CLM-P810

Toroidal pinches

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ABSTRACT

The theory of toroidal pinches in particular reverse field pinches and spheromaks is reviewed. The key area of ideal and resistive stability of the pinch has been explored for nearly three decades whereas the non-linear consequences of these instabilities has only recently received attention. The spontaneous ability of pinches to relax and produce a reversed toroidal field was noted in early experiments and numerous theoretical ideas were developed to explain this process. The theory of plasma relaxation is now well developed and provides a general basis for explaining many of the phenomena observed in pinches. The theory of anomalous transport is in its infancy but the invariance properties of the underlying equations do provide important information on the transport scaling.

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HISTORICAL INTRODUCTION

The theory of toroidal pinches really began some 3 to 4 decades ago. The theory of the pinch effect itself is much older. Evidence for the magnitude of the pinch effect was first noted early this century when currents of a few tens of kA's associated with lightning passed down a hollow lightning conductor. These early observations [1] demonstrated the unstable nature of the pinch effect in a current carrying conductor. Research on toroidal pinches began in the late 1940's and was concentrated in a few countries, eg UK, USSR and the USA. In 1946 G P Thomson filed a secret patent for a toroidal reactor using DD (see Table 1 (b) for parameters) which is interesting to compare with present reactor designs. Experiments were started at this time with a toroidal facility at Imperial College [2]. Thonemann also proposed the idea of thermonuclear reactions in a toroidal pinch, probably based upon the ideas of Tonks [3] and Bennet [4] in America.

In the early 1950's much of the theoretical work was classified and we can now look back on it with some interest when comparing the activities in the different countries. Pinch theory evolved along very similar lines in the USA, USSR and the UK. Thin skin pinch models were topical at that time and the work of Rosenbluth,[5] Tayler [6] and Shafranov [7] were very similar in this respect, at least they all got the same results! The work by Rosenbluth at Los Alamos was however a little different in that he also considered particle effects using the adiabatic invariants of the motion. The result was an instability criterion involving anisotropic pressure which if $p_\perp > p_\parallel$ was more unstable [5].

A very significant step forward in the development of the theory of toroidal pinches occurred at the 1958 Geneva conference on the Peaceful Uses of Atomic Energy. This was the first real opportunity for the countries actively involved in fusion research to jointly discuss their results in an open manner. There were numerous contributions on the theory of pinches. One reviewer [8] noted that "the theory papers showed that in the hydromagnetic approximation, complete stability could not be predicted for the magnetic field configurations so far observed in these pinch discharges". However the seeds of what was to come had already been sown, Rosenbluth in his paper on the stability and heating in a pinch,[5] noted that if the sharp surface layer of previous analyses was given finite thickness, this led to the important result that stability could only be achieved if the toroidal field was reversed outside this layer. He also noted that stability would only be transient due to diffusion and heating to high β and that a new helical equilibria might be set up which could in turn generate a reversed toroidal field as required for stability. Diffusion would make the plasma unstable again and the cycle would be repeated. This picture is not dissimilar to that which was observed much later in the fast reverse field pinches built in the early 1970's. At the same time the spontaneous reversal of the toroidal magnetic field had been observed on the ZETA device [9] and this time can truly be said to be the birth of the reverse field pinch. His early study of a pinch included a proposal for a reactor, see Table 1(a). It is a much smaller device than that originally proposed by Thomson and Blackman in their secret patent, which is shown for comparison in Table 1(b). At the present time when our large tokamaks approach conditions of substantial α particle heating, it is interesting to compare these proposed parameters of some 3 decades ago.

The early classification problems led to some difficulties in pinch research and Academician Lev Andreivich Artsimovich noted that "the question of whether a given neutron belongs to the noble race of descendants of thermonuclear reactions or whether it is a dubious offspring of a shady acceleration process is something that may worry the press men but it should not, in the present stage of development of our problem, ruffle the composure of the specialist". Artsimovich was of course to play a very important role in the further development of pinch devices.

STABILITY

The problems of stability encountered, both theoretically and experimentally in these early 'stabilised' pinch studies led researchers in the USA and the UK to study the hardcore geometry where MHD stability could be ensured. These hardcore pinch experiments showed all the characteristics of instability previously seen in the stabilised pinch experiments, and were not in agreement with simple hydromagnetic theory. This stimulated a re-examination of the theory and a number of workers studied the effects of finite electrical conductivity. A number of authors quickly came up with what we now know as the tearing mode, in particular Whiteman,[10] Rebut, [11] and Furth[12] culminating in a fine paper on resistive instabilities, by Furth, Killeen and Rosenbluth [13]. This theoretical work dealt a hard blow to the pinch experiments as it seemed difficult to see how they could be stable and many were discontinued. However, some work continued at a low level in the UK on the ZETA device, where it was noted that field reversal was accompanied by a quiescent period characterised by a striking reduction in the level of fluctuations. This prompted a careful examination of stability theory and it was established that field reversal was a necessary condition for obtaining ideal MHD stability,[14] though at that time the issue of tearing mode stability remained. The radial variation of the pitch of the magnetic fields - Fig 1 was critical in controlling that stability, in particular minima giving rise to instability. Work on pinches was also devoted to studies of the hydromagnetic turbulence present and which was later to turn out to be of considerable value in understanding the loss processes present in such devices.

At the Dubna conference in 1969 the results from the T-3 tokamak in the USSR on the electron temperature by Thomson scattering were presented which was to have a profound effect on fusion research and its direction. At the same meeting, there were a number of presentations on the theory of toroidal pinches which led the theoretical reviewer, Marshall Rosenbluth, to note that the contributions by a particular author [15] were perhaps the most important of the conference! These theoretical results established the detailed conditions necessary for the ideal MHD stability of a reverse field pinch indicating that central β values up to 30% were possible [16]. In addition stimulated by toroidal stability calculations in the Soviet Union, results were presented for the on-axis stability criterion for a toroidal pinch, $\frac{R^2}{2} qq'' < -4/9$ where q is the safety factor, which showed that the criterion was unchanged provided $q(o) < 0.5$.

Work in the early 1970's shifted more towards growth rate calculations which allowed more detailed comparison with experiment and the concept of sigma stability was introduced [17]. At the same time there was renewed interest in resistive instabilities. It was established that a wide range of tearing mode stable reversed field pinch configurations did exist [18] with currents diminishing to zero near the conducting wall - Fig 2. Thus the pinch could be stable without resorting to the high current density at the wall necessitated by the so called Bessel function model (or minimum energy configuration). However although the tearing and rippling mode (with high $\chi_{\parallel}$) could be stable, the resistive g mode, ie the mode driven by the pressure gradient, could not be stabilised. The tearing mode stable current distributions could support an ideal pressure limit $\beta(o) \lesssim 17\%$ at the high pinch parameter, $\theta \lesssim 3$ ($\theta = B_{\theta}(a)/\bar{B}_{\phi}$) though with a 4% vacuum region this fell to 12%. The growth rate calculations of resistive modes were also fashionable at this time particularly by Killeen and co workers [19]. Such calculations did permit detailed identification of pitch minimum, kink, tearing and g modes in fast pinch devices.

