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by

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Abstract

The effect of electromagnetic turbulence on electrons and ions under tokamak conditions is considered using a kinetic description. Taking the magnetic fluctuation spectrum as given, the density fluctuation spectrum is self-consistently calculated taking account of quasi-neutrality. The calculation is valid for arbitrary collisionality and appropriate to low frequencies typical of experiment. In addition to the usual enhancement of the radial electron energy transport, it is found that the turbulent fluctuations can heat the plasma at rates comparable to ordinary ohmic heating under some conditions. Interestingly, electromagnetic turbulence appears to imply only an insignificant correction to the toroidal resistance of the plasma as estimated from Spitzer resistivity. The scalings of anomalous transport, density fluctuations and heating with temperature and plasma volume are investigated. Although the turbulent heating is small compared to the ohmic power in TFR, the assumption that the magnetic fluctuation spectrum of the turbulence is invariant leads to the interesting result that the turbulent heating can be comparable or larger than the ohmic power in JET-like plasmas.

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1. Introduction

It is generally believed that turbulence in tokamaks is responsible for the observed anomalous transport of particles and energy. The purpose of the present paper is to investigate the effects to be expected from a given spectrum of magnetic turbulence in the presence of ohmic heating in tokamaks. We consider a quasi-neutral, kinetic description of electrons and ions subjected to turbulence with frequencies small compared to ω_{ci} and perpendicular wavelengths of the order or larger than the ion larmor radius. Collisions are fully accounted for using appropriate Fokker-Planck collision model operators. In the past, attention has generally focussed on the electrostatic and electromagnetic losses due to turbulence. In the present paper we give a more complete account of the energetics and in particular exhibit explicitly the local heating effects of the turbulence. We also discuss the global consistency of the formulation as a whole by relating it to the ohmic input (IV). Under certain conditions, subject to hypotheses relating to the turbulent spectrum, we show that the heating effects of turbulence can to some extent counterbalance the losses due to the turbulence and result in interesting observable consequences for the temperature profiles.

In previous papers, we have discussed the magnetic turbulence interpretation of anomalous electron thermal diffusivity from both the two-fluid (HAAS and THYAGARAJA 1984) and test-particle (THYAGARAJA et al 1985) points-of-view. In the latter, the discussion was restricted solely to the effect of specified magnetic fluctuations on the energy transport of <u>electrons</u>. Thus, ignoring all drifts, we were able to calculate the contribution to $\chi_{\perp e}$ from magnetic fluctuations under TFR-like conditions. The discussion did not include ion dynamics and the effects of quasi-neutrality. Thus we were not able to account for ambipolar particle transport and the possible heating effect of the fluctuating electric fields doing work on the fluctuating currents.

Here, we further develop the test-particle approach to formulate a kinetic description for both ions and electrons. By taking the appropriate moments we are led to a form of radial energy balance which shows both ohmic and turbulent heating terms. To assess the relative magnitudes of these terms we apply our theory to TFR and JET. Assuming the magnetic fluctuation spectrum to be given we derive levels for the density fluctuations which agree with those in TFR. Then using this magnetic fluctuation spectrum we evaluate the electron thermal diffusivity $\chi_{\perp e}$ and find this to agree with the $\chi_{\perp e}$ as determined by the TFR group from their transport codes. We find the turbulent heating term to be small compared with the ohmic dissipation. However, making the heuristic assumption that the magnetic fluctuation spectrum is the same in JET, we find that the turbulent power can be comparable or larger than the ohmic power. The magnitude of the effect is principally a function of temperature, the turbulent term scaling like $\frac{1}{T_e^{1/2}}$ while the ohmic power scales like $\frac{1}{T_e^{3/2}}$. The effect of this term is to off-set to some extent the turbulent losses expected for the system. For these conditions toroidal resistivity is essentially Spitzer. Section 2 describes the basic model while Section 3 considers the perturbation analysis. Numerical results are presented in Section 4, and Sections 5 and 6 contain the discussion and conclusions respectively.

2. Model

For present purposes we consider a periodic cylinder to be adequately representative of tokamak geometry. The electromagnetic turbulence spectrum is assumed to be of low frequency, typically of order the drift-frequency ω^* ($\lesssim$ 1/2 MHz), with mode numbers m, n such that $\frac{m}{a}\,\rho_i \sim 1$, where ρ_i is the ion-larmor radius. The fluctuations are assumed to be essentially in the poloidal plane, that is, $\delta B_z \ll \delta B_r$, δB_θ, where r, θ, z are the usual cylindrical coordinates. Experimental evidence for plasma turbulence having the above properties is cited by LIEWER (1985). In our previous paper (THYAGARAJA et al (1985)) assuming the magnetic fluctuation spectrum as given, we calculated the density fluctuations and found them to be consistent with the results observed in TFR. We subsequently found that this choice of magnetic spectrum also yielded the electron thermal diffusivity profile, $\chi_{\perp e}(r)$, obtained by the TFR group from transport code studies. Thus encouraged we shall assume the same magnetic turbulence spectrum to be appropriate in the present study.

Taking quasi-neutrality to apply at all times, the electron and ion distribution functions $f_{i,e}(\underline{r}, v_{\parallel}, t)$ are assumed to depend only on the parallel velocity components. The perpendicular velocities of both species are obtained from the leading-order drift approximation, which for electrons is

$$\underline{v}_{de} = \frac{c}{B^2}\left(\underline{E} + \frac{\nabla p_e}{en}\right)\times\underline{B}, \tag{1}$$

and a similar expression for the ions. With these approximations the Fokker-Planck equations take the following forms

$$\frac{\partial f_e}{\partial t} + v_{\parallel}\,\nabla\cdot(\underline{b}f_e) + \nabla\cdot(\underline{v}_{de}f_e) - e\frac{E_{\parallel}}{m_e}\frac{\partial f_e}{\partial v_{\parallel}} = C(f_e,f_e) + C(f_e,f_i) \tag{2}$$

$$\frac{\partial f_i}{\partial t} + v_{\parallel}\,\nabla\cdot(\underline{b}\,f_i) + \nabla\cdot(\underline{v}_{di}f_i) + e\frac{E_{\parallel}}{m_i}\frac{\partial f_i}{\partial v_{\parallel}} = C(f_i,f_i) + C(f_i,f_e) \tag{3}$$

