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Abstract

In this paper we re-examine the transport of momentum in a collisional plasma rotating at sonic speeds. The slowing down rate due to collisional transport is found to be classical ($\tau_m^{-1} \sim \nu_{ii}\rho_i^2/a^2$) and the presence of impurities does not change the order of magnitude of this theoretical result. Thus collisional transport theory cannot explain the anomalously high loss rates observed experimentally. Some existing theories of collisional momentum loss are discussed in detail.

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I Introduction

It is well documented that many Tokamaks exhibit anomalously high rates of toroidal momentum loss. Electron, and possibly ion, energy are also lost at rates greater than predicted by neo-classical theory and it is common to seek an explanation for this in terms of turbulent transport from some micro-instability. It is therefore natural to suppose a similar mechanism is responsible for the loss of toroidal momentum and one such, ion temperature gradient turbulence, has been suggested, (MATTOR, DIAMOND and LEE (1986)).

However, STACEY and SIGMAR (1984 & 1985) have advanced the hypothesis that, in a torus, classical gyroviscosity allied to inertial effects due to sonic flows, can account for the experimental observations. This contrasts with an earlier discussion by HOGAN (1984) of the same problem based on the full Braginskii stress tensor which led to a Pfirsch-Schlüter enhancement of classical perpendicular viscosity.

We have therefore reconsidered this problem and show in Section II that, for an up-down symmetric Tokamak, classical perpendicular viscosity (without a Pfirsch-Schlüter enhancement) is appropriate. This result is consistent with kinetic treatments of this problem for low flow velocities by ROSENBLUTH et al (1971) and HAZELTINE (1976) and for sonic flows, by HINTON and WONG (1985),(see also COWLEY and BISHOP (1986)).

Because of this confusion in the literature an important contribution in this paper is the discussion, in Section III, of the physical interpretation of our calculation and the origin of the different results obtained by Stacey and Sigmar and by Hogan.

II Calculation of Collisional Toroidal Momentum Loss

In this section we derive a momentum diffusivity from Braginskii's stress tensor (BRAGINSKII (1965)) given in Appendix A, so that our result is valid for a collisional plasma in which λ_{mfp}, the mean free path for all species, is small compared with the macroscopic scale size. For simplicity we first consider the case with no temperature gradient or impurities; the modifications due to temperature gradient are discussed in Appendix B and the role of impurities is discussed in Appendix C.

In this situation the plasma is described by the following set of equations for each species α:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot n_\alpha \underset{\sim}{V}_\alpha = S_\alpha, \tag{1}$$

$$m_\alpha n_\alpha \left(\frac{\partial \underset{\sim}{V}_\alpha}{\partial t} + \underset{\sim}{V}_\alpha \cdot \nabla \underset{\sim}{V}_\alpha\right) = -\nabla p_\alpha + Z_\alpha e n_\alpha \left\{ -\nabla\phi + \underset{\sim}{E}_I + \frac{\underset{\sim}{V}_\alpha \times \underset{\sim}{B}}{c} \right\}$$

$$-\nabla \cdot \underset{\approx}{\pi}_\alpha + \sum_\beta m_\alpha n_\alpha \nu_{\alpha\beta} (\underset{\sim}{V}_\alpha - \underset{\sim}{V}_\beta) + \underset{\sim}{P}_\alpha, \tag{2}$$

$$\nabla \times \underset{\sim}{B} = -\frac{4\pi}{c} \underset{\sim}{J} = -\frac{4\pi}{c} \sum_\alpha Z_\alpha e \underset{\sim}{V}_\alpha, \tag{3}$$

$$\nabla \cdot \underset{\sim}{E} = \rho = \sum_\alpha n_\alpha e Z_\alpha, \tag{4}$$

and

$$\frac{\partial \underset{\sim}{B}}{\partial t} = -\nabla \times \underset{\sim}{E} = -\nabla \times \underset{\sim}{E}_I \tag{5}$$

where the electric field $\underset{\sim}{E} = -\nabla\phi + \underset{\sim}{E}_I$ with $\underset{\sim}{E}_I$ the inductive component. $\nu_{\alpha\beta}$ is the collision frequency between species α and β, while S_α is the particle source and $\underset{\sim}{P}_\alpha$ the momentum source for species α and the other symbols have their usual meaning. The viscosity tensor $\underset{\approx}{\pi}$, given in Appendix A, has a decomposition

$$\underset{\approx}{\pi}_\alpha = \sum_{i=0}^{4} \underset{\approx}{\pi}_{i\,\alpha} \tag{6}$$

where $\underset{\approx}{\pi}_{0\,\alpha}$ is the dominant parallel component characterised by a coefficient of parallel viscosity $\eta_0 \sim n_\alpha T_\alpha / \nu_{\alpha\alpha}$, $\underset{\approx}{\pi}_{1\alpha}$ and $\underset{\approx}{\pi}_{2\alpha}$ are the gyroviscous terms with viscosity coefficients $\eta_{1,2} \sim (\nu_{\alpha\alpha} / \omega_{c\alpha})\eta_0$ and $\underset{\approx}{\pi}_{3\alpha}$ and $\underset{\approx}{\pi}_{4\alpha}$ are the perpendicular components with $\eta_{3,4} \sim (\nu_{\alpha\alpha}/\omega_{c\alpha})^2\eta_0$. Here we have introduced the gyro-frequency $\omega_{c\alpha} = eB/m_\alpha$ and it is also convenient to define the thermal velocity $V_{th\alpha} = (2T_\alpha / m_\alpha)^{1/2}$ and gyro-radius $\rho_\alpha = V_{th\alpha} / \omega_{c\alpha}$.

