CLM-P848

CLM-P848

Plasma current drive by helicity injection in relaxed states

J.B.Taylor
M.F.Turner

UK ATOMIC ENERGY AUTHORITY

Culham Laboratory

Plasma current drive by helicity injection in relaxed states

by

J B Taylor and M F Turner
Culham Laboratory, Abingdon, Oxon
(Euratom/UKAEA Fusion Association)
United Kingdom

ABSTRACT

The sustainment of plasma discharges (current-drive) using helicity-injection and relaxation is discussed. It is shown that toroidal plasmas may be sustained by applying only poloidal voltage and current. Some of the configurations which can be maintained in this way are calculated.

1. Introduction

In the theory of relaxed states [1], a turbulent plasma relaxes to a configuration of minimum-energy subject to the constraint of constant magnetic helicity. In a toroidal system with $\underset{\sim}{B}\cdot\underset{\sim}{n} = 0$ on the boundary, the (gauge-invariant) helicity is

$$K = \int \underset{\sim}{A}\cdot\underset{\sim}{B}\, d\tau - \oint \underset{\sim}{A}\cdot d\ell \oint \underset{\sim}{A}\cdot ds \tag{1}$$

where $\underset{\sim}{B} = \nabla \times \underset{\sim}{A}$ and the line integrals $\oint d\ell$ and $\oint ds$ are along paths the long and short way around the surface of the torus. In a spherical system, such as a spheromak, the helicity is simply

$$K = \int \underset{\sim}{A}\cdot\underset{\sim}{B}\, d\tau \tag{2}$$

The relaxed state satisfies the equation

$$\nabla \times \underset{\sim}{B} = \mu \underset{\sim}{B} \quad , \quad \nabla\mu = 0 \quad , \tag{3}$$

but its full determination depends on the boundary conditions and is different for toroidal and spherical systems.

In a toroidal system with a perfectly conducting boundary, on which $\underset{\sim}{B}\cdot\underset{\sim}{n} = 0$, the relaxed state is determined by two parameters. The first is the already mentioned magnetic helicity. The second is the toroidal flux Ψ, which is invariant on account of the boundary conditions. When both K and Ψ are specified, the parameter μ and the normalised field profile are determined by the dimensionless ratio K/Ψ^2 . The value of μ varies continuously with K/Ψ^2 but is always below the lowest eigenvalue of Eq. (3). This eigenvalue corresponds to a solution of Eq. (3), satisfying the boundary condition $\underset{\sim}{B}\cdot\underset{\sim}{n} = 0$, for which the toroidal flux Ψ vanishes. It can also be described more succinctly as a solution of the equation

$$\nabla \times \nabla \times \underset{\sim}{a}_i = \lambda_i \nabla \times \underset{\sim}{a}_i \tag{4}$$

with $\underset{\sim}{a}_i = 0$ on the boundary.

For a spherical system (with the same boundary conditions), the relaxed state is determined in a different manner. One again has the invariant magnetic helicity, but one cannot define an invariant toroidal flux (since the system is simply-connected). However, in a simply-connected region, the <u>only</u> solutions of Eq. (3) which satisfy the boundary condition are the eigenfunctions themselves. Consequently for a spherical system μ must equal the lowest eigenvalue and is thus determined solely by the size and shape of the boundary. The helicity then determines only the magnitude of the field.

The distinction between toroidal and spherical systems is, of course, related to the fact that in a spherical system there is no vacuum field which satisfies the boundary condition. If we write the field as the sum of a vacuum field $\underset{\sim}{B}_v$, which carries the toroidal flux, and a plasma field $\underset{\sim}{B}_p$ which carries no toroidal flux, then Eq. (1) becomes

$$\nabla \times \underset{\sim}{B}_p - \mu \underset{\sim}{B}_p = \mu \underset{\sim}{B}_v \quad . \qquad (5)$$

In a toroidal system this equation is inhomogeneous, with $\underset{\sim}{B}_v$ acting as the source, but in a simply connected system $\underset{\sim}{B}_v = 0$ and only the eigenvalue solutions exist (note that $\underset{\sim}{B}_p$ satisfies the eigenfunction boundary conditions).

2. <u>Helicity Injection</u>

Although we may treat K as invariant during the fast relaxation, it will subsequently decay, along with the magnetic field, on the resistive timescale. Usually, the field departs from the relaxed profile as it decays and this leads to secondary relaxations which restore the relaxed profile but at a lower field strength. It seems, therefore, that if the helicity lost through resistivity were continuously replaced, the plasma might be indefinitely maintained close to its original state. Helicity injection and relaxation together therefore provide a possible means for "current drive" in resistive plasmas.

Two methods for injecting helicity have been proposed. The first is by applying oscillating toroidal and poloidal voltages to a toroidal discharge [2] and is not considered in this paper. The second form of helicity injection [3] involves only static voltages, but requires the boundary condition to be changed so that there is some magnetic flux penetrating the boundary. Although our main interest is in toroidal systems, it is convenient to discuss first the use of helicity injection in the flux-core spheromak (FCS), shown in Fig 1. This is a modification of the simple spheromak in which flux penetrates the boundary through the polar caps and a static voltage is applied between them.

3. Flux Core Spheromak

Before we can discuss "static" helicity injection we must reconsider the definition of helicity, since if $\underset{\sim}{B}\cdot\underset{\sim}{n} \neq 0$ on the boundary the expressions (1) and (2) are no longer gauge invariant. In the FCS the simplest way to restore gauge invariance [1,4] is to imagine that the field inside the sphere is extended outside it as a vacuum field, defined by $\underset{\sim}{B}\cdot\underset{\sim}{n}$ at the boundary surface. Then the total integral of $\underset{\sim}{A}\cdot\underset{\sim}{B}$ inside <u>and outside</u> the sphere is gauge invariant. If the boundary is perfectly conducting, so that the normal component of field is "frozen in", changes in the interior do not affect the hypothetical field outside. Thus one can define the difference in helicity between two plasma configurations, each with the same assumed extension outside, in gauge invariant form. An alternative expression of gauge invariant helicity has been given by Finn and Antonsen [5]. In this case, one introduces a vacuum field $\underset{\sim}{B}_v$ <u>inside</u> the system, again defined by the normal component of the field on the boundary. Then a gauge invariant helicity can be defined as

$$K = \int(\underset{\sim}{A} + \underset{\sim}{A}_v)\cdot(\underset{\sim}{B} - \underset{\sim}{B}_v) \qquad (6)$$

where the integral is over only the internal volume. This form can also be used in a toroidal system if the reference field B_v also carries the correct toroidal flux.

