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THE LIMITS OF THE DIFFUSIVE THEORY OF ELECTRON CYCLOTRON HEATING

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INTRODUCTION

The use of diffusive operators to model physical processes is widespread. In particular, in numerical codes which model the effects of radio-frequency heating on particle distribution functions in magnetically confined plasmas [e.g. 1,2,3,4], the heating is represented by an operator which is diffusive in velocity space. The validity of the diffusive approach rests on two assumptions: firstly that the particle motion in the wave fields is represented adequately by unperturbed orbit theory, and secondly that changes in particle speed resulting from interaction with the waves are small compared to the scale length of the distribution function f in velocity space [5]. In this paper the operator appropriate when these conditions are violated is discussed, and the resulting equation for f is solved numerically for cases in which the first approximation is valid but the second is not, thus allowing investigation of the validity of diffusive theory. Electron Cyclotron Resonance Heating (ECRH) is discussed here, although the theory is readily generalised to Landau Damping, ICRH, and other processes usually modelled with diffusive operators.

THE NON-DIFFUSIVE HEATING OPERATOR

The diffusive operator may be derived either by using quasilinear theory [6], or by considering the motion of single electrons and then generalising to the ensemble using Fokker-Planck theory (eg. [1]). We use the latter method and follow the approach in ref. [7], limiting the treatment to uniform background magnetic field. If initially $\mu = \mu_i = v_\perp^2/2$, then the change in μ due to one transit through the ECRH beam is [1,7]

$$\Delta\mu = K\sqrt{\frac{\mu_i}{2}}\cos\psi + \frac{K^2}{8} \qquad (1)$$

Here ψ is the initial phase difference between the wave and the electron, $v_\perp$ is the component of velocity perpendicular to the magnetic field and K, the "kick" received by the electron, = $(eE_-\tau)/m$ with E_- the component of the wave electric field rotating with the electrons and τ a measure of the time spent in resonance in the beam. This form for K is for extraordinary mode heating at the fundamental resonance: for ordinary mode or harmonic heating other expressions for K are appropriate [1,7]. For harmonic heating the term of order K^2 in eq(1) is different; however for both modes and all resonances the following phase-averaged equality applies [5],

$$\langle\Delta\mu\rangle = \frac{1}{2}\frac{\partial}{\partial\mu}\{\langle(\Delta\mu)^2\rangle\} \qquad (2)$$

We now generalise to the ensemble writing [5]

$$\left.\frac{\partial f}{\partial t}\right)_{ECRH} = \frac{1}{\Delta t}\{\int P(\mu-\Delta\mu,\Delta\mu)f(\mu-\Delta\mu)d(\Delta\mu)-f(\mu)\} \qquad (3)$$

where $P(\mu,\Delta\mu)$ is a kernel giving the probability of change from μ to $\mu+\Delta\mu$ in time Δt, and we have assumed loss of phase information between successive resonances with the beam. In Fokker-Planck theory, Taylor expansion of the integrand in eq(3) yields, using eq(2) [7],

$$\left.\frac{\partial f}{\partial t}\right)_{ECRH} = \frac{\partial}{\partial\mu}\left[\frac{1}{2\Delta t}\langle(\Delta\mu)^2\rangle\frac{\partial f}{\partial\mu}\right] \qquad (4)$$

We refrain from making this diffusive approximation and retain the integral mapping of eq(3) for our study of the limits of diffusive theory. The kernel $P(\mu,\Delta\mu)$ may be derived from eq(1) [7],

$$P(\mu,\Delta\mu) = \frac{1}{\pi}\frac{1}{\sqrt{\frac{K^2\mu}{2} - (\Delta\mu-\frac{K^2}{8})^2}} \qquad (5)$$

Equation (5) can hold even if the inequality $|\Delta\mu|\ll\mu$ is not satisfied. Figure 1 shows the results of a calculation by a particle following code [7], in which 1000 particles, identical in initial conditions other than their phase, pass through an ECRH beam. The dashed line, given by eq(5), reproduces the final distribution. However at higher electric fields, due to non-linear physics associated with the change of the electron mass due to heating or cooling, the kernel P differs from the analytic form (Fig 2). In the latter case eq(3) is still valid, although with this different form for P.