We solve Eqs (1) to (5) by an expansion in ρ_i/a where a is the minor radius, treating λ_{mfp}/a as a subsidiary expansion parameter, and seeking an evolution of toroidal momentum on a timescale

$$\frac{\partial}{\partial t} \sim 0 \left(\frac{\nu_{ii}}{\omega_{ci}} \frac{V^2_{thi}}{a^2\omega_{ci}}\right). \tag{7}$$

as will be justified a posteriori. Therefore with ε labelling order in ρ_i/a,

$$\underset{\approx}{\pi}_\alpha(\underset{\sim}{V}) = \underset{\approx}{\pi}_{0\,\alpha}(\underset{\sim}{V}) + \varepsilon(\underset{\approx}{\pi}_{3\,\alpha}(\underset{\sim}{V}) + \underset{\approx}{\pi}_{4\,\alpha}(\underset{\sim}{V})) + \varepsilon^2(\underset{\approx}{\pi}_{1\,\alpha}(\underset{\sim}{V}) + \underset{\approx}{\pi}_{2\,\alpha}(\underset{\sim}{V})) \tag{8}$$

Momentum sources do not change the order of magnitude of this answer if $|\nabla \cdot \underset{\approx}{\pi}_0| \gg |\underset{\sim}{P}_i|$ and, since this is always satisfied, we consider the case $\underset{\sim}{P}_i = 0$ for simplicity.

a) Lowest order flows

In an axisymmetric torus $\nabla \cdot \underset{\sim}{B} = 0$ implies that the magnetic field can be expressed as

$$\underset{\sim}{B} = \nabla\psi \times \frac{\underset{\sim}{e}_T}{R} + I\,\frac{\underset{\sim}{e}_T}{R} \tag{9}$$

where R is the major radius of the torus, ψ is the poloidal flux function and $\underset{\sim}{e}_T$ is a unit vector in the toroidal symmetry direction.

When the equilibrium flow velocity is of order the sound speed the lowest order ion momentum equation yields the familiar result

$$\frac{\underset{\sim}{V}_i^0 \times \underset{\sim}{B}}{c} = -\underset{\sim}{E}^0 = \nabla\phi^0 \tag{10}$$

and the electrons obey an identical equation. The magnetic field is therefore frozen into the zeroth order flow. By taking the scalar product of Eq.(10) with $\underset{\sim}{B}$ we deduce that $\phi^0 = \phi^0(\psi)$. Since $\nabla\phi^0$ is perpendicular to the flux surface $\underset{\sim}{V}^0$ is in the surface. We resolve $\underset{\sim}{V}_i^0$ into two components, one parallel to $\underset{\sim}{B}$ and one in the toroidal direction (see Fig. 1.). It follows from Eq.(10) that

$$\underset{\sim}{V}_i^0 = \omega(\psi) R\underset{\sim}{e}_T + \lambda_i \underset{\sim}{B} \tag{11}$$

where $\omega = c\partial\phi^0/\partial\psi$ and λ_i is arbitrary. For stationarity on this fast timescale the continuity equation (Eq.(1)) constrains this flow to be of the form

$$\underset{\sim}{V}_i^0 = \omega(\psi) R \underset{\sim}{e}_T + \frac{K_i(\psi)}{n_i} \underset{\sim}{B} \tag{12}$$

This form of the flow is discussed from a physical point of view in Section III. We will refer to the toroidal part of the zeroth order flow as the rigid rotator part. The parallel flow contains a poloidal component which is strongly damped by parallel viscosity (also known as magnetic pumping, (HASSAM and KULSRUD (1978)). On the long timescale of Eq.(7) we therefore expect K_i to be small. This is demonstrated mathematically by considering the ion momentum equation in next order of the ν_{ii}/ω_{ci} expansion. The $Z_i e n_i^0 (\underset{\sim}{E} + \frac{\underset{\sim}{V} \times \underset{\sim}{B}}{c})^1$ term is annihilated by the operation $\int\frac{d\ell}{B_p n_i^0} \underset{\sim}{B}\cdot$, where $d\ell$ is an element of length in the poloidal direction and B_p is the poloidal magnetic field. Physically, we are computing the flux surface average of the work done by the parallel force on the parallel flow. Then, writing

$$\langle A \rangle = \int \frac{d\ell}{B_p} A \tag{13}$$

we obtain the equation (temporarily retaining the effect of temperature variations)

$$m_i \left< \frac{\partial}{\partial t} (\underset{\sim}{B} \cdot \underset{\sim i}{V}) \right> + m_i \left< \underset{\sim}{B} \times \underset{\sim i}{V} \cdot \nabla \times \underset{\sim i}{V} \right> = \left< (\underset{\sim}{B} \cdot \nabla T_i) \ln n_i \right> - \left< \frac{\underset{\sim}{B}}{n_i} \cdot \nabla \cdot \underset{\approx i}{\pi} \right>$$

$$+ Z_i e \left< \underset{\sim}{B} \cdot \underset{\sim I}{E} \right> + m_i n_i \left< \nu_{ie} \underset{\sim}{B} \cdot (\underset{\sim i}{V} - \underset{\sim e}{V}) \right> \qquad (14)$$

Since the induced electric field $\underset{\sim I}{E}$ is of order $\eta \underset{\sim}{J}$ and the resistivity $\eta \sim m_i \nu_{ie}/ne^2$ the last two terms in Eq.(14) are comparable and give no contribution in zero and first orders, while the time derivative also appears only in second order. In zero order we insert $\underset{\sim i}{V}^0$ into Eq.(14) noting that the inertia term vanishes identically. Using the form of $\underset{\approx 0i}{\pi}$ given in Appendix A we find that $\underset{\approx 0i}{\pi}\left(\omega R \underset{\sim T}{e}\right) = 0$ so that, ignoring temperature gradients, Eq.(14) reduces to

$$\left< \frac{\underset{\sim}{B}}{n_i} \cdot \nabla \cdot \underset{\approx 0i}{\pi} \right> = 3K_i(\psi) \left< \eta_0 \left(\underset{\sim}{b} \cdot \nabla \frac{\underset{\sim}{B}}{n_i} \cdot \underset{\sim}{b} - \frac{1}{3} \underset{\sim}{B} \cdot \nabla \frac{1}{n_i}\right)^2 \right> = 0 \qquad (15)$$

where $\underset{\sim}{b} = \underset{\sim}{B}/B$. The parallel viscosity therefore constrains the parallel flow to be zero at this order. The relaxation of K_i^0 to zero clearly occurs on a timescale $\varepsilon^2 V_{thi}/a^2 \nu_{ii}$. We discuss the physical mechanism responsible for this relaxation in Section III.