Once the helicity is suitably defined, it can readily be shown that it is constant if the boundary is equipotential and the plasma perfectly conducting. It can also be shown that if a voltage V_e is applied between the electrodes, and the plasma has resistivity η (so that $\underset{\sim}{E}\cdot\underset{\sim}{B} = \eta\ \underset{\sim}{j}\cdot\underset{\sim}{B}$,) then helicity changes at a rate [3]

$$\frac{dK}{dt} = 2[V_e\Psi_e - \int\eta\underset{\sim}{j}\cdot\underset{\sim}{B}\ d\tau] \tag{7}$$

where Ψ_e is the magnetic flux through each electrode. The first term represents the 'injection' of helicity and the second its resistive dissipation. Thus, by applying the appropriate voltage across the polar caps, the helicity can be maintained.

When the Flux Core Spheromak is maintained by helicity-injection, then, unlike the simple Spheromak, the parameter μ does <u>not</u> have to be an eigenvalue. It is controlled by the current I_e through the electrode, since in the relaxed state $\mu = I_e/\Psi_e$. (Note that the configuration is therefore completely determined by Ψ_e and I_e.) When μ is varied the behaviour of the system as μ approaches the lowest eigenvalue is interesting. As $\mu \to \lambda_o$ the current on field lines linked to the polar electrodes (the electrode-linked current) becomes confined to a thin filament along the axis and almost all the plasma current is generated by relaxation. (Just as reverse field is generated in a toroidal pinch.)

The generation of current by relaxation (the plasma dynamo) during helicity injection leads to an increase in the impedance between the electrodes. Qualitatively this can be illustrated by considering a plasma of uniform resistivity. Then helicity balance requires

$$V_e\Psi_e = \eta\mu\int B^2 d\tau \tag{8}$$

so that the impedance $Z = (\eta/a)\ \hat{Z}$ and

$$\hat{Z}\ (\mu a) = \frac{a\int B^2 d\tau}{\Psi_e^2} \tag{9}$$

Note that $\hat{Z}$ depends only on μa and $\hat{Z} \sim a^4/\delta^4$, where δ is the radius of the central filament of electrode-linked current. As $\mu \to \lambda_0$, $\delta \to 0$ and the impedance $\hat{Z}$ becomes infinite. However, it is important to realise that this increase is not simply a resistive effect due to the constriction of the electrode-linked current; this would produce only a much smaller impedance $\sim a^2/\delta^2$. In the relaxed state, helicity and energy are extracted from the electrode-linked current and distributed throughout the plasma by the relaxation (dynamo) process. This is reflected as the excess impedance over that due to constriction of the electrode-linked current channel.

It is convenient to discuss helicity-injected relaxed states in terms of the eigenfunctions of the associated eigenvalue problem (Eq 4). If we assume these form a complete set we can write [3]

$$\underset{\sim}{B} = \underset{\sim}{B}_v + \underset{\sim}{B}_p = \nabla \times \underset{\sim}{A}_v + \Sigma\, \alpha_i \nabla \times \underset{\sim}{a}_i \tag{10}$$

where $\underset{\sim}{B}_v$ is the interior vacuum field defined by the normal flux on the surface of the system (so that $\underset{\sim}{B}_p$ satisfies the same boundary conditions as the eigenfunctions $\underset{\sim}{a}_i$). The eigenfunctions are orthogonal and can be normalised so that

$$\int \underset{\sim}{a}_i \cdot \nabla \times \underset{\sim}{a}_j \, d\tau = \delta_{ij} \operatorname{sgn}(\lambda_i) \tag{11}$$

(note $\underset{\sim}{a}_i$ then has dimensions of inverse length). Then

$$\alpha_i = \frac{\mu I_i}{(\lambda_i - \mu)} \operatorname{sgn}(\lambda_i) \qquad I_i = \int \underset{\sim}{a}_i \cdot \underset{\sim}{B}_v \, d\tau \tag{12}$$

and the impedance becomes

$$\hat{Z} = \frac{a}{\Psi_e^2} \left[\int B_v^2 \, d\tau + \mu^2 \sum \frac{|\lambda_i| I_i^2}{(\lambda_i - \mu)^2} \right] \tag{13}$$

The impedance increases with μ and diverges like $(\lambda_o - \mu)^{-2}$ as $\mu \to \lambda_o$.

4. Toroidal Systems

We noted earlier that relaxed states in a torus are determined in a different way to those in a simply-connected system, even though the governing equation is the same in both cases. Similarly, helicity injection in a torus has features which differ from those in a flux core spheromak.

A toroidal configuration superficially similar to the flux core spheromak is shown in Fig. 2. If V_e and Ψ_e again denote the voltage and magnetic flux between the upper and lower electrodes, then the rate of change of helicity is again given by Eq (7). Thus, in a torus, as in a spheromak, helicity can be maintained by the application of static voltages provided that the magnetic flux through the boundary is not everywhere zero. Similarly, the value of μ is again controlled by the ratio I_e/Ψ_e. However, in a toroidal system there is an additional parameter which affects the relaxed state maintained by helicity injection. This is the toroidal flux Ψ_t - which is independent of Ψ_e. Consequently in a toroidal system the steady-state for a given current and flux through the electrodes is not unique.

A toroidal system can be analysed in a similar way to the FCS. We again separate the field inside the torus into vacuum and plasma parts, but there are now two vacuum contributions to consider, one related to the external flux Ψ_e and the other to the toroidal flux Ψ_t. Thus $\underset{\sim}{B} = \underset{\sim}{B}_p + \underset{\sim}{B}_e + \underset{\sim}{B}_t$ where $\underset{\sim}{B}_e$ is the vacuum field corresponding to the normal field at the electrodes and $\underset{\sim}{B}_t$ is the vacuum field corresponding to the toroidal flux. We then expand $\underset{\sim}{B}_p$ in terms of the eigenfunctions defined by Eq (4). [Note that the eigenvalue boundary condition $\underset{\sim}{a}_i = 0$ ensures both zero normal field on the boundary <u>and</u> zero toroidal flux, as required for $\underset{\sim}{B}_p$.]

It is convenient to introduce the coefficients

$$u^2 = \frac{a \int \underset{\sim}{B}_e^2 \, d\tau}{\Psi_e^2} \quad : \quad v^2 = \frac{a \int \underset{\sim}{B}_t^2 \, d\tau}{\Psi_t^2} \qquad (14)$$

$$g_i = \frac{\int \underset{\sim}{a}_i \cdot \underset{\sim}{B}_e}{\Psi_e} \quad : \quad h_i = \frac{\int \underset{\sim}{a}_i \cdot \underset{\sim}{B}_t}{\Psi_t}$$

which are independent of μ, $\underset{\sim}{B}_e$ and $\underset{\sim}{B}_t$. Then

$$\underset{\sim}{B} = \underset{\sim}{B}_e + \underset{\sim}{B}_t + \sum \alpha_i \nabla \times \underset{\sim}{a}_i \tag{15}$$

where

$$\alpha_i = \frac{\mu \ \mathrm{sgn}(\lambda_i)}{(\lambda_i - \mu)} (g_i \Psi_e + h_i \Psi_t) \tag{16}$$

If we again take uniform plasma resistivity then the impedance of the system (normalised to η/a) is

$$\hat{Z} \left(\mu a, \frac{\Psi_t}{\Psi_e}\right) = \left(u^2 + v^2 \frac{\Psi_t^2}{\Psi_e^2}\right) + \sum \frac{\mu^2}{(\lambda_i - \mu)^2} \left(g_i + h_i \frac{\Psi_t}{\Psi_e}\right)^2 \tag{17}$$

where it has been assumed that $\underset{\sim}{B}_e$ and $\underset{\sim}{B}_t$ are orthogonal, as is the case in an axisymmetric system.