The resistive interchange or g mode arises in the pinch because the average curvature is unfavourable. It was thought that high temperature effects such as finite Larmor radius, perpendicular viscosity could easily stabilise the mode. Furthermore, even if the mode were unstable the diffusion coefficient in the non-linear phase would be comparable to the classical value so that its effect would not be serious. Both conclusions however, were found to be incorrect, in particular the magnetic perturbations associated with the g mode lead to substantial transport because they produce magnetic islands which readily overlap and lead to the stochastic field lines. In addition it was found that the electromagnetic mode with finite Larmour radius effects in cylindrical geometry was unstable at significant values of β (few %) [20]. Parallel viscosity however was found to be stabilising and this led to critical pressure profiles which could be grossly stable when the average value of β was significantly below 10% [21]. Such modes were probably detected in the fast toroidal reverse field pinches - Fig 3 shows the magnetic perturbation and its associated magnetic island structure.

The early work by Rosenbluth and others [22] on the diffusion due to stochastic fields produced by magnetic islands associated with resistive instabilities led to the consideration in more detail of the losses associated with the magnetic perturbations induced by current or pressure driven resistive modes and this initiated detailed studies of their non-linear behaviour in toroidal pinches. These were recognised as probably determining the limiting value of β and the transport in a pinch and also linked to the processes which control the reversed field generation or dynamo action in the pinch.

Stabilised pinches, that is toroidal pinches without field reversal received early attention, in particular the force-free paramagnetic model was studied for its ideal and resistive stability properties. For a pinch configuration parameter $\theta \lesssim 1.1$ it was shown to be stable to both ideal and resistive tearing modes. It also supports small plasma pressure $\beta \sim 5\%$. Such a configuration has significant current density at the conducting wall and cannot support a vacuum region between the edge of the plasma and the conducting wall. Recently this configuration has received more attention, particularly in TORIUT-6 where discharges with $q \sim 0.6$ were sustained apparently relatively stabily [23]. This type of operation had previously been noted in the setting up phases of reverse field pinches where "magic numbers" were noted as the value of the safety factor passed through various rational values at the boundary. These magic numbers were associated with bursts of strong MHD activity which affects the impedance of the discharge and manifests itself in the current and voltage waveforms. Ideal toroidal stability calculations for configurations with q in the region .3 to .7 do indicate regions of low growth when the region of pitch minimum falls close to an irrational q value.

A more recent theoretical question relates to thin shell operation which is desirable in future reverse field pinches and spheromaks. This is particularly relevant to a reactor as the wall time constant is significantly shorter than the sustainment time of the pinch. Thin shell operation permits accurate control of plasma position using the vertical and radial magnetic fields and possible shift and tilt instability control using separate feed back coils. Configurations which are stable with a perfectly conducting wall become unstable with modes growing on a timescale for the helical field to penetrate through the resistive wall. If the configuration is only marginally stable then growth on a hybrid timescale occurs, $\tau \sim \tau_W^{1/3}\,\tau_A^{2/3}$ where τ_W is the wall time constant and τ_A the Alfvén transit time [24]. Investigations of the effect of a resistive wall on both ideal and resistive modes [25] show that the wall destabilises both modes but that the tearing mode growth can be stabilised by sufficient toroidal rotation. Ideal modes are de-stabilised when the rotation becomes close to Alfvénic as noted by Rosenbluth [26] long ago (who assumed a perfect wall) and are not affected by the wall. The reduction in mode rotation at finite amplitude due to wall dissipation suggests that there may be problems with non-linear tearing modes in thin shell pinches eg the SLINKY mode observed on OHTE may be an example [27].

ANOMALOUS TRANSPORT

Some of the earliest work in this area was motivated by the failure of the experimental field configurations in pinches to satisfy the requirements of ideal and resistive MHD stability. This led to a general study [28] of the turbulence induced in the ZETA device by these instabilities. Their work both theoretically and experimentally covered fully developed MHD turbulence, which was characterised by close proximity to two dimensional turbulence. The measured triple correlation was close to that expected for such turbulence - Fig 4. They also considered the competition between convective transport losses and those associated with magnetic field fluctuations associated with strong magnetic turbulence present in the pinch. Rusbridge [29] in his tangled discharge model showed that the fluctuations would lead to rapid heat loss along the field lines which could explain the temperature profiles and also led to field configurations which produced reversal. The alternative explanation of the dynamo action in the pinch involving a small number of helical instabilities through the $<\underset{\sim}{v} \wedge \underset{\sim}{b}>$ correlation was also considered sufficient to account for the observations.

The key problem in toroidal pinch research is the anomalous energy loss. The conventional approach has been to calculate the anomalous transport by identifying an instability and then performing a non-linear calculation to obtain a saturated amplitude and deducing the resultant transport in this state. An alternative approach has also been adopted based upon the invariance properties of the equations describing the plasma [30]. Nearly all the theoretical approaches of anomalous loss in the pinch have been associated with resistive fluid turbulence. These losses can be either convective or associated with transport along the stochastic field lines, created by the resistive turbulence itself. The Rosenbluth Rechester [31] expression

$$D_m = \left(\frac{\delta b_r}{B}\right)^2 L_c$$

where L_c is the correlation length, is a key element in these calculations.

The correlation length is a complex issue [32] where in strong turbulence L_c can depend directly on D_m. In the collisional limit some theories including the invariance properties give $\delta B_r/B \propto 1/S^{\frac{1}{2}}$ (where S is the Lundquist or magnetic Reynolds number, $S = \tau_\sigma/\tau_A$ where τ_σ is the field diffusion time and τ_A the Alfvén transit time). For resistive 'g' modes the collisionless loss due to transport along stochastic magnetic fields is of the form

$$D \sim \eta \left(\frac{M}{m}\beta\right)^{\frac{1}{2}} f_0(\nabla p, s)$$

where f_0 is a function of pressure gradient and shear and η is the resistivity. The convective losses must have the form $D \sim \eta\, f_1(\nabla p, s)$ thus we have the result that conduction exceeds convection if $\beta > (m/M \times \beta_c)^{\frac{1}{2}}$ where β_c is the ideal ballooning limit in a tokamak and the Suydam limit in a pinch. If $\beta \sim 10\%$ and if $\beta/\beta_c > 0.05$ conduction losses will dominate. In the collisional limit this expression becomes $\beta > (m/M \times \beta_c{}^3)^{\frac{1}{4}}$. As β_c is typically 10% in the pinch then heat flow due to transport along stochastic magnetic field lines will be more important. The scaling of these losses in the pinch is such that $\beta \sim (m/M)^{1/6} \beta_c{}^{\frac{1}{2}} \sim 8\%$. An interesting point arising from these estimates is the relative performance of the tokamak and the reverse field pinch. If the tokamak and the pinch both have the same critical $\beta_c \sim 10\%$ and are subject to localised resistive pressure gradient driven turbulence then the confinement in the reverse field pinch and the tokamak (with ohmic heating) should be the same!

The impact of the electromagnetic resistive g mode in the reversed field pinch has been studied including the effect of the resultant stochastic field lines on the confinement [33]. For β values $\sim 5\%$ the resistive g mode has island forming parity and creates a substantial magnetic perturbation throughout the plasma. In this quasilinear treatment each individual mode tends to saturate by flattening the equilibrium pressure gradient in the vicinity of the singular surface. Mixed helicity interactions between the m=1 and m=0 modes play an important part in destroying magnetic surfaces as indicated in Fig.5. The fluctuation level is calculated to decrease with S in a manner similar to that given by the invariance arguments and also to increase with increasing plasma pressure. These modes give a temperature scaling $T \sim I^{0.7}$ where I is the current and it has been assumed that $I \propto N$, where N is the area density [33,35]. This is slightly more pessimistic than the temperature scaling $T \sim I$ implicit in the constant β result for the low β localised resistive g mode.