To calculate the slowing down of the toroidal rotation $\omega_o(\psi,t)$, we will still require $K_i^1(\psi)$, the longitudinal part of the first order flow. $K_i^1(\psi)$ is obtained from the ion momentum equation in next order of the ν/ω_c expansion, but we delay the calculation until certain equilibrium properties have been established, in particular the nature of the poloidal variation of $n_i(\psi, \ell)$.

b) Equilibrium

Adding the momentum equations (Eq.2) for the ions and electrons we obtain the zeroth order pressure balance equation

$$-m_i n_i \omega^2 \frac{\nabla R^2}{2} = -\nabla p + \underset{\sim}{J}^0 \times \underset{\sim}{B} \tag{16}$$

where $p = n_i(T_i + T_e)$ and $\underset{\sim}{J}^0 = Zn_i e(\underset{\sim}{V}_i^1 - \underset{\sim}{V}_e^1)$, and to this order $-4\pi\underset{\sim}{J}^0 = c\nabla \times \underset{\sim}{B}$. Taking the scalar product of Eq.(16) with $\underset{\sim}{B}$ we obtain,

$$n_i = \overline{n}\ (\psi)\ \exp\left\{\frac{\omega^2 R^2}{c_s^2}\right\} \tag{17}$$

where $c_s = \left(2(T_i + T_e)/m_i\right)^{1/2}$ is the constant sound speed. The density variation in the surface provides a parallel pressure gradient which balances the component of the centrifugal force in the surface. The toroidal component of Eq.(16) yields $\underset{\sim}{J}^0 \cdot \nabla\psi = 0$, which with Eq.(7) implies $I^0 = I^0(\psi)$. The $\nabla\psi$ component of Eq.(16) yields the equilibrium equation

$$-\left[\frac{\partial\overline{p}}{\partial\psi} + R^2\overline{p}\frac{\partial}{\partial\psi}\left(\frac{\omega^2}{c_s^2}\right)\right]\exp\left\{\frac{\omega^2 R^2}{c_s^2}\right\} = \frac{\Delta^*\psi}{R^2} + \frac{I}{R^2}\frac{\partial I}{\partial\psi} \tag{18}$$

where $\overline{p} = \overline{n}\ (T_i + T_e)$ and Δ^* is the usual Grad-Shafranov operator,

$$\Delta^*\psi \equiv R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial Z^2} \tag{19}$$

When temperature gradients are included in the problem (see Appendix B) $\underset{\sim}{B}\cdot\nabla T^0 = 0$ and Eq. (18) is unchanged except that $c_s = c_s(\psi)$. The equilibrium specified by Eq.(18) is determined by the three free flux functions $\overline{p}$, ω/c_s and I (MASCHKE and PERRIN (1980)), which evolve through the transport equations.

c) Evolution of Toroidal Angular Momentum

We derive an equation governing the evolution of toroidal angular momentum by annihilating the leading order part of the momentum equation (Eq.(16)). This annihilator is $\int\frac{d\ell}{B_p} R\, \underset{\sim}{e}_T\cdot$, and, applying this to a plasma with a single ion species, we obtain.

$$< m_i \frac{\partial}{\partial t} \left(n_i \omega R^2\right)> + m_i \frac{\partial}{\partial\psi} < n_i \omega R^2 \underset{\sim}{V}\cdot \nabla\psi>$$

$$= < \underset{\sim}{J}\cdot \frac{\nabla\psi}{c} > - \frac{\partial}{\partial\psi} < R\underset{\sim}{e}_T\cdot \underset{\approx}{\pi}_i\cdot\nabla\psi >. \qquad (20)$$

Physically the terms in Eq.(20) represent, from left to right: the rate of change of angular momentum, the convection of angular momentum, the toroidal $\underset{\sim}{J}\times\underset{\sim}{B}$ force and the viscous drag between surfaces.

First we show that the toroidal $\underset{\sim}{J}\times\underset{\sim}{B}$ and convection terms can be neglected. It follows from Gauss's law and charge conservation that the toroidal $\underset{\sim}{J}\times\underset{\sim}{B}$ force is also equal to the rate of change of charge inside the surface. Thus

$$< \underset{\sim}{J} \cdot \frac{\nabla\psi}{c} > = < \frac{\partial \underset{\sim}{E}}{\partial t} \cdot \frac{\nabla\psi}{c} > \simeq < \frac{\partial\omega}{\partial t} \frac{|\nabla\psi|^2}{c^2} >. \tag{21}$$

where we have used Eqs.(9) and (12). Comparing the $\underset{\sim}{J} \times \underset{\sim}{B}$ force to the rate of change of angular momentum we find

$$\frac{< \underset{\sim}{J} \cdot \frac{\nabla\psi}{c} >}{< m_i \frac{\partial}{\partial t} (n_i \omega R^2)>} \simeq \frac{V_A^{\ 2}}{c^2} \tag{22}$$

where V_A is the Alfven velocity $V_A = B/\sqrt{4\pi\, m_i n_i}$. Since $V_A/c << 1$ in most experiments, the toroidal $\underset{\sim}{J} \times \underset{\sim}{B}$ force will be ignored.

We may express the convection term (which involves the second order radial flow) in terms of the lower order flows by taking the scalar product of the ion momentum equation with $\omega R^3 \underset{\sim}{e}_T$ and flux surface averaging. To second order we obtain

$$\frac{e}{c} < n_i \omega R^2 \underset{\sim}{V}_i^2 \cdot \nabla\psi > = -m_i < \omega R^2 \nabla\cdot(\underset{\sim}{V}_i^1 \omega R^2 n_i) > - < \omega R^2 \nabla\cdot \underset{\approx}{\pi} \cdot \underset{\sim}{e}_T R >$$

$$+ m_i < \omega R^2 \nu_{ie} n_i (\underset{\sim}{V}_e^1 - \underset{\sim}{V}_i^1) \cdot \underset{\sim}{e}_T >. \tag{23}$$