ELECTRON DISTRIBUTION FUNCTION

A code has been written to solve the time dependent electron Boltzmann equation for $f(v_{\parallel},v_{\perp})$, in which the effects of collisions (with stationary, massive ions and an electron Maxwellian) are balanced by those of the ECRH, using eq(3) or eq(4). The discretisation of eq(3) conserves particles and gives the standard diffusive discretisation for small K. Also, for the same f, eqs(3) and (4) give the same power absorption, as may be shown analytically. We now examine the validity of the diffusive approximation. If the diffusion coefficient $D = K^2/(16\Delta t)$ is large compared with collisional diffusion ($\upsilon_{ee}v^2$, with υ_{ee} the collision frequency of the heated electrons) then the (steady state) distribution will be flat in the $v_{\perp}$ direction ("quasilinear flattening") and the diffusive approximation will become valid. This behaviour is produced by the code; i.e. if D is "large", the distributions produced using eqs(3) and (4) as the heating operators are similar. However, if D is small then differences between the two approaches are observed. This does not necessarily mean that K is small compared to $v_e = (2T/m)^{1/2}$, only that collisions are sufficiently strong to keep f close to Maxwellian. We expect that in cases in which the diffusive approach is invalid differences will be apparent in the deviation of f from Maxwellian, a measure of which is the driven current J.

In the table below we show J as a function of K for heating localised in velocity space around $v_{\|} = v_e$. We have varied Δt to maintain $D = 0.0025\, \upsilon_o v_e^2$ with $\upsilon_o = (n_e e^4 \ln\lambda)/(4\pi\varepsilon_o^2 m_e^2 v_e^3)$, and thus the absorbed power is independent of K. The current is normalised to give J=1 for the diffusive limit (K=0). J differs from 1 for $K \gtrsim 2v_e$ (ie. $\Delta v_{\perp} \gtrsim v_e$) although even for $K = 4v_e$ the difference is only 16%.

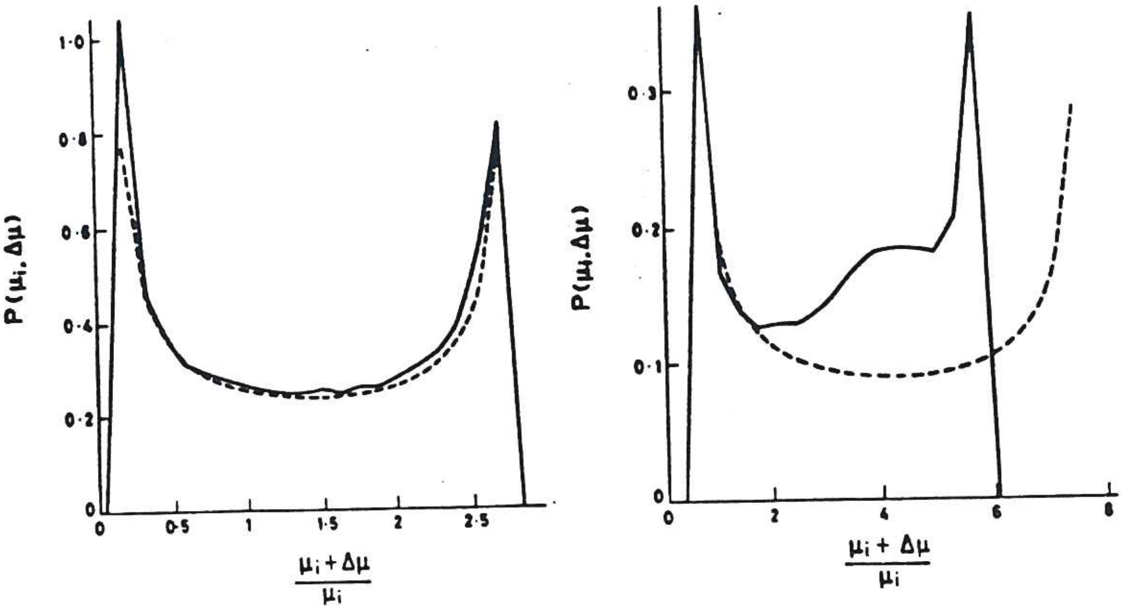
K/v_e	J
0.2	1.001
2.0	1.04
4.0	1.16

Figures 3 and 4 show contour plots of the deviation of f from Maxwellian for K=0.2 and K=4. The dashed (solid) lines indicate f is less (greater) than the Maxwellian. The contour plot for $K=4v_e$ extends to higher $v_{\perp}$ and the two distributions differ noticeably.