The impedance becomes infinite as μ approaches the lowest eigenvalue λ_o, but it also depends on Ψ_t/Ψ_e. When μ is small the torodial flux always increases the impedance between the electrodes. However, for μ close to the eigenvalue λ_o the effect of toroidal flux depends on the parity (right-handed or left-handed) between the fluxes Ψ_e and Ψ_t.

An added complication is that in an axisymmetric torus the lowest eigenvalue is often not coupled to the vacuum fields [6]; ie the coefficients g_i and h_i vanish for $i = 0$. This is the case for example, [7], when the lowest eigenmode is itself non-axisymmetric ($\sim \exp(im\phi)$). The $i = 0$ term in (17) would then be zero for $\mu < \lambda_o$ and indeterminate for $\mu = \lambda_o$. More realistically we should consider the coupling coefficients

g_o and h_o to be small; then the rise in impedance as $\mu \to \lambda_o$ is very sudden, corresponding to a hard limit on the current. (There is a similar hard limit on the toroidal current in a conventional axisymmetric pinch [8].)

5. Computation of Relaxed States with Helicity Injection

If we restrict our attention to axisymmetric systems and write

$$\underset{\sim}{B} = \frac{\underset{\sim}{e}_\phi \times \nabla\chi}{R} + \frac{\underset{\sim}{e}_\phi f}{R} \qquad (18)$$

then the relaxed states are solutions of

$$\frac{\partial^2\chi}{\partial Z^2} + R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial\chi}{\partial R} + \mu^2\chi = 0 \quad ; \; f = \mu\chi + \text{Const.} \qquad (19)$$

The boundary value of χ is specified, up to an irrelevant constant, by the normal field $\underset{\sim}{B} \cdot \underset{\sim}{n}$ and in a simply-connected domain χ must be regular as $R \to 0$. This is sufficient to determine χ completely. However in a toroidal system there is no condition on χ as $R \to 0$ and the value of $\int\chi dS$ (the toroidal flux Ψ_t) must be specified in order to determine χ completely. For the toroidal system it is convenient to solve equation (19) in two steps, corresponding to the separation of the vacuum field into $\underset{\sim}{B}_e$ and $\underset{\sim}{B}_t$. Thus one first computes (19) with boundary condition χ = constant and $\int\chi dS = \Psi_t$. Then one computes a second solution with χ having the specified value on the boundary and $\int\chi dS = 0$. The most general solution, for a given configuration and a given μ, is then a linear combination of χ_1 and χ_2.

In practice the computations were done using a finite difference system on a 36 x 36 square mesh, with a standard SOR method. For each case χ is fixed on the boundary ($= \chi_b$) and μ is adjusted during iteration until $\mu \int\chi^+ dS$ (where χ^+ denotes that the integration is only over the region in which $\chi > 0$) converges to a specified value and all residuals are reduced to a minimum. This method is more stable than the direct one of fixing μ during iteration. When a particular value of μ is required it can be obtained by adjusting the specified value of $\mu \int\chi^+ dS$.

In the toroidal system, the second solution, with $\int\chi dS = 0$, was not in fact computed directly because such solutions were difficult to obtain with the method used. This is in part because the constant in the boundary value of χ could not be specified in advance for this case. The solution with $\int\chi dS = 0$ is therefore obtained by first finding a solution with χ_b arbitrarily fixed, but giving the correct $(\underset{\sim}{B} \cdot \underset{\sim}{n})$. For this solution $\int\chi dS \neq 0$ and an appropriate multiple of the first solution, with χ_b = constant, is therefore subtracted to give $\int\chi dS = 0$.

6 Flux Contours for Helicity Injected Relaxed States

A Spherical

Figs (3) and (4) show flux contours computed for the flux core spheromak of fig (1). When μ is less than the lowest eigenvalue λ_o = 4.49, the configuration has a central core of flux linked to the polar electrodes and carrying the electrode current. This is surrounded by nested toroidal surfaces similar to those of an ordinary spheromak. [However, unlike the ordinary spheromak, the toroidal field is not zero on the outermost flux surface; in this respect the surfaces resemble those of a toroidal pinch.] As already mentioned, when $\mu \to \lambda_o$ the electrode-linked current contracts to a narrow filament along the axis and the impedance becomes infinite - as does the ratio of plasma current to electrode current. For $\mu > \lambda_o$ the electrode current flows over the outer surface and surrounds toroidal surfaces like those of an ordinary spheromak.

B Toroidal

As an illustration of toroidal systems with helicity injection we have taken the torus shown in fig (2) with inner radius $R_i = 1$, outer radius $R_o = 2$ and height $h = 1$. The electrodes cover 20% of the upper and lower surfaces. It is useful to note that in the normal relaxed state of this torus the "field-reversed" point (ie $B_\phi = 0$ on boundary) occurs at $\mu = \mu_R = 4.48$; the flux contours for this situation are shown in fig (5).

(i) $\mu < \mu_R$, Non-reversed Pinches

Figs (6-11) show flux-contours for typical helicity-injected states when μ is less than μ_R . When there is no externally produced toroidal flux and μ is small, as in fig (6) ($\mu = 2$, $\Psi_t = 0$), there are no nested toroidal surfaces - although there is a toroidal field which reverses sign on the surface indicated by a broken line. Nested toroidal surfaces are created when sufficient toroidal flux is added. Thus in fig (7) ($\mu = 2$, $\Psi_t/\Psi_e = 0.22$) there are nested toroidal surfaces with the electrode-linked flux passing inside them (ie on the smaller toroidal radius side). When the direction of the toroidal flux is changed, as in fig (8) ($\mu = 2$, $\Psi_t/\Psi_e = -0.61$), the electrode-linked current passes outside the nested surfaces. The nested surfaces in figs (7 and 8) represent examples of very weak toroidal pinches sustained by helicity injection.