Diamond, Rosenbluth and colleagues [34] studied the non-linear interaction of tearing modes in the pinch and also estimated the thermal transport resulting from these modes. In this case non-linear generation and coupling to m=2 modes was advanced as the main mode saturation mechanism and the dynamo process was attributed to the m=1 tearing modes which sustained the configuration. Using renormalised turbulence theory (an approach similar to that used earlier in investigations of magnetic fluctuations on the ZETA device) it has been shown that coupling by interaction with neighbouring m=1 modes to stable m=2 modes balances the growth due to the relaxation of the current profile. This theory like the electromagnetic g mode yields $T \sim I^{0.7}$ by balancing the heat loss with ohmic heating and also assumes that $I \propto N$. The problem has been examined more recently for the non-linear tearing mode in a reverse field pinch [35] and a non-linear saturation mechanism dependent on cascading to smaller scales was proposed and used to estimate the profile evolution. The interaction with the m=0 mode was also included. The behaviour appears to be similar to the inertial range in ordinary fluid turbulence, with a broadening of the magnetic spectrum from coupling of the different m=1 modes leading to m=0, n=1 and m=2, activity. These new results indicate that $\delta B_r/B$ is virtually independent of S with the linear instability growth rate being unrelated to the instability drive in the non-linear regime. These results like the electromagnetic g mode indicate a temperature scaling $T \propto I^{0.7}$. Further examination [35] of resistive pressure driven gradient turbulence shows that more accurate calculations of the fluctuation level lead to a thermal diffusivity which is enhanced beyond the simple result given above, leading to a slow degradation of the poloidal β as the plasma current increases. It was also been noted [35] that ion temperature gradient driven modes can arise in a reverse field pinch with a similar scaling.

It is quite possible that with future large devices and higher temperatures that the tearing mode behaviour associated with the dynamo action will become less important, that the pressure gradient driven turbulence will become weak due to collisionless effects and that the $\beta_\theta \sim$ constant scaling will dominate.

RELAXATION AND RECONNECTION

A key element in toroidal pinch research is the relaxation of pinch configurations to a minimum energy state as propounded by Taylor [36]. Many of the theoretical predictions are in good agreement with the experimental data [37]. The self reversal of the toroidal reversed field in a pinch is explained as the natural tendency for the plasma to relax by a process involving dissipation and reconnection to a minimum energy state with a reversed toroidal magnetic field, if the pinch parameter $\theta > 1.2$, where $\theta = B_\theta(a)/\bar{B}_\phi$, $B_\theta(a)$ is the poloidal field at the wall and $\bar{B}_\phi$ the average toroidal field. This final minimum energy state is characterised by a single invariant, the helicity $K_0 = \int_{V_0} \underset{\sim}{A}.\underset{\sim}{B}.d\dot{\tau}$* where $\underset{\sim}{A}$ is the vector potential, the volume here being bounded by a flux conserving shell. The relaxed state is then defined by a force-free configuration $\underset{\sim}{J} = \mu\underset{\sim}{B}$ where μ is independent of position. In cylindrical geometry this gives rise to the Bessel function model which has a reverse field for $\theta > 1.2$ and for which the minimum energy state becomes helical when $\theta > 1.56$. The ability of a plasma to relax to this state is portrayed by comparing the predictions with experiment using an F-θ diagram, in which the field reversal ratio $F = B_\phi(a)/\bar{B}_\phi$, $B_\phi(a)$ being the toroidal field component at the wall, is plotted as a function of θ - Fig 6. The experimental points of all the toroidal reverse field pinches to date lie somewhat to the right of this minimum energy curve, in part due to the fact that the current density near the boundary tends to be small and also due to the fact that the configuration can support a significant plasma pressure. Tearing mode stable reverse field configurations have $\mu(r)$ decreasing with radius and give F-θ curves similar to the observations. Field diffusion will make the configuration move away from this minimum energy configuration leading to instability which tends to counteract the diffusion so that the configuration can be maintained and this explains the quasi-stationary nature of the reverse field pinch observed in the experiments. The nature of the processes involved in generating the field configuration and the associated plasma losses are not yet well understood and are clearly critical in determining the confinement properties of the system. Some theories have suggested that the scaling of the magnetic field fluctuations to provide this dynamo action is $\delta B/B \propto S^{-\frac{1}{2}}$. There is experimental evidence to support such a relationship [38].

*In multiply connected domains care must be exercised in evaluating K.

The helicity has a practical interpretation at constant toroidal flux - it is proportional to volts seconds stored in the discharge. The helicity has a rate of change given by $dK_1/dt = 2V_\ell \psi$ where ψ is the toroidal flux. A fascinating consequence of relaxation theory is the possibility of current drive by oscillating the driving voltages. If the toroidal and poloidal voltages are applied out of phase then they generate helicity and this implies an effective unidirectional voltage which is available then to sustain the plasma current [39]. This is receiving careful attention experimentally and theoretically at the present time [40].

Given the positive stability of the minimum energy state then some window for finite stable β should be present. This has been calculated in a number of ways in particular by inflating the equilibrium in a flux conserving manner to determine the ideal MHD β limit for such configurations - this is shown in Fig 7 where values of central $\beta \sim 10\%$ are obtained.

The properties of the relaxed state are found to depend crucially on topology be it toroidal or spherical and on the boundary conditions. This is particularly important when comparing the results from different experiments such as the reverse field pinch, OHTE [41], multi-pinch [42] and spheromaks. The process of reconnection almost certainly involves the presence of resistivity which no matter how small allows the topology of the field lines to no longer be preserved. As the resistivity becomes small, the region over which it acts becomes smaller with larger gradients there. A very similar process is involved in resistive instabilities and magnetic reconnection at X points.

The general theory of relaxed states in toroidal systems has also been discussed [43]. In the cylindrical relaxed state when $\theta > 1.56$ the eigen function is not axisymmetric and consists of a symmetric part and a helical component with m=1 and Ka = 1.25. This is true also in other cross-sections, but with more convoluted cross-sections such as the multi-pinch, the lowest eigen function may be axisymmetric. The multi-pinch has a cross section which is rather like Doublet. As μa is increased the initial configuration is a symmetric one with B_ϕ the same sign everywhere, whereas when $\mu a \sim 2.2$ the configuration becomes anti-symmetric with B_ϕ of opposite sign in the 2 halves of the configuration. For this particular case full reversal and current saturation are almost coincident. These particular features of the relaxed state have been clearly demonstrated in the multi-pinch experiment. Relaxation is also important in the spheromak [44] Fig 10. There are toroidal surfaces as in the toroidal pinch but the confining shell is now spherical. There is no central aperture for toroidal field coils, consequently the toroidal field is zero at the boundary and thus it resembles a toroidal pinch at the point of field reversal. In the spheromak there is only a single invariant, K rather than the two invariants in a toroidal device, K, and ϕ the toroidal flux. In this case the value of μ is determined by the shape alone. For a sphere $\mu a = 4.49$. The eigen-value may not be axisymmetric depending on the shape of the container. For example for a cylinder of height h and radius a; with $h/a > 1.67$ there is a non-axisymmetric mode [45] the 'tilting' mode. As in reverse field pinch discharges the measured field configurations are in agreement with the relaxed state predictions and the $\mu(r)$ profiles are approximately constant. The flux core spheromak is an interesting variant of the conventional spheromak which is obtained by introducing a core of externally produced flux along this line of symmetry. In this case the helicity is not well defined and a relative helicity is introduced. This demonstrates that helicity can be injected or extracted from the flux core spheromak which provides a means of sustaining the relaxed state against resistive decay. A good example of a flux core configuration is the CTX experiment [46] where it has been possible to sustain the spheromak at high currents (~ 0.5 MA) for several msec. The possibility that relaxed states can be maintained in this manner has led to investigations of new configurations where the source of the helicity is more distant from the confinement region [47]. It is very important to note that all relaxed states are stable against the perturbations which leave the helicity variant and this includes resistive tearing modes thus relaxed state theory provides a full non-linear description of the resistive tearing mode.