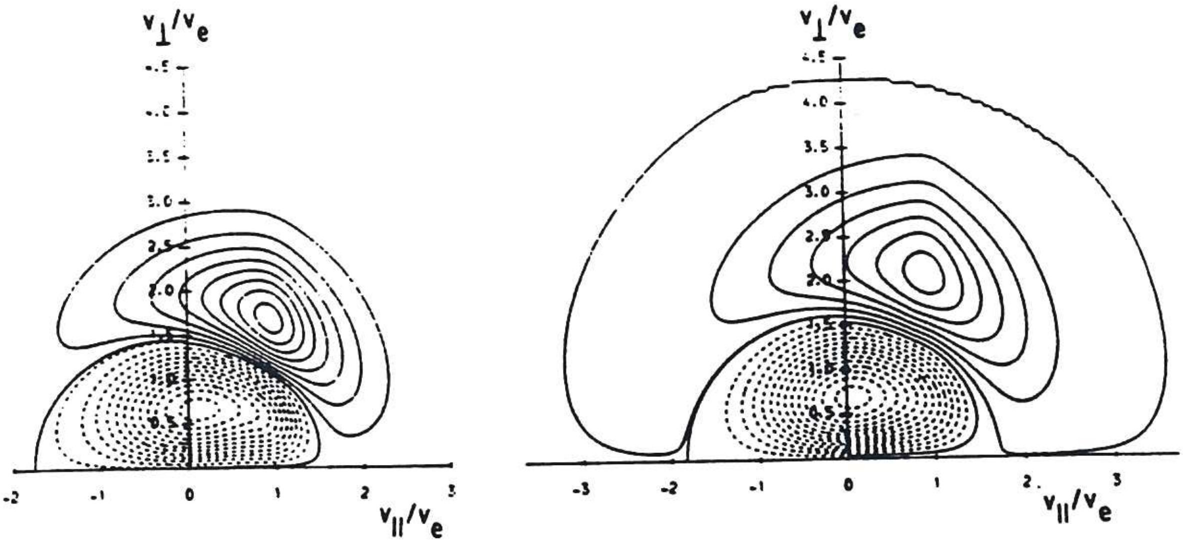
In conclusion the diffusive approximation appears valid even for quite large "kicks" $\Delta v_{\perp} \sim v_e$. Non-linear effects on the single particle motion, leading to violation of eqs(1) and (5) (cf. Fig 2), may be more likely to invalidate the diffusive model than the "kick" being comparable to the thermal speed. However systems with trapped particles (beyond the scope of the present code) may show larger effects. In such systems the deeply trapped and passing populations would be coupled directly, not just through diffusion at the trapped/passing boundary, possibly leading to lower currents than predicted by diffusive models.

REFERENCES

[1] M R O'Brien, M Cox and D F H Start, Nucl. Fus. 26 (1986) p.1625
[2] G D Kerbel and M G McCoy, Phys. Fluids, 28 (1985) p.3629
[3] C F F Karney and N J Fisch, Nucl. Fus., 21 (1981) p.1549
[4] I Fidone et al., Nucl. Fus., 27 (1987) p.579
[5] A J Lichtenberg and M A Liebermann, Regular and Stochastic Motion, Springer-Verlag (1983)
[6] C F Kennel and F Engelmann, Phys. Fluids, 9 (1966) p.2377
[7] A W Taylor, R A Cairns and M R O'Brien, to be published in Plasma Physics and Controlled Fusion



Figs 1 and 2 Graphs of velocity distributions of 1000 electrons after resonant heating by 28GHz radiation. The electrons had identical initial conditions ($v_\perp = v_\parallel = 10^7 ms^{-1}$) except for their phase with respect to the waves. $E_- = 7.5 \times 10^3$ V/m (Fig.1) and $E_- = 2 \times 10^4$ V/m (Fig.2). The width of the beam was 10cm. The dashed line is the analytic result (eq(5)).



Figs 3 and 4 Contours of deviation of f from Maxwellian for $K=0.2v_e$ (Fig.3) and $K=4v_e$ (Fig.4). Solid (dashed) lines represent positive (negative) deviation. Contour values are uniformly distributed between the most negative and most positive deviations.