The first order flow $\underset{\sim}{V}_i^1$ can be calculated from the first order ion momentum equation

$$- m_i n_i^0 \omega^2 \frac{\nabla R^2}{2} = -\nabla p_i + n_i^0 e \left(-\nabla\phi^1 \times \frac{\underset{\sim}{V}^1 \times \underset{\sim}{B}}{c}\right) \tag{24}$$

together with Eq.(17). We first obtain the potential ϕ^1 from the parallel component of Eq.(24) and Eq.(17)

$$\frac{e\phi^1}{T_e} = \frac{1}{2}\frac{m_i\omega^2R^2}{(T_e + T_i)} \tag{25}$$

This potential provides an electron density variation which ensures charge neutrality when the ions are expelled by the centrifugal force. The perpendicular component of Eq.(24) and the condition of stationary density in the continuity equation at this order yield

$$\underset{\sim}{V}_i^1 = \frac{K_i^1(\psi)\underset{\sim}{B}}{n_i} + \frac{cT_i}{e}\left(\frac{\bar{n}'}{\bar{n}} + \frac{m_i\omega\omega' R^2}{T_i}\right) R\underset{\sim}{e}_T \tag{26}$$

where prime denotes differentiation with respect to ψ. Note that the diamagnetic toroidal velocity has a contribution due to velocity shear.

One may verify that substituting $\underset{\sim}{V}_i^0$ and $\underset{\sim}{V}_i^1$ into Eq.(23) yields, to second order,

$$< n_i\omega R^2\underset{\sim}{V}_i\cdot\nabla\psi > = 0 \tag{27}$$

where we neglect the friction term because $\nu_{ie}/\nu_{ii} \sim (m_e/m_i)^{1/2}$. Thus we conclude that the toroidal $\underset{\sim}{J} \times \underset{\sim}{B}$ and convection terms in Eq.(20) may be neglected.

The slowing down is therefore determined by the viscous term in Eq.(20); evaluation of this term is aided by two identities:

$$\nabla\psi\cdot\underset{\approx}{\pi}_{0i}\ \underset{\sim}{e}_T \equiv 0 \tag{28}$$

for any velocity, and

$$\nabla\psi\cdot\ \underset{\approx}{\pi}_{3i}(\omega R\underset{\sim}{e}_T)\cdot\ \underset{\sim}{e}_T = \nabla\psi\cdot\underset{\approx}{\pi}_{4i}(\ \omega R\underset{\sim}{e}_T)\cdot\ \underset{\sim}{e}_T = 0 \tag{29}$$

Thus the viscous term in Eq.(20) is, to second order,

$$\begin{aligned}\langle\ R\nabla\psi\cdot\underset{\approx}{\pi}_i\cdot\underset{\sim}{e}_T\ \rangle &= \langle\ R\nabla\psi\cdot\underset{\approx}{\pi}_{1i}(\omega R\underset{\sim}{e}_T)\cdot\underset{\sim}{e}_T\rangle + \langle R\nabla\psi\cdot\underset{\approx}{\pi}_{2i}(\omega R\underset{\sim}{e}_T)\cdot\underset{\sim}{e}_T\rangle \\ &+ \langle\ R\nabla\psi\cdot\underset{\approx}{\pi}_{3i}(\ \underset{\sim}{V}_i^1)\cdot\underset{\sim}{e}_T\ \rangle + \langle\ R\nabla\psi\cdot\underset{\approx}{\pi}_{4i}(\underset{\sim}{V}_i^1)\cdot\underset{\sim}{e}_T\rangle\end{aligned} \tag{30}$$

so that the gyroviscosity contributes at the same order as the perpendicular viscosity. The slowing down rate can therefore be calculated once K_i^1 is determined as we do not require $\underset{\sim}{V}_i^2$.

K_i^1 is determined from the parallel work done at first order using Eq.(14). In this order the friction term and time derivative in Eq.(14) are negligible, the inertia term again vanishes identically, and we obtain,

$$\langle\ \frac{\underset{\sim}{B}}{n_i}\cdot\ \nabla\cdot\underset{\approx}{\pi}_{0i}(\underset{\sim}{V}^1) + \frac{\underset{\sim}{B}}{n_i}\cdot\nabla\cdot\underset{\approx}{\pi}_{3i}(\omega R\underset{\sim}{e}_T) + \frac{\underset{\sim}{B}}{n_i}\cdot\ \nabla\cdot\underset{\approx}{\pi}_{4i}(\omega R\underset{\sim}{e}_T)\ \rangle = 0 \tag{31}$$

At this point we exploit the fact that the collisionality parameter, $\lambda_{mfp}/\ a$, is small in the collisional regime. The contribution to Eq.(28) from the diamagnetic parts of $\underset{\sim}{V}_i^1$ in $\underset{\approx}{\pi}_{3i}$ and $\underset{\approx}{\pi}_{4i}$ is smaller than that from the perpendicular viscosity terms, $\underset{\approx}{\pi}_{1i}$ and $\underset{\approx}{\pi}_{2i}$, by a factor $\lambda_{mfp}/\ a$

and can be ignored. However K_i^1, driven by gyroviscosity in Eq.(31), is given by

$$K_i^1 = \frac{\frac{\omega'}{3} \langle \eta_3 \left(\frac{R^2}{n_i} \underset{\sim}{B}\cdot\nabla B - \frac{2I^2}{Bn_i} \underset{\sim}{B}\cdot\nabla \ln n_i\right)\rangle}{\langle\frac{\eta_o}{n_i^2} \left(\underset{\sim}{b}\cdot\nabla B - \frac{2}{3} \underset{\sim}{B}\cdot\nabla \ln n_i\right)^2\rangle} \tag{32}$$

where η_o and η_3 are given in Appendix A. Thus

$$\frac{K_i^1 B}{n_i} \sim \omega R\left(\frac{\rho_i}{a}\right)\left(\frac{a}{\lambda_{mfp}}\right) \tag{33}$$

so that K_i^1 gives a contribution to Eq.(30) through the gyroviscous term comparable to that from the rigid rotation in the perpendicular viscosity.

