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Fine structure in the distribution of ion heating in a rotating toroidal plasma

R. O. Dendy

UK ATOMIC ENERGY AUTHORITY

Culham
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R. O. Dendy

Culham Laboratory, Abingdon, Oxfordshire OX14 3DB, England
(Euratom/UKAEA Fusion Association)

Abstract

In the laboratory frame, the helical path followed by an ion contained in a rotating toroidal plasma is determined to leading order by three factors: its velocity $v_{\parallel}$ parallel to the magnetic field, the q-factor of the flux surface on which the ion travels, and the velocity of bulk toroidal rotation $V_R(q)$ of the flux surface itself. A parameter $q^*(q, V_R(q), v_{\parallel})$ is calculated, which determines whether the helical path in the laboratory frame is closed (rational-q^*) or open (irrational-q^*). In general the spatially localised region where ion heating takes place is fixed in the laboratory frame. Those ions which have rational-q^* paths that pass through the heating region are subject, by frequent repetition, to greatly enhanced heating. The associated fine structure in the spatial and velocity space energy deposition is examined.

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Culham Laboratory
United Kingdom Atomic Energy Authority
Abingdon
Oxfordshire OX14 3DB

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Introduction

The guiding centre motion of the charged particles in a tokamak follows, to leading order, the lines of force of the magnetic field. For a detailed discussion of the magnetic field geometry in a tokamak, we refer elsewhere; see, for example, WESSON (1978). Here, we shall be concerned with the primary characteristic of the magnetic field lines in a tokamak, which is represented by the parameter q. This is the number of toroidal circulations that a field line makes in the course of one complete poloidal circulation. Magnetic surfaces for which q is a rational number are of interest for two classes of reason. First, from the magnetic point of view, any magnetic field perturbation can be expanded in terms of its poloidal and toroidal Fourier harmonics (m,n). It has been recognised for some time (ROSENBLUTH, SAGDEEV, TAYLOR and ZASLAVSKI, 1966) that magnetic field perturbations with given (m,n) resonate with the corresponding rational surfaces $q = m/n$ of the toroidal magnetic field, which are accordingly candidates for linear instability. Second, there is the point of view of particle kinematics, with which we shall be concerned here. Many tokamak heating schemes - for example electron cyclotron resonance heating and neutral beam injection - involve the transfer of energy from a fixed source to the plasma particles, rather than to the magnetic field. This energy transfer takes place within a relatively limited and well-defined region of space. The number of times that individual particles return to this region is clearly of interest, as it determines the degree of heating which they undergo. Since particle motion is governed to leading order by the value of q, so too is this number. A previous study (DENDY, 1985) considered the electron cyclotron resonance heating of a plasma which is at rest with respect to the localised gyrotron radiation source. It was shown how hot closed helical bands of electrons could arise in regions of finite width around rational surfaces $q = m/n$ with m and n low integers. We note that recent soft X-ray observations on JET (WELLER, CHEETHAM, EDWARDS, GILL, GONDHALEKAR, GRANETZ, SNIPES and WESSON, 1987) indicate the presence of "snakes" - persistent closed helical features - at rational-q surfaces during pellet injection. While magnetohydrodynamic explanations have been suggested, these experimental observations also motivate consideration of the role of rational-q surfaces from the point of view of ion kinematics.

The present study of ion kinematics in a rotating toroidal plasma is further motivated by the attempt to identify principles that may be of use when interpreting ion cyclotron emission from beam-heated tokamak discharges. Fine structure in the deposition of energy among the ions might result in corresponding features in the ion cyclotron emission spectrum. In general beam-heated discharges display bulk toroidal rotation, except when the effect is deliberately suppressed through beam balancing. At the centre of the plasma, the rotation velocity may be of order ten per cent of the ion thermal velocity (SUCKEWER, EUBANK, GOLDSTON, McENERNEY, SAUTHOFF and TOWNER, 1981). At the edge of the plasma, the rotation velocity tends to zero. Detailed experimental information on the variation of the rotation velocity with distance from the magnetic axis does not appear to be available. However, there are theoretical grounds for supposing that the velocity of rotation is constant on a given magnetic flux surface (HINTON and WONG, 1985). Thus, it can be written as a continuous function of q. This effect must be included when calculating the frequency with which a given ion passes through the spatially localised region where heating takes place. We shall show how the existence of magnetic surfaces, together with bulk toroidal rotation at a significant fraction of the ion thermal velocity, and the spatial localisation of ion heating combine to give fine structure to the spatial and velocity space energy deposition.

