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CLASSICAL SINGLE PARTICLE DYNAMICS OF THE ANOMALOUS DOPPLER RESONANCE

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CLASSICAL SINGLE PARTICLE DYNAMICS OF THE ANOMALOUS DOPPLER RESONANCE

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Abstract

A gyroaveraged Lagrangian is used to describe the motion of a single electron undergoing anomalous Doppler resonance with a wave that has both electrostatic and electromagnetic components, propagating at arbitratry inclination to the magnetic field. It is shown that the flows of parallel and perpendicular energy are oppositely directed, and have magnitudes in the ratio $1 + \omega/(\Omega/\gamma):1$. The change in perpendicular kinetic energy, or equivalently Larmor radius, is related very simply to the $\underline{E} \times \underline{B}$ drift occurring at Landau resonance. The classical single-particle description of the anomalous Doppler resonance underlies the existing classical collective descriptions, and is the classical counterpart to the well-known quantum single-particle description.

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41.70 + t 52.20.Dq 52.35.Qz

I. INTRODUCTION

The anomalous Doppler resonance[1] plays a fundamental role in plasma physics.[2-15] It is a basic mechanism by which the component of electron kinetic energy parallel to the magnetic field can be transformed into perpendicular energy and into the field energy of an electromagnetic wave, travelling slower than the electron, that satisfies the familiar resonance condition that we discuss below. This mechanism places a theoretical limit on the stability of non-thermal electron velocity distributions. Such a limit is particularly of interest when the plasma current, which is necessary for bulk plasma stability in a tokamak, is supported by an extended superthermal tail. It is believed to explain[4] the relaxation oscillations in the soft X-ray signal that have been observed from low-density tokamak discharges, both Ohmic[7,14] and with lower hybrid current drive.[9,11,14]

In this paper, we give a classical single-particle treatment of the anomalous Doppler resonance. Previous descriptions have been based on quantum single-particle theory or on classical collective theory. The connections between the energy flow in the quantum and collective beam-plasma cases were pointed out by Nezlin.[3] More recently, the parallel and perpendicular energy flow has been demonstrated in the case of a continuous distribution of electron velocities.[15] The existence of a collective classical treatment suggests that there exists an underlying single-particle classical basis for the anomalous Doppler resonance, which is also the classical counterpart to the quantum single-particle treatment. In principle, the fundamental nature of the resonance, in terms of the relations between field energy and parallel and perpendicular electron kinetic energy, is determined at the single-particle level. The collective treatment adds a different class of information: namely, whether a given wave in a given ensemble of electrons grows or is damped - the overall direction of energy flow. This information is not provided by a single-particle treatment, where the initial wave-particle phase information remains, and has not been integrated over a distribution.

The single-particle classical theory presented here is based on the gyroaveraged Lagrangian techniques that have been developed by Littlejohn.[16-19] In our treatment, the magnetic field geometry is simple, so that the gyroaveraging process is relatively easy, and Lie transform techniques are not called for. While it is hoped that the treatment of the gyroaveraged Lagrangian is self-explanatory, we refer again to Littlejohn's papers[16-19] for a more extensive discussion of the topic. Our treatment is developed for waves that have both electromagnetic and electrostatic components. A Hamiltonian approach to the interaction of electrostatic waves with magnetised plasma has already been derived,[20,21] but it has not included a treatment of the anomalous Doppler resonance. The classical Lagrangian approach employed here enables us to demonstrate explicitly for a single particle the energy flow between parallel and perpendicular kinetic energy that is the hallmark of this resonance. In addition, consideration of classical single particle dynamics yields information on the relations between the anomalous Doppler resonance and the Landau resonance. We note that the change in perpendicular energy that is associated with the anomalous Doppler resonance is equivalent to a change in Larmor radius. We then examine the very close links between this isotropic quantity, and the directional $\underline{E} \times \underline{B}$ drift velocity that is induced by the perpendicular component of the electrostatic field at Landau resonance.