Stronger toroidal pinches are obtained when μ is increased. Fig (9) ($\mu = 4$, $\Psi_t = 0$) shows that a weak system of toroidal surfaces can be created by helicity-injection even without any external toroidal flux. When sufficient toroidal flux is added a more substantial toroidal structure is formed. Thus in fig (10) ($\mu = 4$, $\Psi_t/\Psi_e = 0.11$) the toroidal surfaces correspond to a (non-reversed) pinch with field-ratio $F \simeq 0.33$. (Here F is taken as the ratio of the toroidal field on the outer closed flux-surface to the field on the magnetic axis.) The electrode current passes inside the toroidal surfaces. Similarly, when Ψ_t/Ψ_e is sufficiently large and negative, as in fig (11) ($\mu = 4$, $\Psi_t/\Psi_e = -0.18$) there are nested toroidal surfaces with the electrode current passing outside them. These surfaces correspond to a pinch with $F \sim 0.43$. Stronger, but still non-reversed, pinches are obtained when $|\Psi_t/\Psi_e|$ is increased.

(ii) $\mu > \mu_R$ Reversed Field Pinches

Figs (12-14) show flux contours for typical helicity-injected states with $\mu > \mu_R$. Fig (12) ($\mu = 5$, $\Psi_t = 0$) shows that when $\mu > \mu_R$, quite substantial nested toroidal surfaces can be created without any external toroidal flux. Indeed there are two sets of nested toroidal surfaces, one with 'positive' toroidal field lying outside the electrode-linked current and one with 'negative' toroidal field lying inside it.

When sufficient toroidal flux is added, one of the sets of nested toroidal surfaces in fig (12) disappears and the other expands to produce a reversed field pinch with the electrode current passing inside or outside the toroidal surfaces. Thus in fig (13) ($\mu = 5$, $\Psi_t/\Psi_e = 0.31$), the toroidal surfaces correspond to a reversed field pinch with $F \sim -0.03$ and have the electrode current passing outside them. Similarly in fig (14) ($\mu = 5$, $\Psi_t/\Psi_e = -0.48$) one has a reversed field pinch with $F \sim -0.10$ and electrode current passing inside it. [Note that for $\mu < \mu_R$ the current passes inside the pinch for $\Psi_t/\Psi_e > 0$ and outside it for $\Psi_t/\Psi_e < 0$; the converse is true for $\mu > \mu_R$.] Again more substantial reversed-field pinches are obtained when $|\Psi_t/\Psi_e|$ is increased.

(iii) Low-μ, Tokamaks

The main interest in helicity-injection is for pinch-like systems in which $q_o < 1$, (here q_o is the toroidal winding number of the field lines near the magnetic axis). Nevertheless it is worth while noting that tokamak-like configurations with $q_o \sim 1$ can also be created when μ is small. [In toroidal relaxed states $q_o \sim 2/\mu R$.] Thus figs (15) and (16) show configurations with $q_o = 1.03$ and $q_o = 1.12$ respectively. As in all tokamak-like relaxed states q(r) is very flat near the magnetic axis. It increases to ~ 1.23 on the outer closed surface in fig (15) and to 1.4 in fig (16), but this is a consequence of the square cross-sections rather than the current profile.

7 Summary and Discussion

We have considered the sustainment of relaxed state plasmas using helicity injection from poloidal electrodes. When the plasma is in a simply-connected volume, such as a sphere with polar electrodes (fig 1), the configuration is entirely controlled by the current I_e and the magnetic flux Ψ_e through the electrodes. The relaxed state parameter $\mu = I_e/\Psi_e$ and the configuration depends mainly on whether μ is greater or less than the lowest eigenvalue λ_o. If $\mu < \lambda_o$ the electrode current passes along the axis and the flux surfaces are otherwise similar to an ordinary spheromak - but the toroidal field does not vanish on the outer surface. As $\mu \to \lambda_o$ the electrode-current channel contracts and both the impedance and the ratio of plasma-current to electrode current $\rightarrow \infty$. When $\mu > \lambda_o$ the electrode-current flows around the outer surface and surrounds the spheromak-like flux surfaces. However, since the inherent stability of relaxed states applies only when $\mu \leqslant \lambda_o$ this configuration is likely to be unstable.

When the plasma is in a toroidal volume the configuration depends on the toroidal flux Ψ_t as well as on the electrode current I_e and flux Ψ_e. Consequently there are several different configurations which can be maintained by helicity-injection in a torus such as fig 2. They are characterised by $\mu = I_e/\Psi_e$ and $\alpha = \Psi_t/\Psi_e$. If μ is less than μ_R (the value for field reversal in a relaxed state without helicity-injection) and α is sufficiently large and positive, the configuration represents a toroidal pinch with electrode current passing on its smaller radius side. When $\mu < \mu_R$ and α is sufficiently negative the configuration represents a pinch with electrode current passing on the larger radius side. Similarly, if $\mu > \mu_R$ the configuration represents a reversed field pinch ($F < 0$), but in this case electrode current passes on the inside if α is negative and on the outside if α is positive. It is also possible to create tokamak like configurations with $q_o \sim 1$ and low shear.

It is noteworthy that, in a torus, whether the electrode current passes inside or outside the toroidal surfaces is controlled by the sign of Ψ_t/Ψ_e, (rather than by μ as in a sphere). This feature means that either configuration can be reached without μ having to exceed the lowest eigenvalue λ_o. [We have not calculated λ_o for the torus of fig (2) because this corresponds to a non-axisymmetric mode which plays no part in our computations. From reference [9] we estimate that $\lambda_o \sim 5.5$]. In a toroidal system the impedance and the ratio of plasma current/electrode current also depend on Ψ_t/Ψ_e rather than on μ, and both become large as $\Psi_t/\Psi_e \rightarrow \infty$.

An interesting, and perhaps unexpected, feature of the configurations with $\mu > \mu_R$ in a torus is that helicity injection can create toroidal flux surfaces even without the provision of any external toroidal flux. This is an extreme example of the fact that in all the configurations we have discussed, a <u>toroidal</u> discharge is maintained by current and flux through <u>poloidal</u> electrodes.

The general behaviour of pinches and spheromaks is known to follow closely that predicted for relaxed states [1] so that helicity-injection seems particularly attractive for them. Indeed Jarboe et al [10] have already succeeded in maintaining a spheromak-like discharge by helicity injection. [On the other hand, attempts at helicity injection in a toroidal pinch have been unsuccessful [11]. This is believed to be due to the short path which existed between the electrodes and permitted surface current and allowed arcing to occur between them. The configuration considered here should be less prone to this problem.]

Of course, a crucial experimental factor is the amount of electrode current required for helicity injection. In this connection we note that in our computations we have selected parameters such that the plasma-current is less than the electrode current in pinches and comparable to it in Tokamaks. This was done for clarity in the figures; in a torus the ratio of plasma to electrode current is controlled by Ψ_t/Ψ_e and can, in principle, be made large - though at the expense of greater plasma dynamo action and any associated energy loss. Some preliminary analysis of the practical problems of helicity injection, including the use of electron emitting electrodes and the effect of sheath voltages, has been given by Jarboe [12].