Tokamaks also display examples of relaxation towards a minimum energy state but they are intermittent and certainly non-unique in character. The inherent stability of the tokamak to both ideal and resistive modes seems in general to preclude a continuous relaxation towards a minimum energy state. Nevertheless it does highlight the fact that reconnection may be quite different in different circumstances although the final state is a unique one.

The fast pinch devices were ideal facilities for testing stability theory and Taylor's relaxation theory. By fast programming it is possible to force the plasma into unstable configurations and then observe the violent relaxation back towards the minimum energy state. Slow programming was found to accurately follow the F-θ trajectory of the minimum energy state. The relaxation could be studied in detail and large amplitude helical kink stabilities were detected of a sufficient amplitude that the solenoid effect of the helix was large enough to produce the reversed field. This behaviour was in very good agreement with early 3D simulations of self reversal [48]. These fast experiments also demonstrated that when the current was sustained then the field reversal was also sustained, for times longer than the field diffusion time. It can thus be concluded that the tendency to relax to a minimum energy configuration can occur by a variety of phenomena, large amplitude helical instabilities as suggested long ago, by a fine scale instabilities and by turbulence [49].
Self reversal has also been explained in terms of mean field equations which are applied to the MHD turbulence appropriate to the earth's dynamo problem which can drive $j\theta$ currents [50]. The turbulent motions leads to two effects [51], the production of an electric field parallel to the magnetic field, the α effect, and a turbulent resistivity called the β effect. Recent nonlinear tearing mode calculations [52] which calculate the $<\underline{v} \wedge \underline{b}>$ correlation, do demonstrate the maintenance of reversal as well as giving an 'eddy' resistivity [53] which can be more important than classical resistivity. A variety of Ohm's laws for the mean magnetic field have been derived leading to a coefficient of electric current viscosity [54,55] akin to the α effect.

There are a number of simulations using spectral codes, in 3 dimensions [56,57]. These calculations indicate a greater tendency to relax to the minimum energy configuration with compressible motions than incompressible ones. However incompressible calculations [57] with different boundary conditions - Fig 8 indicate an F-θ evolution in time, which is very similar to that which occurs in the fast pinch experiments. For $S \sim 10^4$ and a non-uniform resistivity profile the F-θ curve is very close to that observed experimentally. The calculations show that the dynamo effect, arises from the mean average coupling between the radial field and axial velocity perturbations. The calculations also indicate that μ is non-uniform unlike the minimum energy configuration but varies with radius in a manner similar to that observed experimentally Fig.9.

SPHEROMAKS

Rosenbluth and Bussac [41] used the Taylor minimum energy principle to study the stability of the spheromak which is the tight aspect ratio limit of the toroidal pinch. This is a force-free configuration with flux surfaces similar to the Hills vortex and is of particular note to astrophysicists. Such configurations could simplify the reactor engineering for a fusion device, by removing the necessity for toroidal field coils and no toroidal blanket is required. Creating such configurations does pose rather special problems but the ingenuity of plasma physicists has led to elegant solutions. One method which has been highly successful is to use a coaxial plasma gun and inject into a flux conserving chamber, combinations of θ and Z pinch discharges have also been successful and a slow inductive process has been used in the S1 device.

The stability of the spheromak was studied by noting that the helicity $K = \int \underline{A}.\underline{B}\, d\tau$ within the plasma volume, is conserved for both ideal and resistive modes. It was found that an oblate spheromak was stable to MHD and resistive tearing modes if surrounded by a conducting wall which was within 15% of the edge of the plasma. Such configurations have very little shear and the safety factor q varies decreases from .825 on axis to .72 at the edge. A prolate spheromak is found to be unstable to a tilting mode whereas an oblate spheromak is unstable to a shift-mode. The tilting instabilities are observed experimentally and reducing the height to radius ratio stabilises the mode in agreement with theoretical predictions. As in the pinch the existence of a free boundary associated with a vacuum region surrounding the plasma leads to new instabilities. The Mercier criterion can only be satisfied by such configurations with very low values of $\beta \sim 0.2\%$. With the oblate spheromak more shear is possible and the β value rises to $\sim$ 1%. Resistive g modes will be unstable because of the unfavourable curvature. However it should be noted that in the spheromak, as in the pinch, that the central values of β are smaller than the volume average β. In particular the engineering β taken out to the external field coils can be greater than 10%. As with the pinch a close fitting conducting shell appears to be an essential requirement for stability.

Spheromaks possess very similar properties to the reverse field pinch concerning conservation of helicity during the strong relaxation phenomena which occurs during the setting-up phase. However the magnetic fluctuations ($\sim$ 10%) associated with n=1 modes, appear to be larger than those present in the pinch. The volume average β values obtained in the spheromak are comparable to those in pinches but the confinement time appears to be shorter [58], possibly associated with a higher level of magnetic fluctuations. This could be because the critical β for the onset of ideal pressure driven modes for this configuration is lower than the pinch. Attempts to improve the β value have been made by inserting a copper cylinder along the axis of the spheromak, however this has not led to an improvement in plasma parameters to date. A difficulty in the spheromak is that the flux conserver has a time constant such that significant flux is lost because of the finite resistivity of the wall. By analogy with the thin shell stability results in a pinch, this may also lead to difficulties with enhanced MHD activity. Attempts to detach the plasma from the flux conserver in the spheromak have been made by applying magnetic flux through the wall, though the external flux acts to destabilise the tilt mode and the $n=2$ mode is also destabilised [59]. A critical bias flux is indeed observed to produce instability but the tilting mode could be stabilised using a central conductor.

CONCLUSIONS

Early studies of pinch dynamics and stability theory by Rosenbluth were very important in influencing the evolution of the toroidal pinch. He was the first worker to establish the importance of the reverse toroidal field for stability and then went on with Furth and Killeen to explain the unstable nature of the hard-core pinches in terms of resistive instabilities. This left the toroidal pinches in some disarray and they were largely abandoned in the early 60's in favour of mirror machines, θ pinches and hard-core pinches. The 60's however did see the consolidation of ideal stability theory for the diffuse pinch with the importance of the reverse field becoming clear theoretically and experimentally. The Novosibirsk and Dubna conferences saw the emergence of the tokamak but there was also progress with the ZETA RFP with $\beta \sim 5\%$ and with RFP theory.