II. LAGRANGIAN FORMULATION

We first construct the Lagrangian for an electron of charge $-e$, moving in a uniform magnetic field which is oriented along the z-axis, whose motion is perturbed by a wave that has both electrostatic and electromagnetic components. The wave has wavevector $\underline{k}$, with components $k_{\parallel}$ in the z-direction and $k_{\perp}$ in the x-direction, and has frequency ω. The canonical momentum $\underline{p}$ of the electron is

$$\underline{p} = \gamma m \underline{v} - e\underline{A}/c = m\underline{u} - e\underline{A}/c \tag{1}$$

Here $\underline{v}$ is the electron velocity, m its rest mass, and γ is the usual Lorentz factor. For the magnetic field of interest,

$$\underline{A} = B_0\, x\underline{\hat{e}}_y + (A_{1x}\, \underline{\hat{e}}_x + A_{1z}\, \underline{\hat{e}}_z)\cos(k_{\perp}x + k_{\parallel}z - \omega t) \tag{2}$$

where B_0 is the static magnetic field strength, and $\underline{A}_1$ describes the electromagnetic component of the wave. The electrostatic component of the wave is described by the potential

$$\Phi = \Phi_1 \cos(k_{\perp}x + k_{\parallel}z - \omega t) \tag{3}$$

Then the Lagrangian of the system

$$L = \underline{p} \cdot \dot{\underline{x}} - H$$

$$= \left(m\underline{u} - e\frac{B_0 x}{c}\,\underline{\hat{e}}_y - \frac{e}{c}\,(A_{1x}\underline{\hat{e}}_x + A_{1z}\underline{\hat{e}}_z)\cos(k_\perp x + k_\parallel z - \omega t) \right) \cdot \dot{\underline{x}} - mc^2(1+u^2/c^2)^{1/2}$$

$$+ e\Phi_1 \cos(k_\perp x + k_\parallel z - \omega t) \qquad (4)$$

Next, we express the Lagrangian of Eq.(4) in terms of coordinates which reflect the guiding centre and cyclotron aspects of the electron motion:

$$\underline{u} = u_\parallel \underline{z} + u_\perp \cos\theta\, \underline{\hat{e}}_x + u_\perp \sin\theta\, \underline{\hat{e}}_y \qquad (5)$$

$$\underline{x} = \underline{X} + \frac{u_\perp}{\Omega} \sin\theta\, \underline{\hat{e}}_x - \frac{u_\perp}{\Omega} \cos\theta\, \underline{\hat{e}}_y \qquad (6)$$

where $\Omega = eB_0/mc$. Clearly, θ represents the gyroangle, $\underline{X}$ is the guiding centre position, and $u_\perp/\Omega$ is the Larmor radius. Let us use Eqs.(1), (2), (5) and (6) to write

$$\underline{p}\cdot\dot{\underline{x}} = m\Big(u_\parallel \dot{Z} + u_\perp \dot{X}\cos\theta + \frac{u_\perp \dot{u}_\perp}{\Omega}\cos\theta\sin\theta + \frac{u_\perp^2\dot{\theta}}{\Omega}\cos^2\theta$$

$$-\Omega X\dot{Y} + X\dot{u}_\perp\cos\theta - Xu_\perp\dot{\theta}\sin\theta\Big)$$

$$-\frac{e}{c}\left(A_{1x}\dot{X} + A_{1z}\dot{Z} + A_{1x}\frac{u_\perp\dot{\theta}}{\Omega}\cos\theta\right)\cos\left(k_\perp x + k_\parallel z - \omega t\right) \qquad (7)$$

Similarly,

$$\cos\left(k_\perp x + k_\parallel z - \omega t\right) = \cos\left(k_\perp X + \frac{k_\perp u_\perp}{\Omega}\sin\theta + k_\parallel Z - \omega t\right)$$

$$= \sum_{n=-\infty}^{\infty} J_n\left(\frac{k_\perp u_\perp}{\Omega}\right)\cos\left(k_\perp X + n\theta + k_\parallel Z - \omega t\right) \qquad (8)$$