Defining B_0 to be the resultant equilibrium magnetic field, that is $B_0^2 = B_{0_\theta}^2 + B_{0_z}^2$, then it is convenient to normalise the instantaneous magnetic field with respect to this quantity. Thus we write

$$\underline{b} = \underline{b}_0 + \frac{\delta\underline{B}}{B_0}, \tag{4}$$

where $\underline{b}_0 = \left(\frac{B_{0_\theta}}{B_0}, \frac{B_{0_z}}{B_0}\right)$. Furthermore, defining the vector potential δA_z, the electrostatic potential $\delta\phi$ and the external electric field E_{0_z}, we have

$$\delta B_r = \frac{1}{r}\frac{\partial}{\partial\theta}\,\delta A_z \quad \text{and} \quad \delta B_\theta = -\frac{\partial}{\partial r}\,\delta A_z \tag{5}$$

and

$$\underline{E} = E_{0_z}\,\underline{z} - \frac{1}{C}\frac{\partial}{\partial t}\,\delta A_z\,\underline{z} + \nabla\delta\phi\ . \tag{6}$$

The electrostatic potential is determined through the quasi-neutrality condition. In the work that follows we regard $B_{0_\theta}(r)$, $B_{0_z}(r)$ and $\delta B_r(r,\theta,z,t)$ as specified quantities.

To make further progress we approximate the full Fokker-Planck collision terms by model forms which ensure the correct conservation properties (GREENE, 1973), and also include classical or neoclassical radial transport explicitly. Thus we write

$$C(f_e,f_e) = \nu_{ee}\frac{\partial}{\partial v_\parallel}\left(v_{the}^2\frac{\partial f_e}{\partial v_\parallel} + (v_\parallel - \bar{v}_{\parallel e})f_e\right), \tag{7}$$

with a similar expression for $C(f_i,f_i)$.
In this form

$$\bar{v}_{\parallel e} = \frac{\int dv_\parallel v_\parallel f_e}{\int dv_\parallel f_e} \quad \text{and} \quad v_{the}^2 = \frac{\int dv_\parallel (v_\parallel - \bar{v}_{\parallel e})^2\, f_e}{\int dv_\parallel\, f_e} = \frac{T_e}{m_e} \tag{8}$$

with similar definitions for ions. We write the electron-ion collision term as

$$C(f_e,f_i) = \nu_{ei}\frac{\partial}{\partial v_\parallel}\left(C_{the}^2\frac{\partial fe}{\partial v_\parallel} + (v_\parallel - \bar{v}_{\parallel i})f_e\right) \tag{9}$$

where

$$c_{the}^2 = \left(\frac{m_i - m_e}{m_i + m_e}\right) v_{the}^2 + \frac{2m_i}{m_i + m_e} \cdot v_{thi}^2 + (\bar{v}_{\parallel i} - \bar{v}_{\parallel e})^2 \quad (10)$$

In the above ν_{ee} denotes the electron-electron Braginskii 90° collision frequency; $\nu_{ii} = \left(\frac{m_e}{m_i}\right)^{1/2} \nu_{ee}$, $\nu_{ie} = \frac{m_e}{m_i} \nu_{ei}$ and $\nu_{ei} \sim \nu_{ee}$.

In the perturbation analysis that follows it is convenient to write the distribution functions as

$$f_{e,i} = f_{oe,i}(r, v_\parallel) + \delta f_{e,i}(\underline{r}, v_\parallel, t) \quad (11)$$

where

$$f_{oe,i} = \frac{n_o(r)}{\sqrt{2\pi}} \frac{1}{v_{the,i}(r)} \exp(-x^2/2) \quad (12)$$

with $x = \dfrac{v_\parallel - \bar{v}_{\parallel e,i}(r)}{v_{the,i}(r)}$ and the electron and ion temperatures in the mean-state related to the thermal velocities by $v_{the,i}^2 = \dfrac{T_{oe,i}}{m_{e,i}}$

3. Perturbation Analysis

The perturbation analysis of the Fokker-Planck equations, Eqs(2) and (3), and the subsequent taking of moments, follows very closely the procedures described in our previous studies (THYAGARAJA et al, 1985). Thus averaging the fluctuations over Θ, z and t (HAAS and THYAGARAJA, 1984) leads to the following radial energy equation for the electrons:

$$\frac{1}{r}\frac{d}{dr}(rQ_{\perp e}) + e\langle \delta E_\parallel \int dv_\parallel \delta f_e (v_\parallel - \bar{v}_{\parallel e})\rangle$$

$$= \frac{1}{r}\frac{d}{dr}\left(r \frac{\rho_e^2}{4\tau_e} \frac{1}{2} \frac{d}{dr} p_{oe}\right) + \frac{2\nu_{ei} m_e n_o (T_{oi} - T_{oe})}{m_i + m_e} + \eta_o j_o^2 \quad (13)$$

where

$$\left.\begin{aligned} Q_{\perp e} &= \left\langle \frac{\delta B_r}{B_o} \delta Q_{e\parallel} \right\rangle + \left\langle \delta v_{er} \frac{\delta p_e}{2} \right\rangle \\ \text{and} \quad \delta Q_{e\parallel} &= \frac{1}{2} m_e \int dv_\parallel df_e \, v_\parallel (v_\parallel - \bar{v}_{\parallel e})^2 \\ \delta p_e &= m_e \int dv_\parallel \, \delta f_e \, (v_\parallel - \bar{v}_{\parallel e})^2 , \end{aligned}\right] \tag{14}$$

with τ_e defined to be $\tau_e^{-1} = \nu_{ee} + \nu_{ei}$ and ρ_e is the electron larmor radius. We note that the effective perpendicular (radial) heat flux $Q_{\perp e}$ comprises two terms. The first - proportional to $\delta Q_{e\parallel}$ - is the electron heat transport due to magnetic fluctuations (THYAGARAJA et al, 1985). The second term-proportional to δv_{er} - is due to the effective radial convection of the electrons. The term involving $\delta E_\parallel$ in Eq (13) clearly represents the rate of working of the fluctuating parallel electric field on the electrons; on the other hand, the perpendicular component of the electric field (due to $\delta\phi$) does no work on the electrons, and hence does not appear. Turning to the right-hand side of Eq.(13) we note that the terms represent classical diffusion, classical equilibration and ohmic heating due to mean currents, respectively. In an appendix we consider the parallel momentum balance in the presence of turbulence for conditions typical of tokamaks (eg.TFR & JET) and show that the parallel electric field is given by

$$E_{\parallel o} = \frac{m_e n_o}{\tau_{ei}} (\bar{v}_{\parallel i} - \bar{v}_{\parallel e}), \tag{15}$$

to a good approximation.

