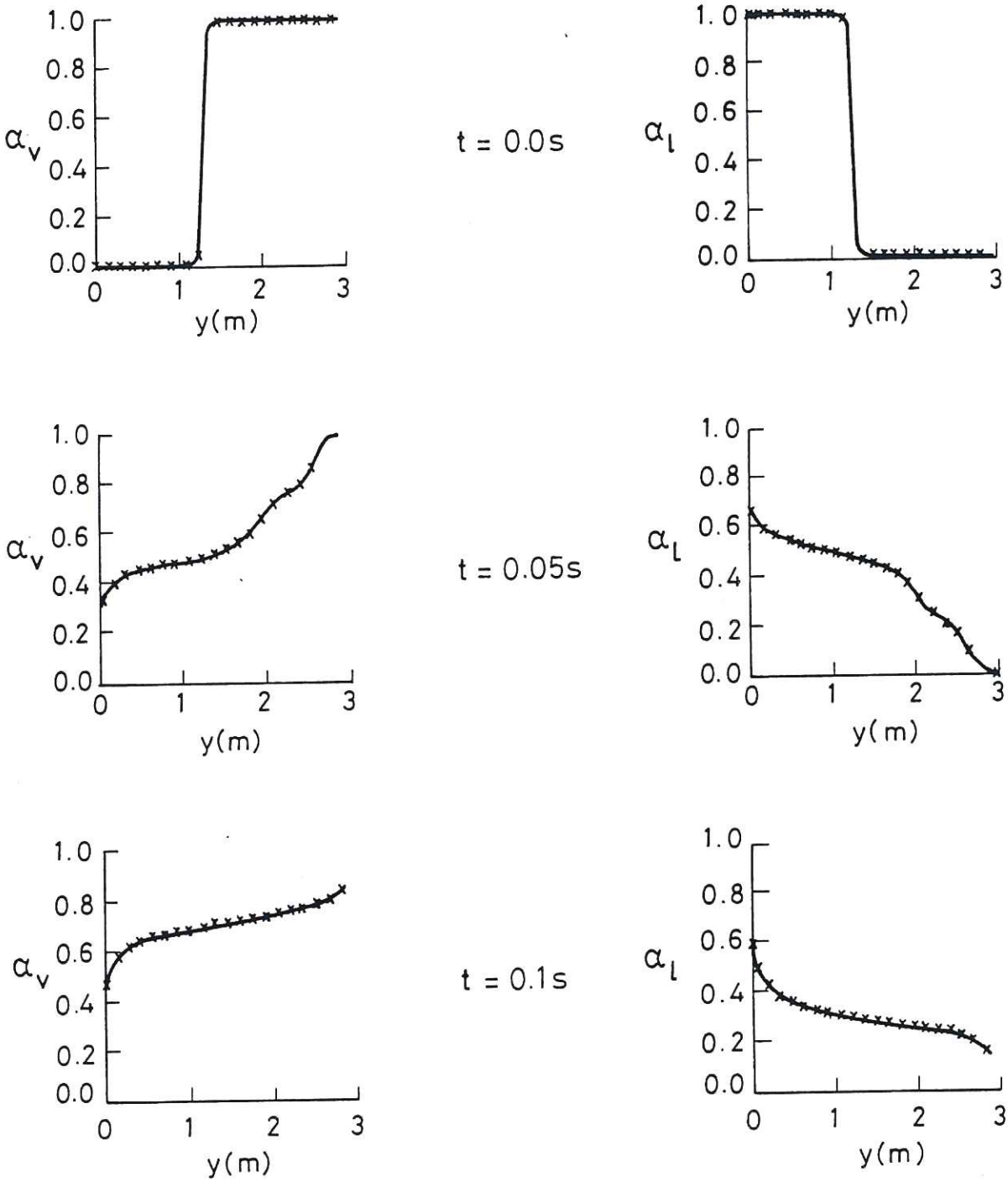


Fig.3a. Water-steam transformation problem