We now aim to construct the gyroaveraged Lagrangian

$$\bar{L} = \frac{1}{2\pi}\int_0^{2\pi} L\, d\theta \qquad (9)$$

The averaging process is relatively simple, and in Eq. (8) the averaging process will select out any resonant terms, while mapping the other terms to zero. In order to preserve generality, we shall not explicitly identify the resonant terms at this stage. Instead, we shall replace the symbol $\sum_{n=-\infty}^{\infty}$ by $\sum_{n}^{res}$, and select out the resonant terms explicitly when convenient. Combining Eqs.(4), (7), (8) and (9), we obtain

$$\bar{L} = mu_{\parallel}\dot{Z} + \frac{mu_{\perp}^2}{2\Omega}\dot{\theta} - m\Omega X\dot{Y} - mc^2\left(1 + u_{\perp}^2/c^2 + u_{\parallel}^2/c^2\right)^{1/2}$$

$$-e\left(A_{1x}\frac{\dot{X}}{c} + A_{1z}\frac{\dot{Z}}{c} - \Phi_1\right)\sum_{n}^{res} J_n\left(k_{\perp}X + n\theta + k_{\parallel}Z - \omega t\right) \qquad (10)$$

The requirement that $\int \bar{L}\,dt$ be stationary for arbitrary variations of the configuration subject to fixed end points gives rise in the usual way to the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial \bar{L}}{\partial \dot{q}_i} = \frac{\partial \bar{L}}{\partial q_i} \qquad (11)$$

where the q_i are X, Y, Z, $u_{\perp}$, $u_{\parallel}$, and θ. For clarity, we shall consider first a purely electrostatic wave with $\underline{A}_1 = 0$. This is the approximation usually employed when studying the stability of non-Maxwellian electron velocity distributions in tokamak plasma.[4-15] Later in this Section, we shall generalise our treatment by including an electromagnetic component $\underline{A}_1 \neq 0$ for the wave field. Setting $\underline{A}_1 = 0$, the equations of motion following from Eqs.(10) and (11) are:

$$\dot{X} = 0 \tag{12}$$

$$\dot{Y} = -\frac{e}{m}\frac{k_\perp \Phi_1}{\Omega} \sum_n^{res} J_n(k_\perp u_\perp/\Omega)\sin(k_\perp X + n\theta + k_\parallel Z - \omega t) \tag{13}$$

$$\dot{Z} = u_\parallel/\gamma \tag{14}$$

$$\dot{\theta} = \frac{\Omega}{\gamma} - \frac{e}{m}\frac{k_\perp \Phi_1}{u_\perp} \sum_n^{res} J_n'(k_\perp u_\perp/\Omega)\cos(k_\perp X + n\theta + k_\parallel Z - \omega t) \tag{15}$$

$$\dot{u}_\parallel = -\frac{e}{m} k_\parallel \Phi_1 \sum_n^{res} J_n(k_\perp u_\perp/\Omega)\sin(k_\perp X + n\theta + k_\parallel Z - \omega t) \tag{16}$$

$$\dot{u}_\perp = -\frac{e}{m}\frac{\Omega \Phi_1}{u_\perp} \sum_n^{res} n\, J_n(k_\perp u_\perp/\Omega)\sin(k_\perp X + n\theta + k_\parallel Z - \omega t) \tag{17}$$

Now the electrostatic wave represents a perturbation on the zeroth-order cyclotron motion. In forming the argument of the sine and cosine functions, we require only the zeroth-order contribution to θ from Eq.(15): $\theta = \theta_0 + \Omega t/\gamma$. Eqs.(12) and (14) integrate to $X = X_0$, $Z = Z_0 + u_\parallel t/\gamma$. Let us define a set of constants

$$\Psi_n = k_\perp X_0 + n\theta_0 + k_\parallel Z_0 \tag{18}$$

Then the remaining equations of interest, which describe the gyroaveraged effect of the electrostatic wave on the electron motion, can be written

$$\dot{Y} = - \frac{e}{m} \frac{k_\perp \Phi_1}{\Omega} \sum_n^{res} J_n(k_\perp u_\perp/\Omega)\sin\left[\Psi_n + (k_\parallel u_\parallel/\gamma + n\Omega/\gamma - \omega)t\right] \tag{19}$$

$$\dot{u}_\parallel = - \frac{e}{m} k_\parallel \Phi_1 \sum_n^{res} J_n(k_\perp u_\perp/\Omega)\sin\left[\Psi_n + (k_\parallel u_\parallel/\gamma + n\Omega/\gamma - \omega)t\right] \tag{20}$$

$$\dot{u}_\perp = - \frac{e}{m} \frac{\Omega \Phi_1}{u_\perp} \sum_n^{res} n\, J_n(k_\perp u_\perp/\Omega)\sin\left[\Psi_n + (k_\parallel u_\parallel/\gamma + n\Omega/\gamma - \omega)t\right] \tag{21}$$