Although the above form of the energy equation has a ready interpretation, it is possible to cast Eq (13) into an alternative form which is far more insightful. In order to obtain this new form, we derive an identity for the $\delta E_{\parallel}$ term. Thus we begin by writing down the linearised electron Fokker-Planck equation, namely,

$$\frac{\partial}{\partial t}\delta f_e + v_{\parallel}\,\underline{b}_o\cdot\nabla\delta f_e + \frac{\delta B_r}{B_o}v_{\parallel}\frac{df_{oe}}{dr} + \delta v_{der}\frac{df_{oe}}{dr} - \frac{e\,\delta E_{\parallel}}{m_e}\frac{\partial f_{oe}}{\partial v_{\parallel}}$$

$$= \frac{1}{\tau_e}\frac{\partial}{\partial v_{\parallel}}\left(v_{the}^2\frac{\partial}{\partial v_{\parallel}}\delta f_e + (v_{\parallel} - \bar{v}_{\parallel e})\delta f_e\right), \tag{16}$$

and where we note that $C_{the}^2 \simeq v_{the}^2$, $\bar{v}_{\parallel e} \ll v_{the}$.

In the work that follows we make this approximation. Multiplying this equation by $\frac{\delta f_e}{f_{oe}}$ and integrating over $dv_{\parallel}$ and averaging over the turbulent fluctuations as before, we derive the identity

$$e\langle\delta E_{\parallel}\int\delta f_e\,(v_{\parallel} - \bar{v}_{\parallel e})dv_{\parallel}\rangle$$

$$= -T_{oe}\frac{d}{dr}\left(\ln\frac{n_o}{v_{the}}\right)\langle\int(v_{\parallel}\frac{\delta B_r}{B_o} + \delta v_{der})\delta f_e\,dv_{\parallel}\rangle$$

$$-\frac{1}{T_{oe}}\cdot\frac{dT_{oe}}{dr}\langle\int(v_{\parallel}\frac{\delta B_r}{B_r} + \delta v_{der})\frac{m_e}{2}(v_{\parallel} - \bar{v}_{\parallel e})^2\,\delta f_e dv_{\parallel}\rangle$$

$$-\frac{1}{\tau_e}T_{oe}\,v_{the}^2\langle\int f_{oe}\left(\frac{\partial}{\partial v_{\parallel}}\left[\frac{\delta f_e}{f_{oe}}\right]\right)^2 dv_{\parallel}\rangle \tag{17}$$

Substituting in Eq (13) we obtain

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{Q_{\perp e}}{T_{oe}}\right) - \frac{1}{n_o}\frac{dn_o}{dr}\Gamma_{\perp e} + \frac{1}{2T_{oe}}\frac{dT_{oe}}{dr}\Gamma_{\perp e} - \frac{1}{T_{oe}}\frac{1}{r}\frac{d}{dr}\left(\frac{r\rho_e^2}{4\tau_e}\cdot\frac{1}{2}\frac{dp_{oe}}{dr}\right)$$

$$= \frac{n_o}{\tau_e\sqrt{2\pi}} < \left(\frac{\delta n}{n_o}\right)^2 > F_e(r)$$

$$+ \frac{2\nu_{ei}}{T_{oe}}\frac{m_e}{m_i} n_o(T_{oi} - T_{oe}) + \frac{\eta_o j_o^2}{T_{oe}}, \tag{18}$$

where $\Gamma_{\perp e}$ is the radial particle flux, that is

$$\Gamma_{\perp e} = <\int\left(v_\parallel\frac{\delta B_r}{B_o} + \delta v_{der}\right)\delta f_e dv_\parallel> \tag{19}$$

The form factor $F_e(r)$ is given by

$$F_e(r) = \frac{<\int \exp(-x^2/2)\left(\frac{d}{dx}\left(\frac{\delta f_e}{f_{oe}}\right)\right)^2 dx>}{<\left(\frac{\int \delta f_e dx}{\int f_{oe} dx}\right)^2>} \tag{20}$$

We now give a discussion of the local and global energetics implications of Eqs.(13) and (18). In order to discuss the global considerations first, it is useful to add Eq.(13) and the corresponding ion radial energy balance equation. If we integrate the resulting equation over the volume, we obtain the following identity:

$$4\pi^2 aR\left[Q_{\perp e}(a) + Q_{\perp i}(a) + Q_{\perp e}^{classical}(a) + Q_{\perp i}^{classical}(a)\right]$$

$$= \int \eta_o j_o^2\, d\tau + \int <\delta \underline{E}_\parallel \cdot \delta \underline{j}_\parallel> d\tau$$

where $Q_{\perp e}$ and $Q_{\perp i}$ are defined as in Eq.(14) and the classical fluxes are as defined in Eq.(13). Using Maxwell's equations we obtain as usual the Poynting relation

$$\frac{c}{4\pi}\int \underline{n}\cdot\underline{B}\times\underline{E}\, dS = \frac{\partial}{\partial t}\int\frac{B^2}{8\pi}\, d\tau + \int\left(\underline{j}_o\cdot\underline{E}_o + \underline{j}_o\cdot\underline{\delta E} + \underline{E}_o\cdot\underline{\delta j} + \underline{\delta E}\cdot\underline{\delta j}\right)d\tau$$

Time averaging this equation, making use of Eqs.(6) and (15), and evaluating the left hand side we get the relation

$$IV = \int \eta_o j_o^{\,2}\, d\tau + \int\langle\, \underline{\delta j}_{\parallel}\cdot\underline{\delta E}\rangle\, d\tau.$$

Substitution in the first equation above demonstrates that the energy flow out of the system equals the total input IV, and hence the global consistency of our formulation. The second relation proves that $\int\langle\underline{\delta j}_{\parallel}\cdot\underline{\delta E}_{\parallel}\rangle d\tau < IV$. The relative size of this term compared with the joule heating term $\int\eta_o j_o^{\,2} d\tau$ depends of course on the amplitude of the turbulent fluctuations. There is no <u>a priori</u> reason why the two integrals cannot be of comparable order. It is apparent that if IV = 0 there would be no turbulence and Eqs.(13) and (18) would be trivially satisfied with all the terms being individually zero.