2. Ion dynamics of a plasma that rotates in the laboratory frame

Operations such as pellet injection and neutral beam heating, which involve interaction with the plasma ions, are localised in space and are fixed in the laboratory frame. In this Section, we shall calculate the frequency with which ions in a rotating plasma return to a fixed point in the laboratory frame. This will enable us to investigate the spatial effects for ions that are equivalent to the rational surface effects for heated electrons that have been discussed elsewhere (DENDY, 1985). The position for ions is complicated by the fact of bulk toroidal plasma rotation at a velocity $V_R(q)$ which is typically of order 0.1 times the ion thermal velocity. We note that V_R is a fraction of one per cent of the electron thermal velocity, and therefore plays a much less significant role in electron dynamics. To leading order, the motion of an ion follows a magnetic field line. At the same time, the magnetic field line is fixed to a toroidal flux surface that is rotating in the toroidal direction with velocity $V_R(q)$. The combination of these two components of velocity, one governed by q and the other by $V_R(q)$, determines the ion motion in the laboratory frame. We shall be concerned throughout with the major proportion of the ions in a tokamak, those that are not magnetically trapped by the radial gradient in magnetic field strength.

Consider a tokamak of major radius R, and a toroidal flux surface characterised by q with a minor radius r. The element of length of a magnetic field line on this surface is

$$ds = (r^2 d\theta^2 + R^2 d\phi^2)^{1/2} \tag{1}$$

where θ and ϕ denote poloidal and toroidal angle respectively. By definition, $q = d\phi/d\theta$, and the inverse aspect ratio of the magnetic surface is denoted by $\varepsilon = r/R$. Then Eq.(1) can be written

$$ds = (1+\varepsilon^2/q^2)^{1/2} qR d\theta \tag{2}$$

The distance which is travelled along the magnetic field line in the course of a single poloidal circulation by an ion is therefore given by L, where

$$L = \int_0^L ds = \int_0^{2\pi} 1+\varepsilon^2/q^2)^{1/2} qR d\theta \simeq 2\pi qR \tag{3}$$

Here, for simplicity, we have used the fact that $\varepsilon^2 << 1$ while q is of order unity. Let us denote the velocity of an ion in the direction of the magnetic field, measured in a frame that is corotating with the magnetic flux surface on which the ion moves, by $v_{\parallel}$. Then the time taken by an ion to complete one poloidal circulation is $|\tau(v_{\parallel},q)|$, where

$$\tau(v_{\parallel},q) = L/v_{\parallel} = 2\pi qR/v_{\parallel} \tag{4}$$

We note that $v_{\parallel}$ may be of either sign, and so therefore may the variable τ. Now the magnetic surface is rotating toroidally with velocity $V_R(q)$, which we shall treat as positive if it is in the same direction as the magnetic field. It follows that the distance which the surface rotates in the toroidal direction in the time taken for a single poloidal circulation is $V_R(q)|\tau(v_{\parallel},q)|$. This quantity augments or decreases the total toroidal angle through which the ion moves, depending on the sign of $v_{\parallel}$ relative to V_R. The algebraic change, expressed as a fraction of a complete toroidal circulation, is

$$\frac{V_R(q)\,\tau(v_{\parallel},q)}{2\pi R} = \frac{qV_R(q)}{v_{\parallel}} \tag{5}$$

where we have used Eq.(4). We now introduce the quantity q^*, which we define as the number of circulations in the toroidal direction that are completed in the course of one circulation in the poloidal direction, as observed in the fixed laboratory frame. From our discussion, and using Eq.(5), it is clear that

$$q^*(v_{\parallel},q) = q\left(1 + \frac{V_R(q)}{v_{\parallel}}\right) \tag{6}$$

We note that our argument was developed for the case where thermal motion along magnetic field lines is more rapid than the toroidal rotation: $|v_{\parallel}| \gtrsim |V_R|$, which is the case for the great majority of ions. The large values of q^* which occur when $|v_{\parallel}| < |V_R|$ reflect the fact that the motion of such ions is dominated by the toroidal bulk rotation of the plasma, with only a small poloidal component due to the slow motion along the magnetic field line. We note also that when $|V_R(q)|$ tends to zero, the value q^* tends to q for all ions, independently of the value of $v_{\parallel}$, as expected. The value of q^* tends to q for all ions for which $|v_{\parallel}| >> |V_R(q)|$, again as expected; see Fig.1. Interesting phenomena arise when $v_{\parallel}$ is opposite in sign and comparable in magnitude to $V_R(q)$. For example, we can have $q^* = 0$, corresponding to purely poloidal motion in the fixed laboratory frame, when the toroidal component of the ion motion is exactly cancelled by the oppositely directed bulk rotation.