The early 70's saw the start of a number of fast RFP experiments in response to both theory and experiment. These used fast programming rather than self-reversal which was little understood at that time. Interest was further stimulated by the work of Taylor who established that the RFP was a minimum energy state and thus stable against MHD and tearing modes. These experiments were unable to achieve sustained high temperature operation and there was a return to the slow mode of operation as obtained on the ZETA device with considerably more success.

In the late 1970's Rosenbluth used Taylor's theory to introduce a new pinch configuration - the spheromak which is being actively pursued as an alternative to the RFP and tokamak.

Throughout the pinch investigations anomalous transport has been a problem, in particular the influence of magnetic perturbations has been highlighted repeatedly by Rosenbluth (and others).

The further development of the theory of toroidal pinches, in conjunction with present and future larger experiments, is a vital element in the progress of the toroidal pinch which I am sure will be forthcoming from the Institute of Fusion Studies at Texas under the guidance of M N Rosenbluth.

TABLE 1 (a)

Possible Characteristics of a Diffusion-limited, Self-heated D-T Reactor

Quantity	Value
Major radius of torus	30 cm (arbitrary)
Minor radius of torus	6 cm
Initial plasma radius (r_0)	1.5 cm
Current	3×10^6 amp
Pressure at wall	400 atm
Initial pinched density (ℓ_0)	$1.3 \times 10^{17}/cm^3$
Burning temperature (Tp)	6 kev
Disassembly time	0.15 sec
Total magnetic energy	4×10^6 joule
Losses (copper torus)	2.5×10^6 joule
Energy produced	3×10^7 joule
Temperature rise of copper surface due to radiation	500°C
Efficiency of burning	10%

TABLE 1 (b)

The Fusion Reactor of Thomson and Blackman

Fuel		D-D
Dimensions	R/a	= 1.30/.30 m
Discharge current	I_p	= 500 kA
Mean number density	n_e	$= 3.5 \times 10^{20} m^{-3}$
*Electron temperature	T_e	= 80 keV
*Ion temperature	T_i	= 500 keV
Particle confinement time	τ_p	= 65 s
Output power	P_{TOT}	= 9 MW (Th)
Heating and current drive	RF	= 2 MW (at 3 GHz)
Neutron output		$= 1.9 \times 10^{19} s^{-1}$

*The patent gives random speeds rather than temperatures

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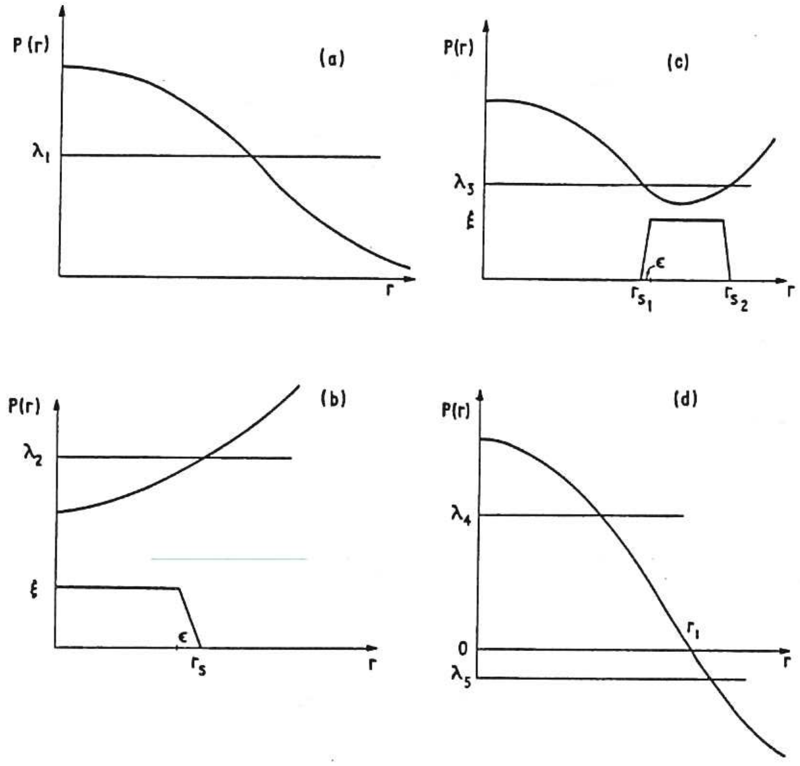
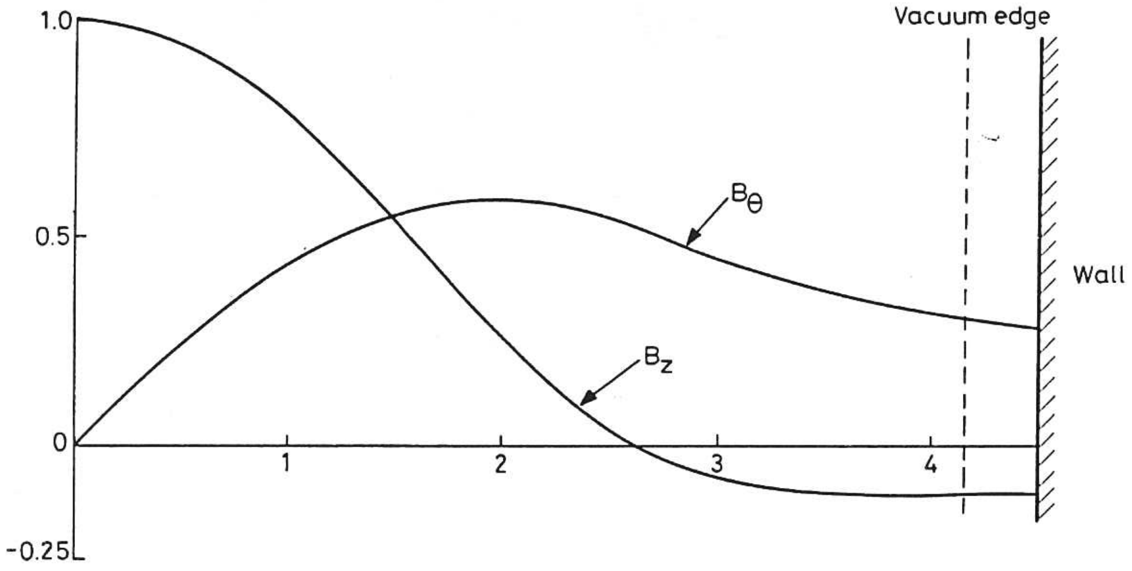


Fig. 1 Four radial variations of the pitch of the magnetic field lines.
(a) characterises a stabilised pinch with currents out to the wall;
(b) the variation similar to that of a tokamak;
(c) that associated with the stabilised pinch surrounded by a vacuum; and
(d) that associated with a reverse field pinch.



Pitch and pressure model equilibrium

Fig. 2 Tearing mode stable reverse field configuration with a vacuum edge.