We now consider the local implications of Eqs.(13) and (18). It is important to compare Eq.(13) with that written by experimentalists. This latter equation is always of the form (for the present conditions)

$$\frac{1}{r}\frac{d}{dr}\left(rQ_{\perp e}^{exp}(r)\right) = \frac{1}{r}\frac{d}{dr}\left(r\,\frac{\rho_e^2}{4\tau_e}\,\frac{1}{2}\frac{d}{dr}\,p_{oe}\right)$$

$$+\ \frac{2\nu_{ei}m_e n_o}{m_e + m_i}\left(T_{oi} - T_{oe}\right)$$

$$+\ \eta_o j_o^{\,2}.$$

The main result of our paper is that locally $Q_{\perp e}^{exp}(r)$ must not be confused with $Q_{\perp e}(r)$ as defined in Eq.(13). This is so, since the term $e\langle\delta E_{\parallel}\int dv_{\parallel}\delta f_e(v_{\parallel}-\bar{v}_{\parallel e})\rangle$ locally can make a significant contribution in

Eq.(13). The main content of Eq.(17) is to show that this term consists of two pieces: a negative definite part which can be identified with the heating effect of turbulence, and another piece related to $Q_{\perp e}$ and the turbulent particle flux $\Gamma_{\perp e}(r)$. Eq.(18) represents this new decomposition of Eq.(13). The structure of Eq.(18) is readily identifiable as the local form of the second law of thermodynamics taking account of ordinary joule heating ($\eta_o j_o{}^2$) and the entropy production due to turbulence coming from the term expressed in Eq.(20). The net effect of this analysis is to show that the transport losses embodied in $Q_{\perp e}$ are off-set to some extent - depending on the turbulent spectrum - by the heating effects of the turbulence. As we shall see later $\int\langle\delta\underline{E}_{\parallel}\cdot\delta\underline{j}_{\parallel}\rangle d\tau$ can be the result of a cancellation between two individually significant contributions.

We now turn to the relation linking $\frac{\delta n}{n_o}$ and $\frac{\delta B_r}{B_o}$ implied by Eq. (16), and the formal expression for the effective electron perpendicular thermal diffusivity. Eq (16) is solved by Fourier analysis where we take $\frac{\delta B_r}{B_o}$ to be

$$\frac{\delta B_r}{B_o} = \sum_{m,n} \int d\omega\, b_{mn}(r,\omega)\, \exp\left(i\left[m\theta + \frac{nz}{R} + \omega t\right]\right) \tag{21}$$

The perturbed electron distribution function δf_e then has the expansion

$$\delta f_e = \exp(-x^2/_4) \sum_{m,n} \int d\omega\, F^e_{mn}\,(r,\omega) \exp\left(i\left[m\theta + \frac{nz}{R} + \omega t\right]\right) \tag{22}$$

where

$$x = \frac{v_\parallel - \bar{v}_{\parallel e}\,(r)}{v_{the}(r)}.$$ Substituting in Eq (15)

F^e_{mn} is given by

$$F^{e}_{mn} = \frac{n_o \tau_e}{\sqrt{2\pi}\, v_{the}} \exp(-x^2/4) \{\bar{X}^{e}_{mn}(r,x,\omega) \frac{d}{dr} \left(\log\frac{n_o}{v_{the}}\right) \delta v_{dre}(r,\omega,m,n)$$

$$+ \bar{Y}^{e}_{mn} \frac{d}{dr}(\log v_{the})\, \delta v_{dre} + \bar{W}^{e}_{mn}\left(\frac{e}{m_e v_{the}} \delta E_{\parallel} + v_{the} \frac{d}{dr}(\log n_o) b_{mn}\right)$$

$$+ \bar{Z}^{e}_{mn}\left(b_{mn}\, v_{the} \frac{d}{dr} \log v_{the}\right)\} \tag{23}$$

where δv_{dre} and $\delta E_{\parallel}$ are the fluctuated forms of Eqs (6) and (1), taking the appropriate Fourier components. The amplitudes $\bar{X}^{e}_{mn}$, $\bar{Y}^{e}_{mn}$, $\bar{W}^{e}_{mn}$ and $\bar{Z}^{e}_{mn}$ are functions of r, ω and the velocity space variable x. They satisfy the following inhomogeneous linear differential system of equations:

$$\left\{\frac{d^2}{dx^2} + 1/2 - \frac{x^2}{4} - i\tau_e \left[\omega + \frac{\bar{v}_{\parallel e}}{q(r)R}\left(m + nq(r)\right) + \left(\frac{m}{r} v_{d\theta e} + \frac{n v_{dze}}{R}\right)\right]\right\}$$

$$\times \begin{bmatrix} \bar{X}^{e}_{mn} \\ \bar{Y}^{e}_{mn} \\ \bar{W}^{e}_{mn} \\ \bar{Z}^{e}_{mn} \end{bmatrix} = \exp(-x^2/4) \begin{bmatrix} 1 \\ x^2 \\ x \\ x - x^3 \end{bmatrix} \tag{24}$$

The ions satisfy a similar system with $\tau_i^{-1} = \nu_{ii}$. This system of equations is numerically solved to obtain the functions $\bar{X}^{e}_{mn}$ etc at every r, m, n and ω. Then substituting in Eq(23) and its ion equivalent, and taking moments, it is possible to obtain the linking relations connecting the Fourier components of $\frac{\delta n}{n_o}$, $\delta\phi$ and b_{mn}. Although these relations are simple to obtain, the algebraic formulae are complicated and offer no insight; they are evaluated numerically in the next section. In contrast to our earlier work, the present theory is fully consistent with quasi-neutrality. Substituting these expressions in Eqs (14) and (20), one can evaluate the heat flux and entropy production due to the assumed spectrum of field fluctuations. The heat-flux $Q_{\perp e}$ naturally takes the form

$$Q_{\perp e}(r) \equiv -n_o \chi_{\perp e} \frac{dT_{oe}}{dr} + n_o T_{oe} U_{Te} \tag{25}$$

which defines $\chi_{\perp e}$. We note from Eq (23) that $\chi_{\perp e}$ essentially arises from the term proportional to $\bar{Z}^e$.