Clearly, when the magnetic field is up-down symmetric $\left(B = B(R, \psi)\right)$, K_i^1 is zero. Thus, in an up-down symmetric tokamak, the viscous damping of toroidal flow comes entirely from the rigid rotator flow in the perpendicular viscosity and Eq.(20) takes the form

$$\langle m_i \frac{\partial}{\partial t} (n_i \omega R^2) \rangle = - \frac{\partial}{\partial\psi} \left\{ \omega' \langle \eta_1 \frac{R^4 B_p^4}{B^2} + 2\eta_2 R^4 \frac{B_p^2 B_\phi^2}{B^2} \rangle \right\} \tag{34}$$

where $B_p = |\nabla\psi| / R$, $B_\phi = I/R$ and $\eta_1 = \eta_2/2 = 3/10\ nT\left(\nu_{ii}/\omega_{ci}^2\right)$ (BRAGINSKII (1965)). The density evolution is given by

$$< \frac{\partial n_i}{\partial t} > = -\frac{\partial}{\partial \psi} \left(\frac{m_i c}{Z_i e} < \nu_{ie} \underset{\sim}{J} \cdot \underset{\sim T}{Re} > \right) \tag{35}$$

and is therefore slower than the evolution of ω by a factor $(\nu_{ie}/\ \nu_{ii})$.

We estimate the damping of toroidal rotation from Eq.(34) as

$$\frac{\partial \omega}{\partial t} \sim \nu_{ii} \left(\frac{\rho_i}{a}\right)^2 \omega \tag{36}$$

and observe that there is no Pfirsch-Schlüter enhancement. One may verify that in an up-down asymmetric tokamak the term resulting from K_i^1 in gyroviscosity gives a momentum diffusivity of order $q^2 \nu_{ii} \rho_i^2/a^2$ times the square of the degree of asymmetry.

III Discussion

In this section we will discuss the physical processes that govern the form of the flow and the slowing down and compare our results with others in the literature.

The lowest order consequence of the momentum equation in a gyroradius expansion is the familiar result $\underset{\sim}{E} + \underset{\sim i}{V} \times \underset{\sim}{B}/c = 0$, Eq.(10). The magnetic field is therefore frozen into the lowest order flow. We may deduce the properties of the flow from the constraints that the magnetic field and density be stationary. Thus there is no flow perpendicular to the flux surface since it would distort the surface. It is convenient to split the flow in the surface into two components: a toroidal flow which, because of axisymmetry, does not perturb the density and a flow parallel to the magnetic field which therefore does not perturb the field. The angular velocity of the toroidal flow, ω, must be a flux function otherwise differential rotation within the surface would 'wind up' the field. To discuss the parallel flow $V_{\parallel i}$ we consider a flux tube through which the fluid flows. The continuity of the particle flux within the tube requires that $V_{\parallel i} n_i \delta A$ be constant (where δA is the cross-section of the tube). Since $\nabla \cdot \underset{\sim}{B} = 0$, δA B is also a constant along the tube so that $V_{\parallel i} = K_i(\psi)B/n_i$. The form of the flow given by Eq.(12) is therefore deducible from relatively simple physical considerations.

The value of K_i is determined by parallel viscosity. The damping of the parallel flow by parallel viscosity can be understood in terms of the work done against $P_\perp$ and $P_\parallel$ the components of pressure perpendicular and parallel to the magnetic field. A fluid element flowing along the field line from the outside to the inside of the torus is elongated along the field and compressed across the field. The rate of doing work against parallel viscosity in a large aspect ratio tokamak is of order $(P_\perp - P_\parallel)\ \varepsilon V_{\parallel i}/a$ (where ε is the inverse aspect ratio). The difference $P_\perp - P_\parallel$ arises from competition between collisionless particle motion which increases the anisotropy and collisions which decrease it and is given by $P_\perp - P_\parallel \sim \varepsilon P V_{\parallel i}/a\nu_{ii}$. Balancing the loss of kinetic energy of the parallel flow with the work done against parallel viscosity leads to a damping rate for poloidal flow of order $\varepsilon^2 V_{thi}/a^2 \nu_{ii}$.

It is interesting to estimate the bulk parallel ion flow resulting from neutral beams by balancing this parallel ion viscosity with the friction between the beams and the ions. Since the frictional force is of order $m_i n_b \nu_{ii} (V_{thi}/V_b)^3 V_b f$, where f is the fraction of beam momentum going into parallel motions, $V_{\parallel i}/V_{thi}$ can be estimated as

$$\frac{V_{\parallel}}{V_{thi}} \simeq \frac{R^2}{\lambda^2_{mfp}} \left(\frac{T_i}{T_b}\right)^3 \frac{\beta_b}{\beta_i} f \tag{37}$$

where $\beta_b/\beta_i = n_b T_b/n_i T_i$ with n_b and T_b the beam density and 'temperature'. In typical tokamak experiments $V_{\parallel i}/V_{thi}$ estimated from Eq.(37) is less than a percent.

Our result Eq. (32) differs from those of STACEY and SIGMAR (1985) and HOGAN (1984). Hogan has argued that momentum diffusion is analagous to thermal diffusion and that a Pfirsch-Schlüter enhancement of momentum diffusivity occurs. To understand this we review the calculation of thermal diffusion. In a non-rotating plasma (see Appendix B)

$$n_i \frac{\partial T_i}{\partial t} \simeq \underset{\sim}{B} \cdot \nabla \frac{K_{\parallel}}{B^2} \underset{\sim}{B} \cdot \nabla T_i - \nabla \cdot \frac{K_{\Lambda}}{B} \underset{\sim}{B} \times \nabla T_i + \nabla \cdot K_{\perp} \nabla T_i \tag{38}$$

where the parallel thermal conductivity $K_{\parallel} \sim n_i T_i / m_i \nu_{ii}$ and $K_{\Lambda}/K_{\parallel} \sim K_{\perp}/K_{\Lambda} \sim \nu_{ii}/\omega_{ci}$. Expanding Eq. (38) in powers of ν_{ii}/ω_{ci} yields to lowest order

$$K_{\parallel} \underset{\sim}{B} \cdot \nabla T_i^0 = 0 \tag{39}$$

and therefore $T_i^0 = T_i^0(\psi)$. T_i^1 is determined in next order by the equation

$$\underset{\sim}{B} \cdot \nabla \left(\frac{K_{\parallel}}{B^2} \underset{\sim}{B} \cdot \nabla T_i^1\right) - \underset{\sim}{B} \cdot \nabla \frac{K_{\Lambda}}{B} I \frac{\partial T_i^0}{\partial \psi} = 0 \tag{40}$$

This is readily solved for T_i^1 which is up-down asymmetric. The evolution equation for $T_i^0(\psi)$ is given by the flux surface average of Eq.(38) in second order,

$$n_i \frac{\partial T_i^0}{\partial t} = < -\nabla \cdot \frac{K_\Lambda}{B} \underset{\sim}{B} \times \nabla T_i^1 + \nabla \cdot K_\perp \nabla T_i^0 > \tag{41}$$

where the contribution from T_i^1 yields the Pfirsch-Schlüter enhancement.