An interesting experiment on helicity injection in pinches is suggested by the fact that if the electrodes are not energised then there is a substantial 'dead-space' surrounding the toroidal pinch in which flux lines intercept the torus wall. It is widely believed that such 'dead-space' leads to a <u>loss</u> of helicity in relaxed states and so to an anomalous loop-voltage [13,14]. (This is caused by the same process of helicity transport as produces the increase in impedance with helicity injection.) Consequently by using one of the toroidal configurations shown here it should be possible to observe the change from an anomalously large loop voltage to an anomalously small one as the helicity injection electrodes are energised. Another possibility is that of using more than one pair of electrodes for helicity injection with different values of I_e/Ψ_e on each pair. Then the extent to which the plasma can relax could be controlled so as to provide insight into helicity-transport, relaxation and the associated energy losses.

Acknowledgement

I am indebted to Dr Tom Jarboe for several useful discussions on helicity injection and for information about his experiments and proposals.

References

[1] A Comprehensive Review of the Theory of Relaxed States and the Experimental Background is given in,

TAYLOR, J.B., Rev. Mod. Phys. 58 (1986) 741

[2] BEVIR, M.K. and GRAY J., Proceedings of Reversed-Pinch Theory Workshop Ed., by Lewis, H.K. and Gerwin, R.A., Los Alamos Report No. LA-8944-C (1981) 176.

[3] JENSEN, T.H., CHU, M.S., Phys. Fluids 27 (1984) 2881.

[4] BERGER, M.A., and FIELD, G.B., J. Fluid Mech. 147 (1984) 133.

[5] FINN, J.M., ANTONSEN, J.M., Comments Plasma Contr. Fusion 9 (1985) 111.

[6] TAYLOR, J.B., Reference 1, Appendix A.

[7] GIMBLETT, C.G., HALL, P.J., TAYLOR, J.B., TURNER, M.F., Phys. Fluids 30 (1987) 3186

[8] TAYLOR, J.B., Plasma Physics and Controlled Nuclear Fusion Research, (Proc. 5th International Conf. Tokyo) Vol. 1 (1974) 161.

[9] TAGUCHI, M., MIYAZAKI,T., KANEKO, S., Phys. Soc. Japan 54 (1985) 2163.

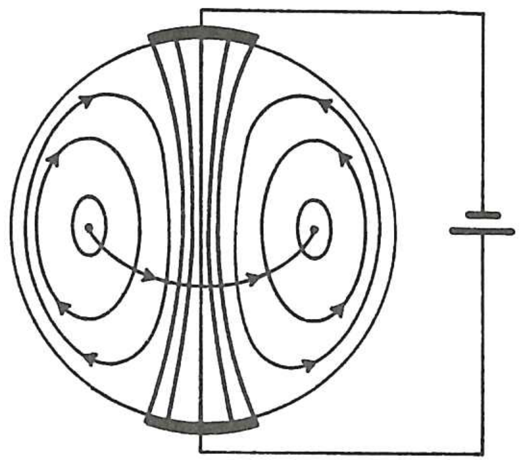
[10] JARBOE, T.R., HENNINS, I., SHERWOOD, A.R., BARNES, C.W., HOIDA, H.W., Phys. Rev. Lett 51 (1983) 39.

[11] JARBOE, T.R., Private Communication.

[12] JARBOE, T.R., Formation and Steady State Sustainments of a Tokamak by Coaxial Helicity Injection, to be published.

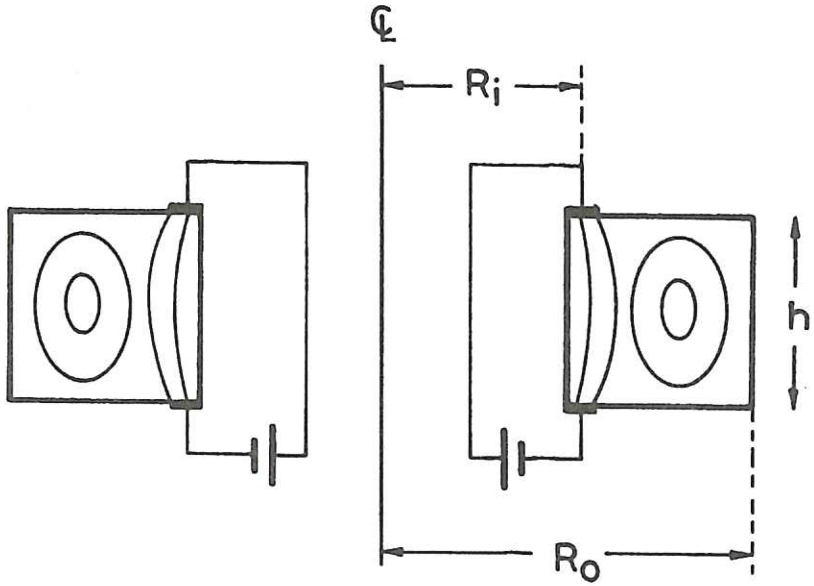
[13] JARBOE, T.R., ALPER, B., Phys. Fluids 30 (1987) 1177.

[14] TSUI, H.Y.W., Contr. Fusion Plasma Phys. 14th Euro. Conf. Madrid (1987) II 473.



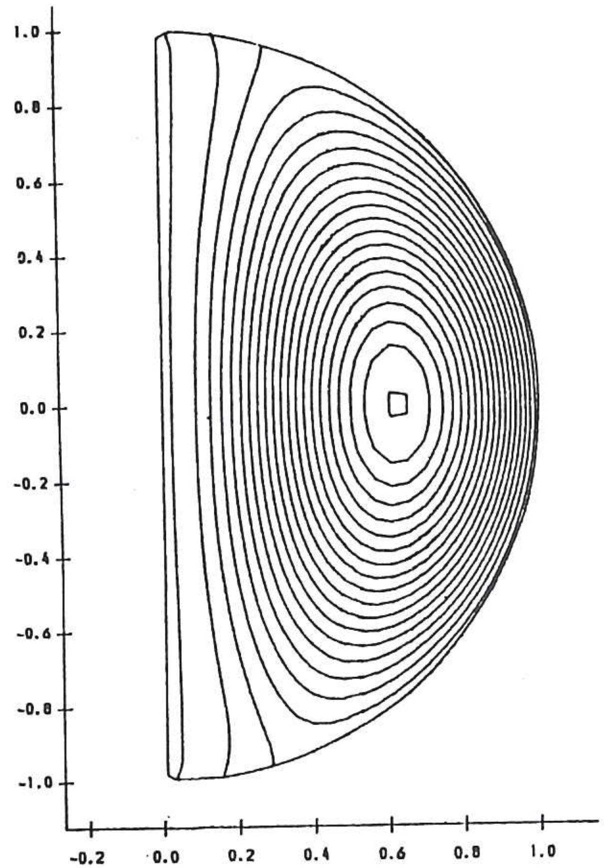
FLUX - CORE SPHEROMAK WITH HELICITY INJECTION THROUGH POLAR CAPS.