4. Numerical Results

We now present results for two strongly contrasted experimental conditions, namely TFR and JET. The magnetic fluctuation spectrum and the q-profile $\left(q(r) = 1 + 3^{r^2}/_{a^2}\right)$ are taken as identical. The fluctuating spectrum is

$$\frac{\delta B_r}{B_o} = \frac{\varepsilon r}{2a} \sum_{m=1}^{m_{max}} \sum_{n=1}^{m} \exp\left(-\frac{1}{4}(m - nq)^2\right)\cos(m\theta - \frac{nz}{R} + m\omega^* t), \tag{26}$$

the form taken in our previous work. The frequency $\omega^* = 5\times10^3$ rad sec^{-1}, $\varepsilon = 3\times10^{-5}$ and $m_{max} = 80$. Other relevant parameters are given in Table I. These together with Eq(26) lead to the results given in Table II. Thus we calculate at $^r/_a = 0.5$, which is a representative point, the following quantities:

$$\chi_{\perp e},\ \left(<(\frac{\delta B_r}{B_o})^2>\right)^{1/2},\ <(\frac{\delta n}{n_o})^2>^{1/2},\ \eta_o j_o^2 \text{ and } P_{turb} = \frac{n_o T_{oe}}{\tau_e\sqrt{2\pi}} <(\frac{\delta n}{n_o})^2> F_e(r).$$

The calculated $\chi_{\perp e}$ (3.9×10^3 cm^2 s^{-1}) for TFR agrees reasonably well with the experimental value of 4.5×10^3 cm^2 s^{-1} (EQUIPE TFR, 1983), indicating that the choice of ε is appropriate for this case. It is then a test of the theory to check whether the predicted density fluctuation level is also in agreement with experiment. This indeed turns out to be the case as comparison with the experimental result of TFR shows. It is useful to note that the ohmic heating power density $\eta_o j_o^2$ due to the mean current is much larger than P_{turb} in TFR, indicating that significant turbulent heating does not occur.

Table I also gives the parameters appropriate to a 5.0 keV ohmic plasma at 2 MA and 5.0×10^{13} cm^{-3} in JET. It is a consequence of the currents and the geometries involved that the ohmic power density is much smaller than in TFR. Table II presents the calculated values of $\chi_{\perp e}$etc for JET, for the same magnetic spectrum and q-profile as in TFR. It is immediately seen that the $\frac{\delta n}{n_o}$ - fluctuation level in JET is slightly lower than in TFR, and yet the $\chi_{\perp e}$ is a factor of 5.0 larger. P_{turb} is of the same order as for TFR, despite the fact that the electron collision time in JET (see Table I) is nearly a factor of 10.0 larger. Thus in JET, the turbulent heating power is very important in balancing the larger values of $\chi_{\perp e}$. From the calculated quantities we can form the non-dimensional ratio $\frac{a^2}{4 n_o T_{oe} \chi_{\perp e}} (\eta_o j_o^2 + P_{turb})$, which is a measure of the self-consistency of the calculation. In both cases this quantity is close to unity, indicating that the assumptions regarding the power spectrum are compatible with the observed density and temperature profiles. Summarising, the results of the above calculations show that the electromagnetic turbulence results in the following effects in a tokamak plasma: (1) It can lead to enhanced perpendicular thermal and particle transport (c.f.Eqs.(14) and (19)), (2) The enhanced energy transport is offset by turbulent heating due to fluctuating electric fields and currents (c.f.Eq.(20)), (3) It apparently does not result in a significant correction to toroidal resistivity as estimated from Spitzer (see Appendix).

In order to examine the scaling of various quantities with temperature at fixed density n, current I and magnetic fluctuation spectrum, we have made a series of runs for JET conditions with central electron temperatures varying from 1.0 to 10.0 keV. The density and temperature profiles are assumed to be gaussian in $^r/_a$. Fig 1 shows plots of various quantities at $^r/_a = 0.5$. We note that P_{ohm} scales like $T_e^{-3/2}$, as expected, but P_{turb} exhibits a maximum at 2.4 keV, as does $\chi_{\perp e}$ (at 4kev).

This is a characteristic - the Knudsen maximum+ - which we have reported previously. The density fluctuation level is virtually constant indicating that the assumption of invariant magnetic and density fluctuation spectra (with respect to variations of T_e at constant n and I) are equivalent. Furthermore, $\chi_{\perp e}$ and the form factor F_e also show maxima. Our results show that the turbulent heating term can be more important than the background ohmic dissipation at higher temperatures; the effect also appears to be greater in larger machines. It should be remembered that we have assumed the power spectrum to be that necessary to correctly interpret TFR. As we shall discuss later on, some supporting evidence for this follows from the ideas of profile-consistency.

We have also considered the scaling of the turbulent heating with temperature, for given I_p, B_{tor}, n_o and power spectrum. This scaling can also be viewed as a variation of collisionality at fixed density. Our study shows that the density fluctuation level at constant magnetic power spectrum is relatively insensitive to changes in electron temperature. The turbulent power and $\chi_{\perp e}$ vary somewhat and have maxima. For large temperature, the turbulent heating scales like $\frac{1}{T_e^{1/2}}$.