We have now established the formula for q^*, Eq.(6). It is the value of q^* that determines how often an ion returns to a given point which is fixed in the laboratory frame - such as, in particular, the heated region. If q^* is rational, then the helical toroidal path of the ion with respect to the laboratory frame is closed. Thus, if $q^* = 1, 3/2, 2, \ldots$, then an ion which starts in the heated region will return to it after one, three, two,... poloidal circulations. Conversely, an ion on the same surface whose closed helical path does not pass through the heated region will never undergo heating. If q^* is an irrational number, the helical path of the ion in the laboratory frame is not closed, and it covers a toroidal surface ergodically. The number of times that the ion enters the heated region, and hence the degree of heating that it experiences, is thus independent of its starting position. It is considerably less than that of the repeatedly heated group of ions having rational q^*, and greater than that of the rational-q^* ions which do not enter the heated region. This argument has already been developed for electrons in terms of q (DENDY, 1985, to which we refer for further details), and the merging of the two regimes for near-rational values of q has been demonstrated. Here, however, we have a significant difference. While q is a function only of which flux surface is considered, q^* is a function of $v_\parallel$ as well as of the identity of the flux surface. Thus, in a rotating plasma, ions on the same flux surface but with different values of $v_\parallel$ have different values of q^*. Conversely, ions on different flux surfaces with different values of $v_\parallel$ may have the same, possible rational, value of q^*.

To fix ideas, let us suppose that $V_R(q = 2) = 0.1\ V_{T\parallel}$, where $V_{T\parallel}$ is the characteristic thermal velocity of the ions parallel to the magnetic field. Then on the rational-q surface at q = 2, the values of q^* for $v_\parallel = -2V_{T\parallel}$, $-V_{T\parallel}$, $V_{T\parallel}$, and $2V_{T\parallel}$ are respectively given by Eq.(6): $q^* = 1.9$, 1.8, 2.2, and 2.1. Thus the values of q^* for typical thermal ions on the same rational-q surface differ by 0.4; furthermore, unlike the value of q itself on this surface, the typical values of q are not low mode-number rational numbers. Conversely, it is only for superthermal ions that q^* has a value close to $q = 2$. Now let us consider neighbouring flux surfaces. For simplicity, we shall assume that $V_R(q)$ does not change significantly over the short distances involved, and retains its value of $0.1V_{T\parallel}$. Then $q^* = 2$ on the following surfaces for the stated values of $v_\parallel$: $q = 1.8$, $v_\parallel = 0.9V_{T\parallel}$;

$q = 1.9$, $v_{\parallel} = 1.9V_{T\parallel}$; $q = 2.1$, $v_{\parallel} = -2.1V_{T\parallel}$; $q = 2.2$, $v_{\parallel} = -1.1V_{T\parallel}$. This is the corollary of the wide range of values of q^* for thermal ions on a given flux surface that has already been demonstrated. We can find thermal ions on a wide range of nearby flux surfaces with particular values of $v_{\parallel}$ for which $q^* = 2$ exactly. The thermal ions on surfaces with $q<q^* = 2$ have $v_{\parallel}>0$; those on surfaces with $q>q^* = 2$ have $v_{\parallel}<0$. If it were possible to localise heating to surfaces with $q>2$, for example, the hot closed helical bands near $q = 2$ would contain ions travelling only in one toroidal direction. It may be possible to exploit this velocity space asymmetry for the purpose of localised current drive. We note also that the phemomena that we have outlined involve the deposition of energy among the plasma ions in a way that is initially strongly localised in real space and in velocity space. Diffusion processes and, possibly, velocity space instabilities will subsequently tend to distribute the energy with greater homogeneity. A detailed study of this nonlinear problem is beyond the scope of this paper. However, we conclude that the associated complex small-scale energy flow in real space and in velocity space will be a further consequence of the spatially localised heating of rotating toroidal plasmas.

3. Conclusions

We have considered the spatially localised heating of the ions in a toroidally rotating toroidal plasma that are not magnetically trapped. In the rest frame defined by the laboratory, the bulk toroidal rotation of the plasma must have a definite direction, and we have examined the consequences for ion motion of the associated loss of symmetry. The existence of magnetic flux surfaces, the fact of rotation, and the spatial localisation of the heating at rest in the laboratory frame are together sufficient to give rise to extensive fine structure in the spatial and velocity-space energy deposition. On an extended range of flux surfaces in the neighbourhood of any rational-q flux surface, the quantity q^* defined in Eq.(6) takes the rational value q for ions travelling with a particular velocity $v_\parallel$. These ions accordingly travel on closed helical paths in the laboratory frame. Those ions whose closed rational-q^* paths pass through the heated region of space are subject by frequent repetition to a greatly enhanced degree of heating. They form a hot closed helical band. The correlation between the position of a flux surface and the value of $v_\parallel$ for which q is rational may be helpful for localised current drive. Finally, we note that the gradients associated with the initially strongly localised deposition of energy among the plasma ions will give rise to complex small-scale energy flow in real space and in velocity space.