While there is some similarity between the thermal conduction and viscosity problems (parallel viscosity is similar to parallel conduction etc.) there is a crucial difference between the role of viscosity in the momentum equation and the role of conduction in the temperature equation. Parallel thermal conduction dominates Eq.(38) whereas parallel viscosity does not dominate the momentum equation (2). In the small gyro-radius ordering employed in this paper the momentum equation is in fact dominated by the Lorentz force ($c\nabla \cdot \underset{\approx}{\pi}_0/VBne \sim \rho_i \lambda_{mfp}/a^2$). In order to determine the constraints imposed by parallel viscosity it is then necessary to annihilate the Lorentz force in the momentum equation by taking the scalar product with $\underset{\sim}{B}/n_i$ and flux surface averaging. Thus K^1 is determined from Eq.(31) by computing the work done over the whole surface, whereas T_i^1 is computed locally on the flux surface. Hogan's calculation neglects the Lorentz force and obtains a <u>local</u> equation for the toroidal velocity $\underset{\sim}{V}^1$ driven by $\underset{\approx}{\pi}_{3,4}(\omega Re_{\sim T})$, analogous to Eq.(39) for T_i^1. The contribution to the toroidal momentum loss from $\underset{\approx}{\pi}_{3,4}(\underset{\sim}{V}^1)$ then yields a Pfirsch-Schlüter factor.

In a series of papers STACEY and SIGMAR (1984,1985) have advanced the hypothesis that classical gyroviscosity can account for the observed anomalous toroidal viscosity. In order to achieve their effect they postulate $O(\varepsilon)$, where ε is the inverse aspect ratio of the tokamak, variations of density n and V_ϕ/R within a magnetic surface, corresponding to non-rigid rotation. In particular these variations must be up-down asymmetric. Gyroviscosity is then able to produce a radial transport of toroidal momentum with a viscosity co-efficient $\mu_\perp \sim \varepsilon^2 D_B$ where D_B is the Bohm diffusion coefficient. It is interesting to note that a similar result for thermal conductivity would arise if one considered the effect of the finite gyro-radius heat flux in the presence of an $O(\varepsilon)$ temperature variation in a surface! To justify these variations in V_ϕ/R and n Stacey and Sigmar use results obtained from momentum equations in which the Braginskii viscous stress tensor is simply replaced by a drag $- n_j m_j \nu_{Dj} \underset{\sim}{V}_j$ (STACEY and SIGMAR (1984)). The arbitrary drag coefficients ν_{Dj} are chosen to simulate the anomalously large toroidal viscosity observed in experiments, and thus greatly exceeds the equivalent value arising from Braginskii's equations. However it grossly underestimates the parallel viscous drag of the classical stress tensor. This is the essential reason for the result obtained by Stacey and Sigmar. It allows much greater parallel flows, ie $K^0(\psi) \neq 0$, than are permitted by the Braginskii anistropic stress tensor. In particular non-rigid rotator flows occur, resulting in up-down asymmetric density perturbations. The gyro-viscous stress then provides the anomalously large toroidal viscosity. The authors can then envisage a bootstrapped situation in which this anomalous viscosity is responsible for the initially assumed anomalous drag. As we have seen the correct form of Braginskii's stress tensor does not permit these effects - one would have to remove the dominant parallel viscosity effects by invoking a greatly enhanced ion collision frequency. These essential points are not significantly affected by complications due to impurities, temperature gradients etc.

IV Conclusion

We have examined the diffusion of momentum in a collisional toroidal plasma with a sonic toroidal flow. We have demonstrated that there is no Pfirsch-Schlüter enhancement of momentum diffusivity in an up-down symmetric tokamak, i.e. the momentum confinement time, as given in Eq.(34), is classical, $\left(\tau_m^{-1} \sim \nu_{ii}\rho_i^2/a^2\right)$. Since experimental slowing down rates are more than fifty times this classical rate (ISLER et al (1986)) and (BRAU et al.(1983)), the momentum loss, like the energy loss, is anomalously fast.

Appendix A The Braginskii Viscosity Tensor

In this Appendix we give the ion viscosity tensor, (BRAGINSKII (1965)). This involves:

Parallel viscosity,

$$\underset{\approx}{\pi}_{0i} = -3\eta_0 \left(\underset{\sim}{b}\,\underset{\sim}{b} - \frac{1}{3}\underset{\approx}{I}\right)\left(\underset{\sim}{b}\,\underset{\sim}{b} - \frac{1}{3}\underset{\approx}{I}\right):\nabla\underset{\sim}{V} \tag{A.1}$$

where $\eta_0 = 0.96\ n_i T_i/\nu_{ii}$ and $\underset{\sim}{b}$ is a unit vector along $\underset{\sim}{B}$;

Gyroviscosity,

$$\underset{\approx}{\pi}_{3i} = \frac{\eta_3}{2}\left[\underset{\sim}{b} \times \underset{\approx}{W}\cdot\underset{\approx}{I}_{\perp} - \underset{\approx}{I}_{\perp}\cdot\underset{\approx}{W} \times \underset{\sim}{b}\ \right] \tag{A.2}$$

and

$$\underset{\approx}{\pi}_{4i} = \eta_4\left[\underset{\sim}{b} \times \underset{\approx}{W}\cdot\underset{\sim\sim}{bb} - \underset{\sim}{b}\,\underset{\sim}{b}\cdot\underset{\approx}{W} \times \underset{\sim}{b}\right] \tag{A.3}$$

where $\eta_4 = n_i T_i/\omega_{ci} = 2\eta_3$, the rate of strain tensor

$$\underset{\approx}{W} = \underset{\sim}{\nabla}\,\underset{\sim}{V} + \left(\underset{\sim}{\nabla}\,\underset{\sim}{V}\right)^T - {}^2/_3\ \nabla\cdot\underset{\sim}{V}\ \underset{\approx}{I} \tag{A.4}$$

and $\underset{\approx}{I}_{\perp} = (\underset{\approx}{I} - \underset{\sim}{b}\,\underset{\sim}{b}\ )$;