It must be clearly noted that the effective $\chi_{\perp e}$ measured by experimentalists for a given external heating source is not necessarily the $\chi_{\perp e}$ calculated from the above theory, especially for JET-like conditions. Thus the experimentalist's energy balance equation is

essentially, (i.e. $Q_{\perp e}^{exp}(r) \equiv - n_o \chi_{exp} \frac{dT_{oe}}{dr}$),

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+In the gas dynamics literature the phenomenon is often referred to as a "Knudsen minimum". The difference in nomenclature arises from the fact that when a flux is a minimum, the associated transport coefficient is a maximum.

$$\frac{1}{r}\frac{d}{dr}(rn_o\,\chi_{exp}\frac{dT_{oe}}{dr}) + P_{ext}(r) = 0 \tag{27}$$

where P_{ext} represents the background ohmic heating, equilibration, radiative loss etc. Knowing n_o and T_{oe} Eq (27) is used to "measure" $\chi_{exp}(r)$. Eq (18) on the other hand, leads to the theoretical form

$$-\frac{1}{r}\frac{d}{dr}(rQ_{\perp e}) + W_{turb}(r) + P_{ext}(r) = 0 \tag{28}$$

where

$$Q_{\perp e}(r) = -n_o\chi_{\perp e}\frac{dT_{oe}}{dr} + n_oT_{oe}\,U_{Te}(r) \tag{29}$$

$$\Gamma_{\perp e}(r) = -D_{\perp e}\frac{dn_o}{dr} + n_o\,U_n(r) \tag{30}$$

$$W_{turb}(r) = P_{turb} + \frac{1}{T_{oe}}\,\frac{dT_{oe}}{dr}\,Q_{\perp e} + \frac{T_{oe}}{n_o}\,\frac{dn_o}{dr}\,\Gamma_{\perp e} - \frac{1}{2}\,\frac{dT_{oe}}{dr}\,\Gamma_{\perp e} \tag{31}$$

where

$$P_{turb} = \frac{n_oT_{oe}}{\tau_e}\,\frac{1}{\sqrt{2\pi}}\,\left\langle\left(\frac{\delta n}{n_o}\right)^2\right\rangle\,F_e(r). \tag{32}$$

Thus we observe that the χ_{exp} is a result of the sum of $Q_{\perp e}$ and the energy flux implied by the turbulent heating terms. This point has two physically important consequences which we note. If there are physical circumstances where P_{turb} is much larger than P_{ohm} or P_{ext} - to leading order the temperature profile is determined by the turbulent loss term balancing the turbulent heating. A temperature profile so obtained would be relatively insensitive to the spatial distribution of P_{ext}. Experiment would then show an "invariant" temperature profile, just as the density profile in the absence of particle sources takes on an invariant aspect. This is a form of "profile-consistency" and could possibly be an explanation of recent experimental observations (FURTH et al, 1985 REBUT and BRUSATI, 1985). In this sense profile-consistency could be a reflection of the deeper fact of the profile invariance of the turbulence spectrum.

A second point of interest which arises from the difference of χ_{exp} and $\chi_{\perp e}$ (as defined in Eqns.(27) and (28)), concerns the time-dependent versions of these equations used, for example, to describe sawtooth heat propagations. The time-scale of this process is short enough to require the inclusion of time derivatives (thermal inertia terms), but long enough with respect to the turbulent fluctuation spectrum for the time-averaging of the theory to be carried out. Experiments in TFTR (FREDRICKSON et al 1986) and JET, for example, suggest that the χ required to fit the sawtooth pulse propagation data can be up to an order of magnitude larger than that obtained from Eq (27), say. In theoretical terms this can be interpreted as follows. Sawtooth propagation inherently involves large spatial gradients of the instantaneous temperature profile. Thus these experiments are very sensitive to the $\frac{dT_{oe}}{dr}$ term in the heat flux, and not at all, or very weakly, to P_{turb} or U_{Te}. This means that the sawtooth data would yield a direct estimate of $\chi_{\perp e}$, rather than χ_{exp}. Furthermore it would suggest that the heat transfer process is diffusive (i.e. proportional to $\frac{dT_{oe}}{dr}$) rather than a mixture of diffusive and convective. Since our numerical simulations of JET suggest that $\chi_{\perp e}$ and P_{turb} can be individually large and yet cancel sufficiently to yield a lower thermal diffusivity, our theory would appear to be capable of explaining the difference between $\chi_{\perp e}$ (heat pulse) and $\chi_{\perp e}$ (stationary).

An interesting feature of our JET simulations is the Knudsen maximum phenomenon observed in both $\chi_{\perp e}$ and P_{turb}. As we have previously noted, a maximum of this type is to be expected when the collisionality parameters $\frac{v_{the}\tau_e}{qR}$ and $\omega\tau_e$ are of order unity. This characteristic of long mean free path phenomena is well-known in rarefied gas dynamics (CERCIGNANI, 1969).

We can also enquire as to whether externally applied magnetic field oscillations at frequencies comparable with the collision frequency can result in plasma heating. This is analogous to collision-time magnetic pumping schemes (STIX, 1962). Leaving aside the question of accessibility of such field oscillations, we have calculated the effect in JET of a single low-frequency mode with m = 2, n = 1 at a frequency of order 100 kHz with a spatial dependence given by

$$\frac{\delta B_r}{B_o} \simeq \frac{r}{a} \exp\left[- 1/4 \ \left(m - nq(r)\right)^2\right] \tag{33}$$

We arbitrarily take the amplitude to be $\frac{\delta B_r}{B_o} = 3 \times 10^{-3}$ at the resonant point. This results in a large value of P_{turb} localised to within 20 cms of the 2,1 resonance. We estimate $\Delta P_{turb}/\left(\frac{\delta B_r}{B_o}\right)^2 = 4 \times 10^{13}$ ergs $cm^{-3}s^{-1}$ and the (ΔP_{turb} heating due to input 2,1 mode) increase in the χ due to the mode which is $\Delta\chi_{\perp e}/\left(\frac{\delta B_r}{B_o}\right)^2 = 1.6 \times 10^{12}$ cm^2s^{-1}. These values suggest that it is possible to heat the plasma at well-defined internal locations and transport the energy from these locations efficiently. Since $\Delta\chi_{\perp e}$ from the calculation is large only where the heating is large, the overall confinement will be unaffected: whether this could be a practical heating scheme is an open question.

Conclusions

Using a model Fokker-Planck theory we have shown the following:

(1) Magnetic turbulence not only enhances radial energy transport but can also result in significant heating effects under JET-like conditions. Thus the net effect of turbulence on the electron energy balance must always be understood as the difference of two possibly large effects. That is, an enhanced outward loss offset by enhanced heating, ohmic heating and any additional sources and sinks.