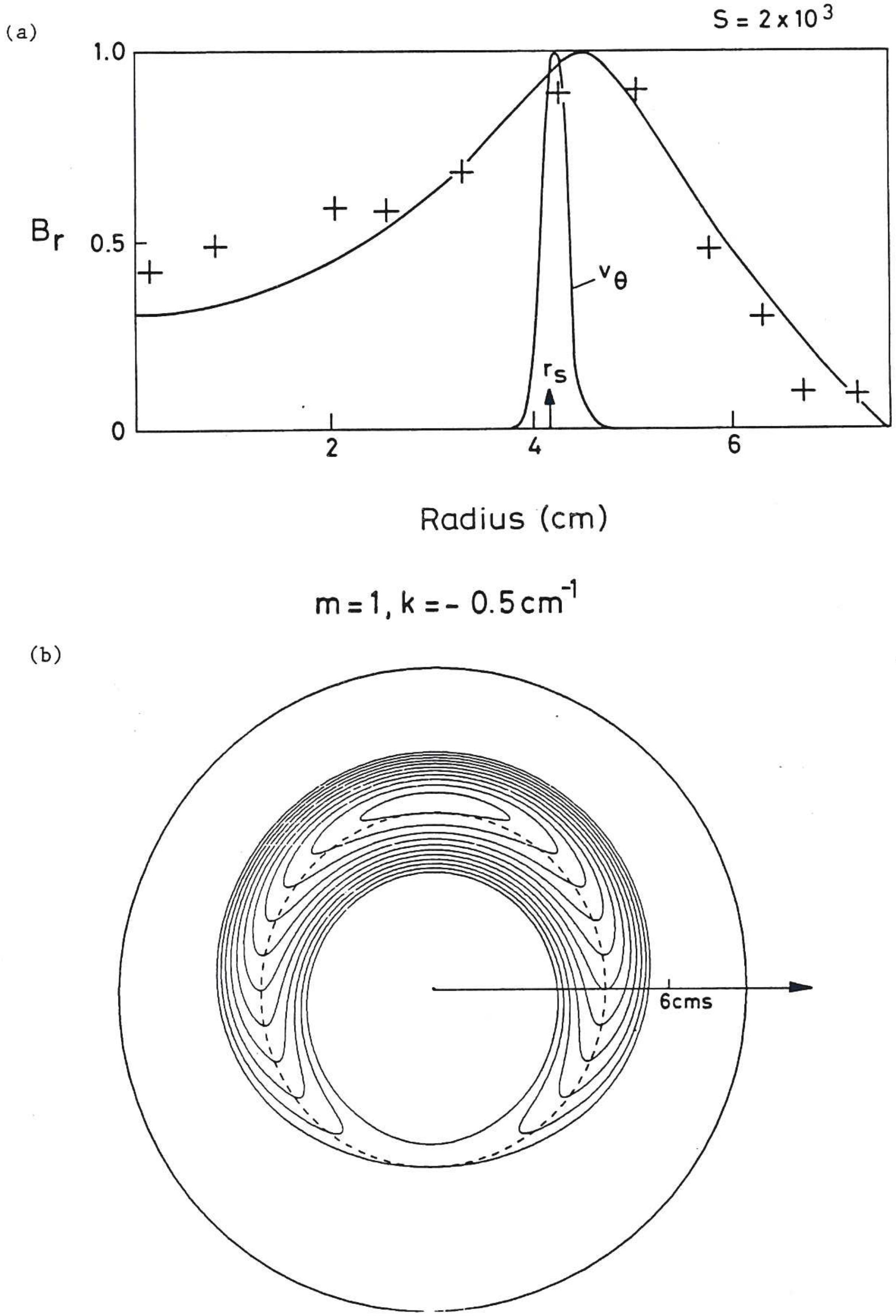


Fig. 3 (a) Measured and predicted radial field perturbation for a resistive interchange or g mode in a pinch.

(b) The magnetic island produced by the resistive interchange mode in the pinch with $\delta B_r/B\theta \sim 7\%$.

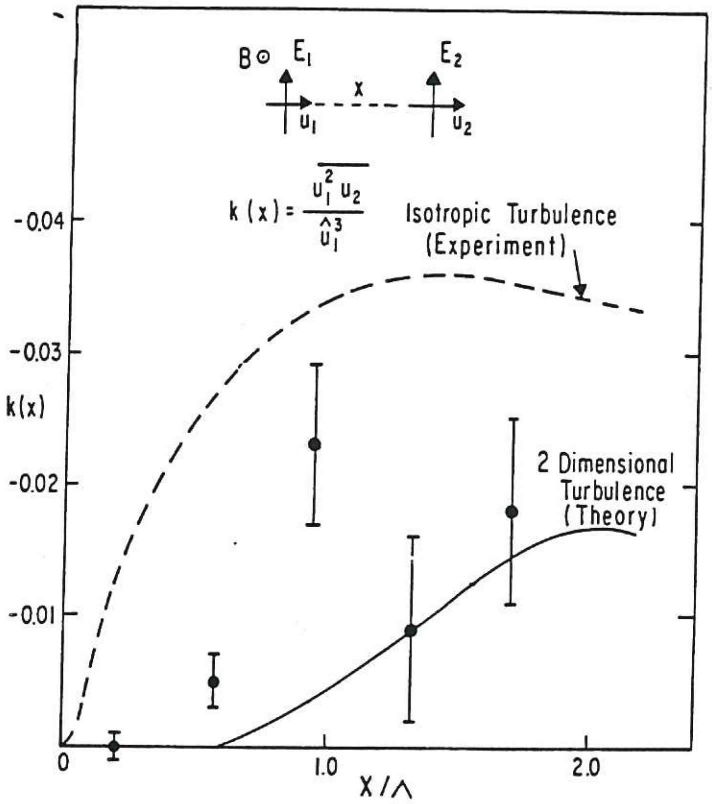


Fig. 4 Triple correlation as a function of the separation normalised to the integral scale length (Λ). The theoretical curve for 2-dimensional turbulence theory is shown and this is compared with experimental results and with experimental curve for isotropic turbulence from experiment.