Perpendicular viscosity,

$$\underset{\approx}{\pi}_{1i} = \eta_1 \left[\underset{\approx}{I}_{\perp} \cdot \underset{\approx}{W} \cdot \underset{\approx}{I}_{\perp} + \frac{1}{2} \underset{\approx}{I}_{\perp} (\underset{\sim}{b} \cdot \underset{\approx}{W} \cdot \underset{\sim}{b}) \right] \tag{A.5}$$

$$\underset{\approx}{\pi}_{2i} = \eta_2 \left[\underset{\approx}{I}_{\perp} \cdot \underset{\approx}{W} \cdot \underset{\sim}{b}\,\underset{\sim}{b} + \underset{\sim}{b}\,\underset{\sim}{b} \cdot \underset{\approx}{W} \cdot \underset{\approx}{I}_{\perp} \right] \tag{A.6}$$

where $\eta_2 = 12/10 \; (n_i T_i \nu_{ii}/\omega_{ci}^2) = 4\eta_1$. Thus

$$\eta_0 : \eta_3, \eta_4 : \eta_1, \eta_2 = 1 : \nu_{ii}/\omega_{ci} : (\nu_{ii}/\omega_{ci})^2. \tag{A.7}$$

The full viscosity tensor is

$$\underset{\approx}{\pi}_i = \sum_{n=0}^{4} \underset{\approx}{\pi}_{ni} \tag{A.8}$$

Appendix B   Temperature Evolution

In this Appendix we discuss the modifications imposed by the presence of temperature gradients. The ion temperature equation is (BRAGINSKII (1965)),

$$\frac{3}{2} n_i \frac{\partial T_i}{\partial t} + \frac{3}{2} n_i \underset{\sim}{V}_i \cdot \nabla T_i + n_i T_i \nabla \cdot \underset{\sim}{V}_i = -\nabla \cdot q_i - \underset{\approx}{\pi}_i : \nabla \underset{\sim}{V}_i + Q_i \tag{B.1}$$

where Q_i is the heat source (we ignore heat exchange with electrons) and $\underset{\approx}{q}_i$ is the ion heat flux,

$$\underset{\approx}{q}_i = -\kappa_{\parallel i} \underset{\sim}{b}\underset{\sim}{b} \cdot \nabla T_i + \kappa_{\Lambda i} \underset{\sim}{b} \times \nabla T_i - \kappa_{\perp i} \nabla T_i . \tag{B.2}$$

The thermal conductivities are,

$$\kappa_{\parallel i} = 3.9 \frac{n_i T_i}{m_i} \frac{1}{\nu_{ii}} , \quad \kappa_{\Lambda i} = \frac{5}{2} \frac{n_i T_i}{m_i \omega_{ci}} , \quad \kappa_{\perp i} = 2 \frac{n_i T_i}{m_i \omega_{ci}} \left(\frac{\nu_{ii}}{\omega_{ci}}\right) \tag{B.3}$$

In lowest order Eq. (B.1) becomes,

$$K_i^0 \left(\frac{3}{2} \underset{\sim}{B} \cdot \nabla T_i^0 + n T_i^0 \underset{\sim}{B} \cdot \nabla \frac{1}{n}\right) = \underset{\sim}{B} \cdot \nabla \frac{\kappa_{\parallel i}}{B^2} \underset{\sim}{B} \cdot \nabla T_i^0 +$$

$$3 (K_i^0)^2 \eta_o \left(\underset{\sim}{b} \cdot \nabla \left(\frac{\underset{\sim}{B}}{n}\right) \cdot \underset{\sim}{b} - \frac{\underset{\sim}{B} \cdot \nabla \frac{1}{n}}{3}\right)^2 \tag{B.4}$$

where we have used Eq.(10). The flux surface average of this equation is Eq. (14) to lowest order. Dividing Eq.(B.4) by T_i^0 we obtain the condition,

$$-\left\langle \frac{\kappa_\parallel}{B^2 T_i^{0\,2}} (\underset{\sim}{B}\cdot\nabla T_i^0)^2 \right\rangle = (K_i^0)^2 \left\langle \frac{3\eta_0}{T_i^0} \left(\underset{\sim}{b}\cdot\nabla\left(\frac{\underset{\sim}{B}}{n}\right)\cdot\underset{\sim}{b} - \frac{\underset{\sim}{B}\cdot\nabla\frac{1}{n}}{3}\right)^2 \right\rangle \qquad (B.5)$$

Since both averages in Eq. (B.5) are positive definite we conclude that

$$K_i^0 = 0 \ , \ \underset{\sim}{B}\cdot\nabla T_i^0 = 0 \qquad (B.6)$$

The order of the slowing down time can now be determined from Eq.(20). Using Eqs.(28) and (29) and $\underset{\sim}{V}_i^0 = \omega(\psi) R \underset{\sim}{e}_T$, we find that the temperature gradient does not effect the order of magnitude of the slowing down. If we wished to evaluate the precise effect of temperature gradients on the slowing down we would integrate Eq.(B.1) to determine $\underset{\sim}{B}\cdot\nabla T_i^1$ and then obtain K_i^1 from Eq.(14). To calculate the correct coupling of the momentum loss to the temperature gradient we must include the contributions to the viscous stress from heat flow (MIKHAILOVSKII and TSYPIN (1971)and TSYPIN (1985)) which were not included by Braginskii. Since the order of magnitude of the result is unchanged by the temperature gradient and the correct result is given in HINTON and WONG (1985), we will not pursue the matter further here.