(2) The levels of turbulence which need to be assumed to get a self-consistent energy-balance appear not to significantly modify the toroidal resistance as calculated from Spitzer resistivity.

(3) We have investigated the scaling with respect to size and temperature of the turbulent heating term, and conclude that, if the magnetic fluctuation spectrum is invariant, it can be more effective than background ohmic heating in JET-like plasmas. However, it must be remembered that the heating effects of turbulence are always off-set by turbulent losses, and it is the net effect which is physically significant.

(4) We have also shown that the relation between δn and δB-fluctuations is relatively insensitive to temperature variations over a large range, thereby showing that the invariance of the density fluctuation spectrum is physically equivalent to that of the magnetic fluctuation spectrum.

(5) This relatively broad view of the effects of turbulence on gross energy transport is shown to result in <u>observable</u> consequences such as the possibility of profile-consistency being related to the invariance of the turbulence power spectrum.

(6) The same features can also explain the observed difference in effective electron thermal diffusivity determined by two different methods, namely, sawtooth pulse propagation and steady-state transport analysis.

(7) We have considered the effects of externally excited single modes and show that in principle they can heat the plasma locally without necessarily destroying global confinement.

Finally, we envisage the development of electromagnetic turbulence in a tokamak in the following qualitative terms. For specified external sources of particles, momentum and energy, purely collisional processes alone presumably lead to very unstable profile configurations. In consequence of the instabilities generated the profiles evolve to new, more relatively stable configurations in which stationary small amplitude turbulence can be sustained. The resulting transport must necessarily be very different from that due to purely collisional processes. In this paper we have been concerned with the effects of a given turbulence spectrum on plasma properties. The complementary problem of successfully characterising the turbulence spectrum in terms of the plasma properties remains open.

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TABLE I

Parameter	TFR	JET
R_o(cms)	100	300
a (cms)	20	125
B_{tor}(tesla)	4.0	3.0
q_a	4.0	4.0
I_p(MAmps)	0.2	2.0
$n_o(o)(/cm^3)$	1.0×10^{14}	5×10^{13}
T_{oe}(o)(keV)	1.6	5.0
τ_e(o)(sec)	2.7×10^{-6}	3.1×10^{-5}
$j_{tor}(o)(MA/_{M}2)$	1.6	0.4
$P_{ohm}(erg/cm^3/sec)$ at r=o	5.4×10^7	5.9×10^5

TABLE II

Calculated Magnitudes at r/a = 0.5

Quantity	TFR	JET
$\chi_{\perp e}(cm^2/sec)$	3.9×10^3	2×10^4
$\langle (\frac{\delta B_r}{B_o})^2 \rangle^{1/2}$	1.1×10^{-4}	1.1×10^{-4}
$\langle (\frac{\delta n}{n_o})^2 \rangle^{1/2}$	1.3×10^{-3}	0.8×10^{-3}
$\eta_o j_o^2$ $(erg/cm^3/sec)$	1.2×10^7	1.3×10^5
$P_{turb}(erg/cm^3/sec)$	3.7×10^6	4.4×10^6

TABLE III

JET RESULTS FOR FIXED, I_p, B_{tor}, n_o, power spectrum.

$T_{oe}(o)$ is the variable. Quantities calculated at $r/a = \frac{1}{2}$. $\left< \left(\frac{\delta B_r}{B_o} \right)^2 \right>^{1/2} = 1.13 \times 10^{-4}$ at this point.

$T_{oe}(o)$ (keV)	$T_{oe}(r/a = 0.5)$ (keV)	P_{ohm} (Erg/cm^3/sec)	P_{turb} (Erg/cm^3/sec)	$\left< \left(\frac{\delta n}{n_o} \right)^2 \right>^{1/2}$	$\chi_{\perp e}$ (cm^2/sec)	$\frac{P_{turb}}{\left(\frac{n_o T_{oe}}{\tau_e} \right) \left< \left(\frac{\delta n}{n} \right)^2 \right>}$
1.0	0.61	1.5×10^6	2.8×10^6	0.96×10^{-3}	8.4×10^3	133
2.0	1.21	0.5×10^6	4.8×10^6	0.89×10^{-3}	1.5×10^4	377
3.0	1.82	0.3×10^6	5.2×10^6	0.83×10^{-3}	1.9×10^4	574
4.0	2.43	0.2×10^6	5.0×10^6	0.79×10^{-3}	2.0×10^4	703
5.0	3.03	0.1×10^6	4.4×10^6	0.77×10^{-3}	2.0×10^4	731
6.0	3.63	1.0×10^5	3.8×10^6	0.78×10^{-3}	1.8×10^4	674
7.0	4.25	0.8×10^4	3.3×10^6	0.77×10^{-3}	1.6×10^4	647
8.0	4.85	0.7×10^4	2.8×10^6	0.74×10^{-3}	1.4×10^4	636
9.0	5.46	0.5×10^4	2.5×10^6	0.74×10^{-3}	1.1×10^4	601
10.0	6.10	0.5×10^4	2.2×10^6	0.73×10^{-3}	1.0×10^4	570