We now select out resonant terms from the sums on the right hand side of Eqs.(19) to (21). This procedure has been postponed from the gyroaveraging which took place at Eq.(10). We need only consider the small argument case for the Bessel functions, as it has been shown elsewhere[15] that ion Landau damping stabilizes the anomalous Doppler effect in plasmas outside this regime. Then

$$J_0(k_\perp u_\perp/\Omega) \simeq 1 \tag{22}$$

$$J_1(k_\perp u_\perp/\Omega) = - J_{-1}(k_\perp u_\perp/\Omega) \simeq k_\perp u_\perp/2\Omega \tag{23}$$

For the Landau resonance, $n = 0$ and

$$k_\parallel u_\parallel/\gamma = k_\parallel v_\parallel = \omega \tag{24}$$

Then Eqs.(19) to (21) give

$$\dot{Y} = - \frac{k_\perp \Phi_1}{B_0} \sin \Psi_0 \tag{25}$$

$$\dot{u}_{\parallel} = -\frac{e}{m} k_{\parallel}\Phi_1 \sin \Psi_0 \tag{26}$$

$$\dot{u}_{\perp} = 0 \tag{27}$$

Now $k_{\parallel}\Phi_1 = E_{\parallel}$, so that Eq.(26) describes the parallel force on the electron arising from the resonant parallel electric field. The sign of this force depends on the initial phase Ψ_0 of the electron with respect to the wave field. This is to be expected in a single-particle treatment of the Landau resonance. Similarly, $k_{\perp}\Phi_1 = E_{\perp}$, so that Eq.(25) describes the $\underline{E} \times \underline{B}$ drift of the electron guiding centre in the direction perpendicular to the plane of the wavevector and of the magnetic field direction. This drift arises becauses, at Landau resonance, the electric field component perpendicular to the magnetic field is constant in the guiding centre frame.

For the anomalous Doppler resonance, $n = -1$ and

$$k_{\parallel}u_{\parallel}/\gamma = \omega + \Omega/\gamma \tag{28}$$

Then Eqs.(19) to (21) give

$$\dot{Y} = \frac{k_{\perp}\Phi_1}{B_0} \left(\frac{k_{\perp}u_{\perp}}{2\Omega}\right) \sin \Psi_{-1} \tag{29}$$

$$\dot{u}_{\parallel} = \frac{e}{m} k_{\parallel}\Phi_1 \left(\frac{k_{\perp}u_{\perp}}{2\Omega}\right) \sin \Psi_{-1} \tag{30}$$

$$\dot{u}_\perp = -\frac{e}{m}\frac{k_\perp\Phi_1}{2}\sin\Psi_{-1} \tag{31}$$

Eqs.(30) and (31) display the fundamental features of the anomalous Doppler resonance. The rate of change of electron kinetic energy parallel to the magnetic field direction follows from Eqs.(28) and (30):

$$mu_\parallel\dot{u}_\parallel = e\Phi_1\frac{k_\perp u_\perp}{2}\sin\Psi_{-1}\left(\frac{\omega + \Omega/\gamma}{\Omega/\gamma}\right) \tag{32}$$

The rate of change of perpendicular kinetic energy is, by Eq.(31)

$$mu_\perp\dot{u}_\perp = -e\,\Phi_1\frac{k_\perp u_\perp}{2}\sin\Psi_{-1} \tag{33}$$

Thus, when the parallel kinetic energy decreases, the perpendicular kinetic energy increases, and vice versa. The magnitudes of these rates of change are in the ratio $1 + \omega/(\Omega/\gamma):1$. This result is identical to that obtained using quantum single-particle[1] or classical collective treatments,[2,3,15] and provides a necessary link between the two treatments. In the collective treatment,[15] the parallel and perpendicular energy flows were calculated using the components at resonance of the bulk quantity $\mathrm{Re}(\underline{j}\cdot\underline{E}^*)$. Here, it has been shown that these flows are a reflection of an underlying classical single-particle effect.