FIG. 1



TOROIDAL SYSTEM WITH HELICITY INJECTION THROUGH POLOIDAL ELECTRODES.

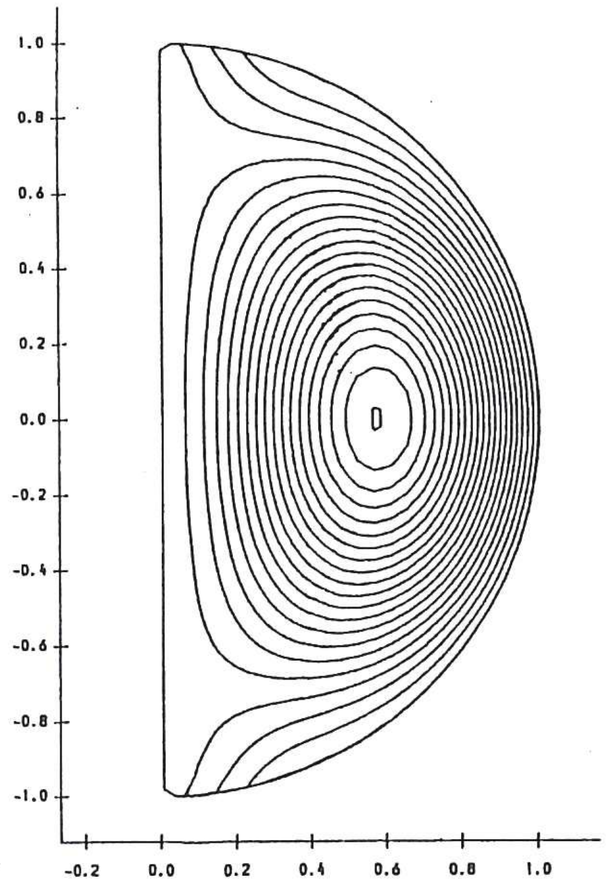
FIG. 2



$\mu = 4.280$

FLUX CONTOURS FOR SPHEROMAK

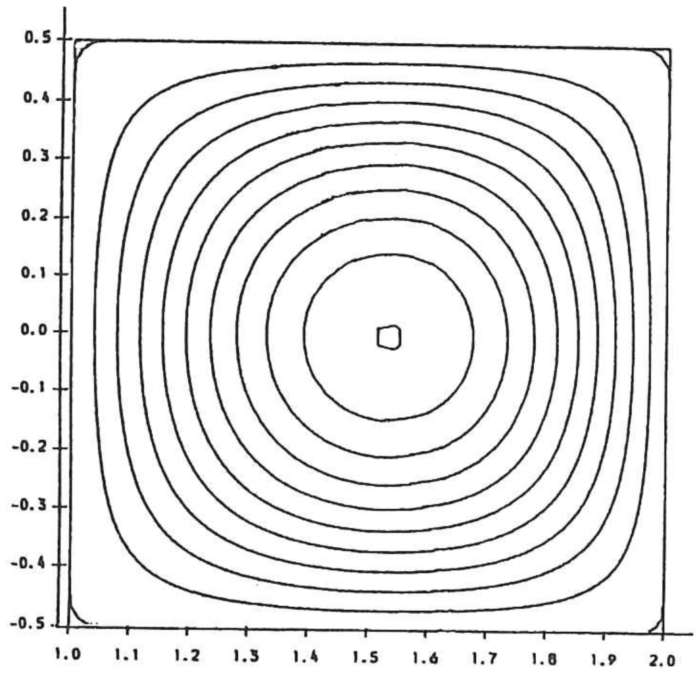
Fig. 3



$\mu = 4.820$

FLUX CONTOURS FOR SPHEROMAK

Fig. 4

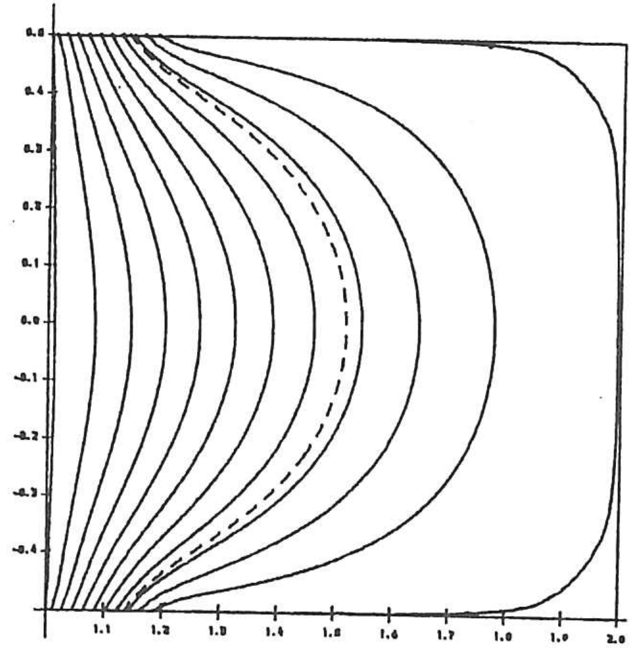


$\mu = 4.480 \quad F = 0$

FLUX CONTOURS FOR TOROIDAL SYSTEM

OF FIG.2 AT FIELD REVERSAL POINT

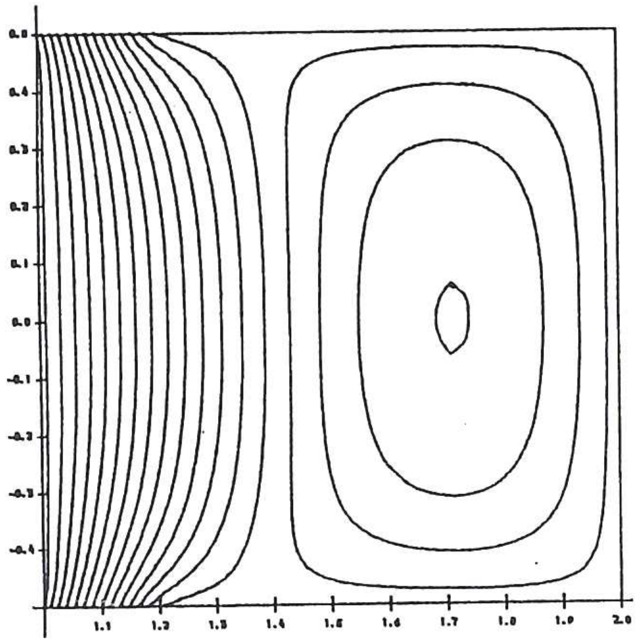
Fig. 5



$\mu = 2.000 \quad \psi_T = 0.0$

FLUX CONTOURS FOR TOROIDAL SYSTEM

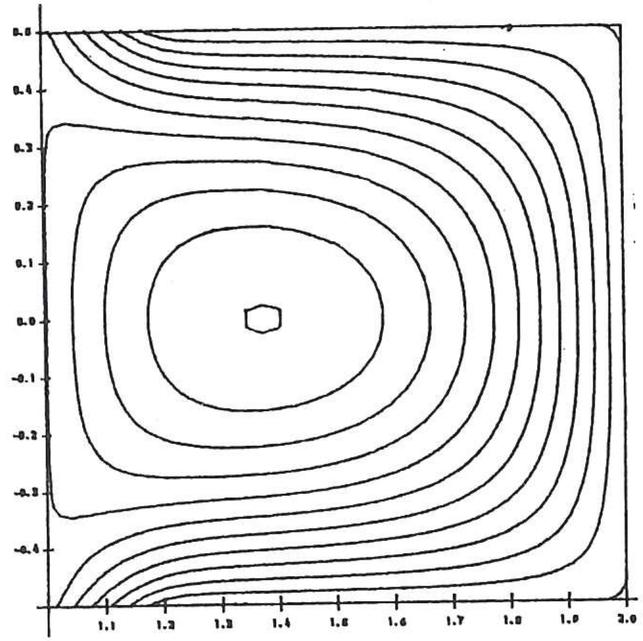
Fig. 6