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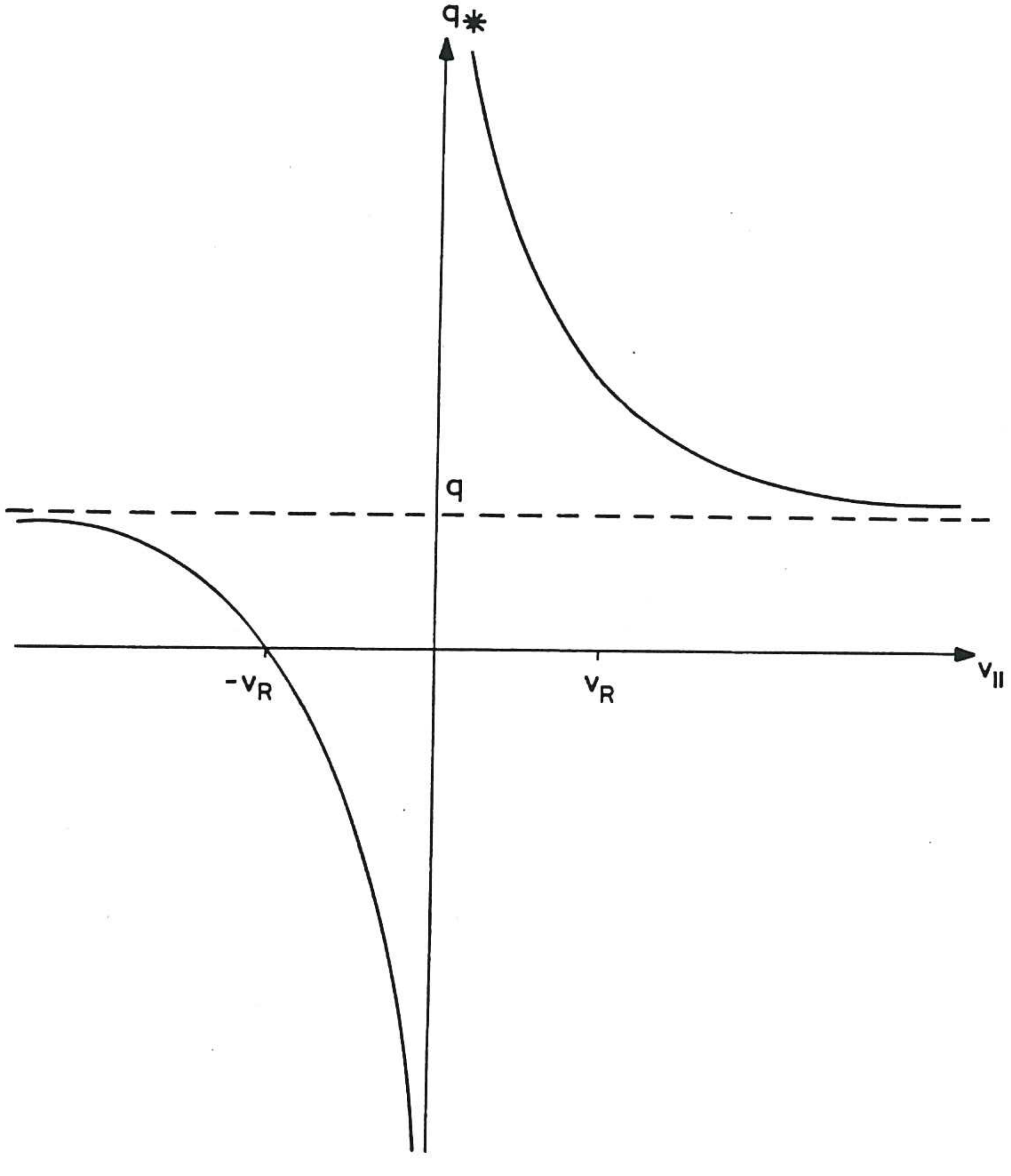

The parameter q^* given by Eq.(6), plotted as a function of $v_{\parallel}$ at fixed q.