Appendix C   The Influence of Impurities

In this Appendix we discuss the modifications to the theory of momentum transport introduced by the addition of an impurity. We shall assume that the impurity density n_I is such that,

$$n_I Z^2 \sim n_i \tag{C.1}$$

In this ordering impurity-ion collisions are comparable with ion-ion collisions. Eq.(10) holds for both species so that the zeroth order flow for both is given by Eq.(11). In calculating the parallel flow from Eq.(14) we must include the friction term. Eq.(14) for the ions becomes,

$$K_i \left[< \frac{\underset{\sim}{B}}{n} \cdot \nabla \cdot \underset{\approx}{\pi}_{i0} \left(\frac{\underset{\sim}{B}}{n}\right) > + < m_i \nu_{iI} B^2 > \right] + K_I < m_i \nu_{iI} B^2 \frac{n_i}{n} > = 0 \tag{C.2}$$

and for the impurities

$$K_I \left[< \frac{\underset{\sim}{B}}{n} \cdot \nabla \cdot \underset{\approx}{\pi}_{I0} \left(\frac{\underset{\sim}{B}}{n}\right) > + < m_I \nu_{Ii} B^2 > \right] + K_i < m_I \nu_{Ii} B^2 \frac{n_I}{n_i} > = 0 \tag{C.3}$$

so that $K_I = K_i = 0$. The zeroth order flow is the rigid rotator, just as in the case without impurities. The viscous slowing down time is now determined from Eq.(20), and again arises from the rigid rotator flow in perpendicular viscosity (and $\underset{\sim}{V}^1$ in gyroviscosity in the asymmetric case). The order of magnitude of the slowing down time is thus not changed by the presence of impurities (i.e. it is still given by Eq.(31)) and so it is unnecessary to evaluate the details here.

Finally we note that the poloidal density variations can be obtained from the parallel component of the zeroth order momentum equations;

$$n_e = \bar{n}_e(\psi)\exp\left(-\frac{e\phi^1}{T_e}\right)$$

$$n_i = \bar{n}_i(\psi)\exp\left(\frac{m_i\omega^2R^2}{2T_i} + Z_i e\frac{\phi^1}{T_i}\right)$$

$$n_I = \bar{n}_I(\psi)\exp\left(m_I\frac{\omega^2R^2}{2T_I} + Z_I\frac{e\phi^1}{T_I}\right) \tag{C.4}$$

The poloidal variation of ϕ^1 is determined from charge neutrality. If $Z_I \gg 1$, $Z_i = 1$ and Eq.(C.1) holds, then ϕ^1 is given approximately by Eq.(25). In this limit the impurities play a role in the collisional dynamics but not in charge neutrality.

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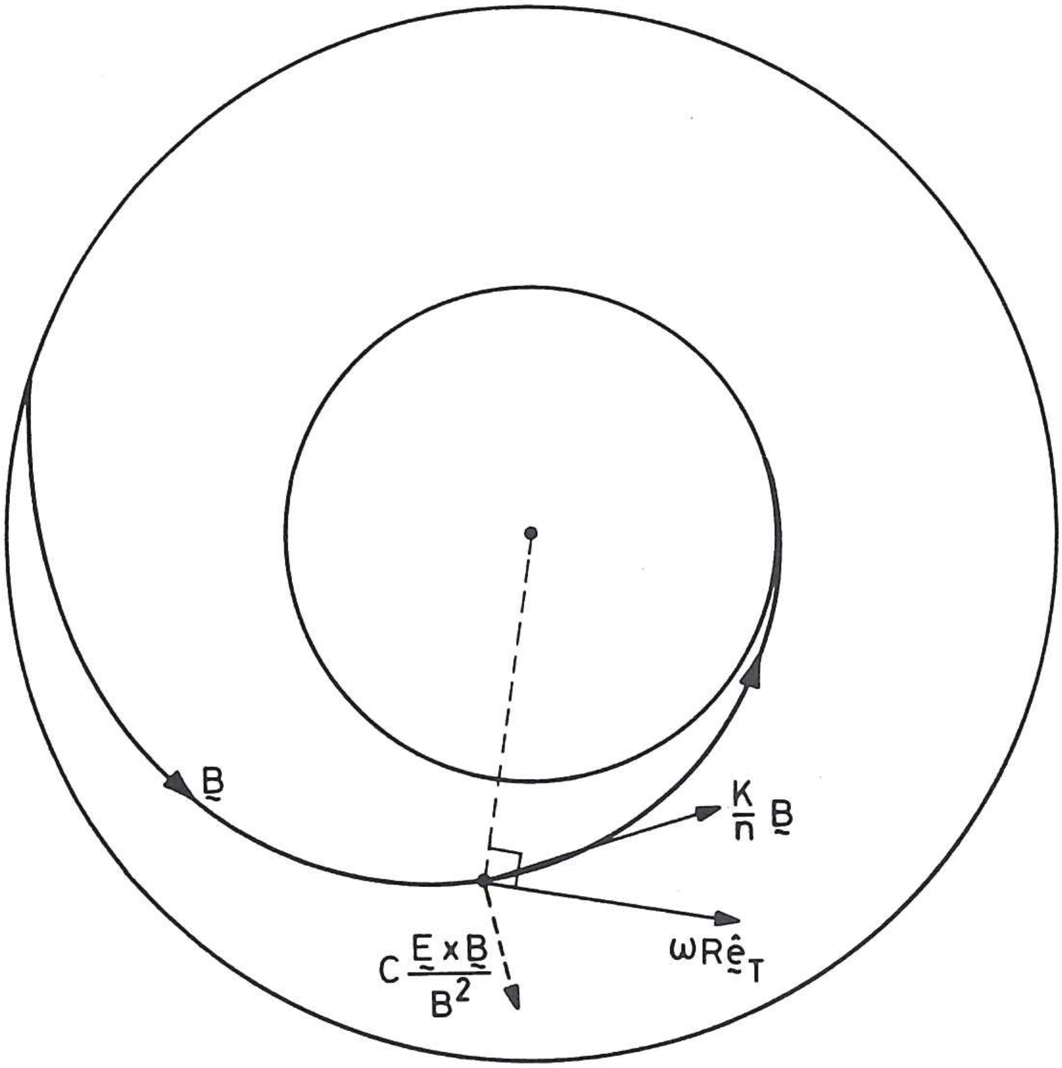

Fig. 1 The Lowest Order Flow

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