$\mu = 2.000 \quad \psi_T/\psi_E = 0.22$

FLUX CONTOURS FOR TOROIDAL SYSTEM

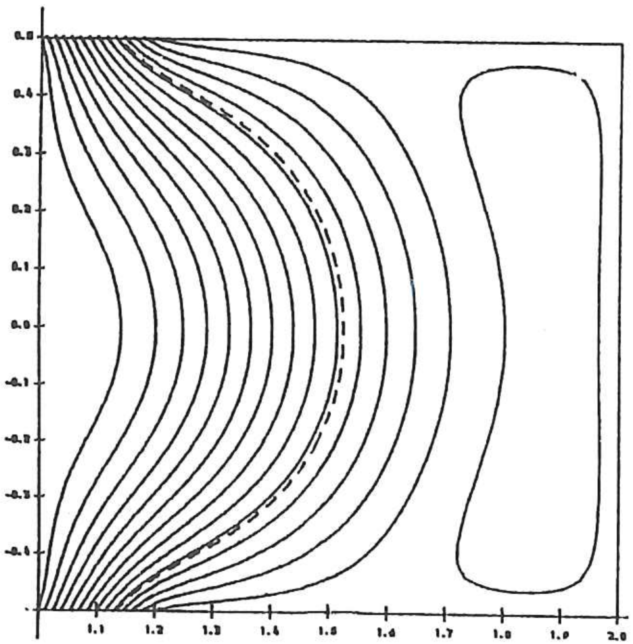
Fig. 7



$\mu = 2.000 \quad \psi_T/\psi_E = -0.61$

FLUX CONTOURS FOR TOROIDAL SYSTEM

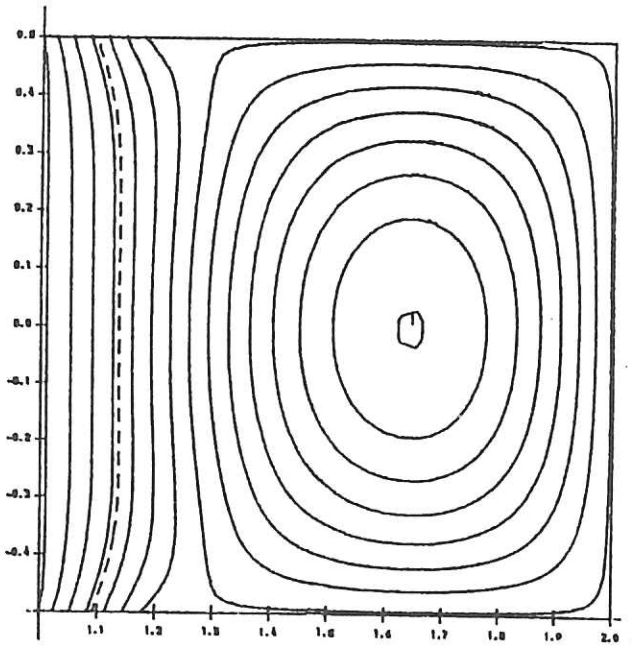
Fig. 8



$\mu = 4.000 \quad \psi_T = 0.0$

FLUX CONTOURS FOR TOROIDAL SYSTEM

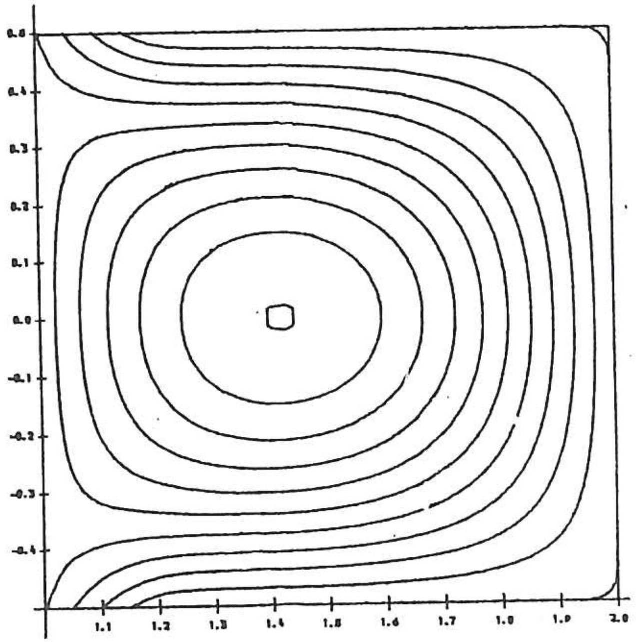
Fig. 9



$\mu = 4.000 \quad \psi_T/\psi_E = 0.11$

FLUX CONTOURS FOR TOROIDAL SYSTEM

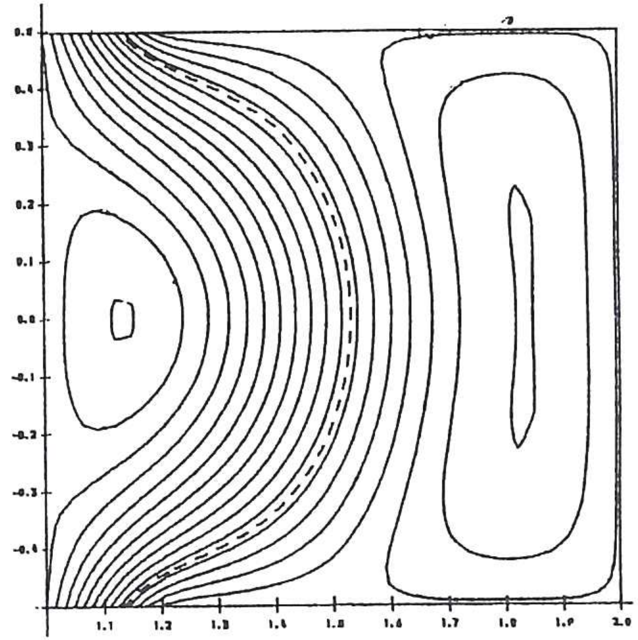
Fig. 10



$\mu = 4.000 \quad \psi_T/\psi_E = -0.18$

FLUX CONTOURS FOR TOROIDAL SYSTEM