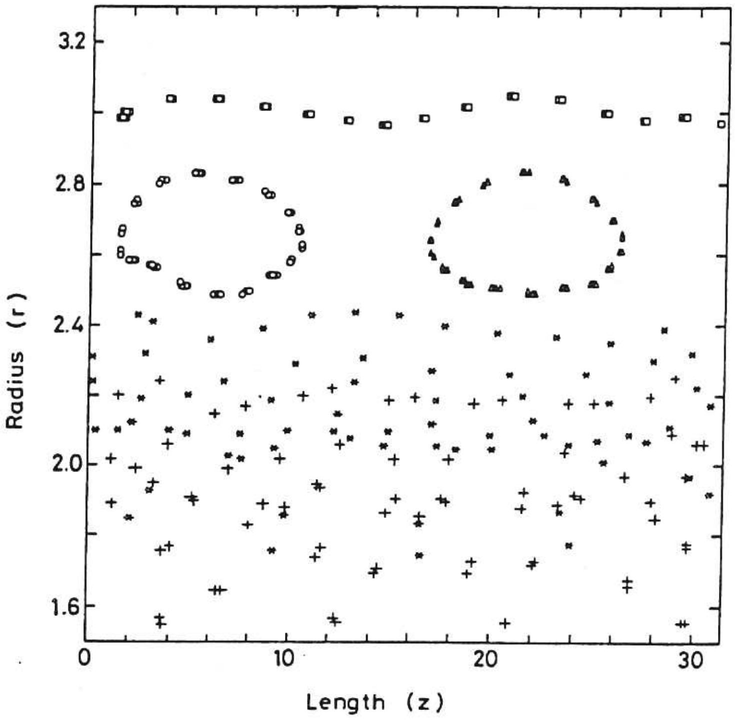
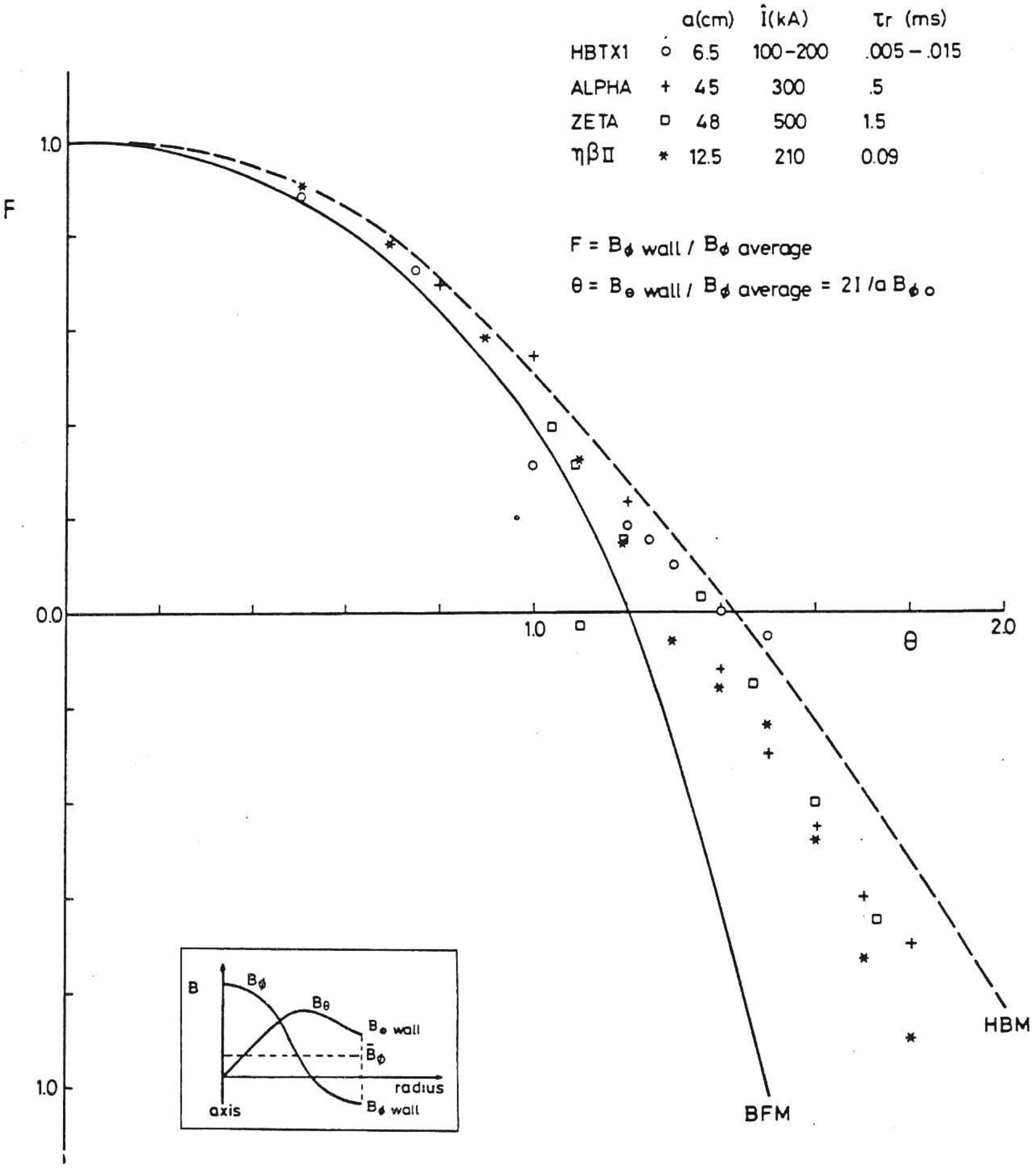


Fig. 5 The destruction of magnetic surfaces in the radius/length plane for a reverse field pinch excited by both m = 0 and m = 1 modes inside the reversal surface. The field reversal radius is at ~ 2.6.



Universal F - θ Curve, showing Data from Four Machines.

Fig. 6 The universal F-θ diagram showing data from 4 machines, BFM refers to the minimum energy configuration and HBM to the minimum energy configuration including pressure up to a value limited by the Suydam criterion.

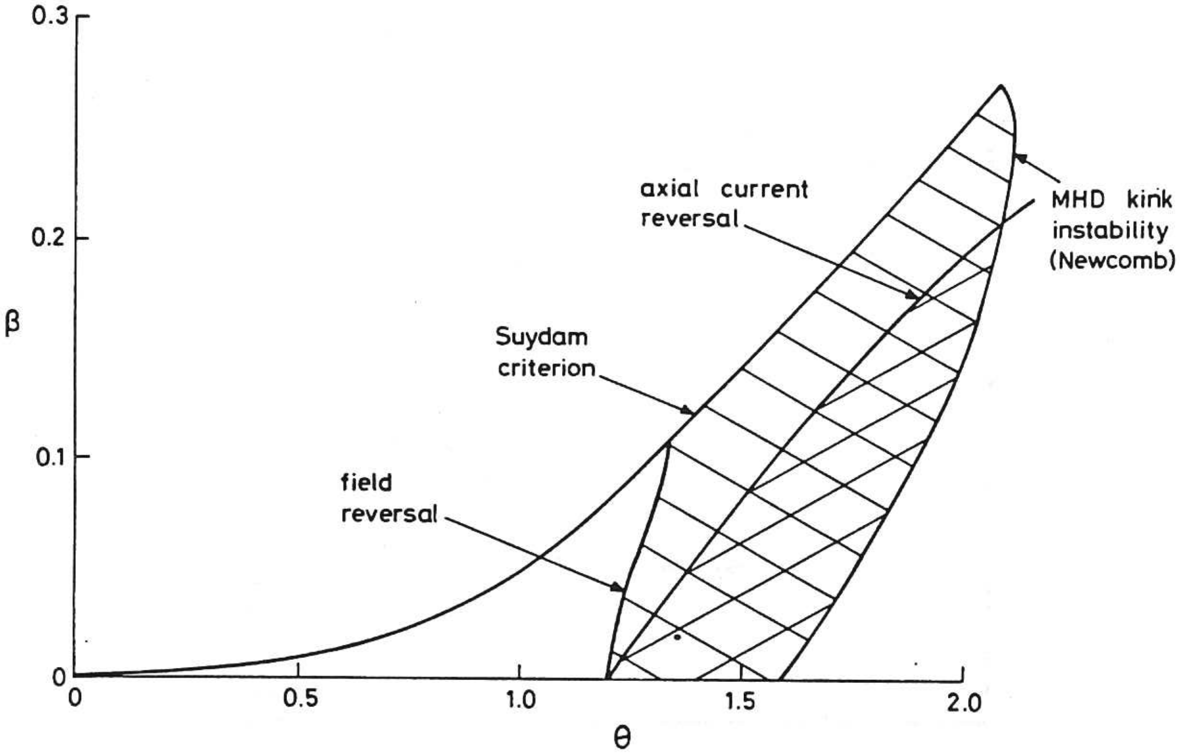


Fig. 7 Stability diagram, showing the limiting β on axis as a function of θ for the minimum energy configuration.