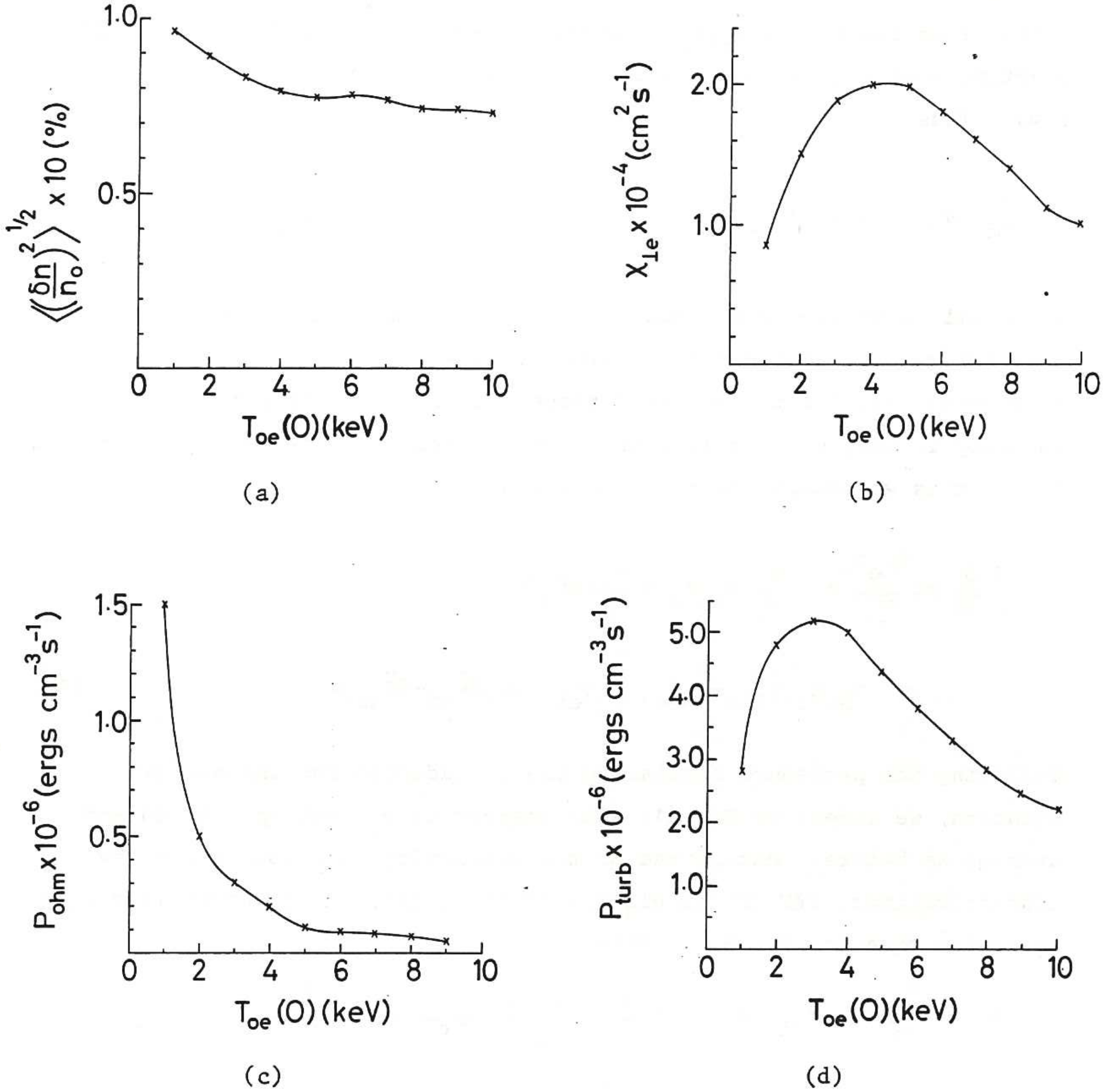

FIG.1 Plots of (a) RMS density fluctuation, (b) turbulent electron thermal diffusivity, (c) ohmic power, and (d) turbulent heating. These quantities are calculated at r/a = 0.5 assuming a magnetic fluctuation level $\langle(\frac{\delta B_r}{B_o})^2\rangle = 1.2 \times 10^{-8}$ at this point. The plasma current, toroidal field, density and temperature profiles, and the power spectrum of the magnetic fluctuations are held fixed, while the central temperature T_{oe}(o)is varied parametrically. The turbulent power for any other fluctuation level can be obtained from the last column of TABLE III.

Appendix

Taking the moment of Eq.(2) with respect to $v_{\parallel}$, making use of definitions and that $C(f_e, f_e)$ conserves momentum, we derive the parallel electron momentum balance equation in complete generality (parallel ohm's law). Thus

$$\frac{\partial}{\partial t}(n\bar{V}_{\parallel e}) + \nabla\cdot\left[n\bar{V}^2_{\parallel e}\,\underline{b} + \frac{p_e}{m_e}\,\underline{b} + \underline{V}_{de}\,n\bar{V}_{\parallel e}\right] + \frac{e}{m_e}E_{\parallel}n = \nu_{ei}n(\bar{V}_{\parallel i} - \bar{V}_{\parallel e}), \quad \text{(A1)}$$

where all quantities are functions of position and time. The mean parallel momentum balance is obtained by space-time averaging as usual. We observe that for tokamak conditions the electron parallel drift velocity is very much smaller than the electron thermal velocity. Making use of this we obtain the mean ohm's law,

$$\frac{1}{r}\frac{\partial}{\partial r}\,r\langle\frac{\delta p_e b_r}{m_e}\rangle + \frac{e}{m_e}E_{\parallel o}n_o + \frac{e}{m_e}\langle\delta E_{\parallel}\delta n\rangle$$

$$= \nu_{ei}n_o\left(\bar{V}_{\parallel io} - \bar{V}_{\parallel eo}\right) + \nu_{ei}\,\langle\delta n\left(\delta\bar{V}_{\parallel i} - \delta\bar{V}_{\parallel e}\right)\rangle \quad \text{(A2)}$$

Following the procedure similar to the one adopted for the energy equation, we linearise Eq.(A1) with respect to b_r, multiply by δn and average as before. Making use of the continuity equations and a few transformations, $\langle\delta E_{\parallel}\delta n\rangle$ is eliminated in Eq.(A2). The result is the following equation for $E_{\parallel o}$. Thus

$$E_{\parallel o} = \frac{m_e}{e}\nu_{ei}\left(\bar{V}_{\parallel io} - \bar{V}_{\parallel eo}\right) + \frac{1}{en_o}\left(\langle\frac{\delta n}{n_o}\nabla\cdot(\delta p_e\frac{\underline{B}_o}{B_o} + p_{oe}\frac{\delta\underline{B}}{B_o})\rangle\right.$$

$$\left. - \frac{1}{r}\frac{d}{dr}\left(r\langle\delta p_e\frac{\delta B_r}{B_o}\rangle\right)\right) \quad \text{(A3)}$$

The first term in Eq.(A3) is clearly Spitzer resistivity. For the conditions considered in this paper - even in JET - the second term is no more than a few percent of the first term, and in practice is likely to be totally obscured by impurity effects.