We note in addition that there is a previously unexplored link between the Landau resonance and the anomalous Doppler resonance. The electron Larmor radius $r_L = u_\perp/\Omega$. Eqs.(25) and (31) indicate that the rate of change of Larmor radius at anomalous Doppler resonance is, on average, equal to half the $\underline{E} \times \underline{B}$ drift velocity acquired by the electron at Landau resonance for the same wave. Both effects are driven by the perpendicular component of the electrostatic field. Physically, the coincidence in magnitudes suggests that the increase in $u_\perp$ - and equivalently Larmor radius - at anomalous Doppler resonance is the counterpart of the $\underline{E} \times \underline{B}$ drift, averaged in all directions by cyclotron gyration. The fact that the $n = -1$ drift is isotropic, whereas the $n = 0$ drift is directional, reflects the basic difference between the two resonances. The wave frequency experienced by the electron at the $n = 0$ Landau resonance is zero, so that a constant $\underline{E} \times \underline{B}$ drift results. At the $n = -1$ anomalous Doppler resonance, the magnitude of the wave frequency experienced by the electron is equal to the electron cyclotron frequency. There is thus a constant phase relationship between the gyrating electron and the perpendicular component of the wave field. This constant phase relationship leads to a steady increase or decrease in the radius of gyration.

Let us now complete our treatment by returning to Eq.(10), and including the electromagnetic wave term in $\bar{L}$. For clarity, we now set $\Phi_1 = 0$. The variation of $\bar{L}$ with respect to Z yields

$$\dot{u}_\parallel = \frac{e}{mc}\left[\dot{X}\left(k_\parallel A_{1x} - k_\perp A_{1z}\right) - \left(n\dot{\theta} - \omega\right)A_{1z}\right] \sum_n^{res} J_n\left(\frac{k_\perp u_\perp}{\Omega}\right)$$

$$\sin\left(k_\perp X + k_\parallel Z + n\theta - \omega t\right) \tag{34}$$

Variation with respect to θ yields

$$\dot{u}_{\perp} = \frac{e}{mc}\frac{\Omega}{u_{\perp}}\left(A_{1x}\dot{X} + A_{1z}\dot{Z}\right) \sum_{n}^{res} nJ_n\left(\frac{k_{\perp}u_{\perp}}{\Omega}\right)\sin\left(k_{\perp}X + k_{\parallel}Z + n\theta - \omega t\right) \tag{35}$$

The results obtained previously in Eqs.(12) and (14) carry over, and as in Eq.(15), $\dot{\theta} = \Omega/\gamma$ to zeroth order. Using Eq.(18), Eqs.(34) and (35) give

$$mu_{\parallel}\dot{u}_{\parallel} = -\frac{e}{c}\left(n\Omega/\gamma - \omega\right)A_{1z}u_{\parallel} \sum_{n}^{res} J_n\left(\frac{k_{\perp}u_{\perp}}{\Omega}\right)\sin\left[\Psi_n + \left(k_{\parallel}u_{\parallel}/\gamma + n\Omega/\gamma - \omega\right)t\right] \tag{36}$$

$$mu_{\perp}\dot{u}_{\perp} = \frac{e}{c}\,\Omega A_{1z}\dot{Z} \sum_{n}^{res} n\,J_n\left(\frac{k_{\perp}u_{\perp}}{\Omega}\right)\sin\left[\Psi_n + \left(k_{\parallel}u_{\parallel}/\gamma + n\Omega/\gamma - \omega\right)t\right] \tag{37}$$

Using Eqs.(14), (23), (36), and (37), it is easy to recover the familiar hallmark of the $n = -1$ anomalous Doppler resonance, this time for an electromagnetic wave field. The energy flows parallel and perpendicular to the magnetic field direction are opposite in sign, and their magnitudes are in the ratio $1 + \omega/(\Omega/\gamma):1$.

III. CONCLUSIONS

We have given a classical single-particle description of an electron which is in anomalous Doppler resonance with a wave propagating at arbitrary inclination to the magnetic field. The wave field has both electrostatic and electromagnetic components. This single-particle description underlies the existing collective classical descriptions,[2-15] and is the classical counterpart of the well-known quantum single-particle description.[1] We have shown how, at the single-particle level, the flows of parallel and perpendicular electron kinetic energy are oppositely directed, with their magnitudes in the ratio $1 + \omega/(\Omega/\gamma):1$. In addition, the change in perpendicular energy, or equivalently Larmor radius, which occurs at the anomalous Doppler resonance, has been related to the $\underline{E} \times \underline{B}$ drift velocity which occurs at Landau resonance.

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