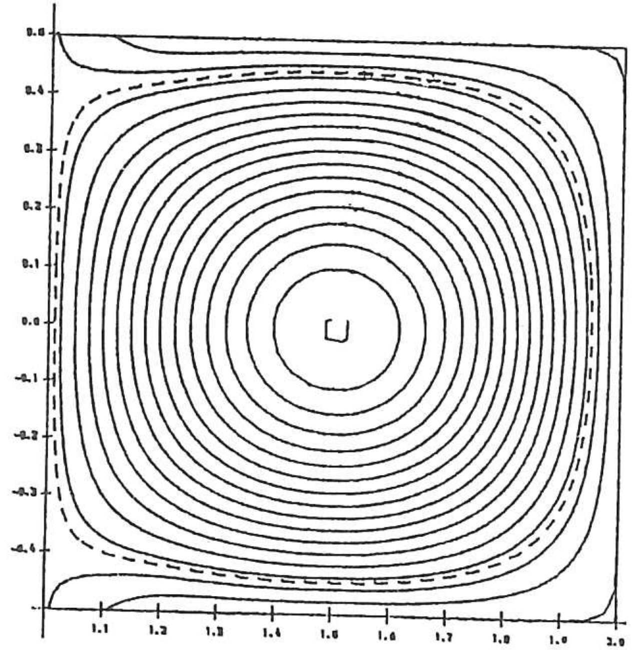
Fig. 11



$\mu = 5.000 \quad \psi_T = 0.0$

FLUX CONTOURS FOR TOROIDAL SYSTEM

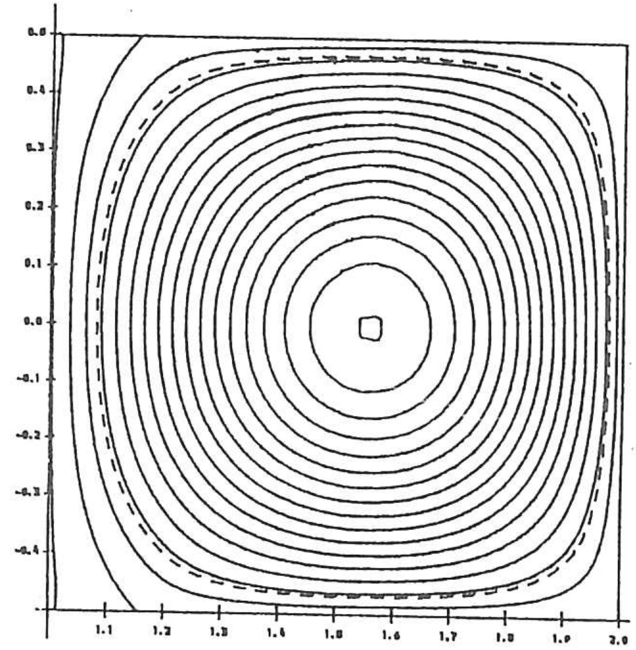
Fig. 12



$\mu = 5.000 \quad \psi_T/\psi_E = 0.31$

FLUX CONTOURS FOR TOROIDAL SYSTEM

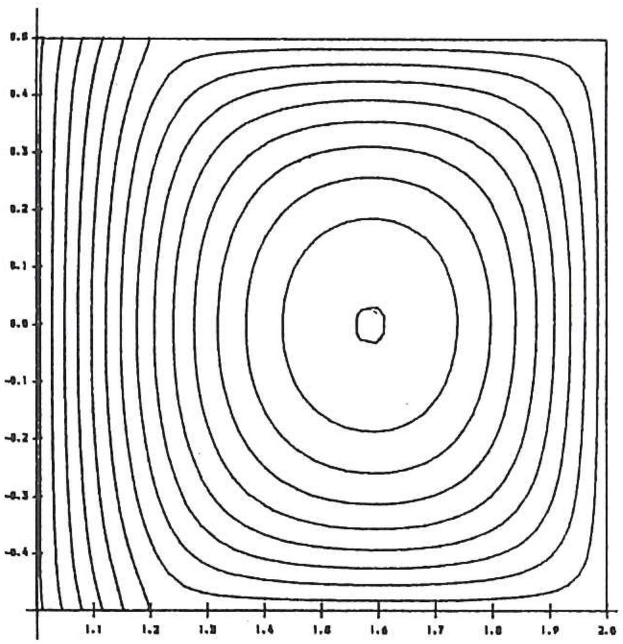
Fig. 13



$\mu = 5.000 \quad \psi_T/\psi_E = -0.48$

FLUX CONTOURS FOR TOROIDAL SYSTEM

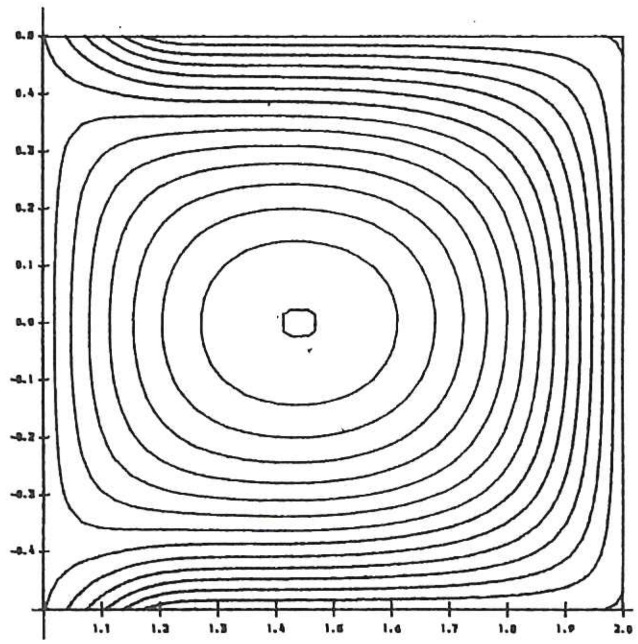
Fig. 14



$\mu = 1.250 \quad \psi_T/\psi_E = 2.44$

FLUX CONTOURS FOR TOKAMAK-LIKE SYSTEM

Fig. 15



$\mu = 1.250 \quad \psi_T/\psi_E = -2.54$

FLUX CONTOURS FOR TOKAMAK-LIKE SYSTEM

Fig. 16