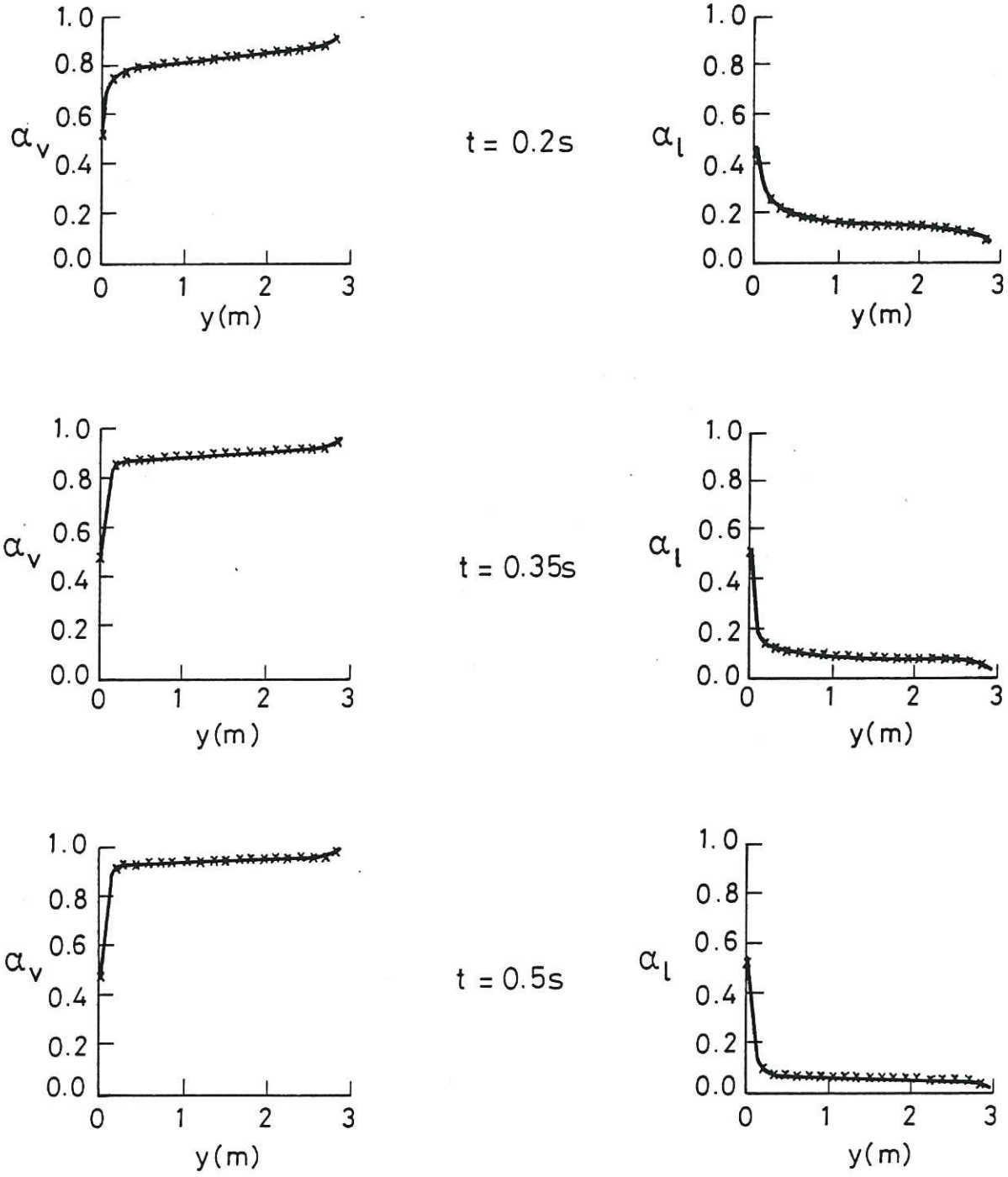


Fig. 3b. Water - steam transformation problem