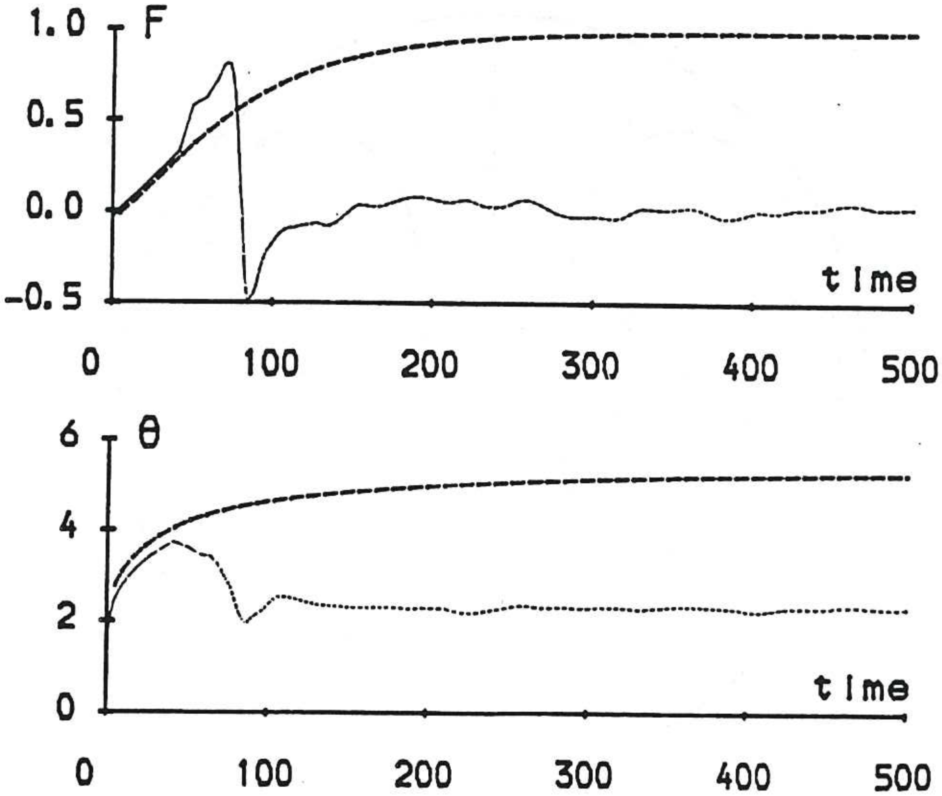


Fig. 8 F and θ as a function of time from 3-D code calculations. The dashed line shows results of 1-D resistive diffusion. In this case $S=10^3$ and η is uniform.

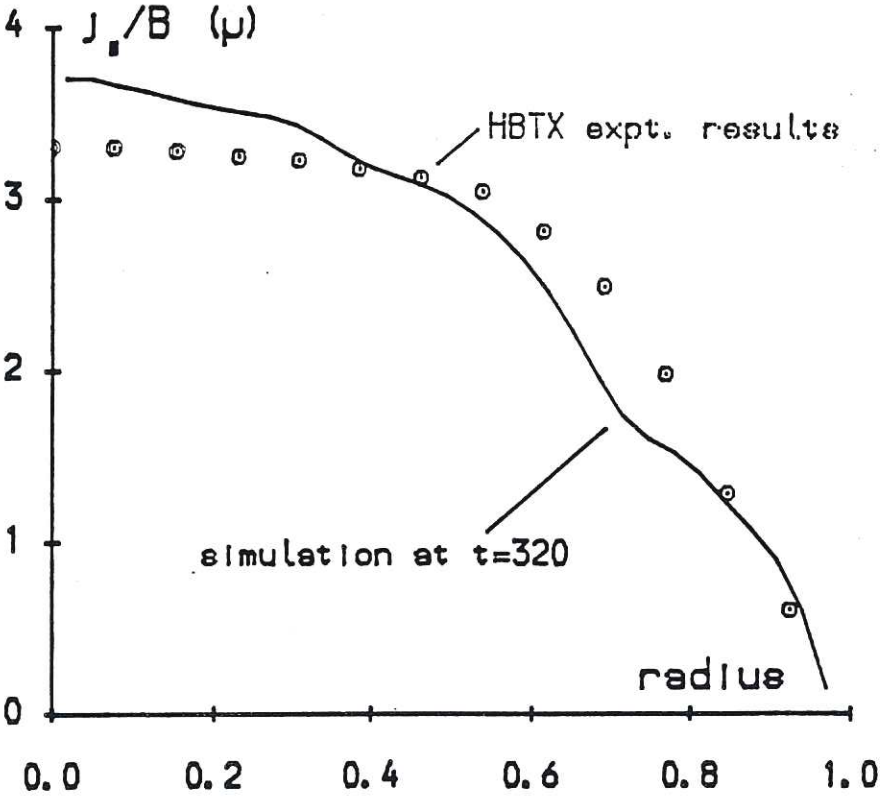


Fig. 9 $J_\parallel/B$ as a function of radius from 3-D code calculations compared with a measured profile from HBTX. In the calculations $S=10^3$.

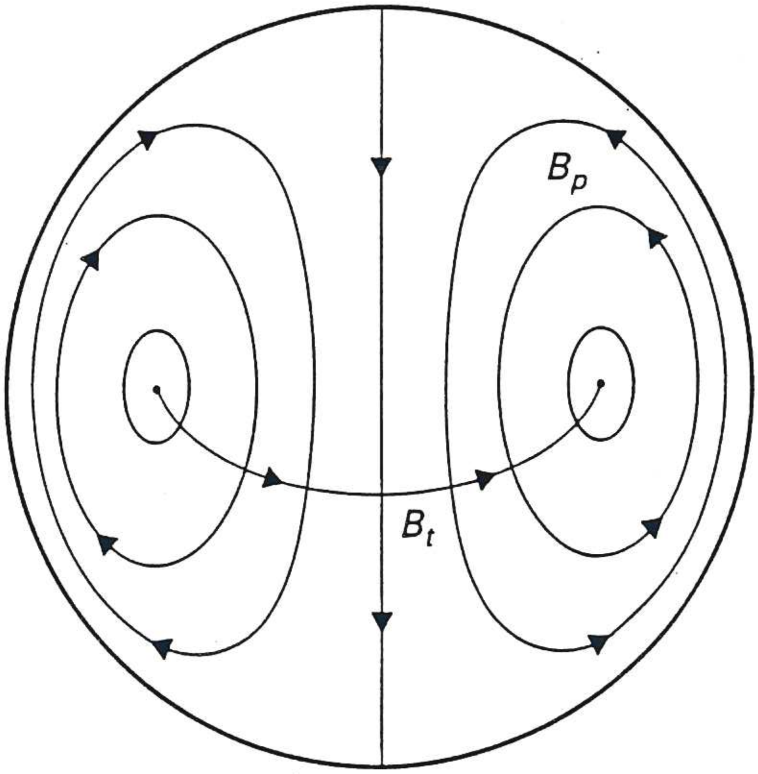


Fig. 10 The classical spheromak configuration showing poloidal and toroidal fields.